Basic Electrical Circuits
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Lecture - 9
Choice of Nodal Versus Mesh Analysis;
Circuit Theorems: Pushing a Voltage Source through a Node,
Splitting a Current Source
Substitution theorem
Superposition

Welcome to lecture nine of basic electrical circuits. In the previous lecture we looked at mesh analysis. It is an analysis method, where we start with Kirchhoff's voltage law around different meshes in the circuit it is a sub case of this loop analysis now we see that it is an alternative to nodal analysis. At the very end of the lecture, we were trying to figure out which one to use nodal analysis or mesh analysis. So, before we get there if there are any questions about any of the previous lectures please ask, so we can discuss those things, any questions about the previous lecture?

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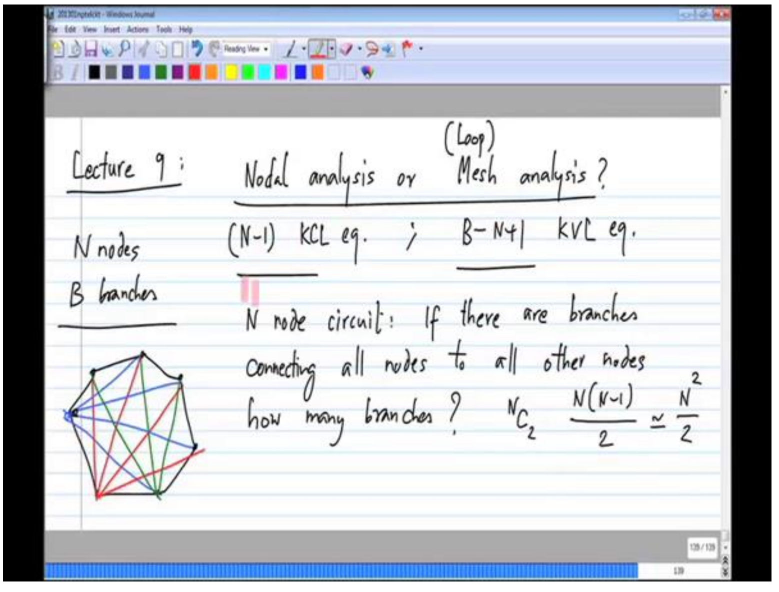

In that case, let us go ahead with the lecture at the end of the previous lecture I asked whether we would use nodal analysis or mesh analysis, I mean I say mesh analysis. So, in general a loop analysis the way to decide is that nodal analysis has n minus 1 KCL equations and loop analysis will have b minus n plus 1 KVL equations, which we have to solve for a circuit with N nodes and B branches. So, what we have to decide is N minus a larger number or B minus N plus 1 a larger number.

So, to answer this I asked you first of all the number of branches depends on very much on the circuit, but what is the maximum number of branches you can have n node circuit. If there are branches between every node and every other node that is if there are branches connecting all nodes to all other nodes, how many branches will be there, please try to answer this question how many branches will be there? There were a couple of answers saying 2 n minus 1 and the other one was n times n minus 1 by 2 n, that is the correct answer.

The correct answer is that it will be n choose 2 or n times n minus 1 by 2, so every branch can be connected to n minus 1 other branches that is why you get this n minus 1 here. Then, you have n possibilities for selecting the starting node and also it get divide by 2 because if node a is connected to b, it is the same thing as connecting node b to node a. Alternatively you can think of while adding branches, you have to pick two nodes, you have n possible nodes and out of that you pick two nodes and you can do that is n choose two ways n times n minus 1 by 2.

So, this I will simply approximate it by n square by 2 for large n, so n minus 1 is approximated by n. So, if every branch is connected to, sorry every node is connected to every other node by branches this number b minus n plus 1 will be of the order of n square by 2 of for large n. The number of KVL equations will be a lot more than number of KCL equations, now in reality it will be somewhere in the middle, it is possible that you have very few branches. You just have a single loop, for instance you can have n nodes and branches only like that.

In this case clearly number of KVL equations would be smaller, alternatively you can have everything connected to everything else, something like this, and in this case you have a lot more KVL equations than KCL equations. Now, reality will be somewhere in between its not that all the nodes will be arranged in a single loop or it is not that every node will be connected to every other node using a branch. So, the number of KVL equations will be smaller than this, but in general it tends to be a little more than the number of KCL equations.

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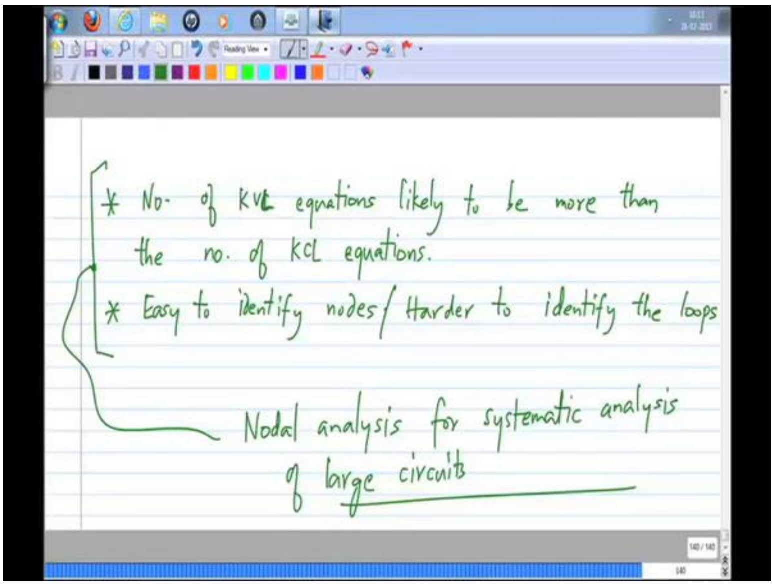

So, that is one reason, although this is not a hard and fast rule and it is possible to have single KVL equation for a n node circuit. If you happen to have single loop. In that case you can choose KVL, now it turns out that this analysis of circuits using a computer that uses a nodal analysis. There is another reason for it that is that it is very easy to identify nodes they are just given to you and this is true even for human beings, you look at a circuit you know what the nodes are, but a little harder to identify all the loops. So, you have to first identify a tree and then start adding links to the tree and form loops.

So, because of these reasons, usually if you want to do systematic analysis of circuits, you end up using nodal analysis, is the audio ok. It seems fine for some people, but maybe for those of you for whom the audio is not good, you have to check the set up at your end. So, this is just to say that you end up using nodal analysis for systematic analysis of large circuits. So, far what we have done is to look at nodal analysis and mesh analysis, we can systematically write down equations for two cases.

If you have only current sources and resistors nodal analysis comes out with a nice structure. Similarly, if you have only voltage sources and resistors mesh analysis equations have a nice structure, but either of these can be used even when you have other sources. You can use nodal analysis when you have voltage sources and mesh analysis when you have current sources in case of nodal analysis with voltage sources. You either define an auxiliary variable or use super node, similarly in case of mesh analysis with current sources you use an auxiliary variable for the voltage across the current source or you use a super mesh.

When you have controlled sources, things tend to become more complicated, but you should be able to figure out those things as well. If you are trying to solve some specific problem, but run into some trouble then please raise it in the class and we will discuss that, so any questions on either nodal analysis or mesh analysis? Then, let us move on to the next topic in our course which certain theorems is involving circuits, what I showed so far was ways of systematically analysing large circuits. So, that is how you use nodal and mesh analysis, now when you are doing hand analysis of small circuits, you tend to use a number of ad hoc ways, and in fact I will discuss many of those things after I discuss the circuit theorems.

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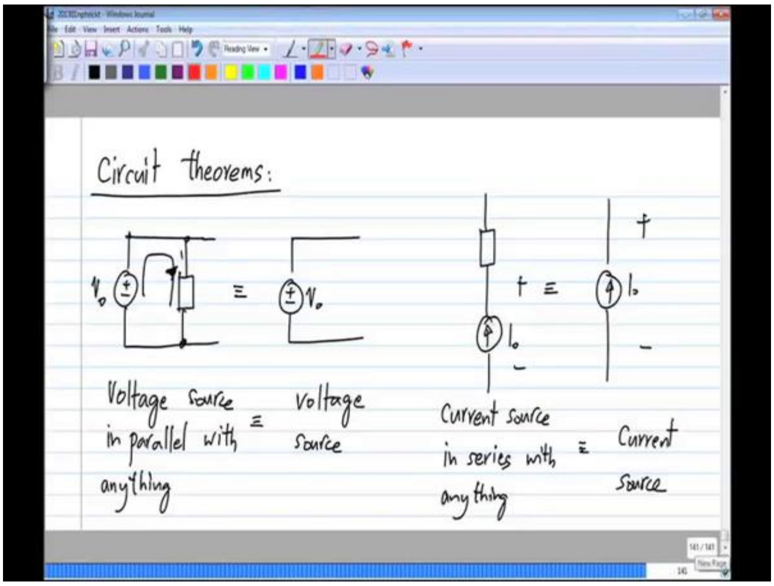

So, we will start with some very simple things first, first of all these are not called circuit theorems, but some transformations that you can use and we have already discussed these, but I will briefly touch upon them. If we have a voltage source in parallel with anything, it is the same as the voltage source, this is just to remind you of some basic results. So, this is V naught and this will also be V naught and similarly if you have a current source in series with anything, it is equivalent to a current source.

I assume that there will be no questions about this, these are really basic results clearly whatever element you have in series the current coming out here will be I naught. So, current source in series with anything will just be a current source, similarly whatever this be across this you have a voltage V naught. So, when you analyse, when I say equivalent, what it means is if you analyse the circuit with this or without this, you will get exactly the same result.

The only difference will be that this may draw some extra current from the voltage source. So, except for the current in the voltage source having this or not having this there will be no difference. Similarly, here except for the voltage across the current source having this or not having this will not make any difference at all, now let us move on to slightly more sophisticated things.

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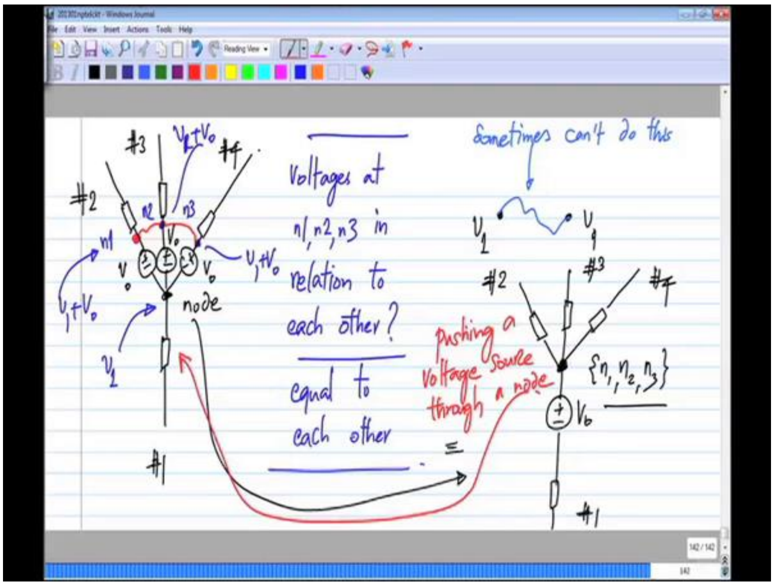

Let us say we have a node with a number of branches and this is the node and this is branch number 1, 2, 3 and 4. So, this is just an illustration and let me assume that in my particular circuit in series with these three branches, there happens to be voltage sources and all of the same value V naught. So, all I am saying is there is a node and in my example I have taken four branches and in three of those branches there is a voltage source with the same consistent polarity and same value V naught.

Let us say this is the circuit, this is just an assumption, now in general you can have n nodes with n minus 1 branches, sorry you can have n nodes with n branches and n minus 1 of those n branches will have the same value voltage source, so this is the circuit. Now, what I would like answers from you is let me call these nodes n 1 n 2 and n 3, what value will be the how will be the voltages at n 1 n 2 and n 3 be related to each other. My question is there will be some voltage at n 1 with respect to reference node of the circuit and n 2 and n 3, so how are these going to be related to each other, please try to answer this.

So, the question is how are the voltages at n 1, n 2 and n three related to each other, so a couple of you have given your answers there will be the same voltage and that is pretty clear because let us assume that the voltage here is some V 1. Now, the voltage at n 1 will be V 1 plus this V 0 and here it will be V 1 plus V 0 again and here also V 1 plus V 0, so they will all be equal to each other. Now, it turns out that if two nodes in a circuit have the same voltage, you can connect them without changing the circuit, so if you have two nodes and you know that the voltages are identical. So, if this is V 1, this is also V 1, then you can short these two without even affecting the circuit.

Now, this can be used done in most cases, but there are some cases where it cannot be done, I will not go into them, now in most general, most regular circuits you can do this. So, what does this mean because the voltage here there and there are the same, I can connect all of them together and in fact if I had more branches with voltage sources just like this, I could connect all of them together. Now, if I observe this circuit, I see that this voltage source and that voltage source and that voltage source are in parallel with each other.

So, these three voltages are in parallel with each other, now if you have a voltage sources with equal value in parallel with each other. First of all, if there are unequal values you cannot even connect them in parallel, but if they have equal value and parallel with each other it is the same as having a single voltage source. So, this is the same as doing this in a single voltage source b naught and this is branch number 1, 2, 3 and 4 and this node represents the union of the three nodes basically of whatever I have shorted and you have a single voltage source.

So, what this whole exercise is showing is that if you have a picture like this with voltage source on one of the branches it is exactly the same as having a picture like this with this voltage source moved to all the other branches, any questions about this? Now, this theorem is sometimes useful to prove other theorems we cannot use directly in this course, but this is known as pushing the voltage source through a node.

Now, I proved it by starting with these multiple voltage sources and combining them in this one, but usually it is more useful to go the other way round. If you have voltage source in a single branch, you can push it into all the other branches connected to that node. So, typically you end up going in this direction and this is a you can call this as pushing a voltage source through a node, so this is about the voltage sources.

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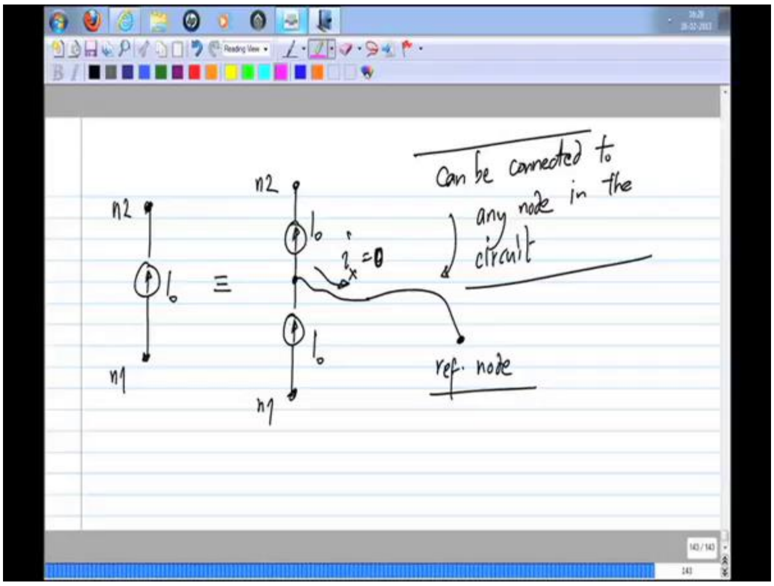

Similarly, for current sources there is something, let us say you have a current source I naught in the circuit and it is exactly the same as having two current sources in series of identical value. So, this by definition is exactly equal to that 1, now if we have formed a new node in the middle and if you take a wire from that node, what will be the current flowing here what is that current going to be whatever be the value of I naught any current source. You take it split it into two current sources in series and form a node in the middle and if you connect a wire to that middle node, what will be the current flowing through that wire? It is pretty clear that this I x will be 0, now this is useful in the following way because the current here is identically going to be 0, you can take this node and connect it to any other node in the circuit.

Now, this is sometimes useful, so if you a have current source connected between let us say some nodes n 1 and n 2, you can split that into two current sources and you can connect the middle to anywhere and usually it is convenient to connect to reference node.

So, instead of a single current source between n 1 and n 2, you will have a current source from n 1 to the reference node and from the reference node to n 2.

Sometimes, this is easier to analyse, again if we will point out if there are examples that make use of this, but this is some general properties that you can use to simplify circuits and visualize certain things about circuit any questions about this? So, these are quite simple, so we can move forward, now the next thing is little more interesting.

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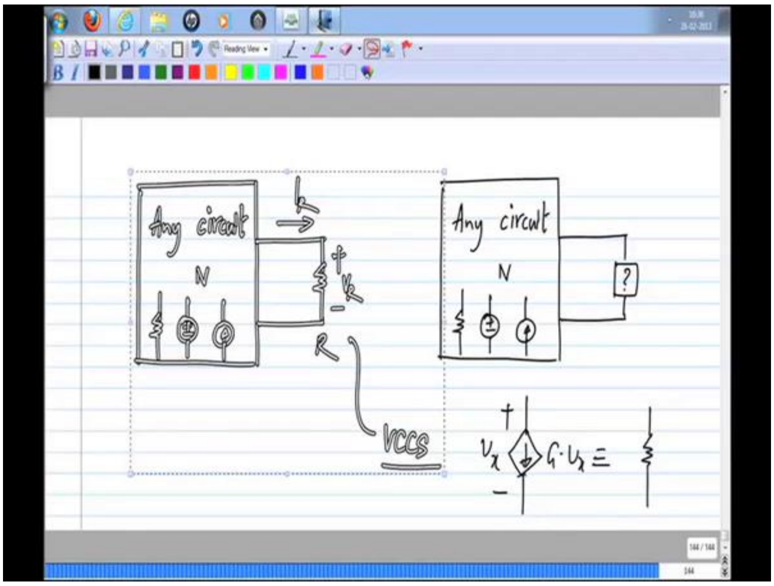

Let us say I have any circuit n, any circuit of network n and there is some element connected to it, I will take a resistor as an example, but this can be any element and it does not even have to be linear. So, let us say I have a resistance R and this has all kinds of things in it, it can have resistors current sources and voltage sources and so on. Now, the question is can I take this circuit remove the resistor and put something else here, some box such that all the voltages and currents in the circuit are exactly the same as voltages and currents in this circuit.

Let me assume that in the original circuit there is a certain voltage across the resistor and there is current through the resistor. So, under these conditions can I replace the resistor with something else so that all the solution to the circuit. All the branch voltages and branch currents in this circuit will be exactly same as the ones in this circuit, so is this possible, so what will I replace it with, please try to answer this.

So, can I replace the resistor with something else without changing any of the branch voltages and branch currents in the circuit n, so please try to answer this, I am not getting any responses. So, because of the set up here, I will not be able to hear your questions over audio, so somebody has raised their hand, but please use the chat window to ask your question. I have only one response which says that this could be replaced by a voltage controlled current source. Now, I am not sure what the intent of this answer, perhaps it means that we have seen that a voltage controlled current source can behave as a resistor if the controlling voltage happens to be across the current source.

Now, this is not my question really I mean this is fine this is true, but this is a resistor, a voltage controlled current source with a controlling voltage across with a controlling voltage across the current source will be exactly the same as the resistor. So, my question was can we replace it by any other element and somebody said voltage source, now that is possible we will see how now it turns out that a resistor can be replaced by either a voltage source or a current source of specific value. Now, please understand that this will not work in general that is you have a circuit n with some particular value of sources inside.

So, in under those conditions you will have some particular current and particular voltage across this resistor, so in that condition you can replace this with a voltage source or a current source if you change the sources inside n. Then, it will not be the same anymore, they will be same only for a particular value of the sources inside the network. So, let us see how we can prove this the proof is very simple and it is involves no algebra just some simple logic, is there a problem with the audio again, so it looks like things are working, so let me copy over this circuit.

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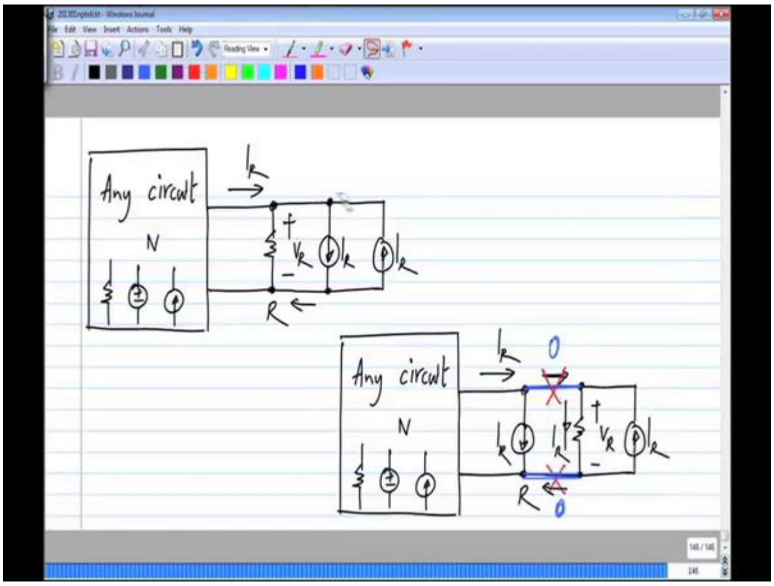

Now, as I said for particular values of some sources inside, this could have been any number of resistors voltage sources current sources even controlled sources and so on. Now, I am showing one particular component here, we could do this with more than one component, now for this set of values, there is a current I R through the resistor and a voltage V R across the resistor. Now, let me do this, what I will do is I will connect a current source of value I R in this direction and another current source of exactly the same value I R in the other direction.

So, I have connected two equal and opposite current sources, which means that I have not really connected anything because the current here will be exactly 0. All I did was to there was nothing connected here and that nothing, I represented as parallel combination of two equal current sources in opposite direction, the value of each current source I took to be the current flowing in the resistor.

Now, what I will do is I will just interchange the positions, you you see that this resistor and this current source and that current source are in parallel. I will put this current source on the left side and the resistor in the middle, this is I R and across this we have V R and the current through resistor will be exactly the same as before because nothing has really changed from my original circuit. I have not made any changes, I added a net of 0 current here and here I just simply changed the way I drew the circuit.

The interesting thing now is what will be the current in this part of the wire this part of the circuit this wire. Let me show that in blue what will be the current in these blue wires, I think all of you can immediately recognized that this current is 0. Now, if a wire is carrying 0 current, there is no point having that wire I may as well cut it off, so this current will be 0.

So, I can cut this off and nothing will change in the circuit because when I have an open circuit and cut wire or a cut wire with exactly 0 current it is exactly the same thing. So, it will not disturb the circuit, so I can cut off those things so what is the bottom line here, my original circuit was like this. After I did all the transformations I described to you and cut off this wire my circuit is like that and all through the steps of logic, we saw that nothing really changes in the circuit.

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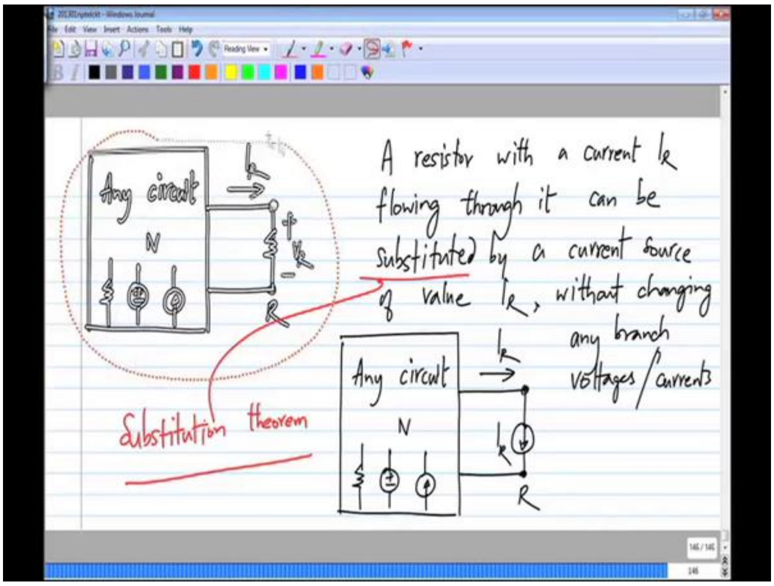

So, the voltages and currents in this circuit will be exactly the same as voltages and currents in that circuit. So, what this means is that a resistor with a current I R flowing through it can be substituted by a current source of value I R in the appropriate direction, of course I R is flowing downwards here. So, this current source has to point downwards and this would be without changing any branch voltages or currents, I hope this is convincing and this particular result is known as the substitution theorem.

So, this it turns out is quite useful in many applications, I mean many applications when I say applications to solve other problems and prove other circuit theorems. So, is this convincing any questions in this chain of reasoning and the proof of substitution theorem? Now, we can also quickly go through another possible substitution what I will do is initially I mean previously I added 0 current and 0 current I represented as I R 2 current sources of value I R in opposite directions in parallel.

Now, what I will do is in series with this particular wire I will break this wire and add two voltage sources of equal and opposite value and value I choose to be V R and what is V R whatever voltage was across this resistor. So, now clearly this voltage will be the same as that voltage and essentially this combination two equal and opposite voltage sources in series its nothing, but a short circuit. So, nothing has changed in the circuit now with reference to some reference voltage, let us call this I have formed this new node here, let me call this n 1 and n 2, how will the voltage at n 1 be related to voltage at n 2?

So, may be maybe I will say value of V n 1 minus V n 2 that is the voltage at n 1 minus the voltage at n 2 what is this value going to be, please try and answer this question. My question is how is this voltage is related to that voltage, now I think it must be pretty convincing to you that I have not changed the circuit by any way. All I did was connect 0 volts in series with a wire, which is like not disturbing the wire at all, but that 0 volt is represented as two equal and opposite voltages of value V R in series with each other.

So, they will be at same voltage, so again this is pretty clear, if we go from here we have a voltage rise of V R and voltage fall of V r. So, these two will be at exactly the same voltage, now given that they are at the same voltage what I can do is connect them up like that. So, my circuit becomes I will redraw it slightly differently, I will have V R across this and you have something hanging from here which is V R and this resistor, but you can see again that the current in this wire is definitely going to be 0.

So, that part can be removed from the circuit, so what we have done is the resistor that was across these two terminals has been substituted by a voltage V r, where V R is the actual voltage across the resistor in the circuit. So, this is another variant of the substitution theorem if you have a resistor with a voltage V R across. It can be substituted by an independent voltage source of value V R without changing the circuit solution is this, so this is another variant of substitution theorem, so in summary what says is that.

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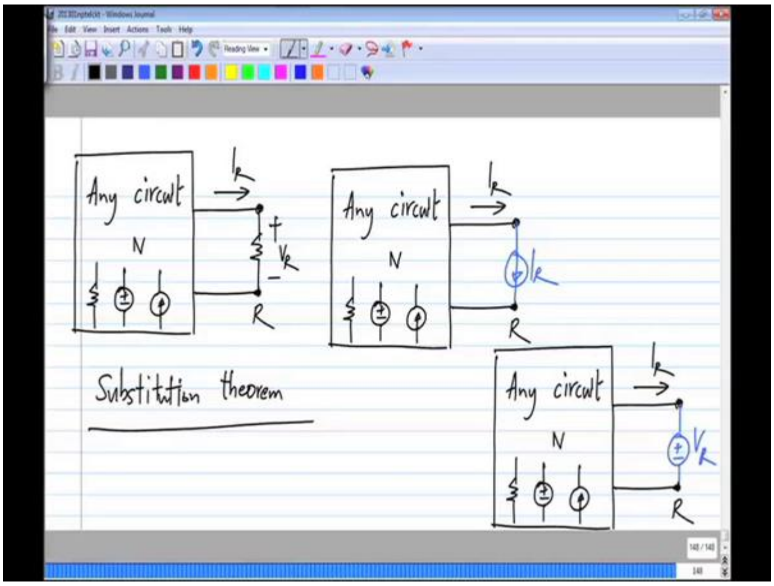

If you have a circuit with a resistor R which has a voltage V R across it and a current I R through it we can substitute that resistor by a current source of value I R in the appropriate direction or a voltage source of value V R in the appropriate polarity. The circuit solutions will be exactly the same that is the branch voltages and currents here, here and here will be exactly the same and this is known as substitution theorem. Any questions about this, any questions about the statement of the substitution theorem or the proof? So, let us move forward one thing I want to point out is we took a resistor and then substituted it. First of all you can do this for multiple resistors in the circuit, it does not have to be only one and also it does not have to be a resistor, it can be any element.

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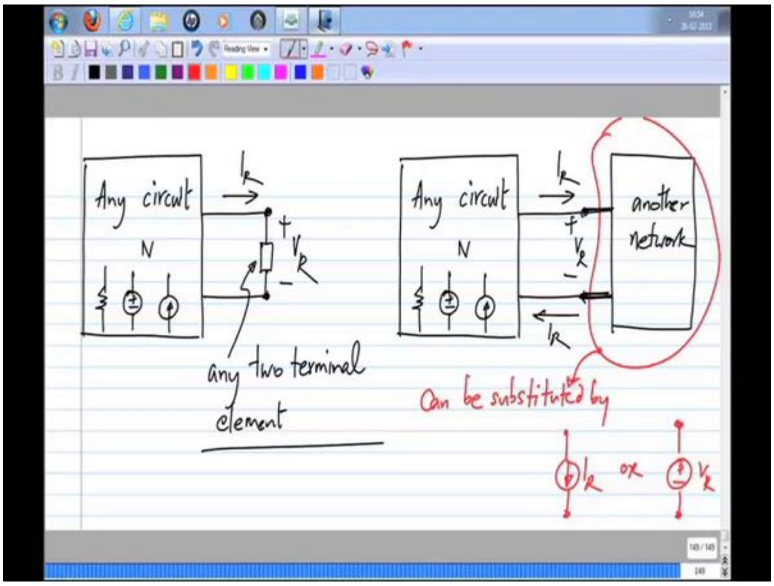

So, instead of a resistor we could have any element, which has a current I R through it and a voltage V R through it I will still call it I R and V R this may not be a need not be a resistor this can be any two terminal element. Also, when you say any two terminal element that itself can be a complicated network with only two terminals exposed. So, this could be another network the only condition is that only two terminals are brought out like this and they are connected and this entire thing can be substituted by let us say the voltage across this two terminals is V R the current flowing that way is I r.

This entire thing can be substituted by a current source of value I R or a voltage source of value V r, so I started with a resistor, but you can work out for yourself that the logic of the proof holds good even if it is non-linear element, it can be any element. In fact, you can substitute a voltage source by a current source and vice versa and also you can substitute a complicated circuit as long as it is only at two terminals.

So, if you have a complicated network with only two terminals brought out you can substitute the entire network by this voltage source or current source, I hope that part is clear. So, there is a question asking about is there any limitation of this theorem, I mean there really is no limitation only thing is this theorem by itself is somewhat limited in that it works for specific values of voltages and currents. I will take an example after that it will become clear, but like I said there is no other limit you can replace any element with a voltage source or a current source and the element, it can be a simple thing like a resistor or some non-linear element like a diode.

It can also be a very complicated network, so as long as only two terminals are brought out, it does not matter what the network is inside it could have thousands of components, but at those two terminals can be replaced by either a current source or voltage source. This current source and voltage source are not arbitrary things, they are the actual currents or voltages flowing in the actual circuit.

So, I hope that is clear that also kind of summarizes the substation theorem that I think someone else asked for and as far as the application of this is concerned, we will see shortly, we will prove other theorems using this theorem. So, there is a question what is superposition theorem superposition we have discussed earlier, we saw that if we proved it with nodal analysis, but you can do it with which any way you want.

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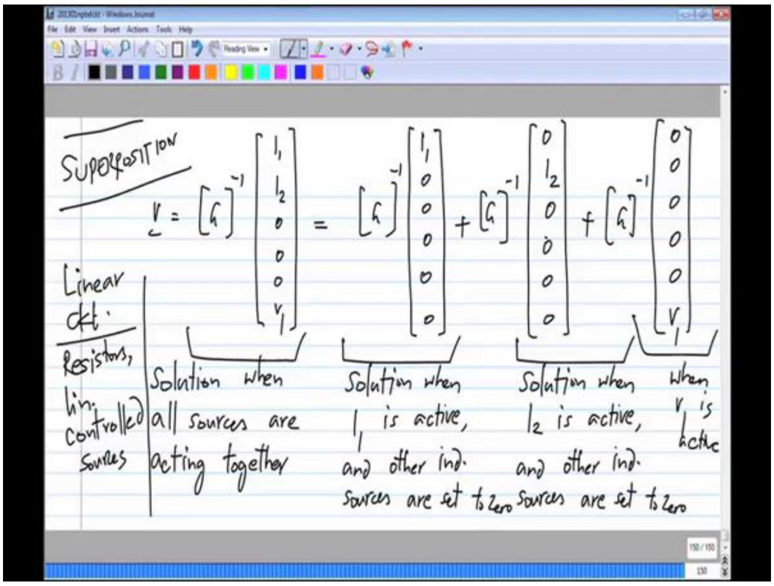

The unknown vector in a nodal analysis in the nodal analysis method will come out to be g inverse times the source vector, I call this I and v, but we know that this contain independent current sources and voltage sources. So, let us say that this independent source vector had two current sources I 1 I 2 and many zeroes and then a voltage source also, this is the same as what I am writing now. So, this is the solution to the node voltages to the unknown vector when all sources are acting together, now this is the solution when I one is active and other independent sources are set to 0.

Similarly, the second one is when I 2 is active and other independent sources are set to 0 and this finally, is when only V 1 is active and other independent sources are set to 0. So, this is superposition theorem what it says is if you have a number of independent sources in a circuit the solution, when all of them are active can be obtained as the solution, sum of solutions, when each one is active and all the others are set to 0.

So, that is you can activate the sources one by one and set all the other sources to 0, in this case you first do the analysis with I 1 only and I 2 only and then V 1 only and add up all the solutions and this works for any circuit that is linear. A linear circuit means it has besides these independent sources it has resistors and linear controlled sources. So, this is superposition, so I hope that part is clear, now let us take a numerical example also to illustrate substitution theorem and to get a bit of practice with circuit analysis.

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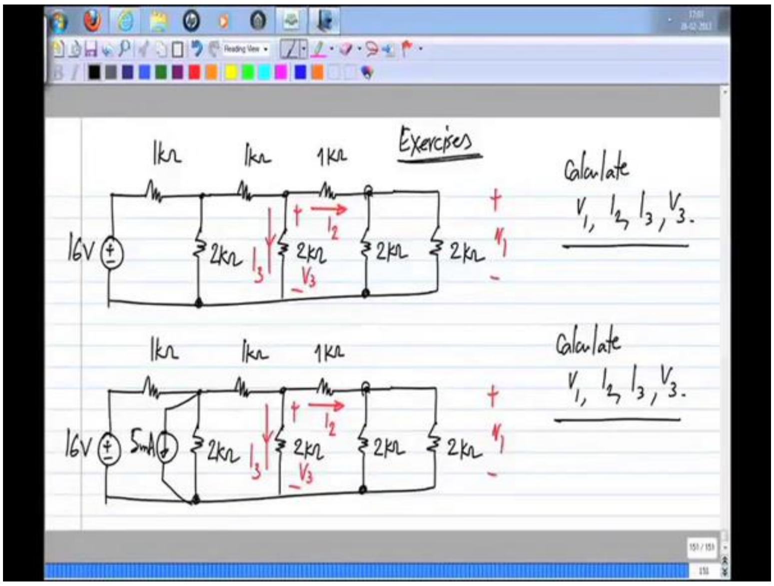

I will take a very simple circuit, let us say it has a single source 16 volts, now what I would like you to find is are these quantities please do not do it right now. In fact, you have to do a little bit of working out this circuit is not very difficult, but you still have to do that. So, please take it as an exercise and do it before next Tuesday and we will discuss this circuit and how to solve it in the lecture. So, please calculate voltage across this, I will call that V 1 and the current through this resistor, I will call that I 2 and the current through this resistor, I will call it I 3 calculate V 1, I 2, I 3.

You can also calculate this V 3 here across this, so please take this up as an exercise and do the analysis before the next lecture and to illustrate superposition, you can also try this. In addition to this, let me say I have a current source of value 5 milli amps, then you can solve for this one, you can do this calculate the same things. So, these are the exercises as usual please go through it please go through the circuit step by step and do it by understanding every step.

The analysis of this is very simple, but you should be able to it with confidence, somebody answered saying V 1 will be 8 volts that is not correct, please do the analysis in detail and then solve for it. Now, let us go to something else we will prove another theorem based on substitution theorem and this is something that is very widely used. In fact, we will use substitution theorem and superposition, so I am glad you brought up the topic of superposition theorem.

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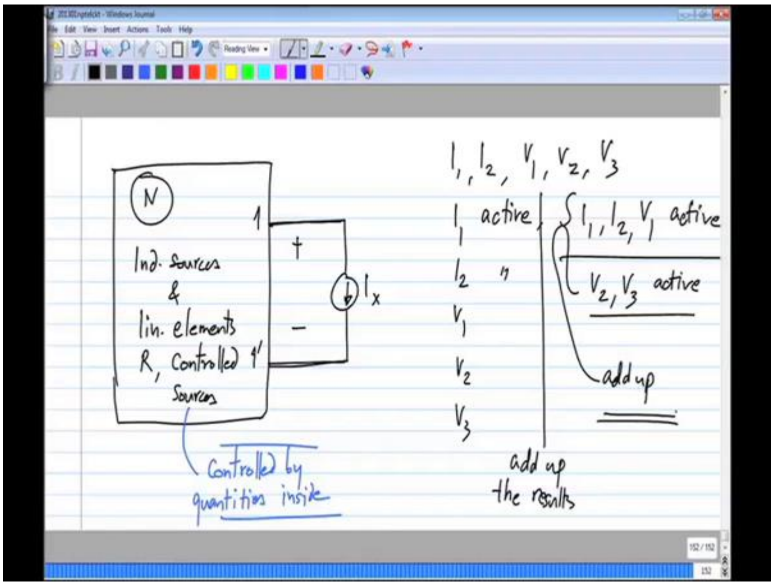

Now, let us say we have a circuit n with independent sources and linear elements, when I say linear elements it has resistors and controlled sources, it can be any network right the connections inside we are not imposing any conditions. So, it will have independent sources and linear elements linear elements means resistors and controlled sources the only condition. I will say it is all the controlled sources have controlling quantities inside n, it is not that any controlled source is controlled by a voltage that is outside somewhere else if it is voltage controlled source, that voltage will be inside n.

Similarly, if it is a current controlled source that current will be inside n, so that is the only condition I will impose. Now, let us say that a current I current source I x is connected to it has two terminals, I will call them one 1 prime that are coming out and I have this current source connected up like this 1 to 1 prime. The definition of circuit I have now the voltage here will be of some form, now what I will do is I will try to solve it by superposition. Superposition means that when I have independent sources, I do not take all of them together, I can take one by one and I do not have to do it one by one.

If I have ten of them, I do not have to it 10 times, I can take 5 once and then the remaining five the other time and so on. So, in this case how I will split it is will do it with only this current source active and everything inside inactive. Then, I will make this inactive the outside current source I x inactive and everything inside active, so that is also valid. So, when I say superposition, we do not necessarily have to take it one by one, so if I have let us say inside the circuit, let us say I have five independent sources, two current sources and three voltage sources.

I can do superposition with I 1 alone active and all other 0, I 2, V 1, V 2 and V 3, then add up the results, instead I can also do it like this, I can take I 1, I 2 and V 1 active and V 2. In that case, I will set V 2 and V 3 as 0, next I will take V 2 and V 3 active and set the rest to 0 and I add up the two results. So, this will give me exactly the same answer, so just you don’t get confused with this, let me show with an example, so please try to solve this.

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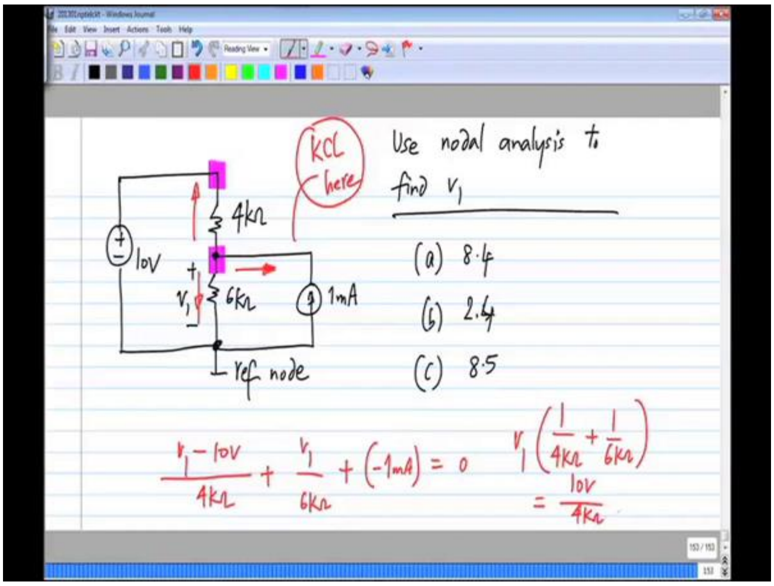

So, let me take a very simple circuit again, so this is the circuit, now first of all you can solve for this in a number of ways, you can either use nodal analysis or mesh analysis and solve for it. So, please do that and let me know the voltage across this 6 kilo ohm resistor, let me call that as V 1 please take this up as an exercise and use nodal analysis to find V 1. So, this is an extremely simple example, there are only two nodes, in fact I suggest that you take this as the reference node. There are only two nodes in the circuit and this node has a voltage source connected to the reference node, so this node voltage is already known.

So, you only have to write one equation one nodal equation and solve for V 1, so please do that please take it as an exercise and let me know the value of this V 1 in this circuit. I hope the question is clear, all I am asking for is this value of V 1 and just do it by nodal analysis by writing the KCL equation at this particular node. I got one response anybody else, so I got three responses all different from each other.

So, hopefully others also have solved it from here you just answer in the poll, I opened a poll with all the responses that I got, please just click on the poll and let me know your answer. So, this is the result of the poll, I mean somebody has voted for all in all of these choices, but the point is only one of these can be correct. So, I will work it out in detail and you can figure out for yourself where you went wrong, this is very important because obviously three of these answers are incorrect. Now, you can figure out where you went wrong and then fix it for the next time, now I said write KCL at this node and from there calculate V 1.

So, the voltage of this node with reference to voltage of this reference node is nothing but V 1, so the current when we write KCL the currents flowing out will be will together be equal to 0. So, the current flowing in the 4 kilo ohm resistor that is flowing away from this node will be V 1 minus 10 volts divided by 4 kilo ohms plus the current flowing in the 6 kilo ohm resistor is V 1 by 6 kilo ohms plus the current flowing here in the current source. So, we know the current flowing in this direction will be minus 1 amperes, 1 minus 1 milli amperes will be equal to 0. So, V 1 if I group all of that together, I will have V 1 times 1 by 4 kilo ohms plus 1 by 6 kilo ohms equals 10 volts divided by 4 kilo ohm plus 1 milli ampere, this is what we will have.

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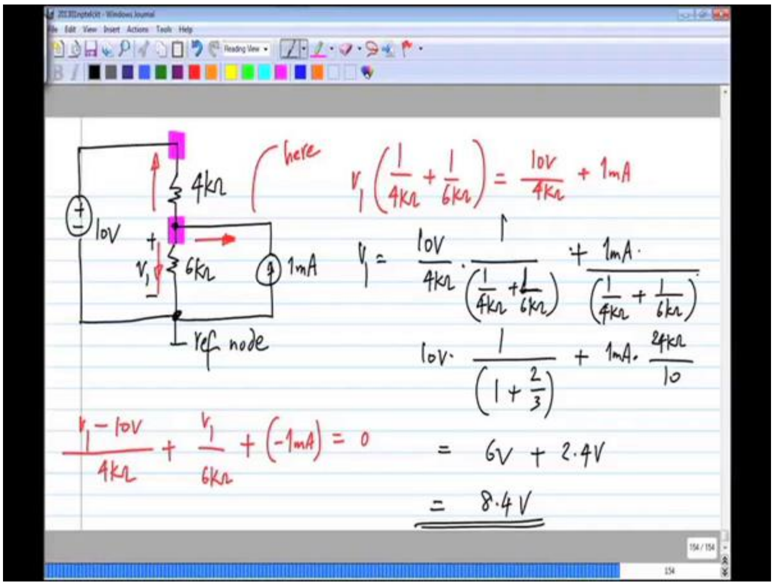

So, V 1will be 10 volt by 4 kilo ohm, 1 by 1 over 4 kilo ohm plus 1 by 6 kilo ohms plus 1 milli ampere divided by 1 by 4 kilo ohms plus 1 by 6 kilo ohms. So, if I take this inside, I will get 10 volt 1 divided by 4 kilo ohm this gives me 1 plus 4 by 6, this gives me two- thirds and this one here if I calculate this I will get 6 kilo ohm plus 4 kilo ohms divided by 4 kilo ohms time 6 kilo ohms. This is basically 10 by 24 kilo ohm in the denominator we have kilo ohm square in the numerator, we have kilo ohm.

So, we have 10 by 24 kilo ohm, so we have 1 milliamps times 24 kilo ohms divided by 10, so you can see that this will be 6 volt that is 10 volts time 3 by 5 plus this will be 1 milliamps time 2.4 kilo volts, which is equal to 8.4 volts. So, the correct answer is 8.4 volts, so I hope it is very clear how to do this so if you do this nodal analysis, you will get the final answer in one shot, now let me also do it by superposition just to demonstrate to you.

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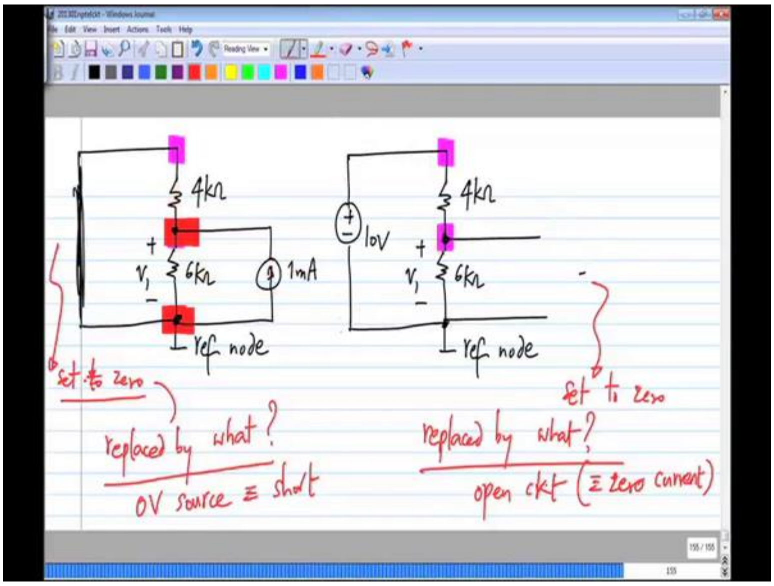

So, what I will do is I will first set this 10 volts independent source to 0 and find V 1 as the result of one milliamp current source and then I will set this to 0, the one milliamp source to 0 and find result of this one as a result of 10 volt source. Then, I add up the values of V 1 from this circuit and that circuit and that will give me the same as same answer as before or it should if superposition is valid. Now, this is a linear circuit we have independent sources and except the independent sources, we have just resistors which are linear.

So, we can apply superposition, now when I set 10 volt source to 0, what does it mean what should I replace the 10 volt source by when I say set to 0, what does that mean what should I replace it by? So, many of you answered that it is replaced by short circuit, so when this is a voltage source when I make the voltage source a 0 volt source, it is a short circuit. So, here I have to replace it with a short circuit, similarly when I set the current source to 0, what should I replace by, what should I replace it by?

So, clearly I have to replace it by 0 current source and 0 current source means no current, which is an open circuit, so there should be no confusion in this, sometimes you here the terms like I remove the sources just try not to use that. It will only lead to confusion, so you just make that source value equal to 0, the voltage source becomes a short circuit if it is a current source, it becomes an open circuit. Now, in the first case, what is the value of V 1 if you observe the 6 kilo ohm and 4 kilo ohm are in parallel. It is drawn in a strange way, but really this voltage, sorry this terminal has two resistors and this terminal also, both the resistors are connected between this node and that node, so that means they are in parallel.

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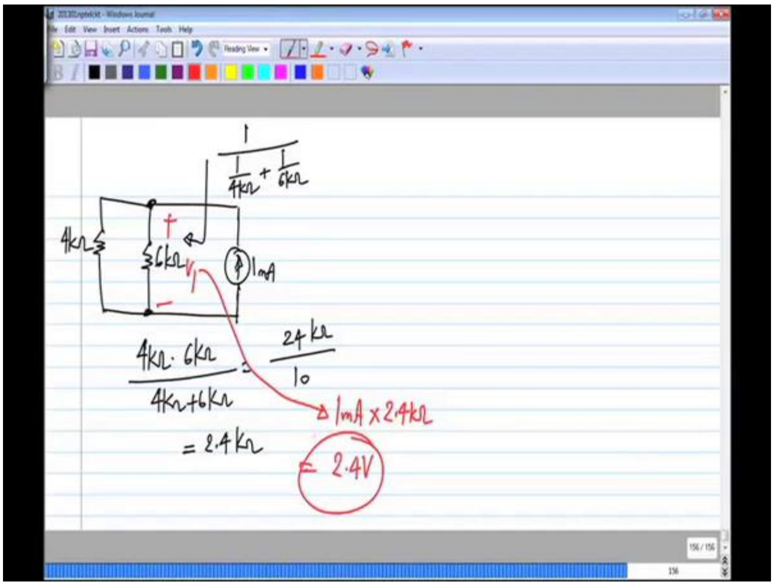

So, what I have really is a one milliamp current source going into this 4 kilo ohm and 6 kilo ohm in parallel and we know what happens when you connect resistors in parallel, it is equivalent to a single resistor of value one by the reciprocal of the resistors. In fact, when you have only two resistors in parallel, you can use the formula R 1 R 2 by R 1 plus R 2, so you will get 4 kilo ohm time 6 kilo ohms divided by 4 kilo ohm plus 6 kilo ohm and this is 24 kilo ohms divided by 10 and which is 2.4 kilo ohms. If 1 milliamp current flows through 2.4 kilo ohms, so this milliamp and kilo ohm multiplied will give you volts.

So, you will get 2.4 volts and by the way when you use superposition you make sure that whatever quantity you to solve for you have to put that in consistent direction everywhere V 1 is the voltage across 6 kilo ohm resistor with upper terminal positive. It is the same here and in this case V 1 will be here across 6 kilo ohm and that is equal to 1 milliamp times 2.4 kilo ohms, which is 2.4 kilo volts.

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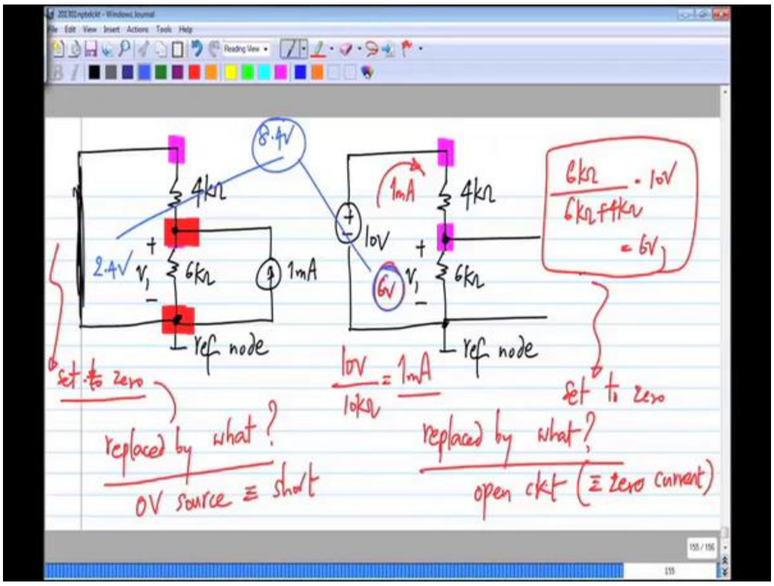

Similarly, if I have this particular case, you can calculate the total current what is the total current flowing here, what is the current flowing in this loop, we have two resistors in series 4 kilo ohms and 6 kilo ohms, and so what is the current flowing here.

Some of you have answered just one always these are quantities with dimensions, so please always give the units along with the number and some of you have answered 1 ampere. So, please be careful of what you are doing and give me the correct answer the series combination of 4 and 6 kilo ohms is a single resistor of 4 kilo ohms plus 6 kilo ohms which is 10 kilo ohms. So, we have 10 volts divided by 10 kilo ohms, which gives you 1 milli ampere, it is not one ampere and don't try not to give answers like 1 because that is only going to get confusion.

So, this is 1 milli ampere and this 1 milli ampere flows through both of these, so the voltage V 1 will be one milliamps time 6 kilo ohm which is equal to 6 volts. So, it is very simple and alternatively you can recognize that this is a voltage divider and use the voltage divider formula. The voltage across the 6 kilo ohm resistor is nothing but 6 kilo ohm divided by 6 kilo ohm plus 4 kilo ohm times the applied voltage source which is 10 volts which gives you 6 volts.

Either way you get the voltage across the 6 kilo ohm resistor to be 6 volts, so here we have 2.4 volts and here we have 6 volts, so the actual total voltage will be the sum of these two which is nothing but 8.4.

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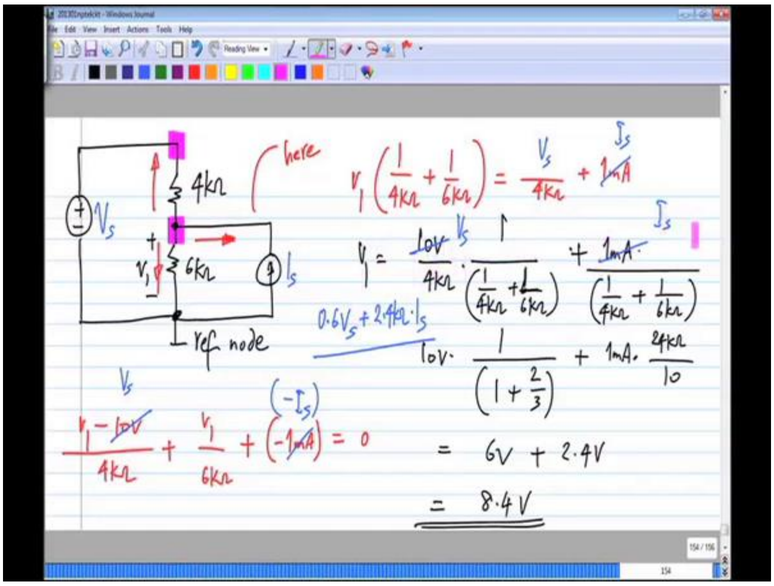

It is exactly the same as what we had here 8.4 volts, in fact the way I have written out the expressions this is the contribution from the voltage source and this is the contribution from the current source. They get added together, now in general if you have multiple inputs that is multiple independent sources, you have any voltage or any current in the circuit will be a linear combination of all the sources. For instance, let me make these variables just for illustrations, let me call this V s and let me call this I s if I write Kirchoffs current law at this node this part will be exactly the same.

Let me instead of 10 volts, I will have V s here and instead of minus 1 milli ampere, I will have minus 1 I s. So, if I take it to other equation, I will end up getting instead of 10 volts, I will have V s and instead of this 1 milliamp, I will have I s, so here I will have V s and there I will have I s. The reason I did that is to just show that this V 1 will be some linear combination of V s and I s that will be some number times V s plus some other number times I s.

If you have many sources, it will always be like this it will be in a linear combination form and it is not only V 1, you take the voltage or current in any part of the circuit, it will be a linear combination of all the independent sources applied to the circuit. So, in this particular case we will get V one to be 0.6 times V s plus 2.4 kilo times I s, so if you substitute V s equal to 10 volt and I s equal to 1 milliamp, you will get the total to be 8.4 volts. So, hopefully this is clear both how to solve circuits like this, this is a very simple circuit and also the general idea of superposition.

So, we solved the circuit with all the sources in one shot and then we did it with one source by one source setting other source to 0 and we got the same answer as we should have got as we expected from a linear circuit. We also showed here that any voltage or current will appear as a linear combination of all the independent sources. So, that is what superposition is, that is how we solve for it, now in this case we had only two sources. If we had more than two, if we had three sources we could have taken two sources at a time or another source taken one source at a time three times and so on.

So, now let us get back to what we were trying to do we have a network n of many independent sources and linear elements linear elements mean resistors and controlled sources. Only condition is that all the controlling quantities are inside this network n and there are two terminals that are coming out that are visible to the outside world 1, 1 and 1 prime. I connect them with source I x from 1 to 1 prime, so to find this voltage across 1 1 prime, let me call this V x, what I can do is many things I can solve the entire circuit, but here I am trying to pull a particular circuit here.

So, I will do it in this way, I will take I x alone and find V x, that is when I say I x alone all the independent sources inside are set to 0 and I will set I x to 0 and use only the independent sources inside. I activate them and find this V x and then I can add up the two results and find the total value, so let me just do that, let me draw the picture corresponding to that and we can end this lecture and continue from here in the next lecture.

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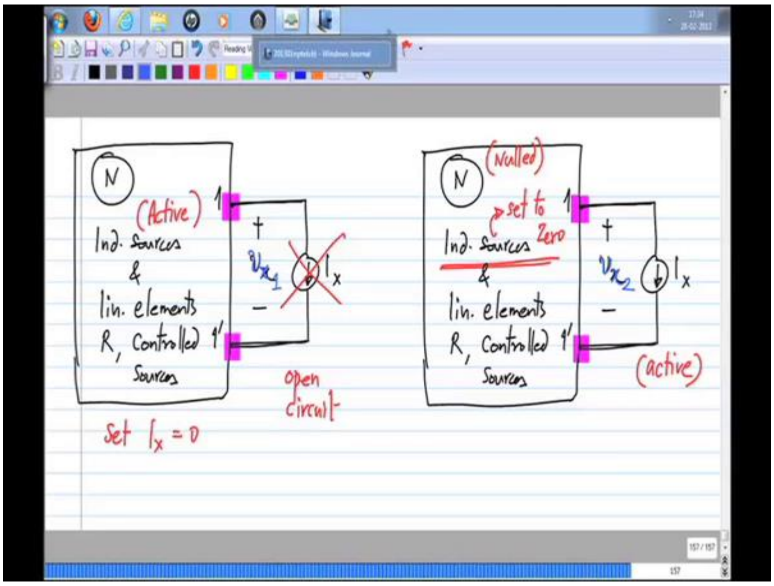

So, first what I will do inside the independent sources will be active and I will set I x to zero, so when I set I x to 0, what happens this is an open circuit and in the next case this I x will be active and this independent sources. Any number of independent sources inside will all be set to 0 and this is known as nulling the circuit. So, I have nulled the networking that means any independent sources I have set to 0, but I x is active, so I will get let us say call this V x 1.

When I x is inactive and this V x 2 when I x is active and I add up the two values of V x to get the final value. So, this we will continue in the next lecture, in fact you can take this also as an exercise and find the general form of the solution that you get for V x 1 and in particular for V x 2 what will be the form of V x 2, what will it be, what will be its dependence on I x? We will take it from there in the next lecture, is there any questions regarding whatever we discussed today or anything else, please let me know I will clear those and end the lecture.

So, there are several questions, one is asking is pushing of voltage sources through a node a new theorem, I am not sure what it is, what is meant by this. I mean certainly I did not invent this, it is a well-known result and a relatively obvious result that is sometimes useful and the next thing is about current division.

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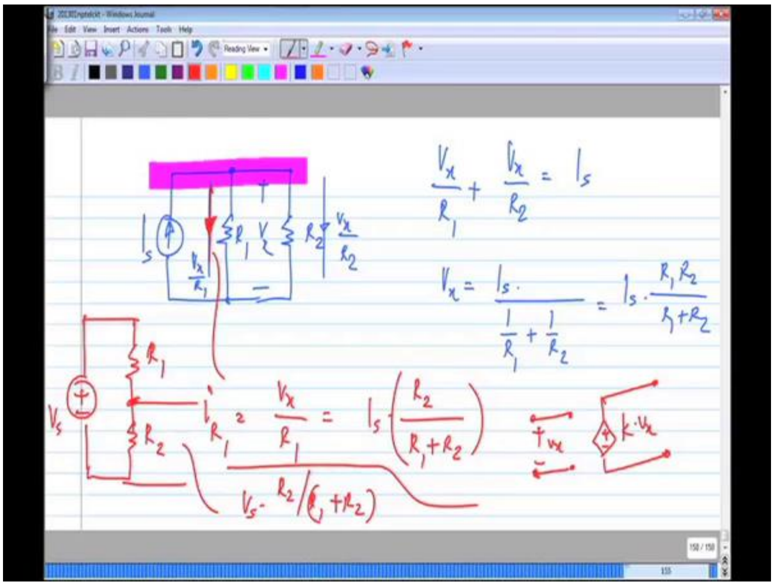

Now, current division is the counter part of voltage division, I will show it by two resistors R 1 and R 2, clearly if you have two parallel resistors R 1 and R 2 the voltage across them will be the same, let me call it V x. The current through this will be V x by R 1 here, it will be V x by R 2 and by applying KCL at this node, we know that V x by R 1 plus V x by R 2 equals let me call this I s. So, V x will be I s divided by 1 by R 1 plus 1 by R 2, which is I s times R 1 R 2 by R 1 plus R 2, I could have written this down directly by noticing that R 1 and R 2 are in parallel, so I s times R 1 parallel R 2 is the voltage V x. Now, if I look at any particular current, say current flowing through R 1, that will be equal to V x divided by R 1.

So, this current I R 1 is V x divided by R 1, which is equal to I s and R 1 cancels here R 2 by R 1 plus R 2 and this is analogous to the voltage divider formula, we have V s the voltage across of this is V s R 2 by R 1 plus R 2. Now, some ratio of resistors in both cases in case of current division the current through R 1 will have R 2 in numerator and in case of voltage divider, voltage division the voltage across R 2 will have R 2 in the numerator that is all.

Now, the next question is actually the question is not clear, I guess the question is what to do with dependent sources when you apply superposition you don't do anything you certainly do not set dependent sources to 0 dependent sources. They will have whatever dependence they have, for instance for controlled voltage source will be dependent on some voltage V x and let us say it is some k times V x. You don't change anything here, you simply do the analysis as usual, so if this is V x that is k times V x and someone asked for the slide showing superposition.

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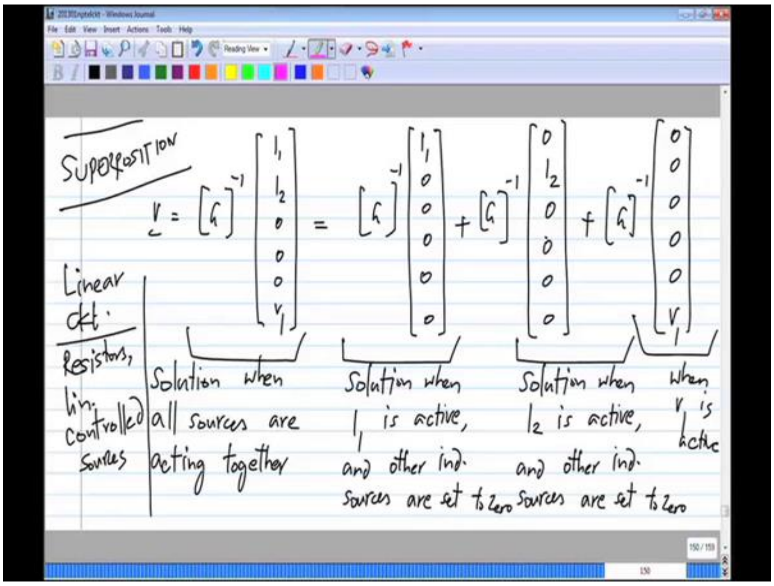

So, it is here all that is saying is the source vector which has many elements can be thought of as sum of one element at a time I 1 here I 2 here and V 1 there and there could have been more things I have not shown it. You see that this basically comes from linearity, so if you have these vectors, you can show it as sum of that one and that one and that one and g inverse times this is g inverse times that plus g inverse times that plus g inverse times that. You can interpret each of them as solution when only one of those sources is active, so I hope that clears up all of those things, then if you have any other questions, then please do feel free to raise them in the forum or in the next lecture.

Thanks for attending I will see on Tuesday.