Basic Electrical Circuits
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Lecture – 10

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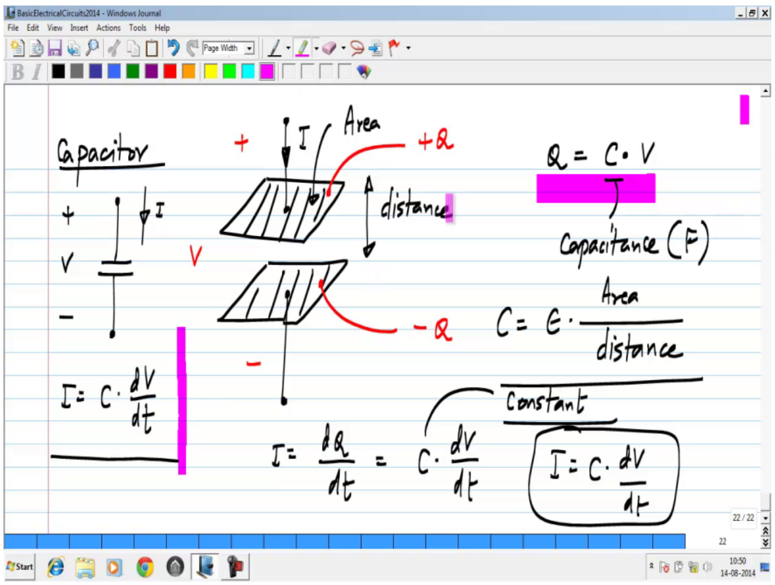

The next basic element is a capacitor and it is given by the symbol as usual. We define a voltage - V and a current - I consistent with passive sign convention. I think from basic physics classes, all of you would be familiar with the prototype structure of a capacitor which consists of two parallel plates with some area and some distance between them. And again from basic physics you would know that if you apply certain voltage - V between the plates that causes a charge plus Q on the plate which is connected to the positive of this voltage –V and minus Q which is connected to the negative of this voltage – V. And the relationship between Q and V happens to be linear with the proportionality constant C which is called the capacitance; C is the capacitance measured in Farads. What it means is that we have a one farad capacitance you apply one volt across it there will be one coulomb of charge on each of the plates every plus one coulomb on the plate which is connected to the positive side of the voltage and minus one coulomb on the other plate. And this C itself is given by some permittivity times area divided by the distance.

Now as before we are not concern with these details, we are not concern with what charges there are, we are not concern with fields inside the capacitor and so on. We only want the relationship between the terminal current and the terminal voltage that is the voltage between the terminals and the current through the terminals. And also this type of formula, this works for a case, where you have two parallel plates separated by a uniform distance. Now if the shapes are odd and the distance keeps changing, you have to evaluate the capacitance using electromagnetic using some integrals and so on. So again that is not a concern at all, the point is if you have two conductors separated by some insulating medium, you will have this linear relationship between charge and voltage, which we will now convert to relationship between current and voltage and we will use that. As far as the circuits are concern, the terminal relationship between the current and voltage is good enough.

Now how do we do that this current – I flowing which is the same as the current- I flowing in this terminal is given by the rate of change of charge that is the definition of current. Right If this Q is increasing that means that there is some current flowing that way, in fact that current is what is causing the Q 2 increase. Now from physics you know that there are these fields and when you have these time varying fields, you will have currents and so on it is known as displacement current. Again we would not worry about that we will have some voltage across this and current can flow in response to this voltage or voltage can be changing in response to the current. Now from this relationship, we have C times dv by dt it is of course assumed that the capacitance itself is the constant; this is given for the all capacitor that we will be using. So we have this relationship I equals C times dv by dt and this is the V I relationship of a capacitor, the current is related to the time derivative of the voltage across the capacitor. Now this comes from the linear relationship between charge and voltage which we have converted to relationship between current and voltage. So for the purpose of analyzing circuits, this is the relationship we will be using.

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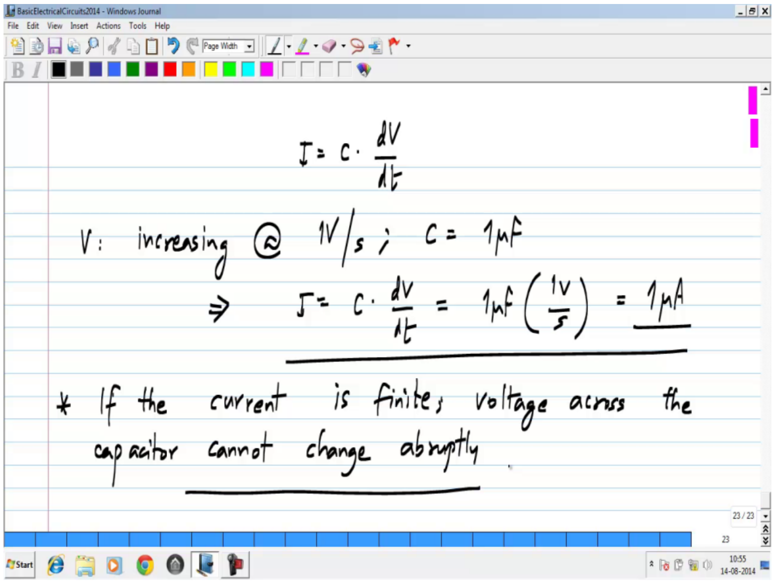

Because the I V relationship, our capacitor consists of a time derivative, we cannot draw a graph of I versus V, so that is possible only if a particular value of V gives a particular value of I or vice versa, in this case it is not possible. Now what does this say if V is increasing at the rate of let us say one volt per second and C is let us say one micro farad, this means that I is C times dv by dt is one micro farad times one volt per second equals one micro ampere, so one micro ampere current will be flowing through the capacitor. It also implies another thing that if the current is finite that is if the circuit is such that the current through the capacitor restricted to a finite value then the voltage across the capacitor cannot change abruptly. So if the currents are finite, the voltage across the capacitor has to be continues. On the other hand, if you do find the discontinuity in the voltage across the capacitor that means that the current had to be infinite at the instant of the discontinuity. Because a voltage changing instantaneously means that the charge on the plates of the capacitor changing instantaneously that means that a current which is the rate of change of charge is infinite, because there was a certain non-zero charge change in a zero time interval so that means an infinite current.

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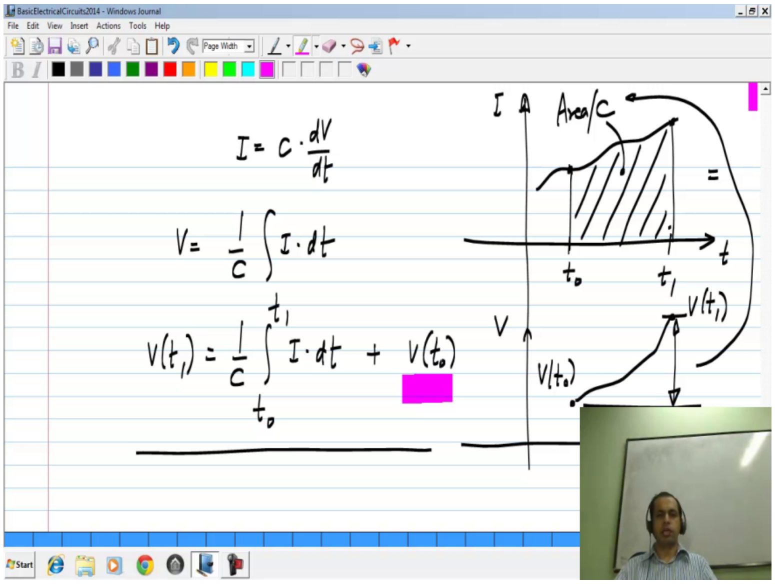

I is given by C dv by dt; and if you invert this, we get the voltage to be 1 over C integral I dt. Now if I carry out the integral from some t naught to t 1, what it says is V of t 1 is 1 over C integral I dt plus V of t 0 that is the change in the volt is between t 0 and t 1 is the integral of current which is the charge accumulated between t 0 and t 1 divided by the capacitance-C. So let say the current follow some wave like this and this is t 0 and this is t 1 I have drawn it versus time and if you plot the voltage, it has some value at t equals t 0, this is the voltage and it will increase in some way; and at t 1, it will be V of t 1. And this increase V of t 1 minus V of t 0 is given by the area under this curve - area under the current curve divided by C. So this whole thing equals the area under the current curve divided by C, so that is what this is saying.

Now, because of this integral relationship, there is some memory right the initial value of the capacitor voltage appears here, so the capacitor is set to be an element with memory, it remembers what happened before. The voltage at a particular instant is not related to the current at that instant but current at all the previous instants, so that is why a capacitor has memory. And this is also apparent from the terminology normally used to describe a capacitor like it stores charge and so on. And this is a contrast to a resistor whose current depends only on the voltage at that instant or which voltage at a given instant depends only on the current at that instant. Resistor has no memory.