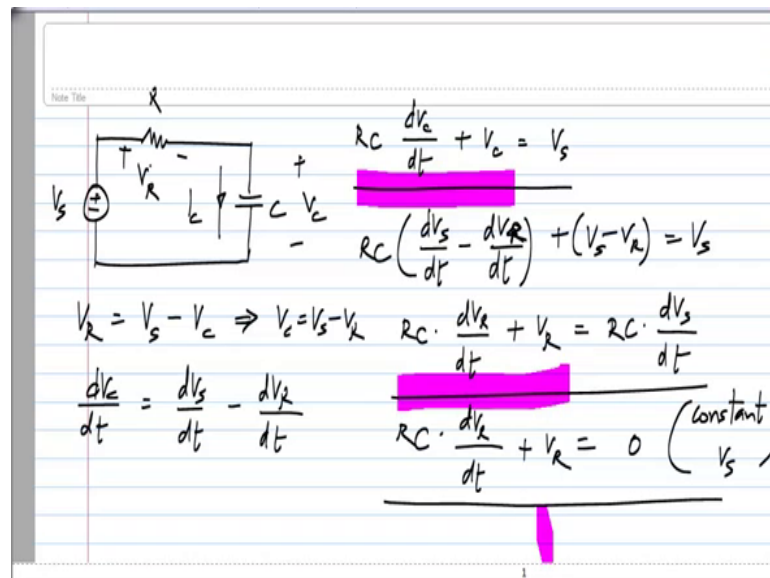


Basic Electrical Circuits
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Lecture – 123

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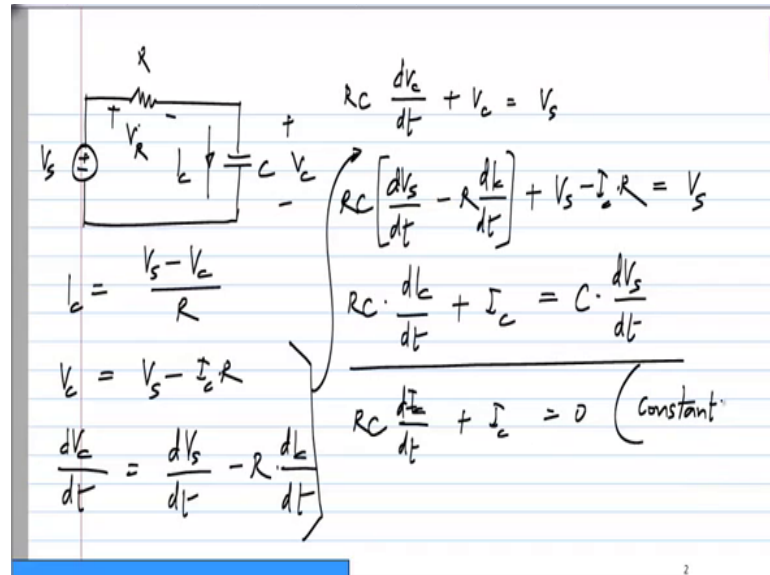
Let me get back to the original circuit what was the differential equation governing this in terms of this V_C by the way I could have return the differential equation in terms of some other variable, let say the current I_C or the voltage V_R and so on. What you will get in that case please do that? Please derive the differential equation with I_C or V_R 's variables and compare it to what I am going to work out by the way the differential equation for this V_C was what was it. So, that is what it was this is the differential equation governing the circuit, then it is return in terms of V_C the capacitor volts.

Now, let us right the differential equation for the circuit in terms of other possible variables such as the voltage across the register V_R or in a current through the capacitor I_C . So, we see that V_R is nothing but, V_s minus V_C , so this comes from Kirchhoff's voltage law. So, then from this we see that V_C is V_s minus V_R . So, now, differentiating both sides what we will get is that the derivative V_C equals the derivative of V_s minus the derivative of V_R and substituting these two relationships into that one we get the following RC times that plus dV_R by dt plus V_s minus V_R equals V_s .

So, this if you rearrange the terms you will see that it is RC times dV_R by dt plus V_R equals RC times the time derivative of V_s . And if we do know that V_s is a constant we

will get $RC \frac{dv_R}{dt} + V_R = 0$. So, this is if you know that V_s is a constant. So, the important thing to note is that the left hand side of that and the left side of that there in the same form. So, the coefficient of first derivative of is RC and coefficient of V_R is 1.

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Now, we can also now we can tried in terms of I_C and it will be similar I_C is nothing but, V_s minus V_C divided by R and if I write V_C as see that it is V_s minus I_C times R . So, time derivative of V_C is time derivative of V_s minus R time derivative of I_C and if I substitute these two in there what will I get RC times dV_s by dt minus R dI_C by dt plus V_C which is V_s minus I_C times R and the right hand side we have V_s and if you rearrange this we will end up with RC dI_C by dt plus I_C equals C times dV_s by dt and again if V_s is known to be a constant we have RC dV_s by dt plus I_C equals 0.

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$$RC \frac{dV_C}{dt} + V_C = V_s$$

$$RC \frac{dI_C}{dt} + I_C = C \frac{dV_s}{dt}$$

$$RC \frac{dV_R}{dt} + V_R = RC \frac{dV_s}{dt}$$

Natural response

$$V_C = () \cdot \exp\left(-\frac{t}{RC}\right)$$

$$I_C = () \cdot \exp\left(-\frac{t}{RC}\right)$$

$$V_R = () \cdot \exp\left(-\frac{t}{RC}\right)$$

Piecewise constant inputs:

$$V_C: V_s + (V_C(0) - V_s) \exp\left(-\frac{t}{RC}\right)$$

$$I_C: I_C(\infty) + (I_C(0) - I_C(\infty)) \exp\left(-\frac{t}{RC}\right)$$

Fine, I have written all three with variables on the left side and the source on the right side. So, what you notice about these three differential equations anything or no.

Student: ((Refer time: 05:13)).

Left hand side is exactly the same the homogeneous parts of the differential equation are the same and this is a general property of any circuit. Now, it is not that earlier we found the solution for the capacitor voltage as $V_s + V_C(0) - V_s \exp(-t/RC)$. Now, if you look at this left hand side is exactly the same so; that means, that the homogeneous parts of everything is the same. So, all of them whether V_C will be something times exponential minus t by RC that is the homogeneous solution to V_C .

Similarly, I_C will also be the same way and we are will also be the same way, these are the natural responses or ((Refer Time: 06:17)) response or the homogeneous solution. Now, this is a general property of any circuit and this is nothing to do with the first order differential equation either. So, if you have it turns out that for higher orders you will get combination of exponentials. So, this just like we have one time constant here for higher order equations you will have multiple time constants and multiple exponential.

So, the natural response of any variable in the circuit whether it is any current or voltage will be exactly the same. So, you do not have to if I had ask you for the current in this loop or the voltage across the register you do not have to solve for the circuit again the solution will be exactly the same form. And also now for piece wise constant inputs everything will also have the same form V_C we already know we will elaborate on this

little V_c of 0 minus V_s and I_c will be basically the final value which I will write as I_c of infinity plus I_c of 0 minus I_c of infinity, this is the general form, the initial minus final value divided by whether time constant RC .

Finally, the V_R also will be the same V_R of infinity which is the final value plus the difference between initial and final values times exponential minus t by RC the ((Refer Time: 07:54)) same time processor. So, it should be able to work out the solution for any variable in the circuit in any complicated circuit.