Basic Electrical Circuits
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Lecture – 130

Now, we will look at an example of a circuit with inductors were inductors currents can undergo instantaneous changes.

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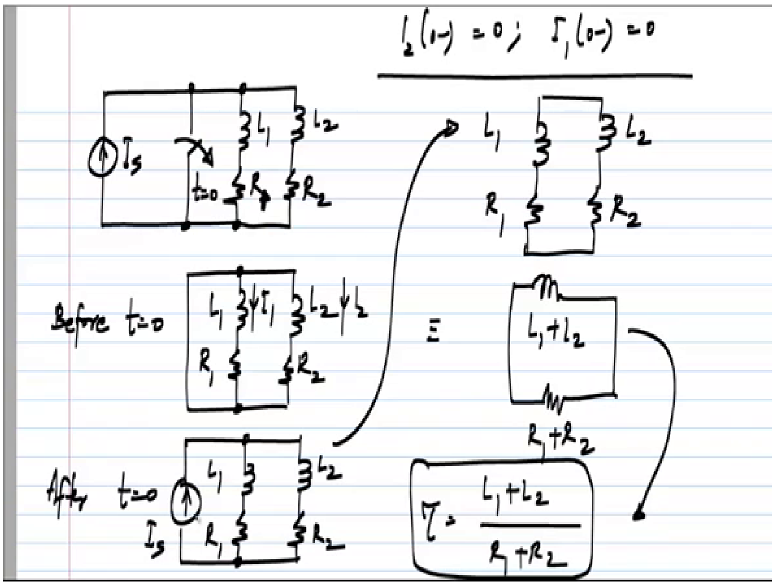

Let say we have a current I s and we a space like this as a mention before if you have switching action that could we equivalent to stepping the values of currents are voltages are in the circuits. And let say this switch is initially short circuited and it open circuits at t equal to 0 and let say we have these two inductors of first off all will determine the order of the circuit, that will come soon.

Now, before t equal to 0 this switch is short circuited and all of this current flows in to the short circuit and all we have effectively is a circuit like this and after t equal 0 we have the current source driving this two parallel branches. So, that is say the value of this is I s, so we can think of basically solving this particular circuit, because a wont you solution for t grade than 0 and in some initial conditions some values of L 1 and L 2, R given at t equals 0.

Let me for simplicity assume that the current through L 1 and the current through L 2 are both 0 just before the step current is applied like said opening this switch is like a applying step current to this particular circuit. So, first of all, what is the order of the circuit, so if I take the source free circuit that is with the current source open circuited, what do I have I have the single loop R 1 R 2, L 1 L 2. And this is the of course, equivalent to we can see that these to inductors are in sires these two resistors are in sires. So, we have L 1 plus L 2 an a single inductor of L 1 plus L 2 across that we have a single resistor R 1 plus R 2.

We know that, if you have a single inductor in a circuit it is a first order circuit and the time constant of the circuit is value of the inductors divided by the value of the resistance, which is across it. So, this will give you the time constant, which is equal to L 1 plus L 2 divided by R 1 plus R 2. So, this is the way calculate the time consistent we will, now the valued formally and verify that this is the indeed the case.

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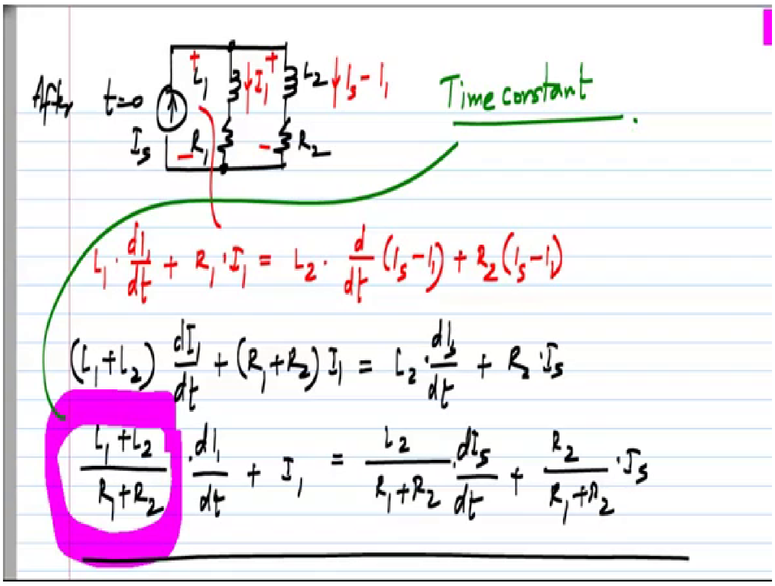

So, let me copy over the this part of the circuit and write the different equation now will choose I 1 as the variable I 1 is the current through well L 1 and by Kirchhoff current law here and these node I know that in the current through L 2 is I s minus I 1. Now, what is the circuit equation i going to it these three branches are in parallel in particular L 1 and R 1 the series branches parallel with the series branches L 2 and R 2.

So, the voltage across these branch and that branch are exactly the same and that is say equation I going to write you can write any way we want you can write the KVL around this 2.1. Now, the voltage across L 1 and R 1 will be L 1 time derivative of I 1 plus R 1 times I 1 and that is the exactly equal to L 2 times the time derivative of the current through L 2, which is I s minus I 1 plus R 2 times the current, which is I s minus I 1. So, if I group all the terms containing the variable I 1 to the left hand side this is what I will and on the right hand side left with.

I will normal said, so that the term without the deli derivative the 0 of term on the left hand side as a coefficient of a unity. So, that lets me read of the time constant, so I divided all terms by R 1 plus R 2. So, now, this is in the standard form and if this coefficient is unity the coefficient here is the time constant is exactly, what we calculated earlier. that is the time constant.

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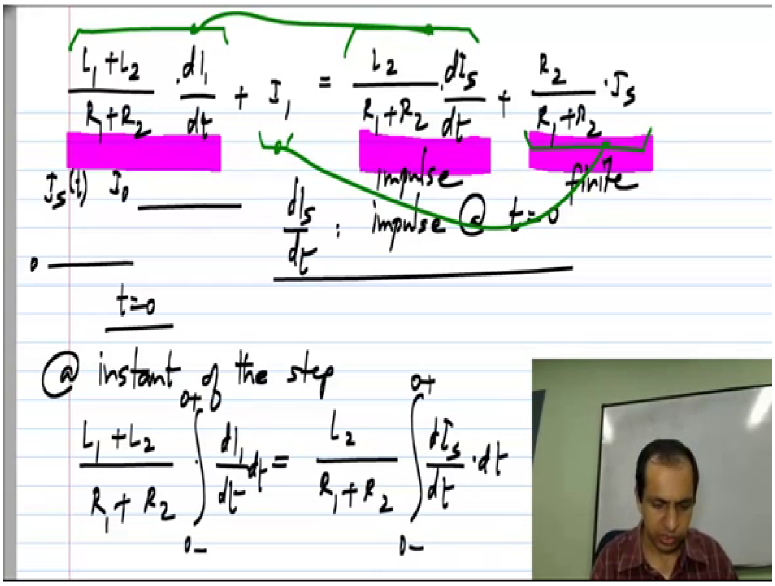

The equation also tells us a number of other things like a said what we have the circuit in which, this current I s is being step at t equal 0 it goes from 0 to some value, let me call that I naught. So, if a think about is I s as a time function it goes from 0 to 0 I naught, so the derivative of I s is an impulse at t equals 0. Now, I do not wont to introducing impulses in to or calculation in and so on and it is not necessary also just to find out what we will not we can simply appreciate the fact that the first derivative of I s is an impulse, so this is an impulse were has this quantity with just as I s is finite.

This already tells you that this I 1 cannot be continues I 1 also must have a jump, because if a I 1 is continues than the slope of I 1 will be finite everywhere and we can never impulse on the right hand side with only finite quantities on the left hand side. So, the only way for this equation is to satisfied is for I want to have a step, which means that the first derivative of I 1, d I 1 by d t also an impulse. So, this part which is an impulse balances this part, which is also an impulse.

So, that is balance by that one and this part, which is finite will balance that part which is also finite which is just at the instant of the step. So, at the instant of the step we have L 1 plus L 2 by R 1 plus R 2 times d I 1 by d t to be equal to L 2 by R 1 plus R 2 d I s by d t. Now, this means that this just the instead of the step if I integrate both sides with respective time this will be equal to each other between the same limits 0 minus and 0 plus.

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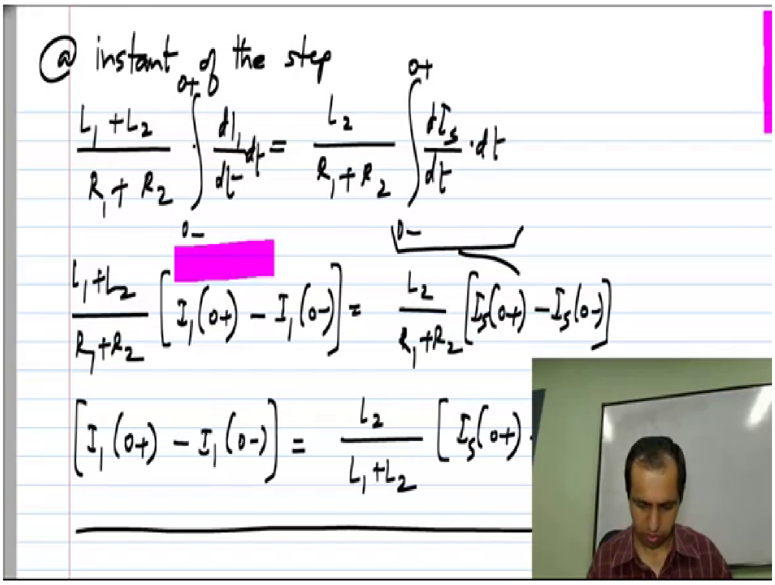

So, if I do this, this quantity with nothing but, I 1 of 0 plus minus I 1 of 0 minus and this quantity here is nothing but, I s of 0 plus minus I s of 0 minus and I will of course, have the same scaling factors on the two sides. So, what this tells you is that R 1 plus R 2 in the denominator will cancel allowed. So, we will have the change in current across the step across the instead of the step will be L 2 by L 1 plus L 2 times the change in the source current.

Now, this is similar to what we have to capacitors if you apply a voltage step across a series combination of capacitors than across each capacitor we will have some capacitor will ratio times the input step. Similarly, if you apply a step current to a parallel combination of inductors than them each inductors you will have step current and the size of the step in each will be related to ratios of inductors.

So, this you can calculate and again just like in case of capacitors when we applied Kirchhoff’s current law at the instant of the step we could ignore the resistors, because if you had infinite currents in capacitors and finite current through the resistors the resistors cot one contribute anything and we can kind of neglected. And; that is what have done mathematically here I would try to balance the left and right sides separately for the impulses and the finite quantities.

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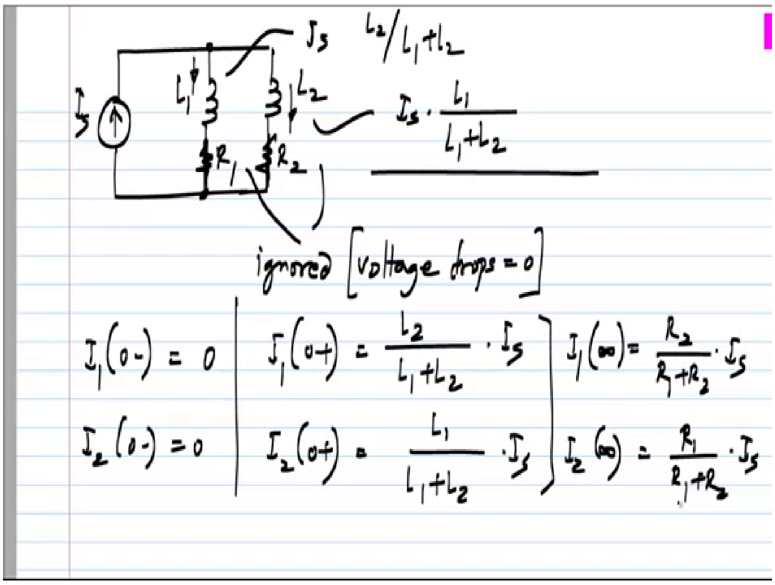

So, just like we ignored resistors in parallel with capacitors in this case we can ignore resistors and series with inductors. So, just for the calculation of current steps whatever R 1 we had here and R 2 we had here these two are ignored while writing KCL; that means that the voltage drops a cross them equal 0; that is they are short circuited. And these makes and again, because if you have a step change and inductor current across the inductor you have and infinitely large voltage and this is finite voltage across the resistor, so we neglected similar to how a neglected restive currents in circuits were capacitors current were infinite.

We have L 1 and L 2 and we have I s and you clearly see that than the current divides in inverse ratios of inductors. So, through L 1 we would have I s times L 2 by L 1 plus L 2 and through L 2 we will have I s times L 1 by L 1 plus L 2. Just like we said that in a series combination of capacitors that R cannot go any were from the intermediate plate the total charge there has to remain the same here also we can state a similar rule were the total flux in case will remain the same in the inductors just across the step.

Remember all of these calculations are valued just calculate the inductor currents immediately after the step after that the resistor voltages will be significant and we have to go from there. So, I 1, which is the current through L 1 at 0 minus was 0 and I 2 as 0 minus was also 0 and just after the application the current step this will be L 2 by L 1 plus L 2 times I s and I 2 of 0 plus is L 1 by L 1 plus L 2 times I s.

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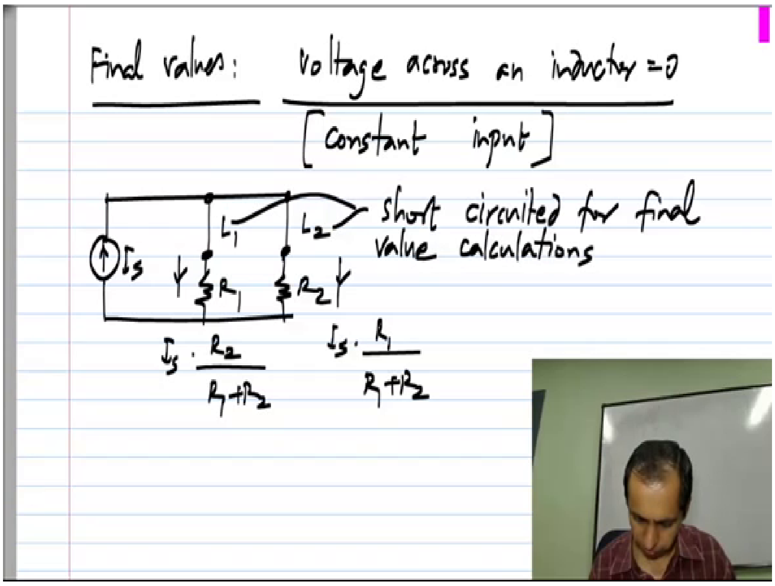

So, from these and the final values, which can also be calculated quite easily for the final values in an inductor circuit, what is that we no voltage across an inductor equals 0 this is of course, assuming constant are fees wise constant in inputs this is for a constant input. Because, than the inductor current also has to be constant and if they inductor current is constant the voltage across it, which is the time derivative of the current will also be 0.

So; that means, that all inductors can be short circuited for the final value calculations when you have constant inputs. So, we have I s and I will short circuit L 1 and also short circuit L 2 and we have R 1 and R 2, so this is the resistive circuit, so we know that the final values will be I s times R 2 by R 1 plus R 2 over there and I s times R 1 by R 1 plus R 2 over there. So, this also we know, we know the time constant, so we can construct the final solution very easily.

So, I 1 of infinity will be R 2 by R 1 plus R 2 times I s and I 2 of infinity would be R 1 by R 1 plus R 2 times I s. So, from these and the time constant we can write down the functions for both I 1 and I 2.

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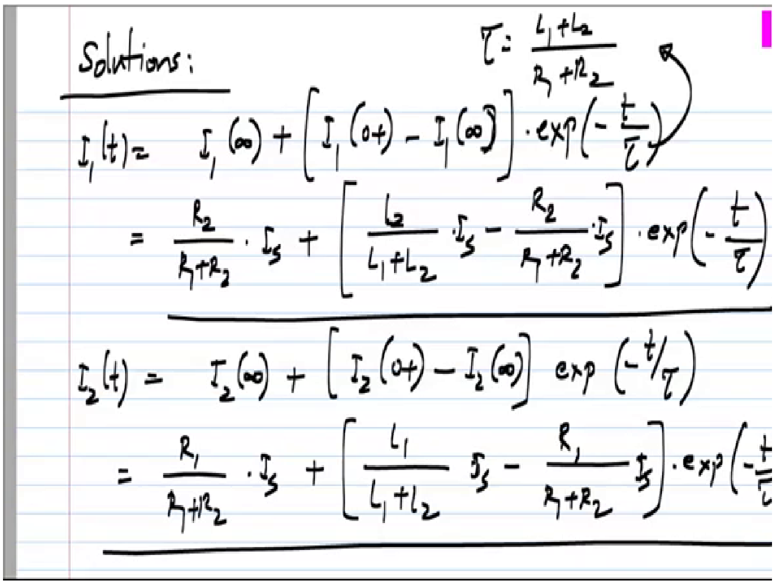

I assume 0 initial conditions, so I 1 of t, which is I 1 of infinity plus I 1 of 0 plus minus I 1 of infinity times exponential minus t by tau term, which this is the of course, the general form for any first of the circuit and I 1 of infinity we found was R 2 by R 1 plus R 2 times I s and I 1 of 0 plus is L 2 by L 1 plus L 2 times I s minus I 1 of infinity, which is R 2 by R 1 plus R 2 times I s exponential minus t by tau I will written the symbol tau, but we know that tau equals L 1 plus L 2 divided R 1 plus R 2, so this is the expression for I 1 of t.

I 2 of t is of course, the same general form I 2 of infinity plus I 2 of 0 plus minus I 2 of infinity exponential minus t by tau, tau is of course, the same and I 2 of infinity is R 1 by R 1 plus R 2 times I s plus L 1 by L 1 plus L 2 times I s minus R 1 by R 1 plus R 2 times I s exponential minus t by tau. So, these are the solutions for I 1 and I 2 as the function of time and if we have numerical values we can substitute these and also we assume 0 initial condition just before the step is applied if we do have non 0 initial conditions than its a still quite easy.

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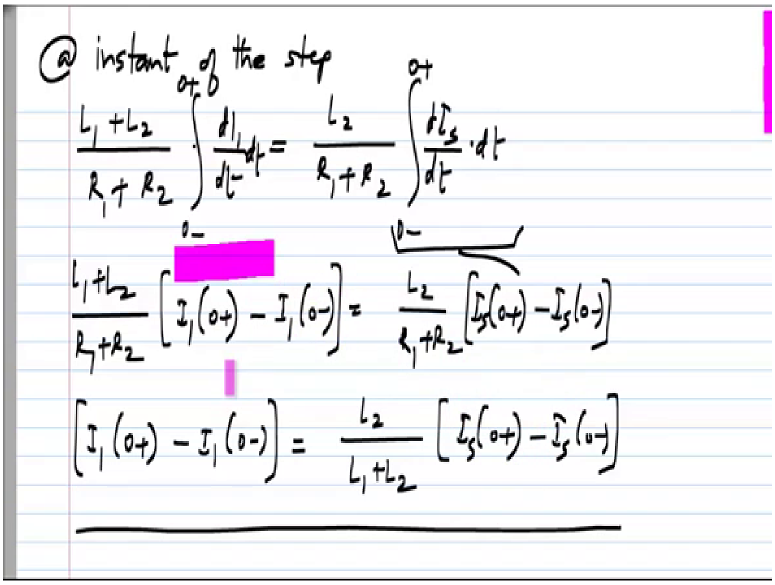

All we have to do subside the right values for I 1 of 0 minus over there, so from there we can calculate everything. So, that is how we calculate currents for any other quantity in a circuit were the inductor current can have step changes.