

Basic Electrical Circuits
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Lecture - 119

In this lecture, we will examine the general form of the response of a first order circuit and divide the response into two parts and try to understand what each part means.

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$RC \frac{dv_c}{dt} + v_c = V_s$: $v_c(t) = V_s + (v_c(0) - V_s) \exp\left(-\frac{t}{RC}\right)$
 Steady state response + Transient response
 Forced response + Natural response
 Particular solution + Solution to homogeneous equation

$RC \frac{dv_c}{dt} + v_c = 0$: $v_c(t) = v_c(0) \exp\left(-\frac{t}{RC}\right)$
 Forcing function
 Zero state response + Zero input response

So, this is the solution to, then we took the homogenous case, where and the solution to that was with zero inputs. So, this part is known as the study state response and this is the natural response. Because, in study state, what is study state mean things are not changing in somewhere, by the way study state does not mean voltages are a constant or current are a constant, that is true when the input themselves are constant.

When the input itself is time varying, the study state could be a time varying quantity as well. But, in this case the study state means a constant voltage and this is the study state response and this is the natural response. In fact, when you have the homogenous equation, you have only the natural response. This, the function on the right hand side is

known as the forcing function. So, what happens is that, initially you will have some characteristics of the circuit that is coming in.

For instance, in this case the forcing function is D C, it is a constant with time and eventually, the solution also become constant with time. But, before it gets there, there is some parts of the solution, that is really a characteristic of the circuit. Here for instance, we do not have forcing function at all and we have an exponential, which has minus 3 by R c. We have an exponential, whose time constant is the some characteristics of the circuit, it similar to the component in the circuit and this natural response will be the same thing.

Now, this natural response will depend on the forcing function, but this is the part that eventually dies out. Now, there is a class of circuits, everything we will discuss will fall in that class, these are called stable circuits and for table circuit, this natural response always dies out. Now, I think you would have examine the solution in some other case also, you can have a spring and then, if you plug it, it will have some natural frequency with it, which it will vibrate.

But, let say you hold it and then you actually force it to move at a certain frequency, you can attach a motor to it and then move it back and forth at a certain rate. So, finally, the spring will only going to move back and forth at the rate at which you are forcing it, because it will not have the natural response any more. But, initially when you start doing it, it may vibrate in it is natural mode and then finally, it will come in to the study state which is related to the forcing function, this is in the same thing.

So, the natural response will eventually die out and we will only have the force response that is left. By the way the study state response is also called forced response and this part is called natural response and this other terminology I am just talk a little bit. And it is, you either call it force plus natural or study state response plus transient response. And I think in the terminology of deferential equations, these things are called particular solution or particular integral and this part is the solution to homogenous equation, some time it is called the homogenous solution.

So, if you look at the homogenous equation, we have this exponential minus 3 by R C scaled by some numbers and that appears in this also and the total solution will be something related to the forcing function plus some homogenous part. All differential

equations will have this characteristic that you will have the force response plus natural response. And sometimes you can also classify this slightly differently, it is done that way.

So, I will write this V_c of t for this case when V_s is present, as I group all the terms with V_s and then, everything with V_c of 0. Now, thinking of these things it is useful also later, we find that even for many, even for complicated circuits as long as you have only one capacitor we will be pretty much able to write down the solution by inspection if you understand these things. So, this is known as the zero state response and this is the zero input response and the names are pretty obvious.

If the state of the capacitor that is the capacitor voltage is zero initially you will get this part of the left of the solution and if the input itself was zero, you will get the right part of the solution. So, these three classifications are the same, I mean it is just different terminology are the same thing, steady state response is the same as forced response which is same as particular solution, whereas this zero state response is not the same as the forced response.

So, the point is now the zero state response also have exponentials which are related to the circuit, whereas in the other case the steady state response or the forced response by definition they do not have this. The exponential of minus t by RC is the natural response; it is like if you plug a spring it is vibrated at some frequency that, is the natural response of that spring. So, this exponential minus t by RC is the natural response of this circuit.