

Basic Electrical Circuits
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Lecture - 117

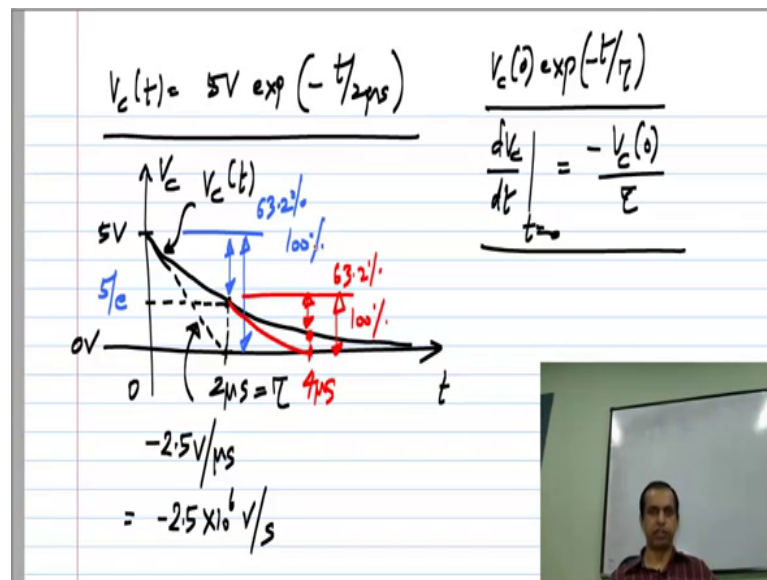
Now, I will consider a numerical example for the case with zero input.

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The image shows a handwritten slide with a circuit diagram and mathematical derivations. The circuit diagram consists of a resistor $R = 2\text{ k}\Omega$ and a capacitor $C = 1\text{ nF}$ connected in parallel. A current of 2.5 mA is indicated flowing into the resistor. The capacitor voltage is labeled as 5 V . The differential equation is given as $\frac{dV_c}{dt} = \frac{I}{C}$, which is solved to give $\frac{dV_c}{dt} = \frac{-2.5\text{ mA}}{1\text{ nF}} = -2.5\text{ V}/\mu\text{s}$. The initial condition is $V_c(0) = 5\text{ V}$. The time constant is calculated as $\tau = RC = 2\text{ }\mu\text{s}$. The final solution for the capacitor voltage is $V_c(t) = 5\text{ V} \exp\left(-\frac{t}{2\text{ }\mu\text{s}}\right)$. A small inset video shows a man speaking.

So, this is zero input, you can imagine that there was a voltage source here which is set to 0 and I will assume that the capacitor voltage at t equal to 0 is 5 volts. We know that with zero input the variables, all variables in the circuit and in particular, the capacitor voltage follows this and the time constant RC for this case is the product of R and C and let say R is 2 kilo ohms and C is 1 nanofarad, then RC is 2 kilo ohms times 1 nanofarad, which is equal to 2 micro seconds. And the initial condition, it is already given to be 5 volts. So, the solution to this is 5 volts exponential minus t by tau, which is 2 micro seconds. Now, it is also useful to always sketch these to find out, how they look like.

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At t equal to 0; obviously, it starts at 5 volts, this is V_c and it has an exponential shape. In the slope at t equal to 0, ((Refer Time: 02:11)) you can find by differentiating this and setting t equal to 0. So, the slope at t equal to 0 will turn out to be minus 5 volt by 2 micro seconds or minus 2.5×10^6 volts per second. Now, this can also be inferred from the circuit, at t equal to 0 we have 5 volts here and 5 volts there, if they are in parallel.

So, a current of 2.5 milliamperes, which is this 5 volts divided by 2 kilo ohms will be flowing that way and we know that, the rate of change of capacitor voltage is the current divided by the capacitance. And the current, if you take the signs for V_c this way and by passive sign convention, the capacitor current is minus 2.5 milli amperes and C is 1 nanofarad. So, this will come out a way minus 2.5 volts per micro second. So, you can say directly from the circuit as well and this is also important to be able to look at the circuit and get the answers.

So, it will start off with a slope, if you calculate the slope of that line, it will be minus 2.5 volt per micro second or minus 2.5×10^6 volts per second. But, of course, the slope will not stay that way, it will keep on continuously reducing and asymptotically, the voltage goes to 0, this is V_c of t . So, one point of interest is, if you draw a tangent at t equal to 0, where does it meet the x axis that is the zero volt line, this is where the voltage is zero volts.

So, this is a tangent and we know that, the slope is minus 2.5 volts per micro seconds.

So, if you calculate this time, it will be 2 micro second and it is exactly the same as tau. I am showing this as a numerical example, but if you use the general form of the expression $V_c(0) e^{-t/\tau}$, then the initial slope is $-V_c(0)/\tau$. So, if you keep on continuing over that line and go down by an amount equal to $V_c(0)$, you will reach tau and at $t = \tau$ if you look at the voltage, you will be easily able to calculate that this is $V_c(0)/e$, where e is the natural exponent.

In general, if you call this as 100 percent or unity, wherever it starts from, then you draw a tangent, it reach the zero line at $t = \tau$ and you will see that, you could have fallen by 63.2 percent over one time constant and you can continue this. I mean for instance you can draw a tangent at this point and where it will hit the zero line will be 1 tau away from this, one time constant away from this. So, this will be 4 micro seconds, because this is at 2 micro second and it is 1 tau away from that one.

Again if you look at how much it has gone down by, if you call this as 100 percent now, this part will be 63.2 percent. So, all these are basically saying in different words that, what we have here is an exponential. So, these properties are all useful to know, again not as memorizing facts, but by understanding them and calculating them for a few example cases.