Basic Electrical Circuits
Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

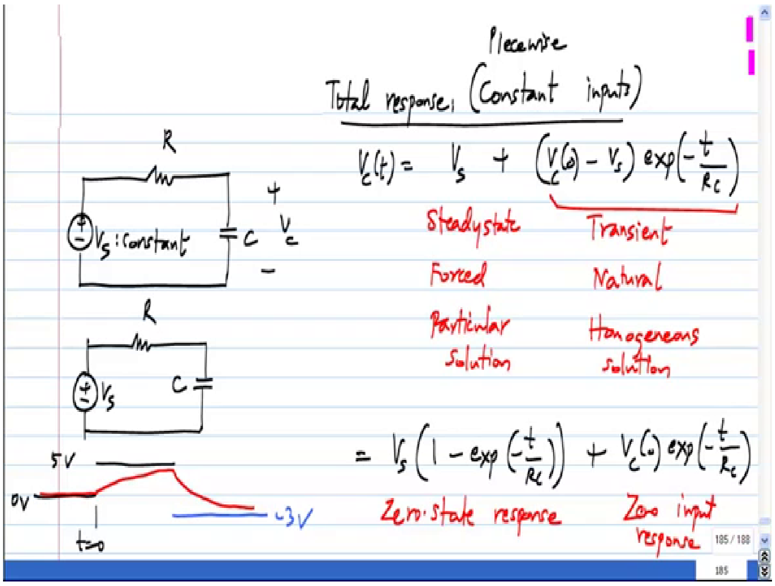
Lecture – 121

In the previous lecture we started analyzing circuits with capacitors, with capacitors and the same thing will applied to inductors as well and the result is that, we have to solve a differential equation. That if we have a single capacitor, we will end up with a first order differential equation and the solution to that turns out to be some explanation response, that is the solution to the homogeneous part of it. And if you have a constant input, the steady state response is also constant and total solution is the sum of steady state and natural response.

The natural response to the exponential, the steady state response for a constant input is a constant. So, combing these, you can find the total solution and so the total response is the sum of study state and natural responses and the natural response comes with some unspecified scaling factor, because that response will satisfy the homogeneous equation with any scaling factor. So, the particular value of the scaling factor has to be found from initial conditions.

Now, with the capacitors you have this property that, it is voltage cannot change instantaneously if the currents are finite. So, that is how you find the initial conditions.

(Refer Slide Time: 01:19)



So, for instance we took this circuit, where there is a constant V s applied to this combination of R and C and this V c has some initial value at t equal to 0 and the total response turned out to be, we could write it in a many different ways, where this is the steady state or forced response or the particular integral as it is called the literature on deferential equations and this part is the transient response. Transient; obviously, means that it is short live and it will go out eventually or the natural response, meaning that this is some propriety of the circuit by itself, not the input that you applied.

Because, you have particular value of R and C, there is an exponential with a certain characteristic with 1 over R C in the argument that is what gives you the natural response or you can also think of it as the solution to the homogeneous equation. And other way of dividing it up is to group all the terms with V s and the terms with the initial conditions and you call this the zero state response. In this case state applies to, the whole state applies to the voltage on the capacitor, the voltage on the capacitor can have a certain value based on it is history, that is what the status is and this is the zero input response.

So, you can think of these two expressions as, here where you have the steady state solution, but there is some residue, which is the difference between initial and steady state. That part decades with time or this also is eliminating. In that, this you can think of as the source voltage building up to it is final value, that is this 1 minus exponential and the initial condition decaying off. So, that is the interpretation of these parts of a solution.

In this particular case, we saw that the transient response or the natural response always rise out, because of that exponential with a negative argument inside it, it always goes to 0 as t tends to infinity. So, you will be left with the study state response. Now, this is the property of what are known as table circuits, pretty much every circuit that we will consider in this course will be a stable circuit. So, that means that finally, you will have a response which is equal to the steady state response.

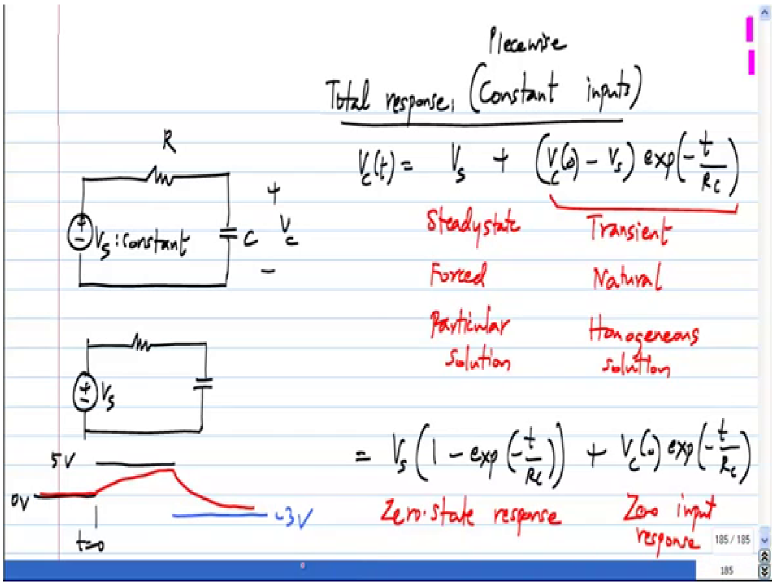
This of course, all these apply with constant inputs and if you have some V s of t, you cannot substitute V s of t into this formula. Now, this can be a model for something like this, where you can say V s changes from some value, let say 0 volts to 5 volts. So, essentially there is a step change in V s that also falls in this category. So, when I say V s is a, when I say a constant input it can be piecewise constant, because let us say we have a step here.

So, you note down the value of this capacitors voltage just before the step and in this case, infinite current cannot flow through the capacitors. So, that value will be preserved just after the step that will form this V c of 0. If we define that instant to be t equal to 0, that will be this value of V c of 0 and this other value is 5 volts will be V s. So, you can find this and let say after some time, this V s changes from 5 volts to minus 3 volts.

How would you find the solution then? So, again the way this equation is written, we have to read from this instance as t equals to 0, but you get the idea. There are some values of V c just before the step and you use that for this V c of 0 and the final value of this minus 3 volts that is what you use for V s. So, when I say constant, it does not have to be absolutely constant, it is as long as piecewise constant you can work out things piecewise piece. So, is this clear?

So, it is pretty; obvious, that what happens is. Let us assume that the capacitor voltage initially was 0, then it will start rising up. Now, we does not reach all the way to 5 volts, but you can compute what that is depending on the amount of time that is elapsed and then it will start heading towards minus 3 volts and if there is a step change again, it will change in some other volts. So, for arbitrary piecewise input, you should be able to find the response of a circuit like this.

(Refer Slide Time: 07:18)



Now, this kind of solution also applies when it is a same type of circuit, but looks very, very slightly different. So, let say this is now I constant a DC source and you have R and C and you once have seen problem of this type, where the switch is either open or closed at some instance of time. So, before t equals to 0, whatever voltages on the capacitor space as it is, because no current can flow through the capacitor, that is an open loop and when t equal 0, this switch is turned on and you get this V s R and C and series. Exactly the same circuit as we had before and the solution will also be the same.

So, the value of the V c of 0 will be whatever was on the capacitor before you close the switch and V s is the value that you apply. So, that is exactly the same thing apply and you could have lots of variance of this and instead of this closing like this, you could have some other switch that is opening and then something else closing and so on. So, you should be able to solve any problem of this type, as knowledge you have only one capacitor that is important.

Because, so far we have learned how to solve differential equations of first order and this is known as the first order circuit. The first order circuit is nothing but, something that follows differential equation of first order.