Basic Electrical Circuits
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Lecture - 122

We will now consider an example with multiple sources switching at different times. Fundamentally no different from any case we have discussed before, but just a little more book keeping involved.

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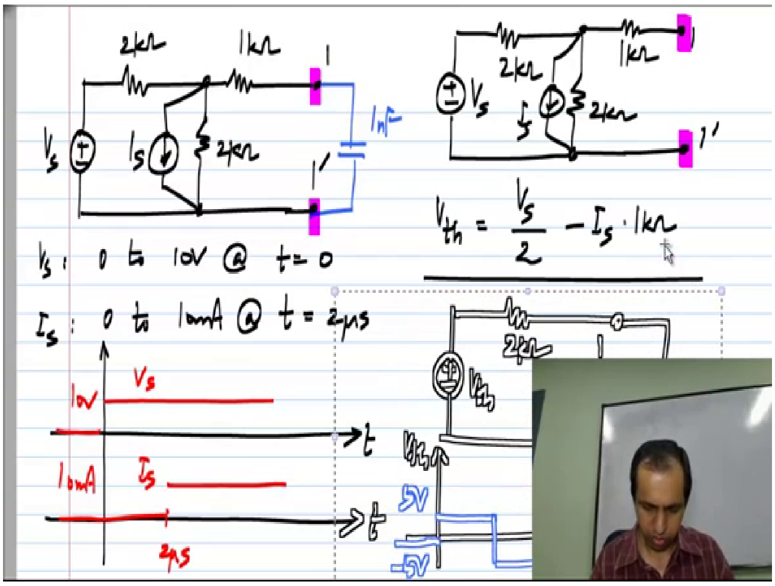

As always a circuit with the single capacitor and let us say, this is 1 nanofarad and these resistors are 2 kilo ohms, 2 kilo ohms and 1 kilo ohm and I also have a current source. Let say it is given that V s switches from 0 to 10 volts at t equals 0 and I s switches from 0 to 10 milli amps at t equals 2 micro seconds. So, if I had to sketch these, this would be V s, the x axis is of course, time and the y axis is either voltage or current and I s would go from 0 to 10 milli amp at t equals 2 micro seconds.

So, looks slightly more complicated, but essentially between the terminals of the capacitor 1 1 prime, we can still represent the whole thing with a single source and series with single resistor or a single current source in parallel with the single resistor. So, either can be used, I will use the Thevenin equivalent here, but you can use the Norton equivalent and verify that, the answers are exactly the same.

So, let me call these V s and I s, let me keep them as variables for now and 1 1 prime, then this is what I have. So, now, again I will not show the details of calculating the Thevenin equivalent voltage, that you can do quite easily and you will see that the voltage between 1 1 prime when it is open circuited, in other words the Thevenin voltage will come out to be V s divided by 2 minus I s times 1 kilo ohm, so that is what it is going to be.

So, now, if you evaluate this, V s is changing at some time I s is changing at some other time, I s is like that. So, if you sketch this entire input to the circuit, so if I calculate V t h in series with R t h and I have my capacitor there, which is 1 nanofarad and if I sketch this V t h, it will look like that. So, this is really all I need to do, so V t h will be 0 before t equal to 0, because both V s and I s are 0. All you have to do is to substitute these values in to that expression and between t equals 0 and t equals 2 micro seconds, the output will step up to 5 volts which is V s by 2 and I s is still 0.

After t equals 2 micro seconds, the contribution of V s will be 5 volts, it is V s by 2 and contribution of I s will be minus 10 volts. So, what happens is, this will switch down and stay at minus 5 volts forever, because we have no other changes that are specified. So, all we have to do is still solve the same circuit, by the way the value of R t h between these two terminals again calculated for yourselves and see that, it comes out to be 2 kilo ohms. So, all we have to do is still solve the same kind of circuit as before, but with the slightly more exotic input.

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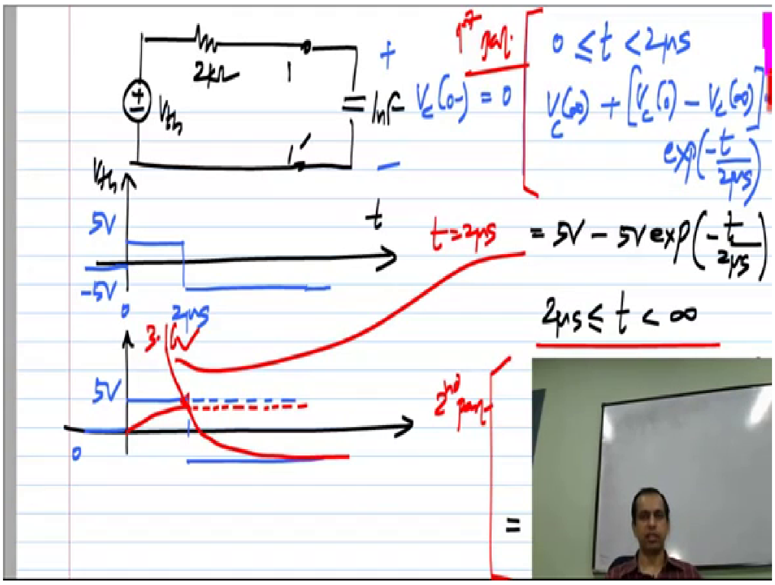

As long as the inputs are piece wise constants this is quite easy, you treat each piece separately as a separate step input with it is own initial condition. So, first we have this step, the input steps from 0 to 5 volts and let me say that, the initial condition just before the first step is applied at 0 minus is 0, it can be anything, but let me assume that it is 0.

So, then for this part of the curve from t equals 0 to t equals 2 micro second, essentially I have only a single step input which goes from 0 to 5 volts; that is, I will be solving the circuit for an input like this. After this it will change, but it does not matter it is effect will only come later, this is 0 volts that is 5 volts that is 2 micro seconds and we know how to do this, we have done this already. So, the solution from t of 0 to 2 micro seconds will be…

We can still use the general form, which is V c of infinity plus V c of 0 minus V c of infinity, the whole thing times exponential minus t by R c and R c here you can see is 2 micro seconds. So, what the output will do is, this V c of infinity is 5 volts as for as this step is concerned, we are only solving with this step input that is the one thing you should get confused about. It is not that V c of infinity will actually reach 5 volts, because the input is being changed again, but if this for the step, if the 5 volts step for first and till take was infinity, it would go to 5 volts. So, it is the final condition assuming only this step is applied; that is an important thing to understand.

So, V c of infinity here is 5 volts plus V c of 0 is 0 and we have minus 5. So, essentially I have 5 volts minus 5 volts exponential minus t by 2 micro seconds and if I sketch that part, it will do something like that and it would eventually have reached 5 volts, but I take was 2 micro seconds we have the other step. So, we can treat that as a separate step and that step is going towards minus 5 volt. So, for the following time period, 2 micro second to infinity, because we have no other changes still infinity.

So, this will be valid and this is the difference step response and as far as this step is concerned, see the input is going to minus 5 volts and at t equals infinity the capacitor will be an open. So, whatever V t h is, that will be the value of V c. So, as far as the second step is concerned, V c of infinity is minus 5 volts plus V c of 0 for this is the initial condition, initial condition is just before the step. I do not want to make the arguments too complicated by writing 2 micro seconds and all of those things and showing this is the combination of sketches and pointing out what happens where.

So, this V c of 0 do not assume it is t equals 0 here, it is the initial condition for whatever step is being applied, so it is at 2 micro second. So, if you calculated it, the first step the 0 to 5 volt step will cause a change like this and it turns out that, the second you calculate for yourself, this is done by substituting t equals 2 micro seconds in this equation and it turns out to be 3.16 volts.

So, V c of 0 for the second part of the response is 3.16 volts minus V c of infinity and that V c of infinity is nothing but, minus 5 volts, because that is for it is eventually going to reach and the exponential is exactly the same, because is a circuit is the same and the time constant is the same. So, essentially each time the change happens you treat that as a new step, the initialization of a new step and solve for it.

So, for this part the solution is minus 5 volts plus 8.16 volts exponential minus t by 2 micro seconds. So, what it will do is the following, so it will eventually reach minus 5 volts it is suppose to reach that the way it will do it is like that. So, the first step will take it to some positive value it does not reach all the way to 5 volts, because we only given 2 micro seconds for it and 2 micro seconds is the time constant of the circuit, it takes many, many time constant for it to reach very close to 5 volts.

So, now it reaches this voltage and then from there it false by to minus 5 volts, because of the application of the second step. Now, one thing I have to be careful about here is this t should be really t minus 2 micro second I have not being very strict with notation here, this t does not refer to the time axis t here, but it is the time elapse after the second step is applied. So, maybe I should have written t minus 2 micro second over there, because this applies only to t more than 2 micro seconds. So, this is the second part and this is the first part.

So, in a similar way you could have a network with many resistor, many independent sources whose values are stepping from one constant to another or you could have fix voltages and currents with some switches being operated, when you change the states of switches what can happen is that it can effectively look like a step input and sometimes when you change the states of switches even the time constants can change.

So, you have to be aware of all of these things, but after each step is applied you treat it as a constant input circuit with certain initial condition and then workout the result. Now, at the time of the next step is applied you calculate the initial condition from the previous solution and proceed. So, even if you have very large number of steps you can do it systematically, you can solve for the voltages or currents in the circuits systematically like this.