Basic Electrical Circuits
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Lecture – 118

Now, you can introduce of course, we solved it with zero inputs that is not the most interesting thing. Sometimes interesting of course, we will start with a sort of zero input circuit with initial starts still in place.

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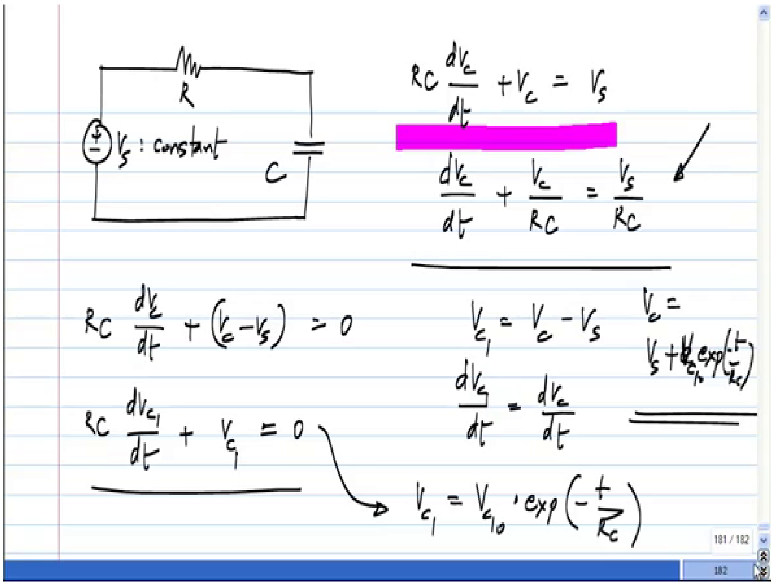

But, you would like to put some input to the circuit, so now, I will say V s is some constant with time. Now, the deferential equation as we already saw was, which also could be written as, some times it is written like this. For us it is not very important, because we are dealing with very simple cases, but many times the deferential equation is written in this normalized form, where the coefficient of the highest derivative is 1. So, what is the solution to this one? How would you solve for this?

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There are many ways to do this. In fact, because it is a first order deferential equation, you can even kind of integrate it. You get that logarithm and then do that, but I will show a slightly different way, which is… Reason to do this is that we know that how to solve it for the case, where the right hand side is 0. So, we go and reducing everything to that case and V s to the constant, so if I call V c 1 as V c minus V s, the derivative here is the same as that; obviously.

So, what is the solution to do this? This we have already found. What is V c 1? Some constant times exponential minus t by R c. So, this is just a change of variable to make the right hand side 0 and get a homogeneous differential equation again and that tells you that for this V c 1 this is the solution. So, what the solution for V c; that is, V s plus this V c 1 0 exponential minus t by R c and then, V c 1 0 has to be again formed from initial conditions.

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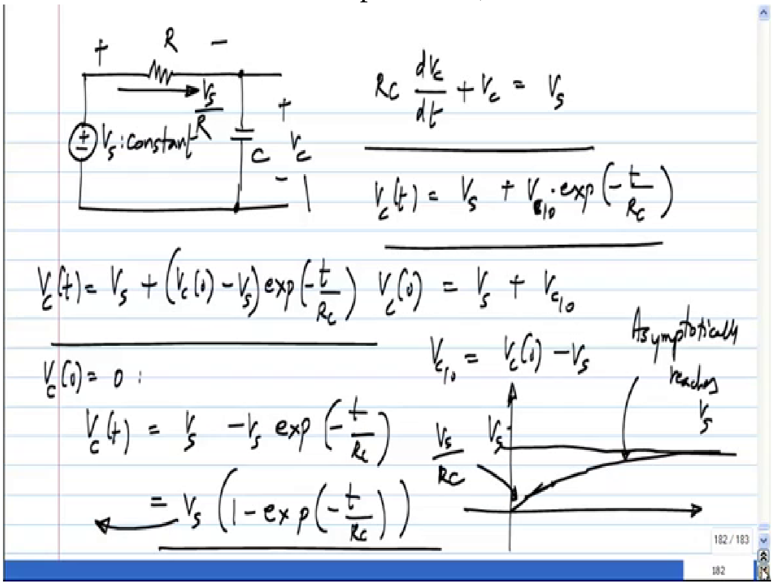
So, I will just call this V c as some V 0 exponential, I mean we let me not change it, V c



1 0 exponential minus t by R c and let us say the initial value on the capacitor was some V c 0 or may be V c of 0. So, what is the value of V c 1 0? So, V c of maybe I think this will again… If I substitute t equal to 0, V c of 0 should be V s plus V c 1 0. So, V c of 0 minus V s is the, this constant V s plus V c 1 0 at t equal to 0. So, the; obviously, V c 1 0 is V c of 0 minus V s. So, this V c of t can be written as V s plus V c of 0 minus V s exponential minus t by R c.

Now, probably the case that you are most familiar with is, when the capacitor starts from a discharge state, when V c of 0 is equal to 0. What is the solution to that? So, V c of t will be V s minus V s exponential minus t by R c, which is... So, I think this you would have seen wherein some, a same I think R l or R c circuits you would have seen this solution. But, in general if you start from a different initial condition, you will get this other solution.

It also make sense, because if V c of 0 equals V s there should be no movement in the circuit; that is, this is equal to be V s and then the current is 0 and there is no change at all and that is what this things says. This exponential completely goes away and it says that V c of t equals V s forever. So, these are all sanity check that you have also do.

So, in general when you solve for something complicated you should have different ways of solving it, we are looking at the circuit you should be able to tell some simple things about solution. So, in this case I know that if V c happens to be equal to be V s at t equal to 0, then there should be no change at all in the capacity voltage. Now, does the solution make sense? If you, the capacitor where initially discharge, what is the current through the capacitor?

If the capacitor is initially 0, what is the current through the capacitor? V s by R, because all of this voltage V s drops across the resistor, so this current has to be equal to V s by R and it is that current that charges the capacitor; that is, the current that is providing charge to the capacitor, so it is voltage builds up. So, what is the rate of increase of the capacitor voltage? It is the current divided by the capacitor, if you have a certain current through a capacitor, current by capacitor is the rate of change of voltage.

Current is the rate of change of charge and charge by capacitance is the voltage, so current by capacitance is the rate of change of voltage. So, initially it has to increase at a slope of V s by R c. You differentiated, do you get that the initial instant, you would do get that, so you get plus V s by R c, so that sanity tests also satisfy. But, as the capacity voltage increases what happens to the current, it decreases. So, the current, because this voltage is ramping up the current will reduce. So; that means, that the rate of increase of voltage falls. It does not do it in a piece wise manner like this, it does it continuously, but I am showing it that way.

It falls further and further and further until it just starts slowly creeping up to the final value. What is the final value? V s, when this final value, when the voltage across the capacitor is V s, the current here becomes 0 and voltages and currents in the circuit stop changing, so and that is known as study state for this circuit with a dc input; that is, V s. For dc inputs in study state, there is no current through the capacitor.

So, this one has asymptotically reaches V s, but this business are finding out what the initial slope is etcetera, it should be able to do even without the solution, just by looking at the circuit. By computing the currents in the circuit, you can tell how much current is going through the capacitor that tells you how fast it is changing, how fast it is voltage is changing.

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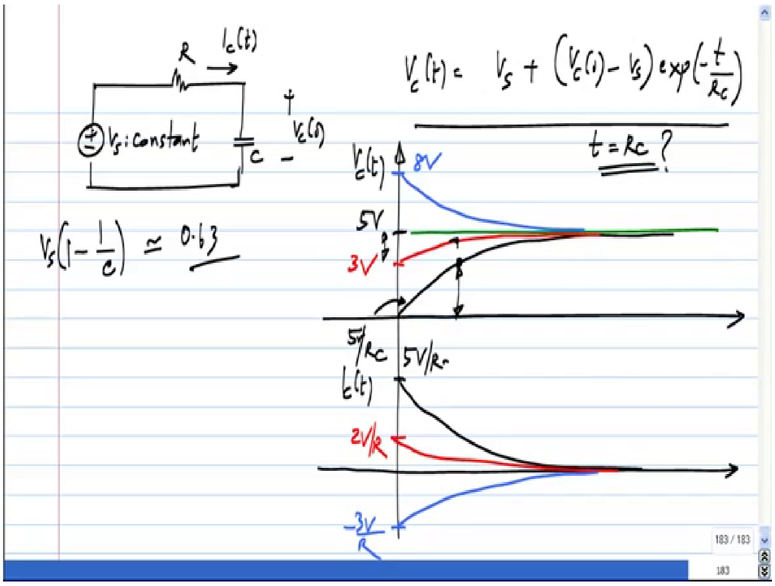

So, what this result says is that, for this circuit if you have V s to a constant, R and C with an initial voltage of V c of 0, the solution was V s plus V c of 0 minus V s exponential minus t by R c. So, if you plot V c of t and let us say V s is somewhere here, this is the value of V s and if you start from 0, it will do that it never reach it, but only does it asymptotically. And if you start from some middle value, let us say let me put numbers here, so let me call this 5 volts. So, the initial slope here is 5 volts by R c and let say, it starts from 3 volts. What happens then? What happens in that case?

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It will still go to 5 volts, but what will be the initial slope, whether it will be the same.

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It will be that is most obvious by looking at to circuit, because here 2 volts across the resistor. So, 2 by R is the current and 2 by R c is the rate of change of voltage. So, it is starts with the smaller slope, but it will asymptotically reach the same value and you can also start from higher voltage, let us say 8 volts something then again the same thing happens expect that now the current is backwards, because a capacitor voltage is larger than this voltage, a current goes that way discharges the capacitor and it discharges to the same 5 volts.

The time constant is same as all cases it is R c. Now, if you start from 0 volts if you take this case which I have shown here, what is the value reached at t equal R c, what is the value of capacitor voltage. So, it will be V s times 1 minus 1 by e, where this is the natural exponent and this is how much is this 0.636 I think, I will remove the last decimal, I am not sure.

So, I will reach some where here and in fact, it is more obvious from the other case, where this case what happens one time constant, what it will be this value 5 by e, 5 by the natural exponent which is about 37 percent of the initial value. So, basically it changes by about 63 percent in one time constant and that is true of where ever it is start from and where ever it finishes, if you define this as 100 percent in one time constant you would a gone 63 percent of that.

So, these are all some useful things to remember, I mean not necessarily memorize, but understand and remember. And what will a current looks like in each of these cases, the currents through the capacitor? When it is start from 0, now what is the initial current, 5 volt by R and current goes to 0 and similarly when it is starts from 3 volts, what is the initial current 2 volts by R and that also goes to 0 and when it is start from 8 volts that is minus 3 volts by R and that also goes to 0 and with the same time constant of course. So, the current through capacitor eventually become zero with constant inputs.