Basic Electrical Circuits
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Lecture – 106

Now, we will discuss solutions to the assignment for week 7 on two port parameters and reciprocity.

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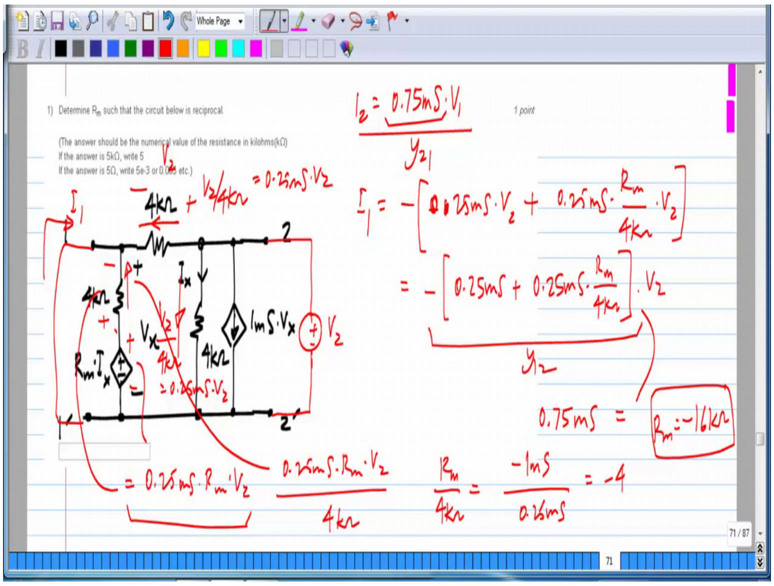

The first question is about a two port network consisting of controlled sources, this means that the network is not necessarily reciprocal, but for certain parameters of the controlled sources it can be reciprocal. The question is to find the value R m; such that it is reciprocal. So, all we have to do is find the two port parameters and specifically we have to find only y 2 1 and y 1 2 or any of the 1 2 1 2 parameters to check for reciprocity.

So, first let say we determine y 2 1 by connecting a voltage source V 1 and finding the current I 2 with port 2 short circuited. So, now, this turns out to be quite easy, this V x equals V 1, so this current here is 1 Millisiemens times V 1. Now, port is 2 short circuited, so no current flows through this 4 kilo ohm resistor and the current through this, the voltage across this is equal to V 1. So, the current that flows there is V 1 by 4 kilo ohms equals 0.25 Millisiemens times V 1.

So, this current I 2 equals this current minus that current, which is equal to 0.75 Millisiemens times V 1, so this number here is y 2 1. Now, we can evaluate y 1 2 by short circuiting port 1 and applying a voltage V 2 to the second port and finding the current I 1. So, again let see what happens. Across this 4 kilo ohms we have V 2; which means that the current here is V 2 by 4 kilo ohms or 0.25 Millisiemens times V 2, across this we all again have V 2 which leads to a current V 2 by 4 kilo ohms or 0.25 Millisiemens times V 2.

Now, this I x here which is 0.25 Millisiemens times V 2 gets multiplied by R m. So, the voltage here is 0.25 Millisiemens times R m times V 2, it is R m times I x, I x is this much. Now, this voltage appears across 4 kilo ohms also because of the short across the second port. So, the voltage this way equals that much, so the current flowing this way is 0.25 Millisiemens times R m times V 2 divided by 4 kilo ohms. So, the total current I 1 equals the negative sum of this current and that current. So, it is equal to negative of 0.25 Millisiemens times V 2 plus 0.25 Millisiemens times R m divided by 4 kilo ohms times V 2, which is basically...

So, this is y 1 2 and that has to be equal to y 2 1, so this whole thing has to be equal to 0.75 Millisiemens. So, from this we get R m divided by 4 kilo ohms to be equal to minus 1 Millisiemens divided by 0.25 Millisiemens equals minus 4. So, R m should be minus 16 kilo ohms, so that is the answer.

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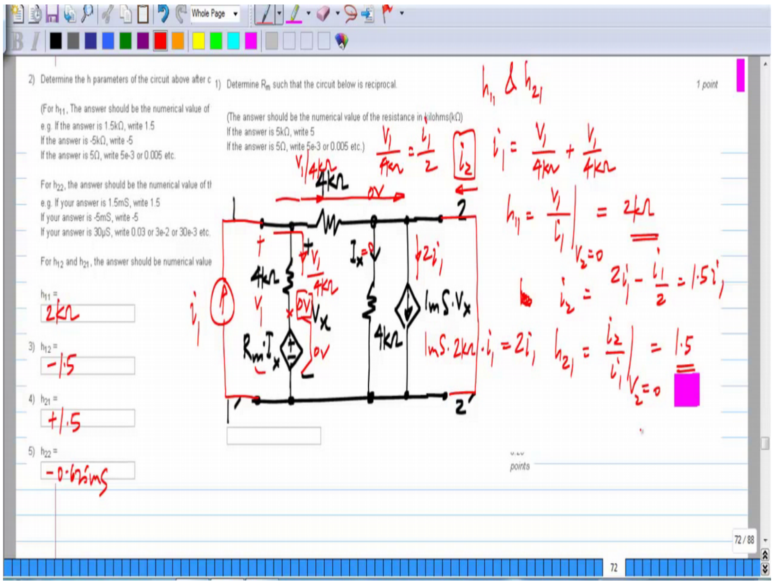

The next question is to determine the h parameters of the circuit it was given before, ((Refer Time: 05:27)) that is here after choosing the right value of R m. So, couple of things can be done, we have already determined a couple of y parameters, so we can go ahead, complete all the y parameters and then convert them to h or we can evaluate h parameters from scratch. So, I am going to evaluate them from scratch by applying a current to the first port and short circuiting the second port, we can find h 1 1 and h 2 1.

So, now, if I apply a current I 1 to this port, the second port is short circuited, so this I x will be equal to 0. So that means that, this voltage source is 0 volts, so this is also at 0. Now, if you see this is at 0 volts, this is at 0 volts, so whatever voltage V 1 you have here, V 1 by 4 kilo ohm flows there and another V1 by 4 kilo ohm flows there and the sum of those two has to be equal to I 1. So, I 1 will be V 1 by 4 kilo ohms plus V 1 by 4 kilo ohms, so h 1 1 which is the ratio of V 1 to I 1 with the second port short circuited will turn out to be 2 kilo ohms.

Now, as far as I 2 is concerned this is I 2, so we have already determined that V 1 is I 1 times 2 kilo ohms. So, this current source here is 1 Millisiemens times 2 kilo ohms times I 1 or 2 times I 1. So, we have 2 I 1 over there and V 1 by 4 kilo ohms, which is equal to I 1 by 2 over there. So, the net current I 2 is given by 2 I 1 minus I 1 by 2 which is 1.5 I 1. So, h 2 1 is I 2 by I 1 with V 2 being short circuited which is equal to 1.5.

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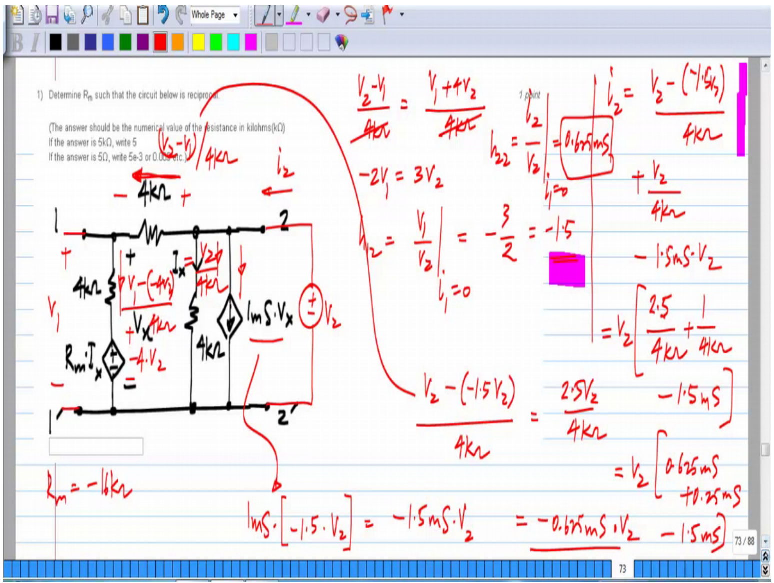

Now, for two more parameters we have to apply a voltage to the second port with the first port left open circuited. So, if you apply V 2 here, this I x will be V 2 by 4 kilo ohms and this R m I x we determined R m to be minus 16 kilo ohms. So, this R m times I x will be minus 4 times V 2. So, now, we have to find V 1, we know that the current flowing this way is V 2 minus V 1 divided by 4 kilo ohms and the current flowing that way is V 1 minus voltage at this point, which is minus 4 times V 2 divided by 4 kilo ohms. So, V 2 minus V 1 by 4 kilo ohms equals V 1 plus 4 V 2 by 4 kilo ohms.

So, this goes away and we will end up with minus 2 V 1 equals 3 times V 2. So, h 1 2 which is V 1 by V 2 with I 1 being 0 is minus 3 by 2 or minus 1.5. Now, this is a useful sanity check, because we have chosen R m such that the network is reciprocal, we see that h 2 1 is plus 1.5 and h 1 2 is minus 1.5. We calculated this independently, but we know that it is reciprocal and h 1 2 has to come out to be minus h 2 1 and finally, we have to determine I 2.

So, we already know two components of I 2, the current flowing here and the current flowing there. So, we need to find the current flowing there, it is 1 Millisiemens times V x which is 1 Millisiemens times V 1, because V x is the same as V 1. So, this current here is 1 Millisiemens times V x which is minus 1.5 times V 2 and this is equal to minus 1.5 Millisiemens times V 2.

So, I 2 is sum of these three components, this is V 2 minus V 1 by 4 kilo ohms, which is V 2 minus minus 1.5 V 2 divided by 4 kilo ohms and this one is V 2 by 4 kilo ohms and this one is minus 1.5 Millisiemens times V 2. So, this will be V 2 times 2.5 by 4 kilo ohms plus 1 by 4 kilo ohms minus 1.5 Millisiemens, which is equal to V 2 times 0.625 Millisiemens plus 0.25 Millisiemens minus 1.5 Millisiemens. So, the answer turns out to be minus 0.625 Millisiemens times V 2.

So, h 2 2 which is I 2 by V 2 with port 1 open circuited is 0.625 Millisiemens ((Refer Time: 12:55)), so h 1 1 is 2 kilo ohms, h 2 2 is minus 0.625 Millisiemens and h 1 2 is minus 1.5, h 2 1 is plus 1.5, those are the answers.

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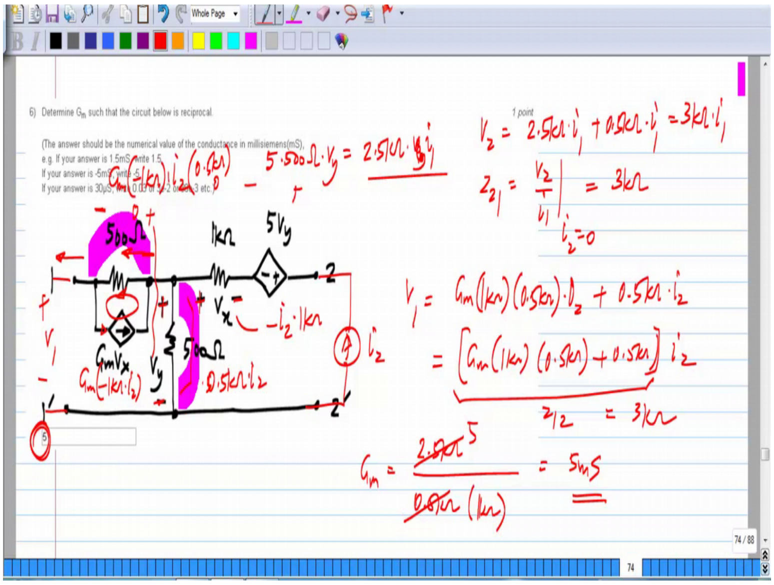

Now, this is another network with controlled sources and you have to find the value of the control source G m; such that the network is reciprocal. In this case you can choose any parameter set, let me choose z parameters. I have to find z 1 2 and z 2 1 and make them to be equal to each other. So, first I will apply I 1 and measure V 2 with port 2 open circuited, they should give me z 2 1. Now, with port 2 open circuited the current in this branch is 0, so V x is 0, so this just goes away.

We see that all of this current has to simply flowing this loop, because nothing can go into that one. So, this V y is nothing but, I 1 times 500 ohms and this voltage is 5 times V y which is 5 times 500 ohms times V y or 2.5 kilo ohms times V y and this V 2 is nothing but, the voltage drop across this which is this one voltage drop by this which is 0 plus the voltage drop across that which is half kilo ohm times I 1.

So, V 2 is 2.5 kilo ohm times I 1 plus half a kilo ohm times I 1 is 3 kilo ohm times I 1. So, z 2 1 which is V 2 by I 1 with port 2 open is 3 kilo ohms, now you compute z 1 2 by applying a current here and open circuiting port 1, if you open circuit port 1 the current through this branch is 0 that is the combination of these two parallel branches that is 0 whatever is coming out of them, then is I 2 flows into this resistor to give you 0.5 kilo ohm times I 2 across it and this V x would be basically minus I 2 times 1 kilo ohm because of the polarity. So, this G m times V x is G m times minus 1 kilo ohm times I 2.

So, the voltage drop in this direction, because this current is simply flowing through that is G m times minus 1 kilo ohm times I 2 times 500 ohms or half kilo ohm. Now, this V 1 is a sum of this voltage drop and that voltage drop. So, V 1 is remember voltage drop from here to there is the negative of this. So, it is G m times 1 kilo ohm times half a kilo ohm times I 2 plus half a kilo ohm times I 2. So, this is z 1 2 and that has to be equal to z 2 1 which is 3 kilo ohms. So, G m turns out to be 3 minus 1.5 which is 2.5 kilo ohms divided by half a kilo ohm times 1 kilo ohm, which gives you 5 Millisiemens. So, G m has to be 5 Millisiemens for this network to be reciprocal.

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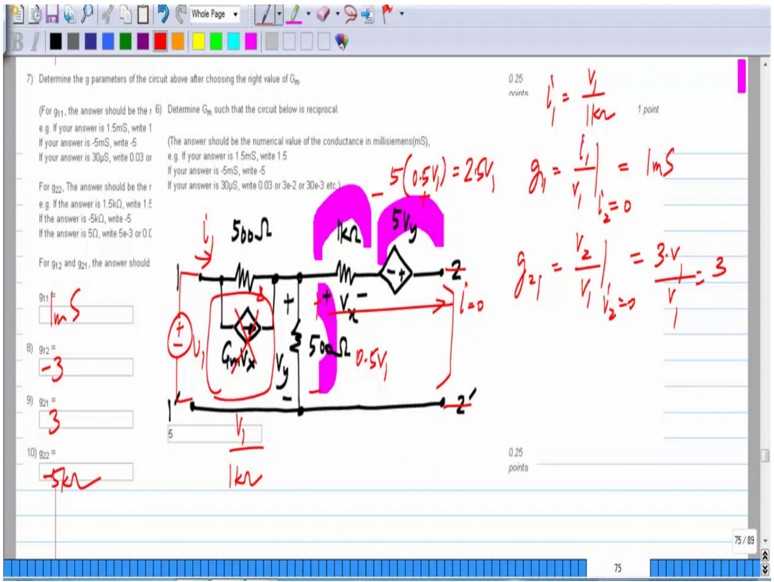

The next question relate to G parameters of this network, again we can either use the y parameters after finishing evaluation of y 1 1 and y 2 2 or evaluate G parameters from scratch will do the later. So, first we apply V 1 and leave port 2 open circuited this gives us g 1 1 and g 2 1. So, if you apply V 1 like this first of all we know that this current has to be 0, because of the open circuits. So, V x is 0, so this current source just goes away and this V 1 simply appears across the series combination of this 500 ohm and that 500 ohm, because no currents flows through there.

So, current V 1 divided by 1 kilo ohm which is a series combination of these two flows this way and V y will be half of V 1 and this voltage here is 5 times V y. So, this is 5 times half of V 1 or 2.5 times V 1. So, first of all the current here I 1 is V 1 by 1 kilo ohm, so g 1 1 which is I 1 by V 1 with port 2 open circuited is 1 Millisiemens and g 2 1 which is V 2 by V 1 with port 2 open circuited is this voltage here is this voltage which is 2.5 V 1 plus that voltage which is 0 plus that voltage which is half of V 1. So, the whole thing is 3 times V 1 divided by V 1 of 3. So, g 1 1 is 1 Millisiemens and g 2 1 is 3.

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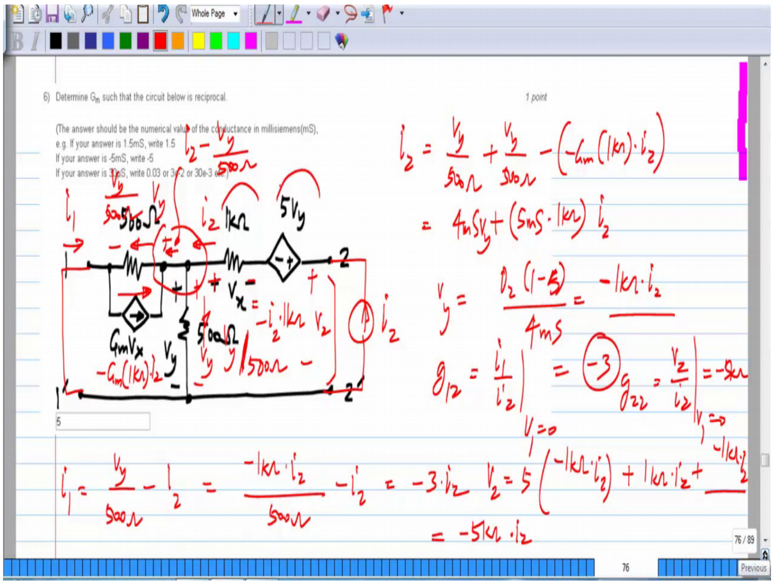

Now, you can evaluate the other two parameters by connecting a current source I 2 short circuiting the first port and finding V 2 and I 1. So, now, we know that this V x equals minus I 2 times 1 kilo ohms, because I 2 is simply flowing through this and this G m V x will be in this direction minus G m times 1 kilo ohm times I 2. So, the current here of course, is I 2. Now, the voltage here let me call this V y as is denoted there, we have a number of relationships this current here is V y divided by 500 ohms.

Similarly the voltage here is also V y because of the short circuit on the first port. So, the current here is V y divided by 500 ohms, so applying KCL here we will get I 2 to be V y divided by 500 ohms plus V y divided 500 ohms minus the current through this G m which is minus G m times 1 kilo ohm times I 2. So, this whole thing gives you 4 Millisiemens plus G m times 1 kilo ohm and we know that G m is 5 Millisiemens 5 Millisiemens times 1 kilo ohm times I 2.

So, now, we take it to the other side and we will find that V y is I 2 times 1 minus 5 divide by 4 Millisiemens or 1 kilo ohm times I 2. So, now, we will look at what I 1 is you can see that I 1 is nothing but, whatever current is flowing here on negative of that and that current is I 2 minus V y by 500 ohms. So, I 1 is V y by 500 ohms minus I 2 or minus 1 kilo ohm times I 2 divided by 500 ohms minus I 2 or minus 3 times I 2. So, g 1 2 which is I 1 by I 2 with port 1 short circuited is minus 3.

Now, earlier we determined g 2 1 to be 3 and g 1 2 is minus 3 we know that this is reciprocal. So, that satisfies sanity check, if you know that it is reciprocal we do not have to independently determine that, but just as an exercise I did that and now confirmed that it is reciprocal again in another way. Now, we have to find V 2 to find g 2 2, so V 2 is nothing but, the voltage drop across that plus the voltage across that plus this. So, V 2 would be 5 V y which is 5 times minus 1 kilo ohm times I 2 plus 1 kilo ohm times I 2 that is the voltage drop across that plus V y itself which is minus 1 kilo ohm times I 2.

So, the whole thing comes out to be basically minus 5 kilo ohm plus 1 kilo ohm minus 1 kilo ohm or minus 5 kilo ohm times I 2. So, g 2 2 is V 2 by I 2 with port 1 shorted which is minus 5 kilo ohms, this would be minus 5 kilo ohms.

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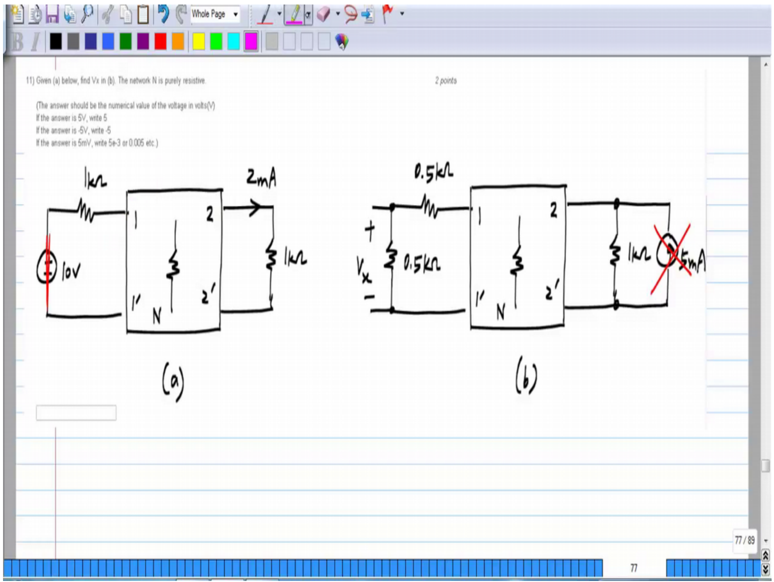

The next problem your given a and your told that N is a purely resistive network they are also some resistive outside, but the part with resistors if you represented as a 2 port will be reciprocal of course, then your asked to find what happens in b. Now, first thing to recognize is that network a and b when you set the independent sources to 0 or exactly the same, because if you set the independent sources to 0 this becomes a short circuit.

So, between 1 and 1 prime you have 1 kilo ohm, here also between 1 and 1 prime you have 1 kilo ohm although it is split up as 051, 0.5 you have 1 kilo ohm there and if you open circuit this on other side also you simply have 1 kilo ohm. So, there are many, many ways of doing this you can write the 2 port parameters for this network N and this at network N and solve it and the condition that you get from the reciprocity of resistive networks is that this N is reciprocal.

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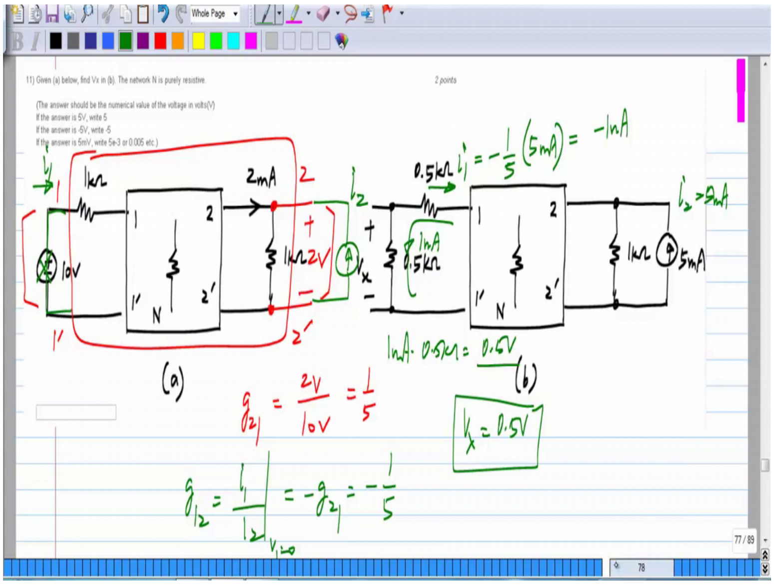

But, the easiest way to do it is define a new 2 port, where this reciprocity can be applied. So, first we know that this voltage here is 2 volts, it is 2 milli amp times 1 kilo ohm. So, now, if I think of this as my 2 port let say this is my 1 1 prime and 2 2 prime, the ratio of this voltage to that voltage that is basically g 2 1 which is given by 2 volts divided by 10 volts or it is 1 5'th. Now, whatever I have enclosed in this the new 2 port I have defined is purely resistive and therefore reciprocal.

So, now, what happens is that let say I remove this I replace this to the short circuit and I apply a current source here and call this I 2 and call this I 1. So, my g 1 2 will be I 1 by I 2 with port 1 short circuited of the new 2 port and this will be minus g 2 1 or minus 1 5'th. So, now, that is exactly the situation I have, I have this current source across this 1 kilo ohm which is 5 milli amps, my I 2 is 5 milli amps and on this I have do not have to do anything it is simply 1 kilo ohm across 1 and 1 prime.

So, I have 1 kilo ohm across 1 and 1 prime. So, this will be my I 1 I am just matching the quantities in this figure with that figure. So, I know that this I 1 in this direction is g 1 2 times I 2 which is minus 1 5'th times 5 milli amps or minus 1 milli amp. So, it means that basically 1 milli amp is flowing in that direction. So, V x it is very easy it is 1 milli amp times half a kilo ohm which is half a volt. So, V x equals half a volt that is the answer.

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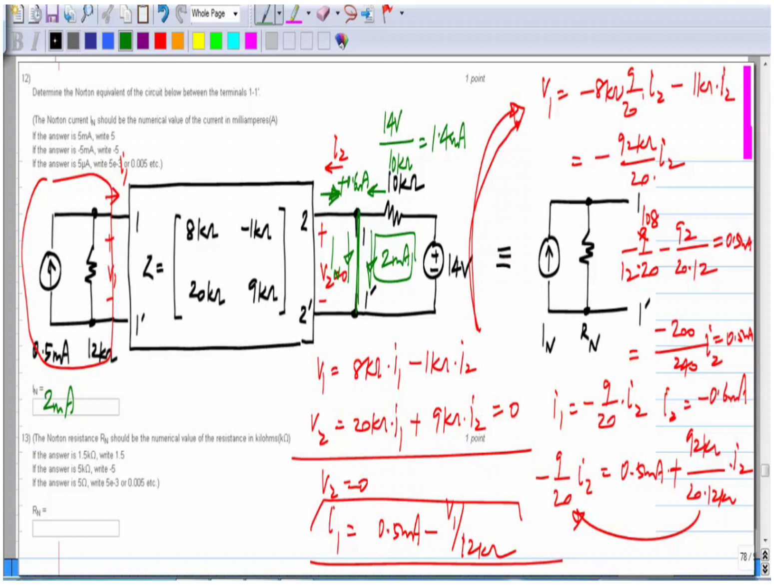

Now, in this case your given some combination of 2 port with some sources and resistors outside and you are asked to find the Norton equivalent between 1 and 1 prime. Now, this may look slightly different in that before you found Norton or Thevenin equivalents of circuits with circuit components, now you have a 2 port, but the definitions are exactly the same. So, you find Norton current by short circuiting between the relevant terminals and finding the current. So, if you short circuit that and find the current in that branch, that will be the Norton current.

So, now, this I N will be whatever is coming there plus whatever is coming from there and we can treat the 2 separately, because we have the port 2 of this short circuited through this or some current will come out of this and this is short circuited through here. So, the current in this will be 14 volts divided by 10 kilo ohms is 1.4 milli amps, so all I have to do is find the short circuit current of this whole thing and that is not difficult.

So, if I use the 2 port notation let me do that this is V 2 which I have set equal to 0. So, this is I 2, this is V 1 and that is I 1 all I have to do is to write out the equations and eliminate the variables. First of all I have the z parameter equations by the way sometimes it may be convenient to convert this z 2 y and so on I will do it in the straight forward way. So, we will write out the 2 port equations first V 1 is 8 kilo ohm times I 1 minus 1 kilo ohm times I 2 and V 2 is 20 kilo ohm times I 1 plus 9 kilo ohm times I 2.

We have the external constraints because of this short circuit V 2 equals 0 and because of this source connected here I 1 is half milli amp minus the current through this which is V

1 by 12 kilo ohms I 1 is half a milli amp minus V 1 divided by 12 kilo ohms from V 2 being 0 we get I 1 2 be minus 9 by 20 times I 2 and substituting that in the first equation we get V 1 to be minus 8 kilo ohms times 9 by 20 times I 2 minus 1 kilo ohm times I 2 which basically turns out to be minus 92 kilo ohms by 20 times I 2 and I put that into this.

So, I 1 is minus 9 by 20 times I 2 which is equal to half a milli amp minus V 1 divided by and V 1 itself is minus 92 by 20. So, 92 kilo ohm by 20 times 12 kilo ohm times I 2 and if I take it to that side I will get minus 9 by 20 minus 92 divided by 20 times 12 equals half milli amp and this comes out to be if I multiply this by 12 I get 108 over there, left hand side will be minus 200 divided by 240 times I 2 equals half milli amp, so I 2 will be minus 0.6 milli amps.

So, the current flowing this way is plus 0.6 milli amps, so the total current that flows here is 1.4 plus 0.6 which is 2 milli amps. So, the Norton current is 2 milli amps and the Norton resistance is found by deactivating the independent sources.

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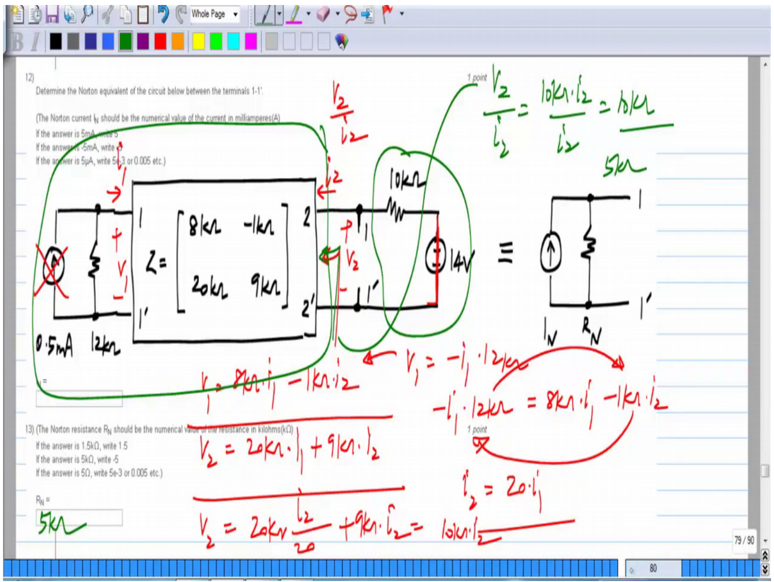

So, we have this current source open circuited the voltage source short circuited and we can find the resistance in here whatever resistance we find that parallel with 10 kilo ohms, because 10 kilo ohms appears directly across 115 is the net Norton resistance and this one again we write the 2 port equations. So, let say this is I 2 and this is V 2. So, the resistance looking then there would be V 2 by I 2 and let say this is V 1 and that is I 1.

So, again we have same the 2 port relationships V 1 is 8 kilo ohms times I 1 minus 1 kilo ohms times I 2 and V 2 is 20 kilo ohm times I 1 plus 9 kilo ohm times I 2. We also have this constraints that V 1 is minus I 1 times 12 kilo ohm, because only a 12 kilo ohm resistance is connected to port 1 nothing else. So, now, if I substitute that in here I will get minus I 1 times 12 kilo ohms equals 8 kilo ohms times I 1 minus 1 kilo ohm times I 2.

If I take this to this side and that to that side I 2 will be 20 times I 1 and I substitute that in here. So, I will get V 2 to be 20 kilo ohms times I 1 which is 20 kilo ohms times I 2 divided by 20 plus 9 kilo ohms times I 2 which simply turns out to be 10 kilo ohms times I 2. So, the resistance looking that way here is V 2 by I 2 which is 10 kilo ohms times I 2 divided by I 2 or 10 kilo ohm itself. So, the net resistance is the 10 kilo ohm offered by this part of the circuit in parallel with this 10 kilo ohm which gives us 5 kilo ohms. So, the Norton resistance is 5 kilo ohms.

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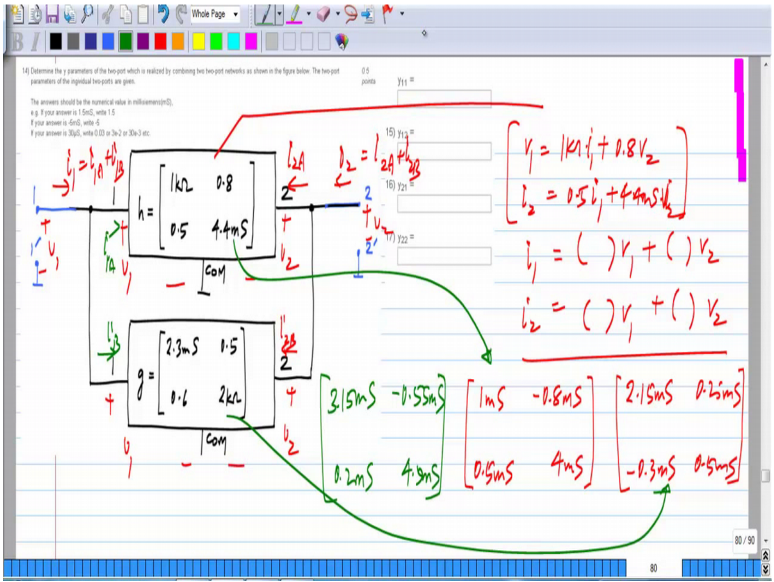

Finally, this question here is it shows to 2 port network connected in some way and you can easily see that they are connected in parallel and h parameters are given for one and G parameters are given for the other one and you should find the y parameters of the combined network. Now, if I call this I 1 A and I 1 B and let me call this V 1 and I 1 the voltage applied to port 1 of the first two port as V 1 second two port is also V 1.

Similarly, the voltage applied it to port 2 of the first two port is V 2, if this is V 2 and this voltage is also V 2. The same voltage is applied to the 2 2 ports and the currents I 1 will be I 1 A plus I 1 B, similarly if this is I 2 this is I 2 A, this is I 2 B, I 2 will be I 2 A plus I

2 B. So, we have same voltages currents adding up, so the 2 ports are in parallel and you can see that if you represent both in y parameter format I won't show this, but you can easily write that and see the y parameters of the two networks will add up to give the overall y parameters.

So, the easiest way is to convert this h parameter to y parameters, this g parameters to y parameters and move on. The way to convert from one parameter set to another is first of all let say I will take the h parameter example, we have V 1 to be 1 kilo ohm times I 1 plus 0.8 times V 2 and I 2 to be 0.5 times I 1 plus 4.4 Millisiemens times V 2. So, you have to rearrange these equations such that we will have I 1 and I 2 on the left hand side and now on the right hand side we have only V 1 something times V 1 plus something else time V 2, similarly for both equations.

So, I will not show this steps of doing this it calculations straight forward, you can go ahead and do that I will just show you the answers. So, y parameters of the first network turn out to be this one, y parameters of the second one turn out to be that one and we have to add the 2 to get the y parameters of the overall network. So, we get 1 plus 2.15 which is 3.15 Millisiemens minus 0.8 plus 0.25 which is minus 0.55 Millisiemens 0.5 minus 0.3 which is 0.2 Millisiemens and 4 plus 0.5 which is 4.5 Millisiemens. So, that is the solution to this particular question.