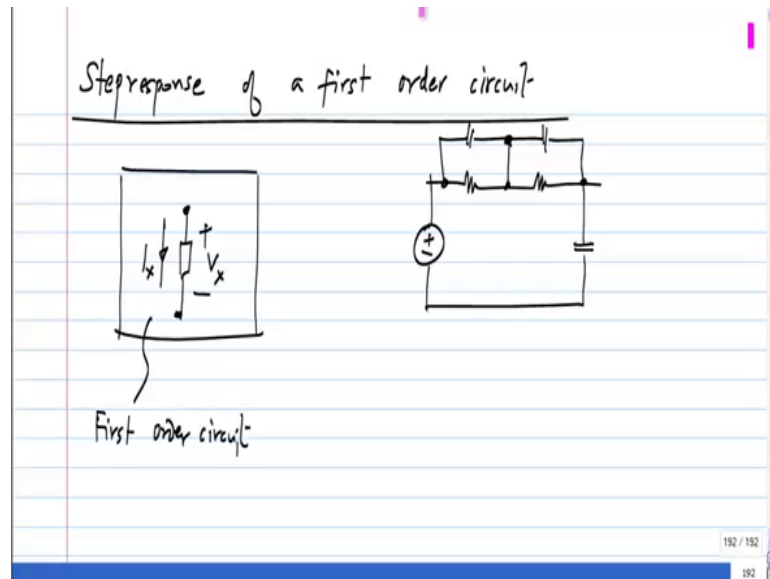


**Basic Electrical Circuits**  
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**Lecture – 128**

So, this is called as step response, because the input is assumed to step.

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So, let say you have a first order circuit; that means, that you have one effective capacitor and you take any branch quantity there, so let say  $V_x$  or the current  $I_x$  and only condition is that it be a first order circuit. So, for instance I could have I could move on with this type I could do this, this is also a first order circuit with some caveat that when you set the source to 0, I will get this capacitor an parallel with series combination. Like I said to series combination always comes with some certainties, but we will ignore those things for, now and call that a first order circuit.

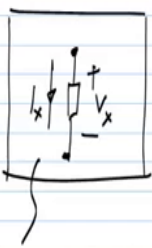
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0: Instant of the step

Step response of a first order circuit?



Step input



$V_x =$

$$V_{x,final} + (V_x(0+) - V_{x,final}) \cdot \exp\left(-\frac{t}{\tau}\right)$$

$$V_x(\infty) + (V_x(0+) - V_x(\infty)) \cdot \exp\left(-\frac{t}{\tau}\right)$$

First order circuit

Initial condition given:  $V_c(0-)$

Input source known:  $V_s(0-)$

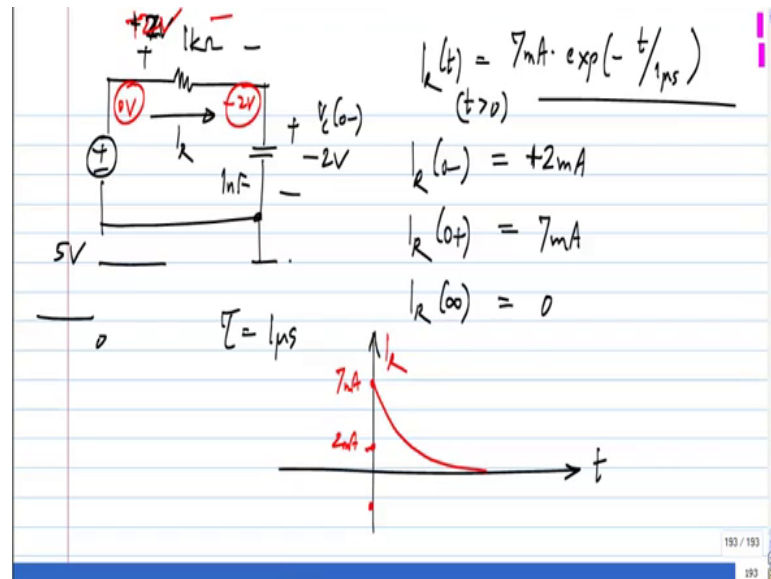
Now, a step response; means that the input or it could have multiple inputs, but let us consider a single input with steps up. The response of any voltage or any current in the circuit it could be capacitor voltage it could be anything else will be of this form I will write it for voltages final plus  $V_x$  initial, which should be taken as 0 plus, because it could be something happening between 0 minus and 0 plus it could be a jump  $V_x$  final, final of course, means infinity. So, I could also write this as  $V_x$  of 0 plus minus  $V_x$  to infinity.

This part this is the part that dice out that is the natural response and this will be multiplied by let me move this little bit exponential minus t by tau. Now, to be able to write this you can personally write down that the differential equation and formulate try to solve it, then you may have to introduce this the derive delta, so derives impulse functions and try to balance the left and right side and solve it that way. But, now that we have solved it for a few cases we can write down some general rules, so to write this response you need all these quantities  $V_x$  of the infinity  $V_x$  of 0 plus and tau.

Now, it is assume that some initial condition is given usually the initial condition number capacitor; that means, that usually the voltage on the capacitor just before this step is applied; that is what it is given. Now, from that you have to calculate, what  $V_x$  is this element may not be the capacitor it could be anything else, for instance in them my R c circuit I could be try to write on the equation for the current through the resistor or the voltage across the resistor. So, it could be any branch module.

So, from the initial condition given and the value of the input at  $t$  equal to 0 minus you can find all the branch voltages just by using a circuit analysis. So, just by using KVL and KCL you should you will be able to get this in a first order circuit you only need one initial condition. So, input source is also known I mean it is known for all time so; obviously, you know it for 0 minus also. So, what I mean by this is let me take an example the simple example.

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Let say this is the step from 0 to 5 volts and the initial condition on this is minus 2 volts this is given and, what I am interested in is the current  $I_R$ , so I want to write an equation for  $I_R$  of  $t$ . So, first of all what is  $I_R$  just before the step is applied, let me say this is 1 kilo ohm, so what is it right it is plus 2 volts, so it is plus 2 milliamps. So, initially I take volts to 0 minus I have 0 volts here with respect to this ground and minus 2 volts there, so it is plus minus 2 volts in that direction.

So, the way I have marked  $I_R$  is plus 2 milliamps, so now, this you know, what is it immediately after you apply the step. So, in this case, so when you go from 0 minus to 0 plus first of all you have to see if there are any loops of voltage source and capacitor alone. In other words the way to do it is why you ignore all the resistors you have only voltage sources and capacitors or sources and capacitors, with that you can solve for any jump in capacitor voltages.

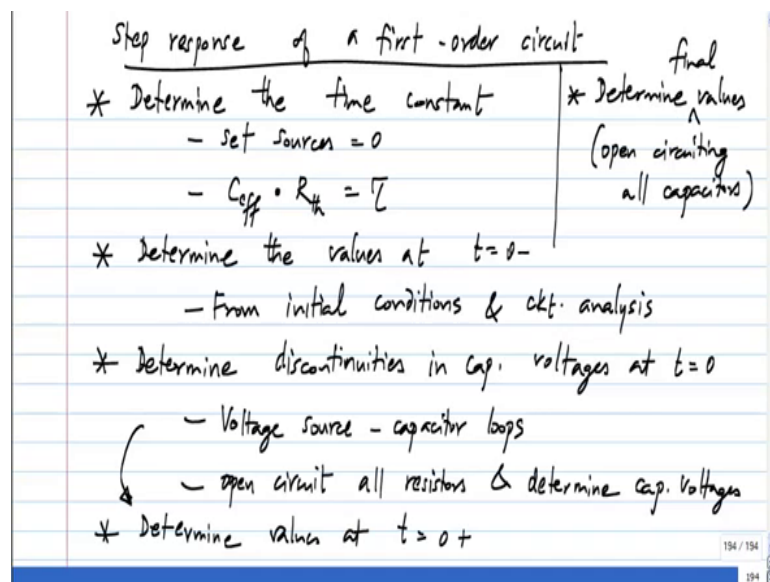
In this case, what is the value of the capacitor voltage just after applying the step it would be minus 2, because there are no loops of voltage sources and capacitors or if I

remove resistors the whole thing open circuit nothing changes. So, from that you can calculate I R or 0 plus, what is that this is 7 milliamps and the final value of I R, what is that, how do find that, you have to open circuit the capacitors, because you are talking about piece wise constant voltages.

So, the steady state also will be constant voltage across the capacitors so; that means, that the current through the will be 0. In this case it is, so happens that I R is same as the current through the capacitors, so this is also 0. So, now, what is the expression for and by the, what is the time constant, let say this is one I R parrot this case is very easy, what is that 1 micro second. So, what is equation for I R t for t greater than 0, 7 milliamps exponential minus t by 1 micro second that is all.

I do not if you have the habit of doing this, but sketching possible solutions gives you a lot of insights it is not very precise, because you are doing hand sketches, but it can give lot of insights. So, what happens to the I R, which is plus 2 milliamps just before t equal to 0 will jump to 7 and then, d k to 0, so that is what it will look like. So, you can do this for any arbitrary circuit the only complication will come the additional complication is when you have loops of voltage sources and capacitors.

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So, the way to go about solving for these things is first you determine the time constant this is, this you set sources equal to 0 and the effective capacitance times determine resistance across it; that is the time constant. And then, you will determine values at t equal to 0 minus this is from initial conditions and some regular circuit analysis and; that

is from  $t$  equal to 0 minus to 0 plus it can jump.

Now of course, you know that it will happen only if you have voltage source capacitor loops. Now, even otherwise you if you not able to identify you simply open circuit all resistors the basis for doing this is the following if you have any discontinuities at all the connects through capacitors will be infinite and the connects through resistors will still be finite as long as all the voltages in the circuit are finite. So, you have these nodes, where you have finite currents from resistors and possibly infinite currents from capacitors.

So, you remove from all the finite currents and it will not change the anything clearly any finite values negligible compare to infinity; that is the idea here. So, you open circuit all the resistors and do the circuit analysis when you have voltage source capacitor loops you have to do it based on chance, which is the same as saying you do it based on the derive delta functions of current. So, you open circuit all resistors and determine capacitor voltages, but any way by inspection of the circuit it should be able to tell whether there are loops containing voltage sources and capacitors or not.

So, this will tell you values set when from this really they also determine this is by this again you open circuit all capacitors, because we are applying piece wise constant inputs we expect voltages across capacitors also would be piece wise constant. So, when the voltage across the capacitors constant the current through; that is 0 that is why we are open circuiting it this is not valid for any other type of input you had input that was going up and down you expect the capacitor voltage also to be constantly changing in that case clearly you cannot open circuit anything there will be currents through that.

So, only we have those piece wise constant nature of the input that you can do this. So, finally, the solution would be of this form I mean I have written it in terms of voltages, but it could be current source this is 0 plus. So, is to find out this that even you have to find out if there are any discontinuities at  $t$  equal to 0. So, you should be able to analyze these things without actually going into details of the differential equation.

Whatever I had a 0 here is the instant of the step and by the way this type you can have multiple steps you tilled as first you apply some step it reach some value not necessarily the final value you determine, what that is use that as the initial condition for the next step that is all.