Basic Electrical Circuits

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Lecture – 131

We have quite exponentially analyze the response a first order circuit to constant inputs. So, now, for any arbitrary first order circuit and some piece wise constant input you can find the response, it is quite hard to find the response for some arbitrary time dependent signal. So, we have started with something that constant, now we will go to something that is a little more complicated.

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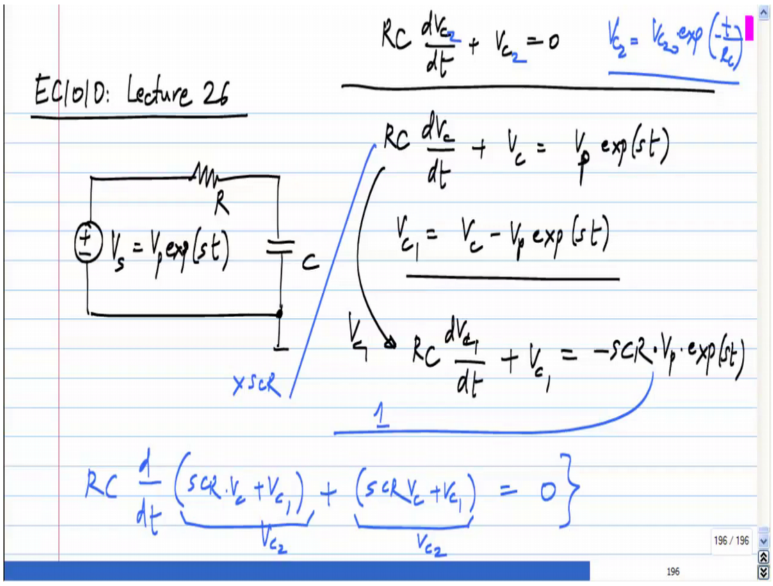

So, let say we have again I will first take this circuit the same thing that we have been using, then this case I will use an exponential as the input. And there a few reasons by using this you will find that any input can be decomposed into a sum of exponential or any integral of exponential with different values of dices.

Student: So, you won't get into that you can assume that for now you will see the details in a course like network sense system.

You can for, now imagine that may be you had a first order system it gives an exponential output and you fed that into another first order system that is why we study this for happens if we do that, so that is the motivation to do something like this. In this case if we write out the differential equation we will get. So, we have defined the value of V c, which will satisfy this and we assume that we solve this for t greater than 0 and so on.

So, how would we go about solving for this, so there is something no other integrating factor which is basically essentially solutions that your mugged up based on previous experience, so you know that if the input is the obvious function the solution will be of that. Let us do the simple miner thing like we how has do in this course, which is that this an a how to solve I will try to reduce this also do that, that is how I solved the constant also right when the constant input I will change the variables.

So, let us try to do that first the simple thing to try to do is, so please rewrite this in terms of V c 1 and compare it to, what I derive we get R C d V c 1 by d t plus V c 1 equals minus S C R V p exponential s t we dint get a homogeneous equation, so what we will do.

Student: We can try a linear combination of these two equations.

This multiply by S C R and this multiply by one what we get is R C that it is just a linear operator I will put this inside that derivative and then, here also I get this.

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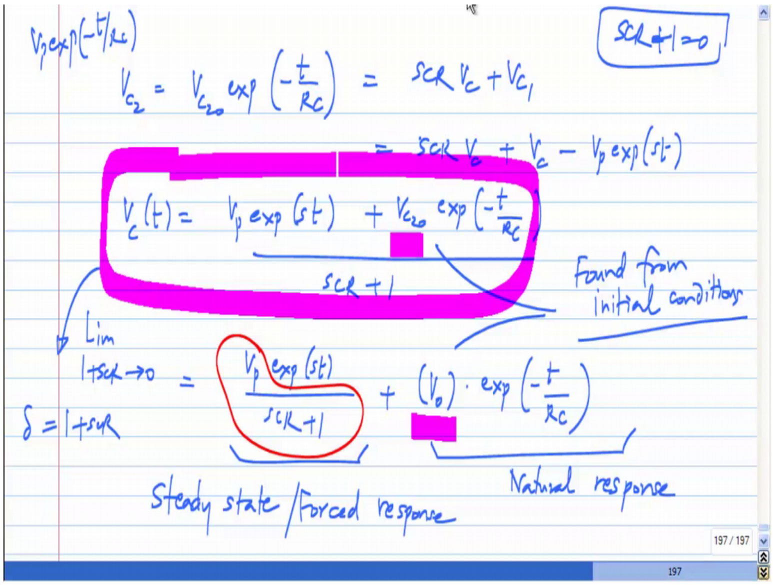

So, now, this new variable it would call that V c 2 if you want what is the solution to V c 2, V c 2 is 0 V c 2 is some constant times exponential minus t by R C. So, now, from this find the solution for V c it is what I want just substitute all the variables and find it only in terms of please do that this just some silly as these two and get the right hand side to be 0 that is all, what is the solution?

Student: V c 2 0 exponential minus t by R C equals S C R times V c plus V c 1.

This V c on itself is nothing but, so what do we have.

Student: V c of t equals V p exponential s t plus V c 2 0 exponential minus t by R C divided by S C R plus 1.

Ok, now you can express this lightly it differently also, so, this V c 2 0 I mean V c 2 itself does not have any particular significant except that it was an intermediate variable that we got while solving this. So, this is some constant, so you can also simply write this as V p exponential s t by S C R plus 1 plus some constant times exponential minus t by R C as usual whether you put it like this or like that this constant has to be found from initial conditions.

So, let say you know that their capacitor voltages some value or t equal to 0 you substitute t equal to 0 and this whole thing and then find that way, so I will simply say some V naught or something like this. So, this has to be found from initial conditions you can put it this way or this way this is just the right definition of the constant, what I want to point out here is, what is this to the response statistic response or poles response or the particular solution and this is the natural response.

The natural response of the circuit remains the same always only thing is this coefficient will change by the way this coefficient will also depend on, what forcing input you apply this is known as the forcing function. Because, if you substitute V c of 0 equal to some value that will also depend on s to adjust this value of V naught some s full comment somewhere. For instance, if V c of 0 was 0 I would get V p exponential 0, which is just V p and this value V c 2 0 would be minus V p to make it equal to 0. So, this constant would be V p divided by 1 plus S C R.

So, this constant itself it depend on the input that you apply it is not the same for any input and for 0 input it will be difference also for 0 input that will simply with initial voltage on the capacitor. So, it will depend on initial voltage on the capacitor and also on the forcing function. So, the natural response does depend on the input the statistic response depends only on inputs that is the definition of the statistic response.

Now, for any stable circuit like the 1 we have this natural response will die out with time and you will be left with only the statistic response as this fine. So, this solution is clearly not valid when 1 plus S C R is 0, so you tell me what might happened, but do you do get any interesting solution, what you can do is you take the limit as 1 plus S C R tends to 0 convenient way of doing it is that is say you will define at another variable some delta equals 1 plus S C R. And then, everywhere you have s you substituted by the appropriate mapping with delta and then you take this limit very simple step you have to use limits and rule and you will get the answer.

So, clearly this as its times is not valid when s equals minus 1 by S C R here, what does it mean, what is the conditions, S C R plus 1 equal to 0. So, what it means is if you excite and R C circuit with an exp1ntial which is same as the natural response of the R C circuit you will gets something weird. So, that is the is not it S C R plus 1 equals 0; that means, that this forcing function is V p exponential minus t by R C exponential minus t by R C is also the natural response of this system.

So, when you excite R C circuit in general any circuit with an exponential, which is also the same as the natural response you will get as slightly different solution you can find the solution. So, you have the statistic or forced response and the natural response that you always have, what is a very, very important thing is that if the input is an exponential, exponential s t the force response is also an exponential is not it.

What is the force response? This is some this whole thing is some constant we apply it V p times exponential s t, what we are getting is some other constant V p by S C R plus 1 may be it is slightly smaller than V p, but the point is if you have a exponential s t as the forcing function the statistic response is also exponential s t. So, it is a very, very important feature of any linear system. So, this complex exponential are either vectors of this linear systems I can vectors I explain briefly, what it means is that if you transform a vector in somewhere, let say using an matrix you will get another vector.

Now, again vectors of a matrix are those vectors for which the transformation is only in the magnitude not in the direction. Similarly, here if you have exponential s t only its magnitude is manipulated I mean it is not become some other function a course like networks and systems will be based on are early is the big part of it will be based on this exponential s t being I can vector of this, so that is way it is very, very useful.

So, combining this idea that the force response to exponential s t is a exponential s t and you can decomposed any input in terms of exponential s t you know that is square were is a some of sin were and harmonic wave at the same frequency 1 kilo at square were will have a sine were at 1 kilo hertz 2 and 3 and so on. So, similarly this can be further generalized into any signal being represented as some linear combination of exponential after all sinusoidal is also exponential wave with x being imaginary.

So, with those two you can come up with this generalized analysis of this system right for any input you will be able to find the output using, what is known as the you won't get into it here you will see that the course. So, now, we have solved the first order circuit for a constant input and for an exponential input of cause these two are not separate cases if I put s equal to 0 I get the other 1 that is pretty obvious if I put s equals to 0 this bottom will become 1 and I have V p and V naught exponential minus t by R C. So, constant is also exponential with this s like frequency the frequency being 0 this fine just like here c R plus omega t omega is the frequency of the sinusoid.

So, this exponential s t s frequency a large value s means that exponential changers very rapidly whether it is positive or negative if negative decay very rapidly if it is positive and large it will blow very rapidly if the value of s is very small again whether positive or negative it will either decay is very slowly are crib of very slowly and weds is 0 it will not changed out and thus same also cos omega t, omega 0 it does not change this fine.

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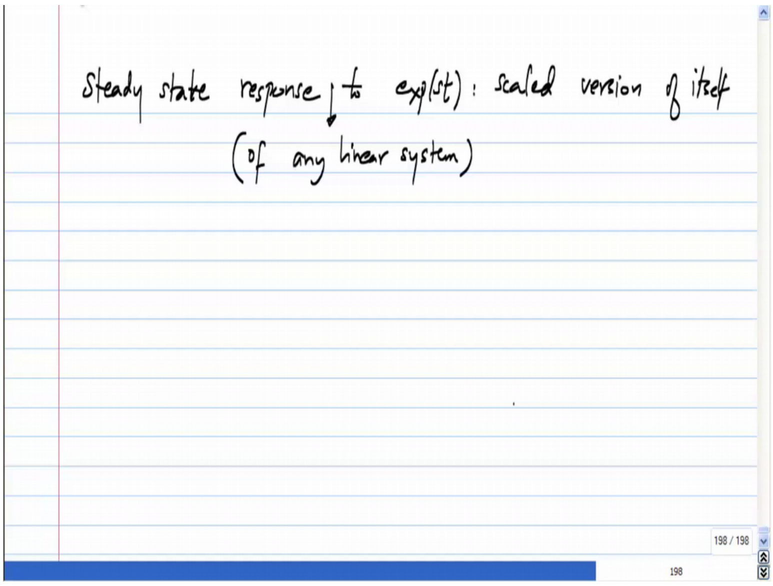

So, the important thing to take 1 from here is that done this first order liner system, but this is true of any linear system. So, if you have second order third order circuit, so still be true the particular find out is first order some.