Basic Electrical Circuits
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Lecture – 124

In this lesson we will summaries the differential equations and responses of first order system and also look at common limit statements such as a DC current cannot pass through a capacitor or we cannot have a DC voltage across an inductor and so on. So, first of all we will take this as an example, but to the statements I make will be by enlarge true for every first order circuit.

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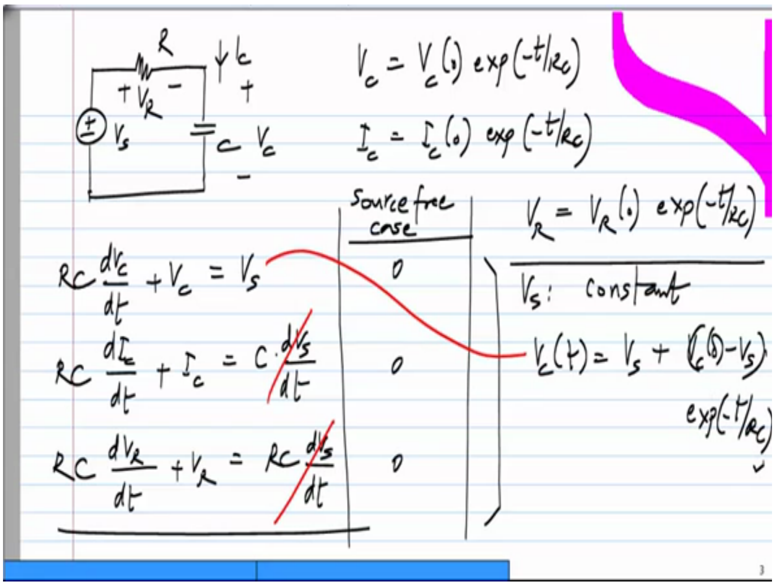

Any circuit with the single capacitor and any number of independent sources and other linear components can be reduced to this form this V s R and c this may be the actual circuit or this V s and R could be the Thevenin equivalent of whatever is connected to the capacitor. As long as you have a single capacitor you will have a first order system at most and this will be the equivalent circuit. Now, let us identify different possible variables I have marked V c the capacitor voltage I c the capacitor current and V R the voltage across the resistor.

Now, we have already seen that we can write the differential equation for this circuit in terms of any of these variables, if you write it in terms of V c this is the differential equation in terms of I c this is the differential equation and finally, in terms of V R this is the differential equation and these are generally true we have not made any assumptions, these are just a differential equation we get if we chose V c as the variable or I c as the variable or V R as the variable.

Now, the thing you can notice which are pointed out earlier is that the left hand side is exactly the same. So, in the source free circuit when V s is 0 all three essentially give you the same differential equation with the right hand sides being 0. If we set V s equal to 0 that is the source free case then the right hand side will be 0 0 and 0. So, you can see that the differential equations are exactly the same, if the right hand sides are 0 only the variables are different.

Now, because the differential equations of the same each of these variables follows the same response. So, the natural response each of these which corresponds to the source free case would be V c would be V c of 0 exponential minus t by R C and for I c also it will be I c of 0 exponential minus t by R C and finally, we have V R of time would be V R at times 0 exponential minus t by R C. So, once you are familiar with the basics of a first order R C circuit you do not even have to write the differential equation, you will be able to write down the natural response simply by identifying the initial condition which you can do a regular circuit analysis.

Now, to understand this properly you need to understand first order differential equations and all the things we have discussed. But, after you have enough experience with it any first order system you should be able to solve simply by identifying the appropriate initial condition and this is true for any variable in the circuit, because all variables will follow the same pattern as you can see over there.

Now, if V s is assumed to be a constant that is the case we have looked at so for then also things are quite easy, if V s is a constant then again these two will be 0. Because, the time derivative of V s is 0 and we know the solution to that as well in fact, it is exactly the same solution I c will eventually go to 0 by the way if you remove this derivative term you will be left with something else just an algebraic equation and that is the steady state solution for constant inputs the steady states of any of these variables is also a constant and the only way these things will be constant is if the derivatives are 0.

So, the steady state equation is basically when the derivatives are 0 steady states for constant inputs and the responses are the same only for V c is it different. Now, again in a general first ordered circuit which could be more complicated than this first order of course, means a single capacitor what you could have any number of resistors controlled sources any number of independent sources and so on. So, you pick any variable it will follow in general some equation like this and what is the solution to this, the way we have written it when V s is constant V c of t is V s plus V c of 0 minus V s times exponential minus t by R C I will elaborate on this a little bit.

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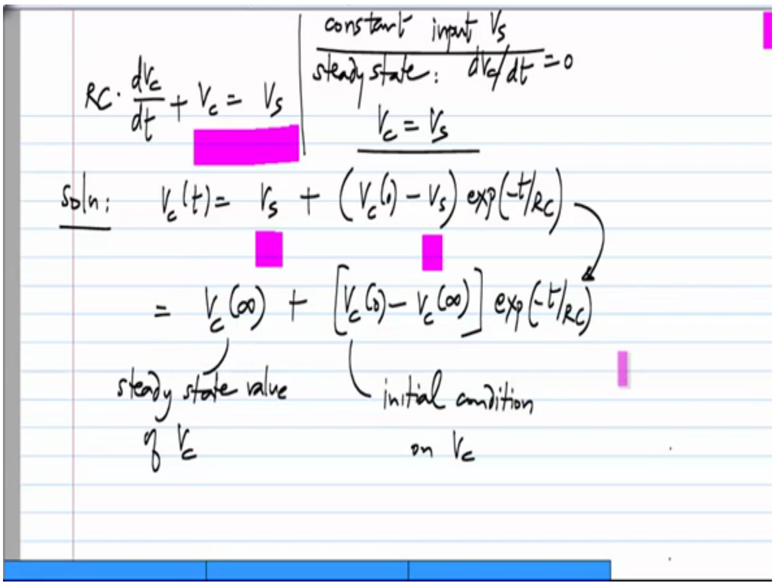

Let me write down the differential equation as well the solution to this is written in the form of the force response plus the natural response. Now, it looks like this V s is specific to this one, but if you think about it for a moment you will realize that whatever appears here is nothing but, the study state solution. So, if you have a constant input V s then the study state is given by d V c being 0 which means that we take only this part of the equation V c equals V s, this is what happens after steady state is reached and like I emphasized every circuit that will consider now will be a stable circuit and this short of steady state will be reach the natural response will eventually die out.

So, this whole equation can be written in a more general way as V c of infinity plus V c of 0 minus V c of infinity times exponential minus t by R C. So, V c of infinity is the steady state value of V c we know that it will asymmetrically reach this value and V c of 0 is the initial condition on V c and this solution is valid when V s is a constant that is very important, if you have V s to be some arbitrary function of time you cannot simply substitute that function of time over here, so this is valid only if V s is a constant.

So, now, this is a general form of solution to any first order circuit we again looking at R C circuit that to with the single R and C, but as long as you have a circuit with either a single capacitor or a single inductor and the rest of it the circuit could be as complicated as you can make it, you could have any number of independent sources and linear element such as resistors and controlled sources.

We know from Thevenin theorem that all of that can be reduce to a single voltage source in series with the single resistor at the terminals of the capacitor and if you have an inductor again the same thing can be done. So, the result is the first order circuit the solution to any variable in such circuits will be of this form.

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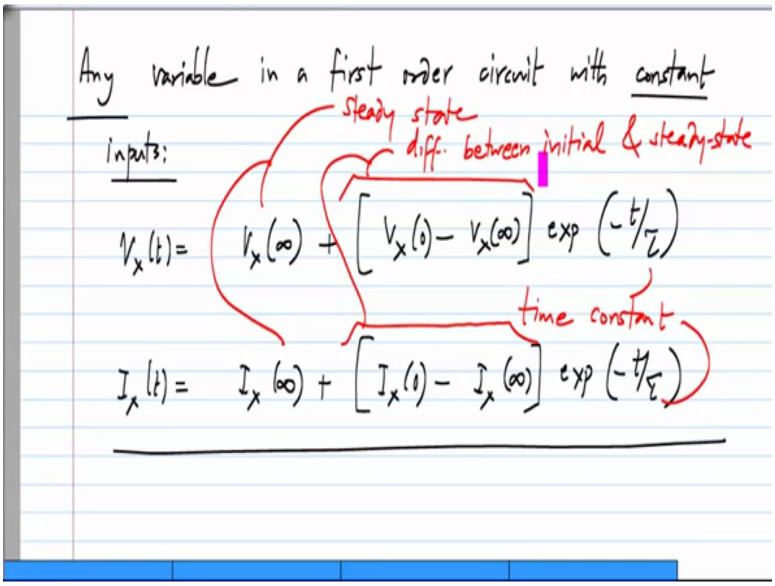

The constant input part is important, let us say it is some voltage V x as a function of time it will be the steady state value plus the difference between initial and the steady state values which decays with time. So, like already pointed out this is the steady state value and this is the difference between initial and steady state and this is the time constant and this is true for any current also in general it will be of this form and of course, it will have the same time constant. So, this is the time constant, this is the steady state value and this part is the difference between initial and steady state values.

What is the point of writing down all this? I have mentioned earlier, if you have a first order circuit with enough experience you should be able to write down the response even without ever writing the differential equation. Now, this is not to say that you should forget everything you know about the differential equation. So, initially you write down the differential equation and make sure that everything is consistent, but after while you will be able to simply identify the steady state values of either currents or voltages I will explain how to do that in a moment and the initial values of currents or voltages.

Once you identify those things the only thing remaining to be identified is the time constant and that also can be identified, because you have either single capacitor or a single inductor and the effective resistance that appears across the capacitor or the inductor in the source free circuit that is the equivalent resistance and the product of the resistance in capacitance is the time constant.

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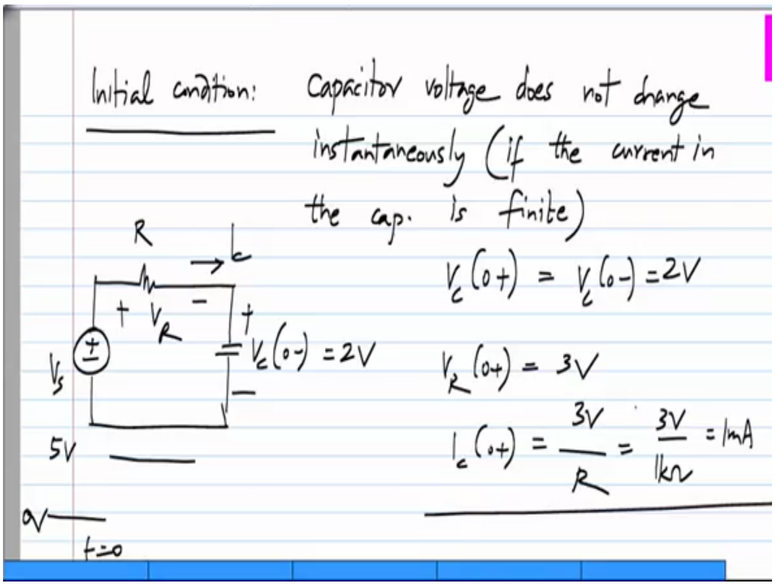

Now, for all the cases that we know this will be modified a little bit later. How do we do these things? So, initial condition it means that before applying the input or before the step changes to some other value, the capacitor has some voltage and usually you find it by assuming that the capacitor voltage does not change instantaneously, this is true if the current in the capacitor is finite based on this you will be able to calculate the initial condition on any variable in a circuit.

So, again let me take a simple circuit like this and let me say that the input V s changes from 0 volts to 5 volts and the voltage on the capacitor a just before the step let us say this step occurs a t equal to 0 this superscript of minus means that it is just before the step and that is equal to 2 volts.

Now, first of all the value of the capacitor voltage if you wanted to find the initial condition on the capacitor voltage that is just after the step which is indicated by 0 plus this will be the same as V c of 0 minus, because we cannot have infinite currents in this circuit, if there is an infinite current through the capacitor then that infinite current also flows through the resistor which means that there has to be in infinite voltage across the resistor we will assume that our V s is finite.

So, we cannot have infinite currents in a case like this later we will see circuits where that can happen, but in this circuit it cannot. So, this will be equal to V c of 0 minus which is same as 2 volts. Instead if you are interested in V R, what is V R the initial condition on V R just as the step is apply V R is nothing but, V s minus V c. So, V s is 5 volts just after the step is applied and V c is 2 volts, because it is continuous V R would be 3 volts.

Finally, if I c you are interested then you can see that this I c is nothing but, the voltage across the resistor divided by R and the initial condition on the voltage we already found out. So, the initial condition on the current is 3 volts by R and if R is 1 kilo ohm it is 1 milli ampere. So, you will be able to find out the initial condition directly from the circuit quite easily.

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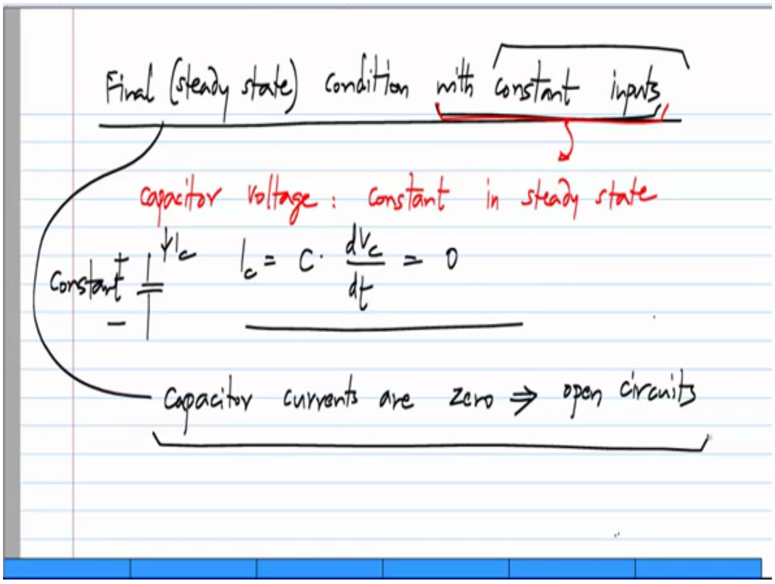

Then, the final condition or the study state condition with constant inputs this is important we are not dealing only with constant inputs. The differential equations we wrote for general and independent of what kind of input you have, but the solutions we have obtain is only for constant inputs. Now, what makes this is it evaluates the following, if you have a constant inputs the voltages across the capacitors will also be constant in steady state.

So, this fact that we have constant inputs means that capacitor voltage is a constant in steady state. So, that gives as a great simplification if the capacitor voltage is constant the capacitor current. So, here we have a constant voltage, so the capacitor current I c which is c times the time derivative of the capacitor voltage will be 0. So, if you have constant inputs and every steady state the capacitor currents will be 0 and which means that they can be treated as open circuits this is fine.

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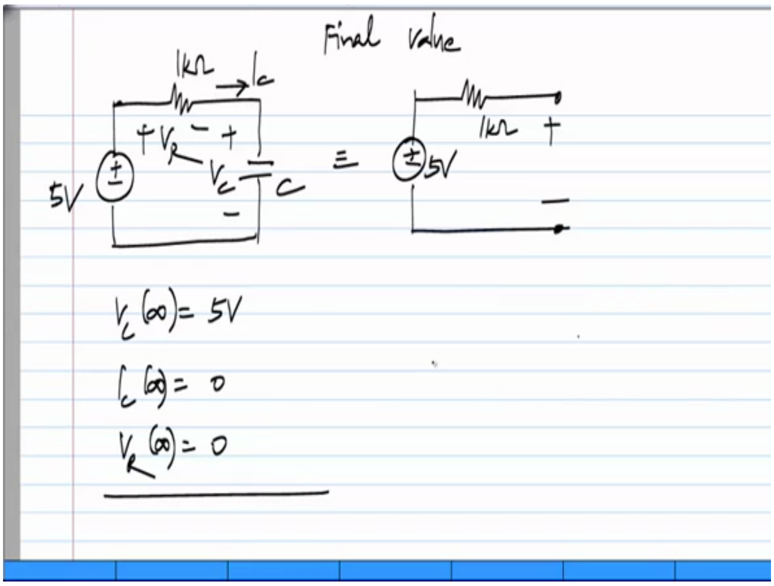

So, again we will take the same example as before, if you are interested in calculating the final value with the 5 volt input and 1 kilo ohm resistor and some capacitor C as far as the final value calculation is concerned we saw that the capacitor current will be 0 and for the final value calculation only we can open circuit the capacitors. And now it is very easy see what is the final steady state voltage, it is equal to 5 volts, because no current flows through this resistor either.

So, if we call this V c, V c of infinity is 5 volts this is the very simple circuit, but in any circuit you can open circuit the capacitor then you will be left with only independent sources and resistors or controlled sources, this is like the earlier circuit analysis without capacitors which you can do quite easily V c steady state value is 5 volts on the other hand if you wanted I c then you can clearly see that, that I c is 0 here because of the open circuit. So, I c of infinity is 0 and similarly if you are interested in V R as that was the variable of interest and you can see that V R is also 0 because there is no current through the resistor. So, this also can be identified quite easily.

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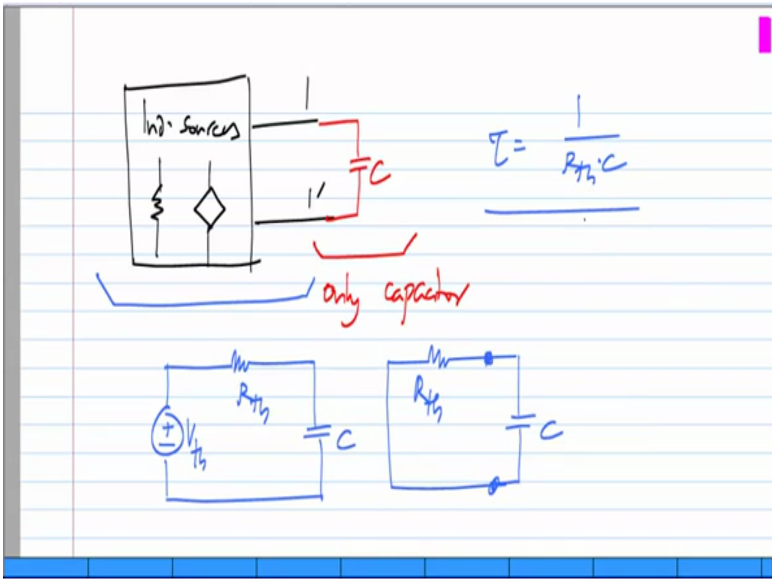

So, we discussed how you can find the initial and the final values by inspecting the circuit or by doing simpler circuit calculations that is you do not have to worry about the differential equation, you may have to solve for some algebraic circuit that is the circuit with independent sources and resistors and controlled sources and so on and finally, calculating the time constant is also very easy.

So, let us say we have any circuit with independent sources then resistors and controlled sources and we have two terminals available to us and across these we connect the capacitor, this is the only capacitor in the circuit. In that case this circuit will be at most of first order and the rest of this can be represented by a voltage source V t h in series with R t h and we have an capacitor C then the time constant of this is found by setting the independent sources to 0 which short circuit is voltage source and we have the C and you have to find the resistance across C and that is equal to R t h.

So, the time constant is 1 over R t h times c, so in other words if you have a first order circuit every variable will follow this type of behavior then you first find I x of 0 this is by assuming that capacitor voltages do not change and this is true if the capacitor currents are not infinite, we will later see examples where it can be infinite. And you find the steady state values that is values at infinity by open circuiting the capacitor, if you have an inductive circuit you have find the steady state by short circuiting the inductor.

Because, the inductor voltages are 0 in steady state with constant inputs, then you have these values are 0 and infinity and the time constant can of course, be found by finding the Thevenin resistance across the terminals of the capacitor in the source free circuit. So, the entire response can be written down by inspection.