Basic Electrical Circuits

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Lecture – 132

We have found the response of an R C circuit or in general a first order circuit to an initial input. Now, the solution we got was general except for one particular case when S C R plus 1 was equal to 0 that is in our usual notation meaning when the exciting exponential as the same time constant as a natural responds of are first order system the solution we got was not defined, because the demo niter went off to 0. So, what were going to do is to take the appropriate limit and see what comes out.

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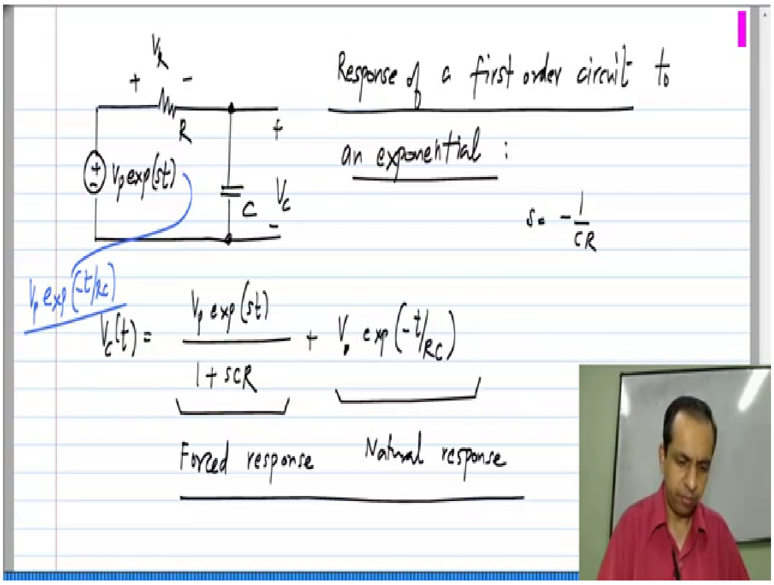

We see let say we are looking for the response of V c, now this could be anything else we could be looking for the response of a V R or the current through the loop. Now, we know that the response of a first order circuit of, which this R C circuit is an example this is the forced response and this is the natural response. Now, when s equals minus 1 by C R, which means that the exciting source is V p experiential minus t by R C that is, which an experiential which is the same as the natural response of a circuit we see that this denominator here goes to 0, so we cannot use this expression as is.

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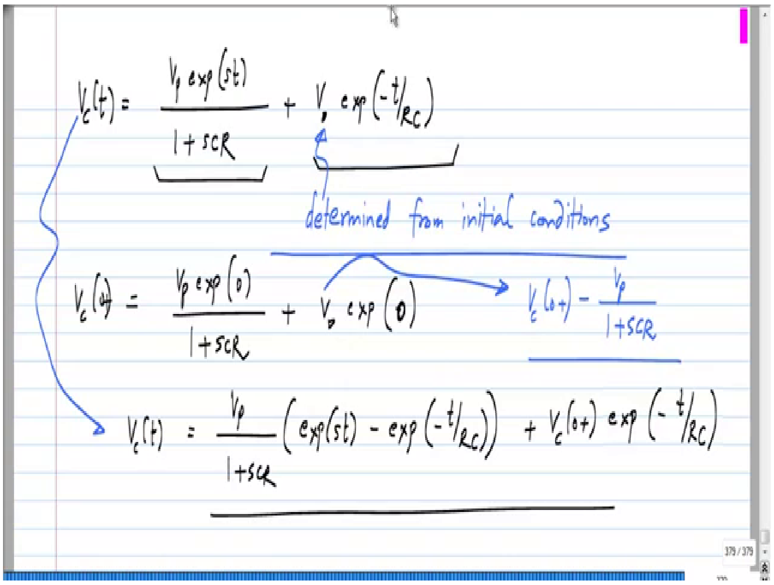

So, will exam in this further we know that in general this constant here is determinate from initial conditions. So, to do that, what we have to do is let say the initial voltage on the capacity V c of 0 of course, it 0 plus this will be equal to V p experiential s time 0 that is just 0 divided by 1 plus S C R plus v naught experiential minus 0 by R C. So, this is also 0. So, we know that this v naught this comes out to be by solving this we get V c of 0 plus minus V p by 1 plus S C R, where this is the initial voltage on the capacitor. So, that is, what this constant v naught has to be.

So, if I use that to complete the expression here, what will I get V c of t equals I have a V p by 1 plus S C R there and V p by 1 plus S C R multiplying that 1. So, V p by 1 plus S C R times experiential s t minus experiential minus t by R C plus V c of 0 plus. So, again this is still undefined for S C R plus 1 equals 0, so to overcome this difficulty, what we will do it is to take the limit of S C R plus 1 going towards 0.

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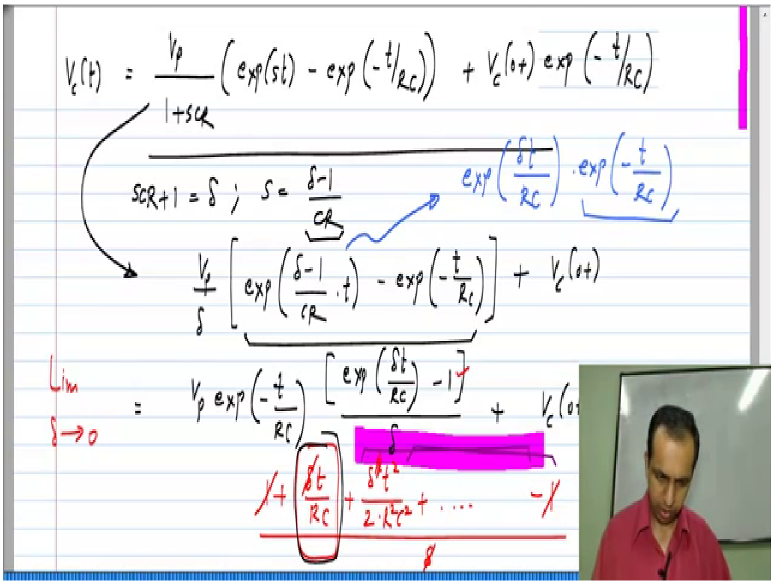

So, first of all let me define S C R plus 1 to be some delta just answer to be easier to do this way you do not have to do it like this of course, then this means that s s delta minus 1 divided by C R. So, this expression becomes V p divided by delta times exponential s t, which is this delta minus 1 by C R. So, that will be exponential delta minus 1 by C R times C minus exponential minus t by R C plus V c of 0 plus and this can be re written as we observe that this is nothing but, exponential delta t by R C times exponential minus t by R C.

So, I will take this exponential minus t by R C, which is common to this term and that term outside. So, I will have this whole thing to be equal to V p exponential minus t by R C and I have exponential delta t by R C minus 1 divided by delta plus V c of 0 plus. And I have to take the limit of this as delta tends to 0 and delta tends to 0 S C R plus 1 tense to 0 or s into minus 1 by C R in this form it terms of quite easy to do that I am sure your probably already familiar with this limit you may have taken it in some other basic class when we learn t about limit and so on, but I will do it here.

So, one of the way to do it is to explain this exponential in power series. So, we get 1 plus delta t by R C plus delta square t square by two times R square C square plus higher volt a terms involving delta d cube delta t to the 4 and so on. And finally, we have minus 1 coming from there the whole thing is divided by delta. So, now, this one just cancels of with this minus 1 and this delta here cancels with this, so this term alone is left out without any deltas or all the others will still here deltas this is delta square becomes delta next delta q becomes delta square and so on.

So, when you take the limit delta going to 0, so all these terms, which had delta with a power more than 1 will go to 0 will be left with only this t by R C.

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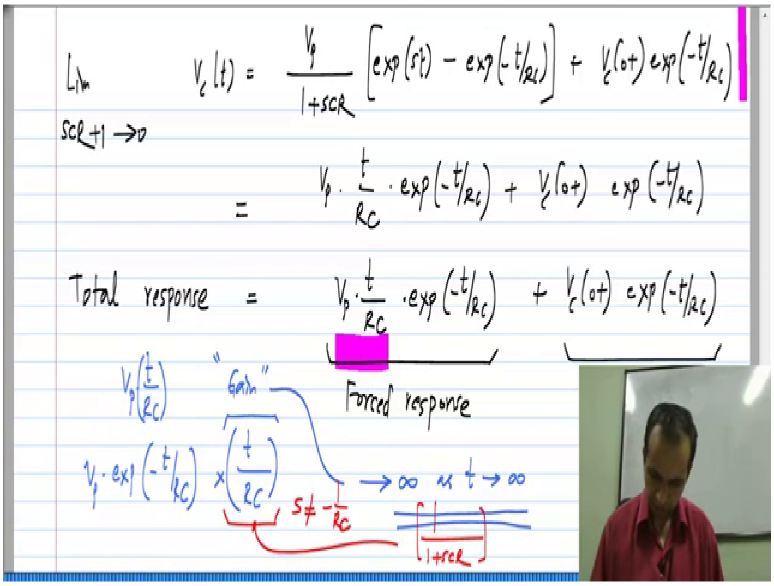

The limit as S C R plus 1 goes to 0 of V c of t, which is really V p by 1 plus S C R exp1ntial s t minus exponential minus t by R C plus V c of 0 plus exponential minus t by R C this whole thing will be equal to V p times t by R C exponential minus t by R C plus V c of 0 plus exponential minus t by R C. So, the total response equals V p times t by R C exponential minus t by R C plus V c of 0 plus exponential minus t by R C this is the forced response and this is the natural response.

So, one thing we can observe is that the co efficient here of the force response is V p times t by R C remember the input was V p exponential minus t by R C and it gets multiplied by t by R C and if you observe this multiplying factor or the gain short to speak of the forced response this goes to infinity as t goes to infinity. Now, this is a general feature that we will see that if you excite a system with it is own natural response, then it will have an infinite gain for that input right a system will have in a infinite gain for it is own natural response now this will be even more evident when we have sinusoidal inputs and circuit with sinusoidal natural response.

Then, you can see what is known as resonates this is also types of resonates, but the point is here the total thing will not go to infinity all though this multiplying factor goes to infinity this experiential false off to 0 much faster than this t by R C goes up to infinity. So, the entire response still goes to 0, so this phenomena is the of resonates is not very evident here. But, the point is that the gain for the input when the input equals the natural response is very large as t goes to infinity the gain also goes to infinity.

Now, remember for some other input when s was not equal to minus 1 by R C this gain instead of this it was 1 by 1 plus S C R. Now, we can if you just substitute S C R equal to minus 1 here you will get infinite, but you see that natural response we have t by R C and as t goes to infinity this number also goes to infinity, but the entire response goes to 0.

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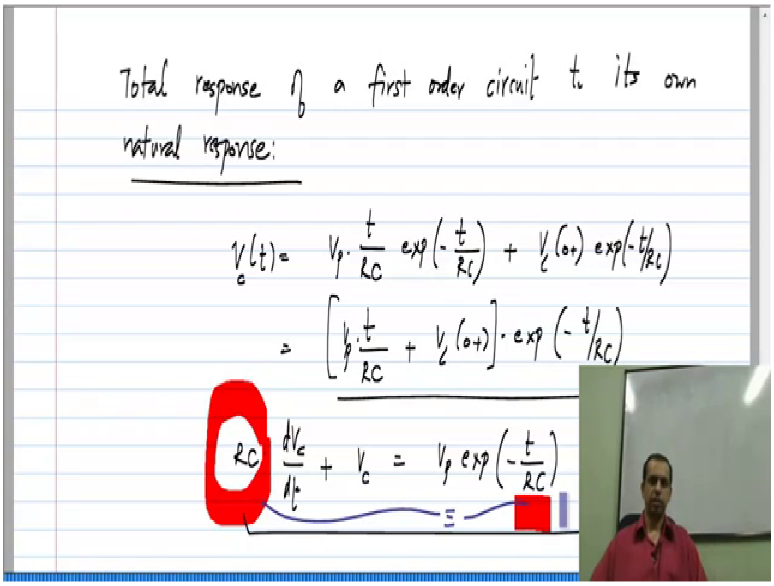

So, in general the total response of a first order circuit to it is own natural response is this, which can also be written as V p times t by R C plus V c of 0 times exponential minus t by R C this is for a differential equation, which is we can also evaluate this for a other differential equation, where different terms are scaled differently. The important thing is here, let say the coefficient of V c is 1, then this coefficient is the time constant and the time constant of this exponential is exactly the same.

So, when these two are the same when the time constant of the differential equation is the same as the time constant of the experiential on the right hand side we will get this solution. Now, some other terms could be scaled V p we could get something else here and that will appear in this position. Now, this is useful in itself to understand and also later when we discuss second order systems particular case of second order systems reduces to this case, where first order system is being given by an exponential, which is equal to its natural response.

So, this is quid in a interesting result and worth fail remembering as well as faring out, how to derive it from the other general expression.