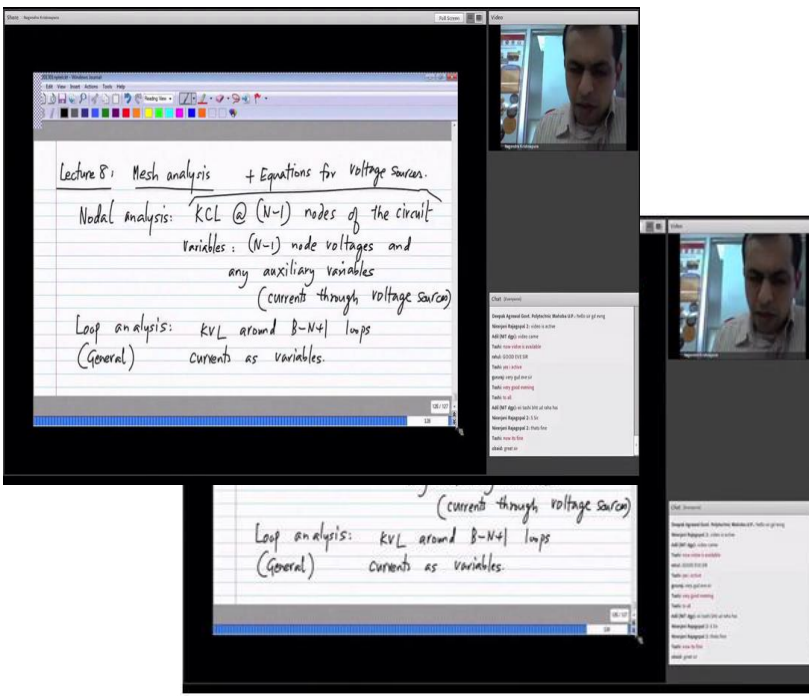


Basic Electrical Circuits
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Lecture No- 08

- 1) Mesh analysis of a Circuit with Resistors and Voltage Sources;
- 2) Comparison with Nodal analysis;
- 3) Mesh analysis of Circuits with Current Sources-Supermesh

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This is lecture 8. In the previous lecture, we looked at Nodal analysis. How to do Nodal analysis, when you have a circuit with current sources and resistors, that was the easiest case

and we extended that to when we have voltage sources. In that case, if you are doing hand analysis you can use a super node. If you are trying to set up the equations for a computer, you can set the current through the voltage source with an extra variable and go ahead with the analysis.

Now, when you have controlled sources, when you have a current controlled sorry voltage controlled current source ~~then, then~~ all it does is to add some terms to the matrix, the conductance matrix, so that it becomes asymmetrical and other cases can also be handled in the same way. You have a voltage controlled voltage source where you will have to again either define a super node, if you are doing a hand analysis that is the preferred method because it reduces the number of equations. Or if you are setting up the equations for analysis by computer ~~then, then~~ you define the current through voltage source with an auxiliary variable and also write down the equation for the controlled source.

In all the cases on the right hand side, you have the source vector \underline{s} which consists of independent current sources and independent voltage sources and on the left hand side you will have matrix times the variable vector. The variable vector will be the node voltages with respect to the reference node plus any auxiliary variable you may have used. If you have the voltage sources, they will be the current through the voltage sources.

Now, before we go on with today's lecture, is there any questions on previous lecture or any aspect of Nodal analysis, I would like to take them up. So, looks like things were pretty clear. So, what we will look at today is alternative to Nodal analysis, that also is sometimes used and that is known as Mesh analysis.

Now, the Nodal analysis this uses K C L at $N - 1$ nodes of the circuit, the circuit has n nodes and you have to write K C L at $N - 1$ nodes and variables are at the $N - 1$ node voltages and any auxiliary variable. And these auxiliary variables are current through voltage sources and the equations come from $N - 1$ K C L equations plus equations for voltage sources \underline{s} [FL]. Now, Mesh analysis is the counter part of this. Now, instead of starting with K V L, first I will tell you about something called a loop analysis.

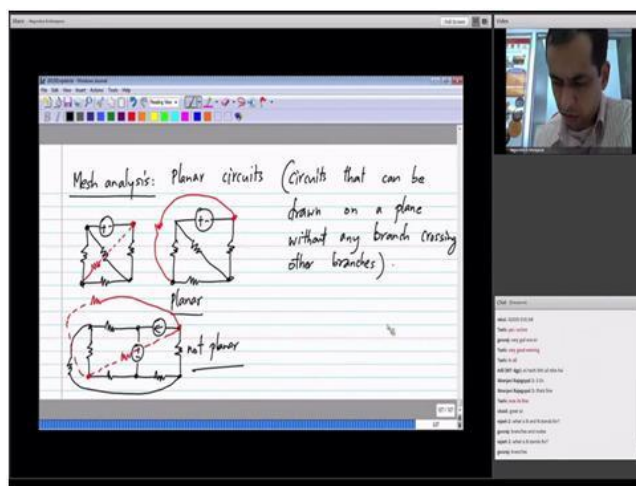
It uses K V L around $B - N + 1$ loops and it uses currents as variables. Now, this loop analysis, the way it is done is you first identify a tree ~~then, then~~ you add each link that is branches that are not in a tree to form a new loop. We took an example of a, this in 1 of the earlier lectures to figure out how many independent K V L equations are there. So, first

identify a tree and then you add links to the tree to form new loops and around each loop you write a K V L equation.

~~Now, this can be done and this is a systematic way of doing it. What we will look at is a subclass of this which is known as Mesh analysis. So, this loop analysis itself is quite general.~~

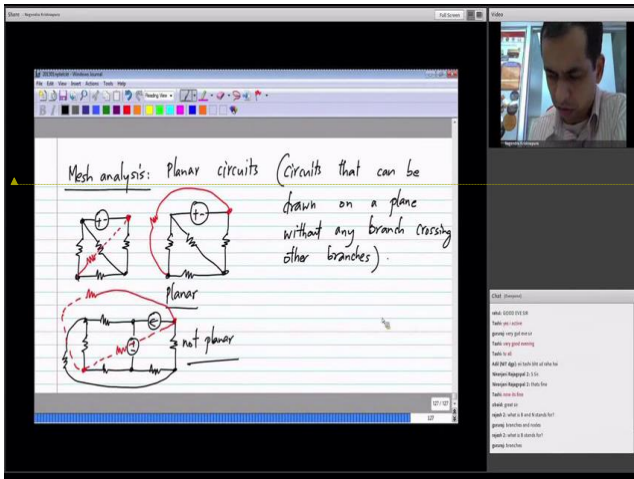
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Now, this can be done and this is a systematic way of doing it. What we will look at is a subclass of this, which is known as Mesh analysis. So, this loop analysis itself is quite general.

Now, the Mesh analysis is a sub case of this loop analysis Now, this can be done and this is a systematic way of doing it. What we will look at is a subclass of this which is known as Mesh analysis. So, this loop analysis itself is quite general.

and is applicable to planar circuits, that is circuits that can be drawn on a plane. So, all the circuits that we have considered so far they can be drawn on a plane. What I mean by this is without any branch crossing other branches. Now, I will give you an example if you have a circuit like this, this is a planar circuit, no branch is crossing any other branch. Now, to this if I add a branch like this, this branch is crossing that branch.

Now, you have to be little careful it is not how you draw the circuits, but how it can be drawn because if I redraw this, if I redraw this even with this red branch, it can be drawn on a plane without any branch crossing any other branch. In this case I have chosen to draw in the middle, but if I draw it outside ~~then, then~~ it would not be crossing anything else, so this is also a planar circuit.

On the other hand, if I have something like this, let us say I have something like that and then I also have something going from this node to that node. Now, if I have any branch between here and there, it has to cross some branch. I can draw it inside here, when in which case it is crossing this branch. Alternatively, I could draw the same thing from outside in which case it will be crossing this branch. So, this circuit is definitely not planar.

So, the method of analysis that I will discuss today, Mesh analysis is not applicable to this circuit, but it is applicable to any planar circuit. Now, I have to emphasize here that the

general loop analysis, which is based on writing K V L in terms of the link currents is applicable to every circuit.

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The screenshot shows a video lecture interface. On the left, a whiteboard displays two circuit diagrams. The first diagram shows a planar circuit with three meshes labeled mesh #1, mesh #2, and mesh #3. It includes voltage sources V_1 and V_2 and resistors $R_1, R_2, R_3, R_4, R_5, R_6$. The second diagram shows a similar circuit with mesh current i_3 and branch currents i_1, i_2, i_3 . Below the diagrams, handwritten notes define mesh analysis: "Mesh: each loop that doesn't enclose any other loop.", "Each mesh: mesh current: (clockwise direction)", and "Current in each branch: Algebraic sum of currents in the meshes that branch is part of". On the right, two video thumbnails show the instructor. Below the thumbnails are two chat windows with text messages.

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You first have to identify a tree and then go on, that is similar to identifying a reference node for Nodal analysis. But this Mesh analysis is simpler and it gives a structure, that is very similar to Nodal analysis and that is what we are going to discuss. So, once you have a planar circuit, so let me again take circuit of this sort and I initially consider a circuit with only resistors and voltage sources. This is a planar circuit, no branch is crossing any other branch

and when you have a planar circuit, you can always identify regions like this, loops like this, which are separate from each other.

So, imagine that you are looking at the map of India obviously, you will be able to identify states on the map and each of the state will be separate from the other state and everything will be on the plane. Now, each of those regions, each of the states is called a Mesh. So, I will call it mesh number 1, mesh number 2 and mesh number 3. So, mesh number 3 is this loop, mesh number 1 is that loop and mesh number 2 is that loop. Let me call it V_1 , V_2 , R_1 , R_2 , R_3 , R_4 . Now, in each mesh we identify a mesh current, let me call this i_1 for mesh number 1 and i_2 for mesh number 2 and i_3 for mesh number 3.

What I mean by this is that, this is also a loop. So, if I take it like this, this is also a loop, but inside this there is another loop. So, that is something that we do not consider a mesh. So, for a planar circuit the definition is unambiguous, each mesh is a loop that does not enclose any other loops and with each mesh we can identify a current.

Each mesh has a mesh current and by convention we will take all of them to be clockwise. You can take all of them to be counter clock wise and you will get the same result, but our convention is to take all of them in clockwise direction. And now, the current in each branch can be written in terms of mesh current, so the current in each branch is the algebraic sum that is, some while taking into account the sign of the current algebraic sum of mesh currents basically.

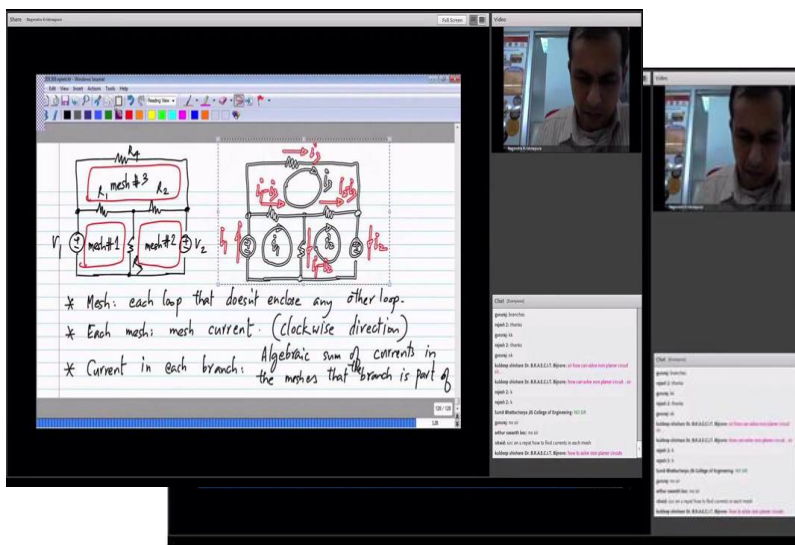
And which meshes do we take? The mesh i_1 ; the meshes that the branch is part of. So, what I mean is, if you take this 1, this branch is part of mesh number 1 and mesh number 3, so the current in this branch would be i_1 flowing from left to right minus i_3 because i_3 is flowing from right to left. If you take this branch that is part only of mesh 3, the current in this will be i_3 . Similarly, if you take this branch, it is branch it is part of only mesh 2, so its current will be i_2 , which will be the current in this will be i_1 minus i_2 and so on.

So, you can any questions regarding the definitions of the mesh and the mesh current and the branch currents in terms of mesh currents. Any questions? I defined planar circuits, meshes and mesh currents and branch currents in terms of mesh currents. Any questions about any of these? There is a question from Obaid asking how to find currents in each mesh, now it is not something that we find yet so far we have not solved for anything, it is a variable that you identify i_1 , i_2 , i_3 . This is like assigning node voltages with respect to reference node. We

assign node voltages v_1, v_2, \dots, v_{n-1} and then, then solve for them.

Similarly, here we identify mesh currents i_1, i_2, i_3, \dots for the number of meshes and then, then we solve for the mesh currents. So, we have to find out what they are. So now, the branch currents must be pretty clear also.

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Just so that it is very clear, I will write that current here is $i_1 - i_3$, current here is $i_1 - i_2$, current here is $i_2 - i_3$ and current in this direction would be i_3 all and here it is i_2 and here it is i_1 . Now, once we defined the mesh currents and find the branch currents in terms of mesh currents, we can write KVL around each mesh.

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Mesh #1: $R_1(i_1 - i_2) + R_3(i_1 - i_2) = V_1$
 Mesh #2: $R_2(i_2 - i_1) + R_2(i_2 - i_3) = -V_2$
 Mesh #3: $R_4(i_3) + R_2(i_3 - i_2) + R_3(i_3 - i_1) = 0$
 not a mesh $i_1(R_1 + R_3) - i_2 R_3 - i_3 R_3 = 0$

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So, if I write K V L around mesh number 1 and let me call it v 1 and this is v 2 if i write k v l around mesh number 1, what do I have? The voltage drop across r 1 plus voltage drop across r 3 equals v 1. And voltage drop across r 1 is r 1 times the current in r 1 which is i 1 minus i 3 plus r 3 times the current in r 3, which is i 1 minus i 2 and that is all that is all the resistors we have and we have an independent voltage source. And just like we put the independent sources on the right side in Nodal analysis. We do the same in Mesh analysis and if we look

at the direction of this i_1 minus i_3 times r_1 is the voltage drop in this direction, i_1 minus i_2 times r_3 is the voltage drop in that direction, so the sum of this plus this equals v_1 , because v_1 is also in the same direction.

Now, for the second mesh what we do is the same the voltage drop across r_2 in this direction is r_2 times i_2 minus i_3 . Now, we always go clockwise around each mesh and we also take the voltage drops in the same direction. So, for this particular branch the ~~one+~~ in the middle we also take a voltage drop with this to be positive and that to be negative. I will erase some of those things. So, while writing the equation for the second mesh we take the drops in that direction while writing equations for the first mesh we take in this direction, for second ~~one+~~ in that direction. ~~A~~and for the third ~~one+~~ again we would do it in the clockwise direction voltage drops in the clockwise directions like that like that and like that.

So, this is just convention even if you write it the other way round you will get the right answers, but if you follow this convention you will be able to, you will be able to if you follow this convention the equation will come out in a systematic manner. Now, 1 of the participants has raised hands, but I think unfortunately today's audio set up is such that I cannot hear your questions. So, whatever questions you have please type it in the chat window.

So, hear the voltage drop across r_2 plus voltage drop across r_3 , ~~-~~ which is if we look at it is the current in this direction times r_3 which is and it is equal to minus v_2 . ~~B~~because if you look at it, this voltage drop is here to there, that is from here to there and that if you look at v_2 it will be minus v_2 , if you look at the appropriate direction or alternatively you can think of this voltage plus that voltage plus v_2 summing to 0. So, that means that if we shift v_2 to the right hand side you will end up getting minus v_2 .

Now finally, for the third loop the voltage drop across this plus the voltage drop there, plus voltage drop there, equals 0, because there are no independent voltage sources in the loop. So, r_4 times i_3 that is the only current flowing through r_4 and for the voltage drop in this direction, I have to take current in that direction which is i_3 minus i_2 plus r_1 times i_3 minus i_1 and this will be equal to 0.

And this is the equation for mesh number 1 K V L equation around mesh number 1, mesh number 2 and mesh number 3. So, I hope this is clear with the procedure for writing the K V L equations around the mesh. So, what you do is to identify mesh current when you represent

each branch current as a sum of 2 mesh currents when I say sum it is sum with direction so it will come out to be the difference between 2 mesh currents, and ~~then, then~~ you write the KVL equations in terms of the mesh currents.

Here you have resistors you will have current times the resistance value and the current is written in terms of the mesh currents. So, you will always have terms like i_1 minus i_3 that is the difference between 2 currents times the resistance that will be on the left hand side, and on the right hand side you will have any independent voltage sources in the circuit.

Now, there was a question from Rajesh asking what is the difference between a mesh and a loop? Like I said a mesh is also a loop, but it does not contain any other loops inside. For instance, in this particular circuit this is a loop and this is also a loop, but the second one is not a mesh because that loop contains a smaller loop inside.

Hopefully that is clear whereas, this is a mesh. So, every mesh is a loop, but every loop is not a mesh. And regarding the direction of mesh currents like I said by convention you always take each mesh current to be in the clock wise direction. Now, that gives you a nice structure because so let us say you have 2 meshes, I am not showing the elements I am just showing the branches in an abstract way. So you take a current like this i_1 and take a current like this which is i_2 now every branch current will be equal to either 1 mesh current or the difference between 2 mesh currents.

So, this branch is common to the 2 meshes and in the left side mesh the current is going downwards and in the right side mesh it will be going upwards. If you choose all your mesh currents to be in the clockwise direction this will always happen. So, every branch current can be written either as a mesh current a single mesh current or as a difference between 2 mesh currents. And this is analogous to if you assign node voltages every branch voltage will be either equal to some node voltage, if the branch is between that node and reference node or it will be the difference 2 node voltages. This is fine?

So, these are the equations in terms of the mesh currents and I will again rearrange them and group the variables together.

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$$\text{Mesh \#1: } R_1(i_1 - i_3) + R_3(i_1 - i_2) = V_1$$

$$\text{\#2: } R_2(i_2 - i_3) + R_2(i_2 - i_1) = -V_2$$

$$\text{\#3: } R_4(i_3) + R_2(i_3 - i_2) + R_3(i_3 - i_1) = 0$$

$$\text{\#1: } i_1(R_1 + R_3) - i_2R_3 - i_3R_1 = V_1$$

$$\text{\#2: } -i_1R_3 + i_2(R_2 + R_2) - i_3R_2 = -V_2$$

$$\text{\#3: } -i_1R_1 - i_2R_2 + i_3(R_1 + R_2 + R_3) = 0$$

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So, I will have $i_1 r_1$, plus r_3 minus i_2 times r_3 , minus i_3 times r_1 to be equal to v_1 that is the equation for mesh number 1 and equation for mesh number 2; that is minus i_1 times r_3 , plus i_2 times r_2 , plus r_3 , minus i_3 times r_2 , that will be minus v_2 . And similarly, for 3 will have minus i_1 times r_1 , minus i_2 times r_2 , plus i_3 times r_1 , plus r_2 plus r_4 equals 0.

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Mesh equations:

$$\begin{aligned} \#1: & i_1(R_1+R_3) - i_2R_3 - i_3R_1 = V_1 \\ \#2: & -i_1R_3 + i_2(R_2+R_3) - i_3R_2 = -V_2 \\ \#3: & -i_1R_1 - i_2R_2 + i_3(R_1+R_2+R_3) = 0 \end{aligned}$$
$$\begin{bmatrix} R_1+R_3 & -R_3 & -R_1 \\ -R_3 & R_2+R_3 & -R_2 \\ -R_1 & -R_2 & R_1+R_2+R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ 0 \end{bmatrix}$$

Diagonal: Sum of resistances in each mesh

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So, these are the mesh equations. Now, let me copy all the mesh equations also and as the

Nodal analysis I will write this in matrix form, I have some matrix multiplying with the vector of variables which is the vector of mesh currents equal to the source vector of independent sources, which is v_1 minus v_2 and 0 and here we will have r_1 plus r_3 .

So, this is what is multiplying i_1 and next would be minus r_3 and minus r_1 . So, this is for mesh number 1, the second equation is for mesh number 2 and you will write minus r_3 r_3 r_2 plus r_3 and minus r_2 and finally, for mesh number 3 minus r_1 minus r_2 and r_1 plus r_2 plus r_4 . So, any questions on this because I had already done Nodal analysis earlier I went a little quickly through this ~~but~~, but if some part is confusing please ask me.

Now, all I did was I first identified mesh currents that is current in each mesh in clockwise direction. Then I expressed every branch current as in terms of mesh currents, and they will

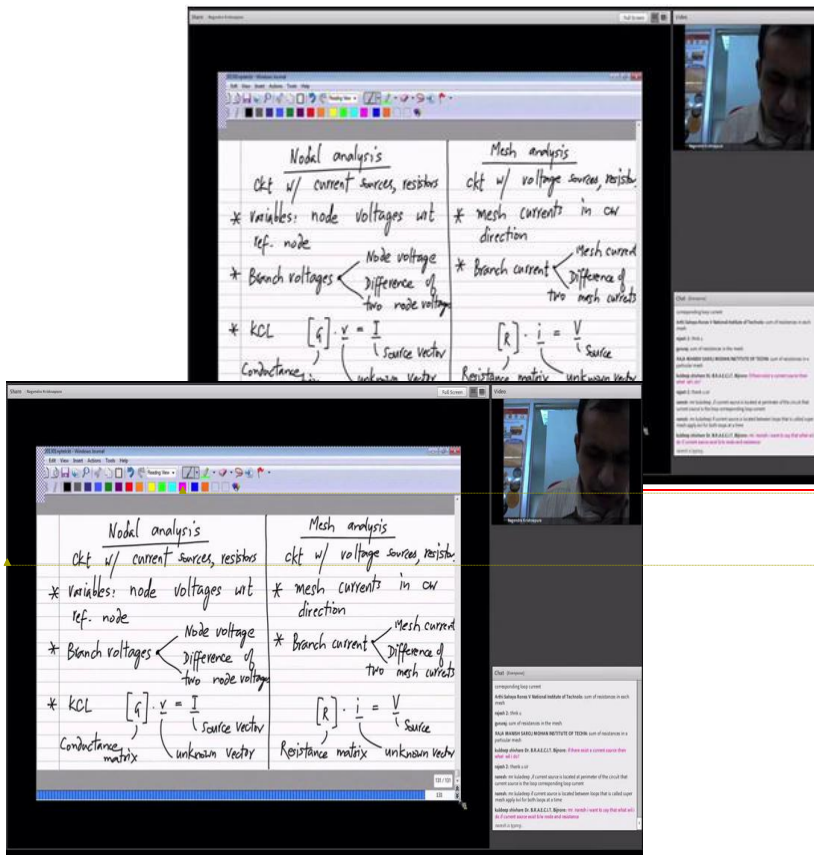
come out to be either the mesh current itself for branches along the periphery of the circuit. And for branches which are common to 2 meshes, it will be difference between 2 mesh currents then I wrote K V L in terms of these mesh currents. And finally, I grouped the coefficients for each variable i_1 i_2 i_3 and then wrote the whole equation as matrix times the vector of unknowns equals the vector of independent sources or a matrix of independent sources.

Any questions so far? Now, you can see the, you can see that this structure is analogous to what we had with Nodal analysis when we had only resistors and current sources. First of all, what are each diagonal elements, what are the diagonal elements, what are the elements of the diagonal of this matrix? Please try to answer this. So, in the diagonal in the first one I see r_1 plus r_3 , second one I see r_2 plus r_3 , and the third one r_1 plus r_2 plus r_4 . What are these? I think many of you were able to easily answer this one. It is the sum of resistances in each mesh.

Now similarly, so the diagonal elements are sum of resistances in each mesh and off diagonal elements you see that this element in the matrix notation this would be called a_{12} and that is the resistance that is common to mesh 1 and mesh 2. So, this is mesh number 1, mesh number 2 and mesh number 3 and this is common to mesh 1 and mesh 2. So, the off diagonal elements are resistances that are common to meshes.

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Now, you can see the analogy with the Nodal analysis very easily so first of all, I will compare Nodal analysis of a circuit having current sources and resistors versus Mesh analysis which of a circuit with voltage sources and resistors. Now, in case of Nodal ~~analysis~~ analysis, the variables are node voltages with respect to reference node and in case of Mesh analysis the variables would be mesh currents in clockwise direction.

Now, all branch voltages will be either some node voltage or difference of 2 node voltages. A branch can be connected between some node and the reference node or between some 2

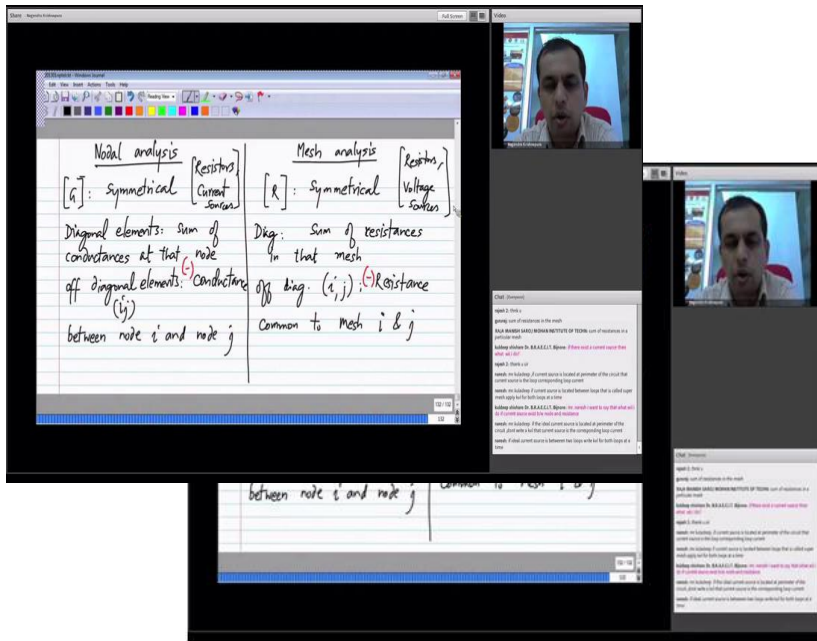
nodes. If it is connected between some node and the reference node then the branch voltage equals the node voltage, if it is connected between 2 nodes then the branch voltage is the difference of the 2 node voltages.

And in case of Mesh analysis, each branch current is either a mesh current. Now, for branches on the periphery, on the boundary of the circuit this will be the case because those branches will be part of only 1 mesh and the branch current equals the mesh current or it will be the difference of 2 mesh currents. So, for branches in the middle again by taking the analogy of the map that will be the borders between 2 states the current in those branches will be the difference between current in the left branch and the current in the right branch. So, the current in 1 will be in 1 direction and the current in the other one will be in the opposite direction and this happens because we choose all mesh currents in the clockwise direction.

And finally, if you write out the K C L equations and group the variables and so on and write it in the neat matrix form you will get conductance matrix times the vector of node voltages, so this is the vector of unknown unknowns. So, this \bar{v} with a bar undersigned denotes a vector and a vector is nothing but; a matrix but; it has only ~~one~~ column equals the source vector.

And in case of Mesh analysis, we will have the resistance matrix which consists of resistances only times the vector of unknown currents that is the mesh currents equals the source vector which consists of voltage sources in each mesh. When I say source vector obviously I mean independent source.

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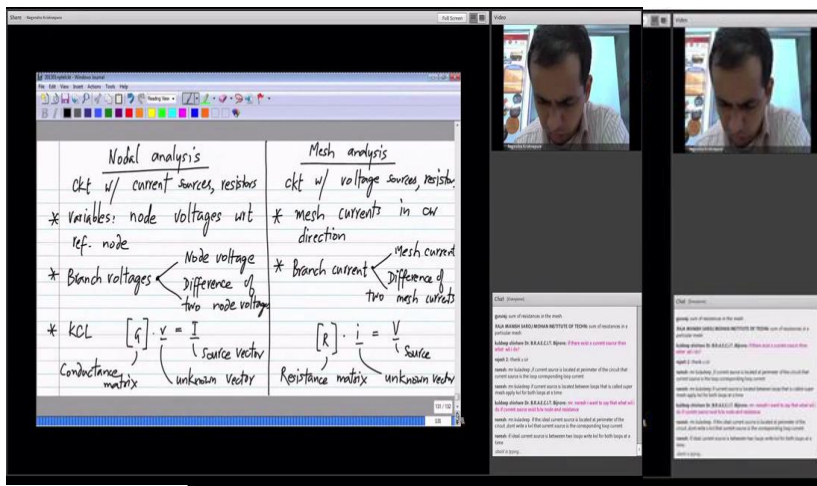
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And continuing the comparison, the conductance matrix G has first of all it is symmetrical, and it has diagonal elements which are sum of conductances and off diagonal elements, which are and when I say off diagonal, if it is element $i j$ in the matrix conductance between node i and node j . And in fact it is the negative of that. Similarly, the resistance matrix is symmetrical and the diagonal elements will be sum of when I say sum of conductances in the Nodal analysis at that node and here it will be sum of resistances in that mesh.

And off diagonal elements, the element i, j would be the resistance common to node sorry mesh i and j , in fact it is the negative of that. So, by the way all this are true when you have only resistors and current sources in Nodal analysis and resistors and voltage sources in Mesh analysis. So, this is just to point out the duality between Nodal and Mesh analysis you start off with K C L in Nodal analysis, K V L in Mesh analysis.

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And in a particular case, when you have a circuit with only current sources and resistors for Nodal analysis, and only voltage sources and resistors for Mesh analysis this is a neat structure for the equations which consists of a symmetric matrix times a vector of unknowns equals the vector of independent sources in the circuit.

And also the matrix elements can be written down by inspection in case of Nodal analysis by looking at conductances at each node or conductances between different nodes. ~~And~~ in case of Mesh analysis the total resistance in each mesh or conductances common to the resistances common to each sorry the total resistance in each mesh or the resistances common to 2 meshes. Any questions so far?

There are some questions about including current source in Mesh analysis I will discuss that shortly. So now, I think the simple case is pretty clear we will just take a numerical example just to get some practice or rather we will reserve the example for later let us have Mesh analysis including a current source.

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Mesh analysis with a current source:

unknown voltage across the current source

(Nodal analysis: unknown current through the voltage source)

Extra variable

Extra equation

$$\text{M1: } R_1(i_1 - i_2) + V_x = V_1$$

$$\text{M2: } R_2(i_2 - i_1) - V_x = -V_2$$

$$i_1 - i_2 = I_0$$

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This is the circuit I had earlier and let us say instead of this resistor r_3 , I have a current source i_1 . Let us say this is the case now as usual for Mesh analysis we have to write K V L, now what is the complication here? I would like to have answers from the participants I want to write I want to go ahead with Mesh analysis for this circuit and I initially took circuit with only resistors and voltage sources.

Now, I have added a current source what is the problem? What is the complication with going ahead with Mesh analysis for this circuit? So, I hope the question is clear now we have already done this, we have addressed the similar question while doing Nodal analysis which includes voltage sources. Nodal analysis we write K C L equations and K C L says sum of currents at every node equals 0.

Now, when you have voltage sources when you have resistors the current is related to the voltage across the resistor when you have voltage sources the current is not related like that any current can flow through the voltage source. That is why we were not able to write the K C L properly because we do not know the current through the voltage source.

Now similarly, my question is if you have Mesh analysis and if you have current sources what exactly is the complication. Again a couple of people have raised their hands ~~but, but~~ today the audio set up is such that I will not be able to hear your questions so all your questions have to be through the chat window.

So, the problem is the following so while writing K V L for this mesh we have to say that voltage across r_1 plus the voltage across the current source equals the voltage across the voltage source ~~but, but~~ a current source can have any voltage across it. For a resistor the voltage across the resistor is the current through the resistor times the resistance value.

So, we can express the voltage source voltage across the resistor in terms of a current through the resistor ~~but, but~~ as a current source by definition can support any voltage across itself. ~~S~~o we cannot express the voltage across the current source as something related to the current value. So, the problem is unknown voltage across the current source, so if you recall when you had Nodal analysis and voltages you had the complication, because of the unknown current through the voltage source.

So, these are the this is the issue now how do we work around this? What did we do in case of Nodal analysis when we had the Nodal analysis with voltage sources what was what was

the thing that we did? How did we get around the problem? So, my question is when we had Nodal analysis with voltage sources what did we do? We do not know the current through it so what did we do? So, what we did was to define an auxiliary variable for the current through the voltage source, now that gave us an extra variable and also an extra equation because of the voltage source. And we were able to write down the equation and also for hand analysis we were able to combine 2 of those equations effectively giving us a super node.

So, here the procedure we follow is exactly the same what we do not know is the voltage across this current source let me call that v_x . So, if I write the KVL equation for mesh number 1, I will have the voltage drop across r_1 which is $r_1 i_1$ minus i_3 because this is common to mesh number 1 and mesh number 3. This is mesh number 3 and here I have mesh number 2, so this is the voltage drop across r_1 plus v_x equals v_1 this is for mesh number 1 and for mesh number 2 we will have $r_2 i_2$ minus i_3 minus v_x equals minus v_2 .

Once v_x is defined with this polarity when you go around the second mesh you will have minus v_x . So, if you look at the drops going clockwise you will have current in r_2 times r_2 minus v_2 minus v_x equal 0 and v_2 is moved to the right hand side sorry plus v_2 minus v_x equal 0, and v_2 is moved to the right hand side. So now, what we have is an extra variable. Now to solve for the extra variable we need an extra equation by the way the equation for mesh number 3 remains exactly as it was before because mesh number 3 is not modified in any way by adding this current source. So, where do we get the extra equation from my question is?

By defining the voltage across the current source to be v_x we have got an extra variable in our set of equations to solve for the extra variable we also need an extra equation. So, where is the extra equation? So, let me call this like I did before this current I will call i_1 and this current I will call i_2 and let me just change this to i_0 just. So, that i_2 do not have 2 i_1 s in the circuit and as Aarti answered this i_1 minus i_2 against this branch current is known that defined by the current source and that will be equal to i_{naught} .

Now, we did exactly this when we had Nodal analysis when a voltage source was connected between node 3 and node 4 let us said we said v_3 minus v_4 equals voltage source value so this was the extra equation. So, the extra equation is nothing ~~but, but~~ the definition of the

current source. So, if I put all of these things together what I will get and I will write it directly in the matrix form.

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The screenshot shows a handwritten matrix equation for mesh analysis. The matrix is:

$$\begin{bmatrix} R_1 & 0 & -R_1 & +1 \\ 0 & R_2 & -R_2 & -1 \\ -R_1 & -R_2 & R_1+R_2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ 0 \\ 0 \end{bmatrix}$$

Below the matrix equation, the text reads: $[R] \cdot \underline{i} = \underline{V}$

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I will have mesh currents and the voltage across the current source as the variables. and for the second mesh we will have and for the third mesh we will have. And the last one will be just the definition of sorry here I would also add plus v x and in the second one minus v x, so that is what we had. I wrote the last entries incorrectly I have plus v x here and minus v x over there and also finally, I will have i 1 minus i 2 equals i naught so that means 1 minus 1 that gives me i 1 minus i 2 and that is equal to i naught. So, in my variable vector I have this

extra stuff and my independent source vector consists of voltage sources and the current source.

And this matrix which I will continue to call the resistance matrix has resistances as well as some dimensionless quantity. So, the set up is still in terms of resistance times the variable vector equals the source vector. Variable current vector equals the source vector ~~but~~, but this variable vector also consists of this auxiliary variable, which is the voltage across the current source and similarly, the right hand side vector consists of independent sources which consists of voltage sources as well as current sources.

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The screenshot shows a video lecture with a central window displaying handwritten notes and a circuit diagram. The notes are as follows:

Mesh analysis with a current source:

Unknown voltage across the current source
 (Nodal analysis: unknown current through the voltage source)

Extra variable

Extra equation

The circuit diagram shows two meshes. Mesh 1 contains a voltage source V_1 and a resistor R_1 . Mesh 2 contains a resistor R_2 and a current source I_0 . The voltage across the current source is labeled V_x . The mesh currents are i_1 and i_2 .

The equations shown are:

$$+I_1: R_1(i_1 - i_2) + V_x = V_1$$

$$+I_2: R_2(i_2 - i_1) - V_x = -V_2$$

The current source is labeled I_0 and the voltage across it is V_x .

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Now, while doing hand analysis this increased number of equations is not a good thing you would like to reduce the number of equations. So, then the answer is very clear again we followed all the steps while doing the Nodal analysis with voltage sources, so I will go through it a little quickly if I sum these to K V L around mesh number 1 and K V L around mesh number 2 what happens is this v_x cancels out.

And this is not a coincidence this will always happen because v_x is defined to be across some branch so on in 1 of the meshes which contains that branch it will appear as plus v_x and in the other mesh it will appear as minus v_x .

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with a

$$R(i_1 - i_3) + R_2(i_2 - i_3) = V_1 - V_2$$

KVL equation around the Super-mesh

Total voltage in the Super mesh

\equiv Dual $\left[\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \right] \left[\begin{matrix} i_1 \\ i_2 \\ i_3 \end{matrix} \right] \left[\begin{matrix} i_1 - i_2 = I_0 \end{matrix} \right]$

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So, it will always cancel out and if I add these 2 equations what I will get is I will have the equation as, let us say this is i_1 i_2 i_3 will have the voltage drop across r_1 , which is $r_1 i_1$ minus i_3 plus I have that plus v_x equals v_1 , but; that v_x cancels with minus v_x and here I have r_2 times i_2 minus i_3 minus v_x which cancels out equals v_1 minus v_2 . And if you observe originally I had the equation K V L equation around this loop of this mesh and the K V L equation around this mesh. So if I add the 2 essentially what I have is a single K V L equation around this bigger mesh which encloses a current source. This is nothing ~~but, but~~ K V L equation around the, this bigger mesh which is called as the super mesh in analogy to super node.

So, if a branch that is common to 2 meshes is a current source what you do is you avoid that branch and you write the equation around a super mesh which encloses that current source. When you do that the current the voltage across the current source will not be in the picture and you will get this K V L equation in terms of all the voltage drops. So, for instances is the voltage drop across r_1 this is the voltage drop across r_2 and on the right hand side you have the total voltage in the super mesh.

This is clear, any questions? This is exactly analogous to what we do with Nodal analysis and voltage sources there we define a super node, which comes out because of defining an auxiliary variable and cancelling out by adding 2 equations. Here we define the voltage across current source as the auxiliary variable, add up the equations for the 2 meshes that auxiliary variable cancels out and you will be left with a single K V L equation around the super mesh.

Now, there is a question about non ideal current sources now we have not discussed non ideal current sources ~~but, but~~ maybe we can quickly do that see as far as the analysis is concerned a non ideal current source will be the same as an ideal current source in parallel with some resistance. So, for instance in this case we would have an extra resistance now if you have that then in case of Mesh analysis you will have an extra mesh and you have to write that. So, when I write a current source like this with this symbol it by definition means an ideal current source. If you have to depict a non ideal current source what you have to do is to have an ideal current source in parallel with a resistor.

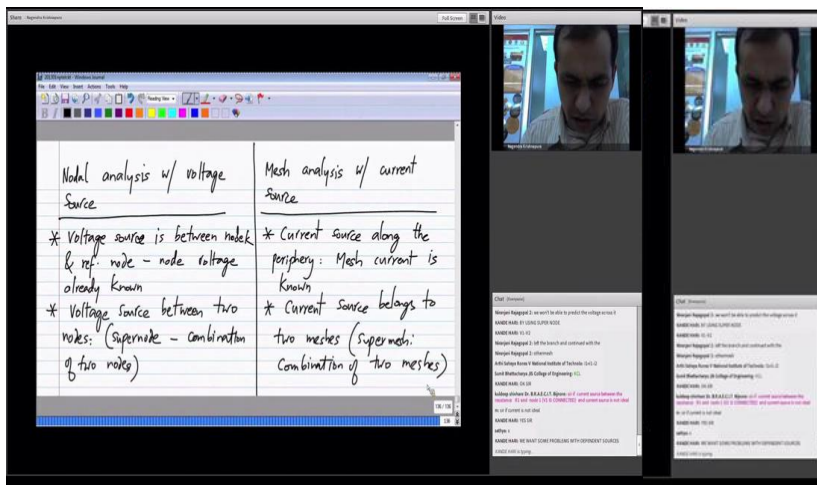
Obviously, now we have increased the number of components in the circuit in case of Nodal analysis the number of nodes remains exactly the same, but; you have an extra element. And

in case of Mesh analysis if you have a resistance across the current source, you will have an extra mesh and you will have to solve for it that is all. So whatever we said so far will apply to this the only thing is if you know that the current source is non ideal you have to model it with 2 components an ideal current source and a resistor.

So, I hope that answers the question. So now, when you write the equation around the super mesh you will have 1 less equation, because for 2 meshes you wrote a single equation and ~~but, but~~ you do still have 3 variables i_1 , i_2 and i_3 . ~~So,~~ you ~~last one~~ ~~st~~ ~~+~~ equation because you defined the combination of 2 meshes as a super mesh and wrote a single equation for these 2 meshes together ~~but, but~~ you will get an extra equation which is the same as before which is the definition of the current source.

So, I hope this is clear. So now, we can again compare Nodal analysis and Mesh analysis, but, when you have an extra when you have a voltage source in Nodal analysis.

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So, when you have Nodal analysis with voltage source first of all if the voltage source is between a certain node, I will call it node k and reference node you already know that node voltage. So, you do not have to write an equation for that and you can write the equation for the rest of the nodes. Similarly, if there is a current source along the periphery then the mesh current is known.

So, again you can remove this variable from the equation and use only the remaining mesh current. So, these are for hand analysis usually for computer analysis what you do is you do not have a different things that you do for different cases. You would like to have a uniform algorithm that covers all the cases so in that case what you do is that you always when you have a voltage source in Nodal analysis you define the current through the voltage source as an auxiliary variable. And then go ahead with it you do not worry about whether the voltage source is between some node and the reference node or between 2 nodes etcetera, but, while doing hand analysis you reduce the number of equations your job only becomes easier so that is why you do this.

And if you have voltage source between 2 nodes you define a super node which is the combination of the 2 nodes, and if you have a current source common to 2 meshes and it is on the periphery it belongs only to 1 mesh, when it belongs to 2 meshes, then you define a super mesh which is say a combination of 2 meshes, so again the analogy between Nodal analysis with voltage source and Mesh analysis with current source is very clear. So you should be able to choose either 1.

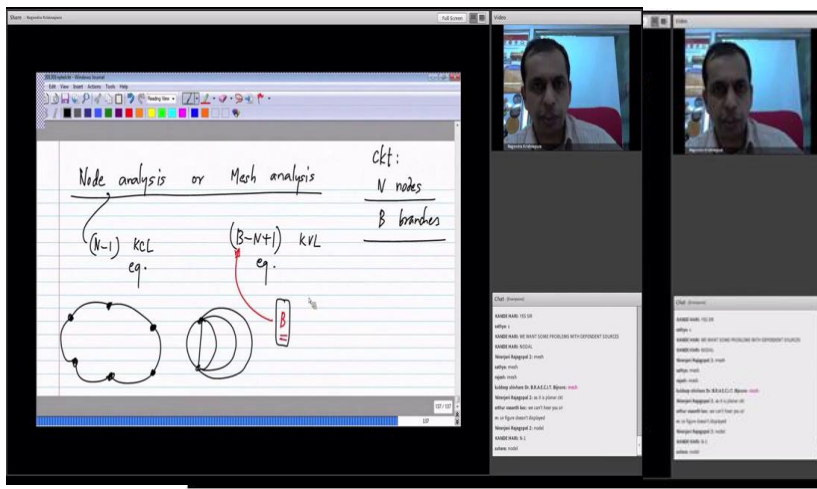
Now, both methods will work for any circuit, for now let us assume that all our circuits are planar circuits, Mesh analysis we know is applicable only to planar circuits. Now my

question is let us say you have a planar circuits which one+ will you choose will you choose Nodal analysis or Mesh analysis I would like opinions of the participants. I opened a poll, you can register your vote for either Mesh analysis or node analysis.

Now, this is a actually I think you can now see the results, so most of you have chosen Mesh analysis and I am a little surprised I would like to hear from you those of you who chose Mesh analysis why you would choose Mesh analysis over node analysis. What is the reason for choosing Mesh analysis? Out of eleven people who have answered nine people have chosen Mesh analysis what is the reason?

Now of course, we are talking only about planar circuits because Mesh analysis the way we defined it is not even applicable to non planar circuits. Now, let us see what would be the basis of choice. Now for node analysis first of all like I said when we do Nodal or Mesh analysis we first write for node analysis write K C L equations and get the node voltages in case of Mesh analysis we get the mesh currents.

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After that we still have some work to do ~~but, but~~ that is rather trivial. In case of node analysis; to find branch voltages you have to take difference between node voltages that you have calculated, and in case of Mesh analysis to find branch currents you have to find differences between mesh currents that you have calculated.

These things are quite easy and I'm assuming it can be done quite easily so the moment you solve the Nodal analysis equations we call the circuit to be solved although we know that a little bit more work is required. Similarly, when you do Mesh analysis you solve for mesh currents and after that there is little bit of more work ~~but, but~~ that is relatively easy so that is solved so the choice is based on number of equations you have to solve.

For node analysis you have $n - 1$ equations so let us take a circuit with n nodes and b branches. So, $n - 1$ KCL equations for Nodal analysis and for Mesh analysis it is $b - n + 1$ KVL equations. So, this is what we need to solve now which of these is likely to be more is $n - 1$ likely to be more or $b - n + 1$ likely to be more.

So, which of these will have more equations, so node analysis has $n - 1$ equations for circuit with n nodes and b branches, Mesh analysis has $b - n + 1$ equations so which is likely to be more. Obviously, if you have to solve you will choose the 1 with smaller number of equations. Again many of you have said Nodal it depends on number of branches it very much depends on b right and also it depends on the circuit for instance I could have a circuit like this.

Now, these are these dots are nodes and the rest are branches you can see that there is a single loop, but, so many different nodes. Alternatively, I could also have things like this I have only 2 nodes ~~but, but~~ so many loops now these are weird cases obviously in this case on the right hand side when you have only 2 nodes you would use Nodal analysis. ~~A~~ and in this case you would use Mesh analysis, there is a single loop ~~but, but~~ what happens in a general case is that this $b - n + 1$. Now when you have $n - 1$ when you have n nodes right it is possible that every node is connected to every other node through some branch.

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The screenshot shows a video conference interface. On the left, a whiteboard displays the following handwritten text:
N node circuit:
If every node is connected to every other node,
how many branches in total?
Below the whiteboard, there are two video thumbnails of a man speaking. At the bottom of the interface, there are two chat windows with text messages.

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Now, this is not very likely, but it is very much possible now if every node is connected to every other node through a branch how many branches will we have? Please try and answer this that is I have n nodes circuits. Now how many branches are there depend very much on the circuit, but obviously the maximum is that every node is connected to every other node that is possible right.

So, in that case how many branches in total will we have? So, I got 1 answer which said the factorial n and now that is not correct so you please think about it. Now, we will stop here and we will continue with this in the next lecture -but, but- in the meanwhile you please think about this if every node is connected to every other node in the circuit how many branches will we have in total? So, what we have done today is to discuss Mesh analysis, which is an analysis based on K V L for planar circuits and we also made an analogy with Nodal analysis.

So, it should be quite easy to understand we also looked at the exceptional case which is Mesh analysis with the current sources. It is analogous to Nodal analysis with voltage sources. So, hopefully you have understood it all then if you have not understood it then please raise your questions in the beginning of the next class and we will discuss them.

It is actually a good point here that if n is greater than 4 or 5 does not the circuit become non planar. It is true it should become non planar let us not worry about node versus Mesh analysis we can talk about node versus loop analysis. Loop analysis can be done even for non planar circuits, I did not discuss that because Mesh analysis gives you a neat structure and that is what is usually in most textbooks, so that is why I did not discuss that but, even a non planar circuit can be analysed with loop analysis and the number of equations you will have to write will be still b minus n plus 1.

So, please think about for this if every node is connected to every other node how many branches in total and also please think about everything we have discussed with Nodal and Mesh analysis if there are any difficulties we will start with that in the next lecture.

-Thank you, I will see you on Thursday.