Basic Electrical Circuits
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Lecture - 7
Nodal Analysis with Voltage sources and Controlled Sources
Brief Introduction to Modified Nodal Analysis
Use of Super node to solve Circuits with Voltage Sources
Superposition Theorem

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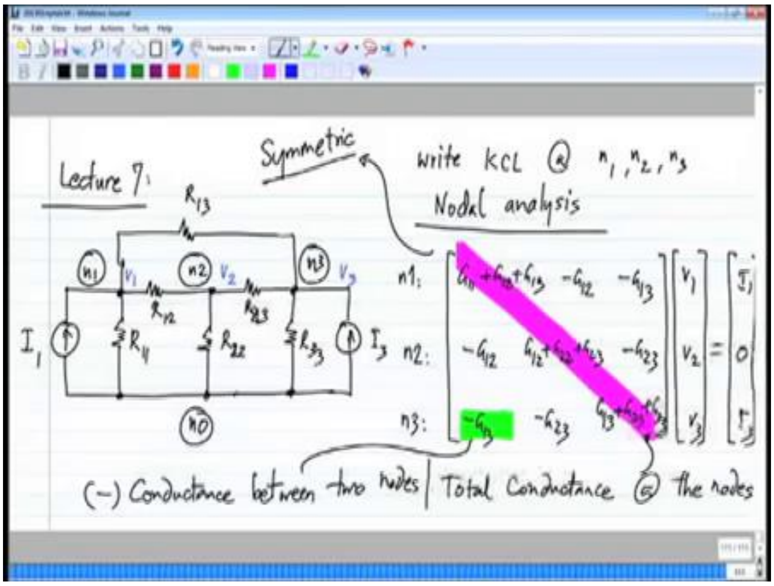

So, let me use the same circuit that I was using last time which was the circuit with the number of resistors and a current source. I call this node n 0, n 1, n 2, and n 3. And the way I would go for doing nodal analysis is I write, I make n 0 in the reference mode, you could pick anyone. In this case, I have picked n 0, and I write down the voltages of each node with respect to n 0 as V 1, V 2, V 3 and express all my relationships in terms of V 1, V 2 and V 3.

Now, so that means the voltage here is V 1, voltage here is V 2, and the voltage there is V 3. And let me call this R 11, R 22, R 33, R 12, R 23, and R 13, this I 1 and this is I 3. And in this case what I first do is, write KCL at nodes n 1, n 2, and n 3. If I do that and I group together all the variables, I group together the coefficients of each of these variables V 1, V 2, V 3. I get the equations in a certain form and I can express that in a neat matrix form, which I am going to write down now.

We saw that, the diagonal elements of these matrix basically correspond to the total conductance connected to each node. So, the first row corresponds to the KCL at n 1, second row to n 2 and third row to n 3. Now, I will use the same ordering for the variables also, that is V 1, V 2 and V 3. And the diagonal elements this element a 11 of the matrix corresponds to the total conductance connected to node 1 which is G 11, the reciprocal of R 11, G 12 and G 13.

Similarly, here this element 22 will consist of G 22, G 12 and G 23. And finally here in this position, we have the total conductance at this node. And the off diagonal elements, this entry a 12 for instance is the negative of the conductance between node 1 and node 2. So, this is minus G 12 and it is a symmetric matrix this is also minus G 12. And here we have the negative of the conductance between node 1 and 3 and here also the same thing.

So, this will be minus G 13 and finally, this will be, these two will be minus G 23. And on the right hand side, we will have the current sources, and in the first one we will have the current source connected to node 1, in the direction that it pumps connected into the node. It is basically the total connect coming into node 1, that is plus I 1 and nothing is, no current source, independent current source is connected to node 2. So, this will be 0 finally, here and a current I 3 is pushing current into node n 3, so this will be I 3. So, this is the, this is what we had and this is what by solving this we get the node voltages, and this is what we call nodal, this is what is termed as nodal analysis.

Now, the circuit is not solved completely, but once you know all the node voltages, you know all the branch voltages. The branch voltages will be either the node voltage itself for instance for this branch and this branch, the branch voltage equals V 1. And if you take this branch for instance R 12 the branch voltage is V 1 minus V 2.

So, that way you can find all the branch voltages. And you can also find the branch current by using branch voltages and the element V I relationships. So, this is what is known as nodal analysis, and other side these will be conductances, total conductance at the nodes and these, the off diagonal forms will be negative of the conductance between two nodes. So, that is all it is there to it, and we get a nice symmetric matrix.

Now, the only restriction is, in this case we have taken a circuit which has resistors and current sources only, we do not have other types of independent sources, that is a voltage source, or we do not have dependent sources yet, we will add them as we go now. So, that is a quick review of what we did in the previous class. Any questions? Any confusion regarding that?

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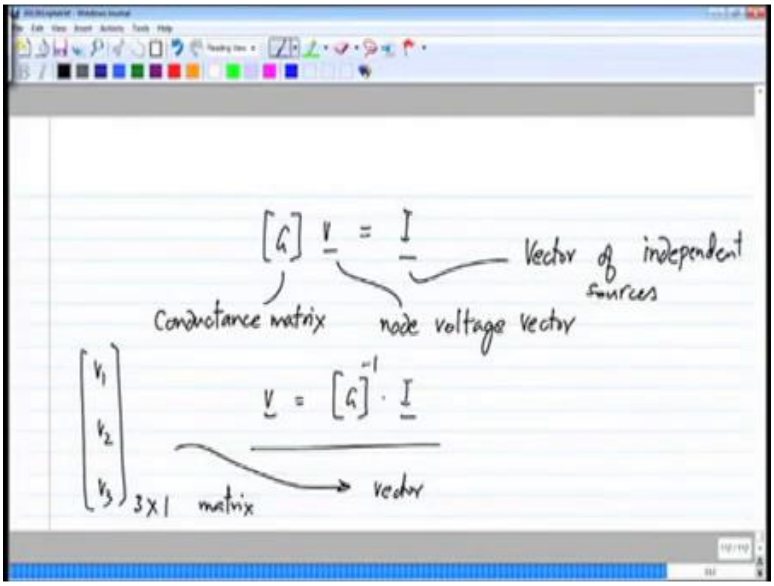

And we have this equation in the form, the conductance matrix times the node voltage vector equals the vector of independent current sources. And this bar below the letter means that it is a vector. And finally, this is the vector of independent sources. And to solve for this, we have to invert the matrix and multiply by the independent sources. Right now I will not worry about solving this, we will see examples of this later, but there are ways to invert the matrix if it is a 2 by 2 matrix or 3 by 3 matrixes, you can do it by hand, beyond that you usually do it using a computer. But the point here is, there is a systematic method of calculating all the node voltages. Your first set of all the node equations and then, there is procedure whereby you will get all of the node voltages.

There is question, why did I call this is a matrix and this is a vector? if you have a matrix with single column, that is usually called vector, a column single column matrix is usually called a vector. So, we have V 1, V 2, V 3, which is you can call it a 3 by 1 matrix, but it is common to call it a vector. So, like I said, I will not worry about solving this right now, it is assume that we will be somehow able to solve it. So, what I will do now, is to add other kinds of elements which we have not added before. So, let me take the same circuit and modify it slightly.

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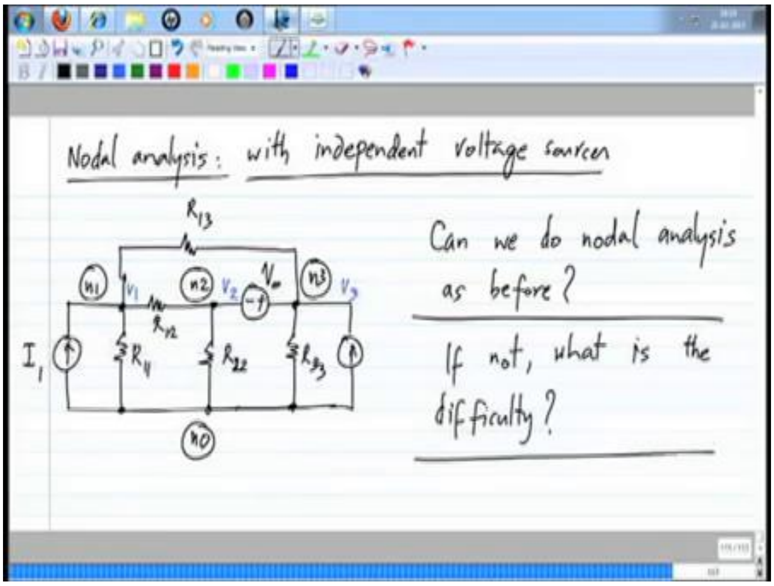

Now, let me remove this resistor R 23 and connect an independent voltage source, and I will call this V naught. So, we want to do nodal analysis, similar analysis as we did before, but with independent voltage sources. So, what I did was, I had a resistor here, I could have put a voltage source anywhere even in parallel with it, but I have just removed that resistor and connected this voltage source. Now my question is, can we go ahead exactly as we did before, if not what is the difficulty with this? The network connection broken and something happened. As some of you recognized the problem is with, the problem is with writing KCL here and there, where the voltage is connected.

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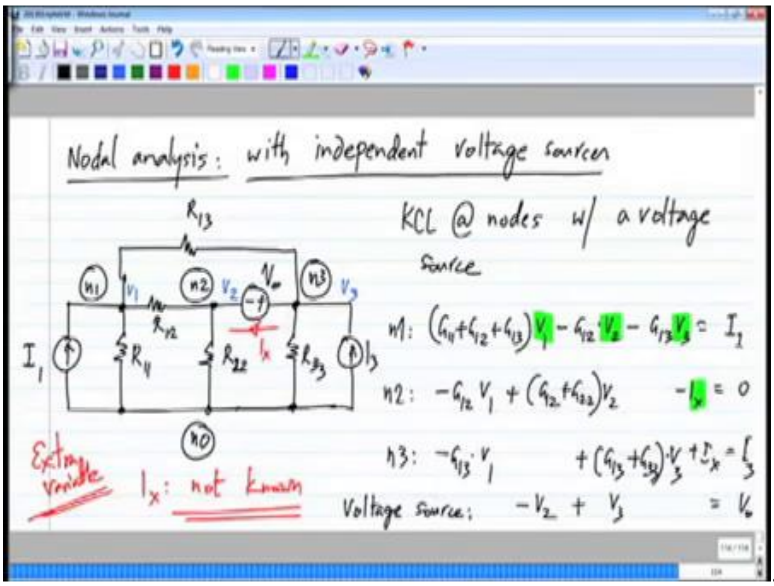

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KCL is still valid, the current going here, plus the current going there, plus the current going there is 0, but the problem is earlier, when we had a resistor we could express the current as this conductance times the voltage difference V 2 minus V 3. Now we do not know what the current is through a voltage source, by definition it will support any value of current.

So, that is the problem in doing nodal analysis, when you have independent voltage sources. So, now what do we have to do? So, we have an extra unknown, let me call that I x and I will use the passive sign convection as always, I will show the current through the voltage source as going from plus to minus. That is as we said this I x is not known, but I can still write the KCL at these 3 equations, if I denote this is I x, and that is what I will do.

So, at n 1, I will write n 1 the equation is exactly the same as before. So, I will write it straight away, at n 2 what we have are the current flowing out, that is V 2 times G 22 V 2 minus V 1 times G 12 and minus I x, I x is flowing in so, minus I x is flowing out. And similarly, at n 3, what we have? We have minus G 13 times V 1 and plus I x, because I x is flowing out of the node and plus G 13 plus G 33 times V 3 equals I 3 so, we have an extra variable I x .

So, we have the first equation is the same as before, and the second one and the third one have this extra variable I x. Now, I totally have four variables V 1, V 2, V 3, and I x and only 3 equations. So, how do I go about solving this? Is there any equation that I am missing? This is the question for the participants. I mean I have now four variables and then only three equations, to solve for four variables, I need four equations right, four independent equations. So, where will I get the extra equation from? So, clearly so, far I have not described the voltage source itself, and that gives me the extra equation.

So, with this difference V 3 minus V 2 will be equal to V 0. Let me write it with the same order minus V 2 plus V 3 equals V 0. Some of you gave the answer that V 2 minus V 3 is V 0 no it is V 3 minus V 2 equals V 0 with the polarity. So, now I have four equations, that is a KCL at the 3 nodes plus the equation for the voltage source, and this is the system of four equations and four unknowns, and I will be able to solve this one. And I will write this in terms of the matrix. So, what I have are.

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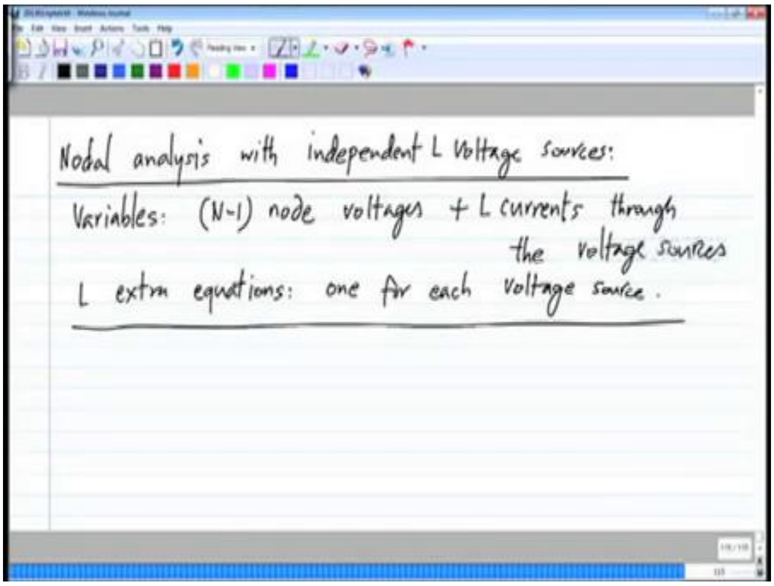

For the case of nodal analysis with independent voltage source. I have the variables to be the node voltages. in general n minus 1 node voltages plus current through the voltage sources. In my circuit, I have a single voltage source, but let us say if I have L voltage sources, I will have L currents through the L voltage sources. So, the number of variables increases by L and the number of equations also increases by L because each voltage source provides a constraint. Now, I will write out the equations in matrix form including the extra variable.

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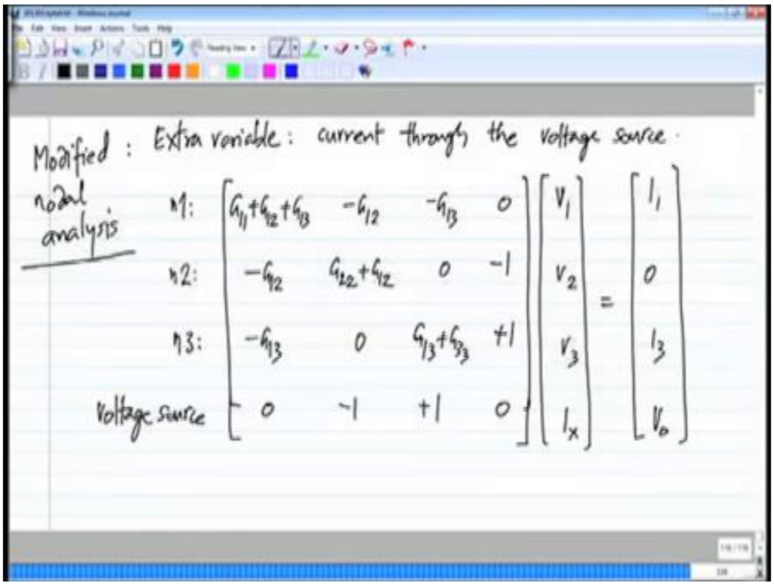

I x this will be G 11 plus G 12 plus G 13 and minus G 12 minus G 13, and if you look at the variable vector, it will be the node voltage V 1, V 2, V 3, and I x. And in the first equation for node n 1, the variable I x does not appear, so the coefficient here is 0. And in this case I will have minus G 12 and here G 22 plus G 12, and here I do not have anything and minus 1 because I have minus I x. This is the KCL equation at node n 2, and at node n 3 I have minus G 13 nothing for V 2 and G 13 plus G 33 and I have plus 1 because plus I x appears in the equation. And I have the constraint for the voltage source, which is that V 3 minus V 2 that is minus V 2 plus V 3 and these things do not appear equal I 1, 0, I 3, these are the currents into node 1 and node 3. And the last equation is minus V 2 plus V 3 equals V 0.

So, this the, this is the nodal analysis expressed in matrix form, and it has been modified to include an auxiliary variable. And that auxiliary variable is the current through an independent voltage source, and this scheme is known as modified nodal analysis. So, whenever you have independent voltage sources, you need to have the current through the voltage source as extra variables, and then you will be able to again set up the equation systematically and solve for that. Any questions about this?

So, the modification includes basically extra variable, which is the current through the voltage source. So, any questions about this, how to deal with, how to deal with an independent voltage source in the circuit? When we have an independent voltage source, we define an auxiliary current, which is the current through the voltage source so, that gives us an extra variable. We get an extra equation from the equation for the voltage source.

Now, obviously if you have many voltage sources the size of the vector will grow, but I have to emphasize here, that the main point of doing this that is being able to write the equation systematically is so, that you can solve it even for large circuits. It is not intended, that you invert this matrix by hand, it is only through a computer. I mean for any circuit of any reasonable size, you will need a computer to do that, but you have to set up your program so, that it looks at the circuit and sets of the equations or this matrix equation correctly. And there are routines for inverting the matrix that you can utilize to solve this.

Now, this matrix, I will still continue to call it conductance matrix, but not all the elements are conductances clearly, the ones in the last row and last column they are not conductances. Now, on the right side we have the vector of independent sources, it is not just independent current sources. Now, we have independent current and voltage sources, independent current and voltage sources. But otherwise the structure exactly the same as before, which is the matrix G, as I said I still continue to call it G, but not all elements in this are conductances.

These are all conductances, but these are basically dimensionless, the last row and last column. G times V that originally was the vector of node voltages, but now it also includes a current. And finally, equals I and again this is vector of independent variables, it has current sources as well as voltage source. So, now I do not want to introduce a new notation so, I am going to keep it as it is.

Now, like I said, this is good for analyzing things for the computer because the procedure is very systematic right. For every voltage source you simply add a new variable, which is the current through the voltage source and also solve for it, and you add equations corresponding to each voltage source. Now, when you are solving it by hand, you would not want to increase the number of equations like this, generally you solve small circuits by hand. And then, going from a 3 by 3 matrix to a four by four matrix is tremendously complicated when you are doing hand calculations. But the computer can handle matrices with thousands of rows and columns quite easily. So, now for hand analysis, the nodal analysis is done with very slight modification. So, let me put down these equations again.

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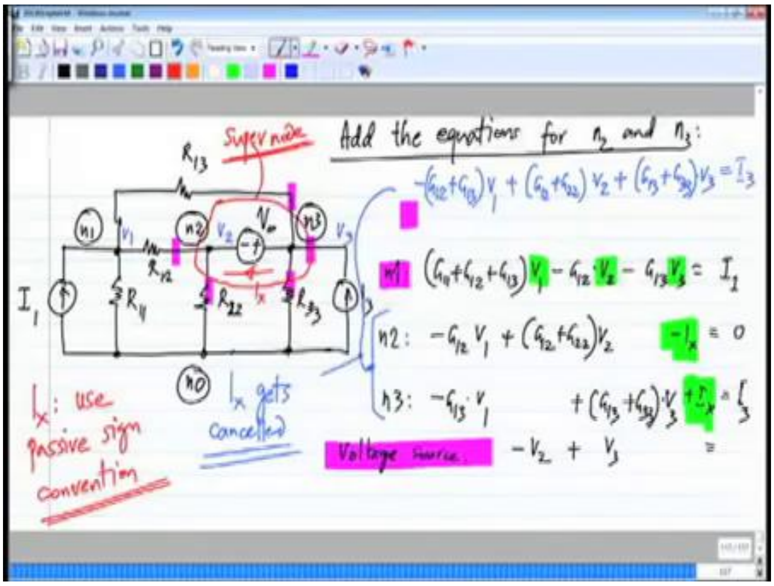

So, we introduced this extra variable I x right. Now, what we do is for instance let us write equations like this, and the voltage source is connected between n 21 and 3. So, I add up the equations, KCL equations for n 2 and n 3. That is I take this equation here plus that equation there, and I add them up to form a new equation, and we get something. The main point is that, this now includes the left hand side of each equation is the total current flowing out of the nodes, that is current flowing out of out of each node in every branch connected to that node.

So, when I add up these two, what it means is, it is the sum of all the current flowing out of n 2 and n 3 together. So, let me do that, I will get minus G 12 plus G 13 times V 1 plus G 12 plus G 22 times V 2 plus G 13 plus G 33 times V 3 to be equal to I 3. So, the important point is, when I add this plus I x here so, the minus I x here cancels with plus I x there. Because I have connected, I have taken the 2 nodes at between which the voltage source is connected, one of them will have plus I x, the other will have minus I x so, they will cancel. So, that is the important part.

So, what does that mean now, this the left hand side here is the total current flowing out of n 2 and n 3, and any current if it is flowing between n 2 and n 3 will get cancelled out because it will have plus in one contribution and minus in the other contribution. So, essentially what we are doing is forming what is known as a super node, that is a combination of multiple nodes, in this case V 2 and V 3 we always combine the two nodes, across which the voltage source is connected. And right KCL in terms of all the currents flowing out of that node, for instance we add that current that is flowing out of the super node.

This current that is flowing out of the super node, this current that is flowing out of the super node a current here and the current there and the current there. So, we write it as, the total current flowing out of the super node to be equal to the total amount of independent current source currents being pumped into that node. Now, you can see that, the current in the voltage source is not flowing out of this super node or into the super node. So, it is not cutting the super node so, it disappears from the picture. Now, this is a technique to reduce the number of equations and not have this auxiliary variable.

So, typically when you do hand analysis, you would use this and by introducing a super node you will end up with cancelling I x and you will have only 1 equation. And we have this extra equation, that is we have still lost one equation, originally we had three for the 3 nodes because of the voltage source and combining these two nodes, we have a single super node equation. So, we have two this is one, and that is the other one.

But we do have the additional constraint imposed by the voltage source. So, finally we have 3 equations and 3 variables. And this is the one fewer than what we did using modified nodal analysis. So, for hand analysis, when you have voltage sources, you introduce super nodes and go ahead with the analysis. Now one very important thing is that, when I say I combine these two into a super node, I am not shorting this to that, I still retain this voltage as V 2 and this 1 as V 3. So, all the currents must be expressed appropriately, that is the current in R 1 2 will be V 2 minus V 1 times G 12, R 22 will be V 2 times G 22 whereas, R 33 would be V 3 times G 33 and R 13 would be V 3 minus V 1 times G 33.

So, the voltages should not be changed it is not like we’re shorting them it is just that we are writing the KCL for the combination of the nodes. Any questions about this? There was a question about, how do we choose the direction of I x? Now, in principle it does not matter because it will always contribute plus I x to one side and minus I x to the other side, but as always use passive sign convention. So, if the voltage source is positive on this side and negative on the other side. That is the voltage source symbol is drawn with the voltage is defined with positive on the right side and negative on the left side, you choose I x also to be flowing from right to left.

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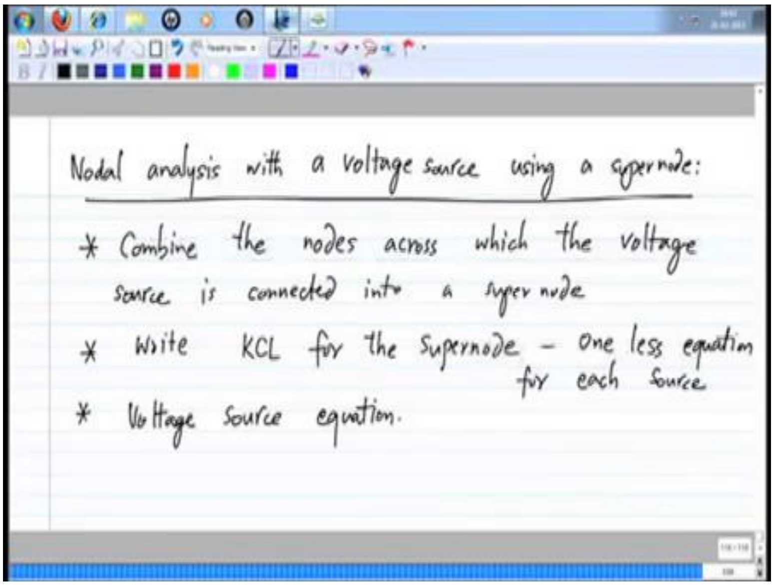

And finally you have the voltage source equation. So, then you can again invert the matrix and get the result. Now, when I say invert the matrix, that is how you have to set it up and solve on a computer. When you are doing hand analysis obviously you will eliminate variables or eliminate variables 1 by 1 sort of by trial and error and so on. But whatever it is if you solve this effectively you have inverted the matrix so, that is what I mean. Now, next we take other components, which we have not so far not included, that is some types of controlled sources. So, let me again have the same, exact same circuit as before, I will use this without the independent voltage source, we can also include that one if necessary.

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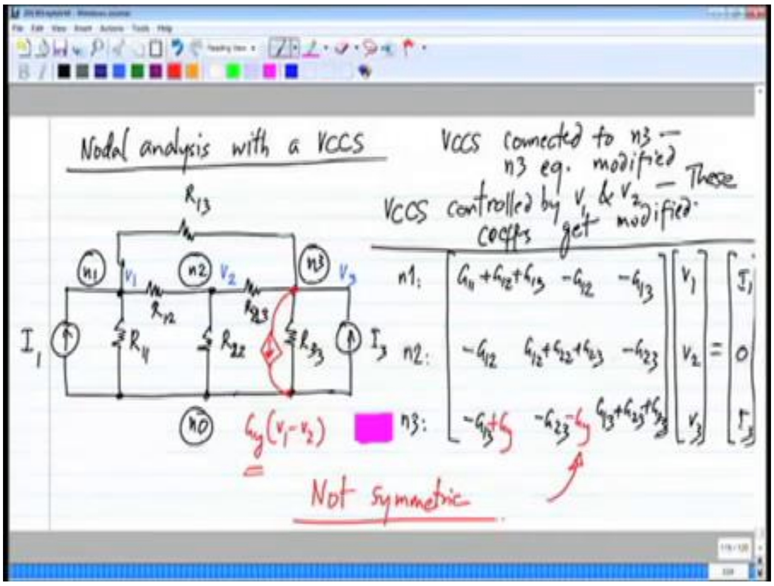

So, this is the resistor R 1 3 right. So, let me take this, I have my original circuit and also the matrix that is set up. So, this is the original step that I had. Now, to this what I will do is, I will add a voltage control current source. And let me say, I add it here, and this current source equals some G y times V 1 minus V 2. So, this is my new circuit, now what I would like to know is. First of all which equations get modified, which rows get modified, I will set up the question.

Yes, Raj please go ahead. The question is, which of the node equations get modified, only for node 1, only for node 2, only node 3, or node 1 and 2 or node 2 and 3 or something else? It is clear that this current source is added to node 3. Now, if it falls between two nodes then, 2 node voltage, 2 node equations would get modified. In this case this extra current source that I added is added only to node 3 so, only n 3 will get modified.

Now the next question is, how will it get modified? Where will this G y appear? Is it in column 1 column 2 or column 3? Column 1 corresponds to coefficients of V 1, column 2 to coefficient of V 2, and column 3 to coefficient of V 3. So, which of the columns get modified because of this controlled source? Is it column a that is coefficient of V 1, column b coefficient of V 2, or column c coefficient of V 3 or is it something else.

So, the question is because of this voltage controlled current source, which of these columns will get modified, is it this one, is it this one or is it that one? a b or c? or is it more than one? Many of you answered that it is the third column that will get, that will get modified, but that is not correct. I think one of you said that, it is these two, that is coefficients of V 1 and V 2, that will get modified and that is correct. Because you do add it to the third row because it belongs to the Kirchhoff's law for node n 3, but if you look at this, it is G y times V 1 minus V 2.

So, the value of the current is related to the voltages here and those are the ones that will get modified. Because if you look at this current, if it was the resistor the current in this would be related to V 3, it would be V 3 times G 33 whereas, this control current source, it the current is G y times V 1 minus V 2 it is related to voltages somewhere else .

So, it is this basically coefficients of V 1 and V 2 that will get modified. I hope that is clear, if there is any questions I will take them? So, the point is that VCCS connected to n 3. So, n 3 equation modified and the voltage current control source is controlled by V 1 and V 2. So, these coefficients get modified and how will that be modified? So, first my question is what happens to this coefficient? Minus G 1 3, originally the coefficient of V 1 in this equation is minus G 13, how will it get modified? What happens to minus G 13?

So, the coefficient of this one will get modified, and this one also. So, how will this get modified? So, one of you answered this, basically the current drawn out of this is G y times V 1 minus G y times V 2. So, because plus G y times V 1 is flowing out of this node because of the controlled current source, this minus G 13 will be modified to minus G 13 plus G y.

Now similarly, the current drawn out has minus G y times V 2. So, that minus G y gets added to this one, and this will be minus G y. Now, this is how you can handle voltage controlled current sources, you do not need any extra variables or equations. Because this voltage controlled current source simply adds currents to one of the nodes, or may be two nodes. In this case, the voltage controlled current source is connected between node n 3 and the reference node, but it could be connected let us say between node 2 and node 3 also.

So, wherever it is connected, it will modify those equations, and which of the coefficients will get modified, that depends on the controlling voltage. In this case, the controlling voltage is V 1 minus V 2, so the coefficients of V 1 and V 2 will get modified. And the signs you choose appropriately, anything on the left hand side is some of current flowing away from the node.

So, G y times V 1 is flowing away from the node. So, plus G y gets added to the coefficient of V 1 minus G y times V 2 is flowing away from the node, so minus G y gets added to the coefficient of V 2. Is this clear, are there any questions? So, one thing you observed is that because of the voltage controlled current source, this matrix is not symmetric. So, if you only independent voltage source sand resistors it would be symmetric, but if you have voltage controlled current sources, it is not going to be symmetric.

So, now we know how to handle these extra cases. First we started with only resistors and independent current sources then, we added independent voltage sources, that you can handle either by an auxiliary variable. Which is easier when you are setting it up for the computer because it is more systematic, or you can identify the voltages to which, the nodes, to which the voltage source is connected as a super node and write a single equation and that is more useful for hand calculation.

And if you have a voltage controlled current source then, you have to modify the matrix, it will become asymmetrical, but in general so, that also you can do. It will modify the equations at nodes to which the controlled source is connected and some coefficients will get modified. So, next we will take another example with a different kind of controlled source which is a voltage controlled voltage source.

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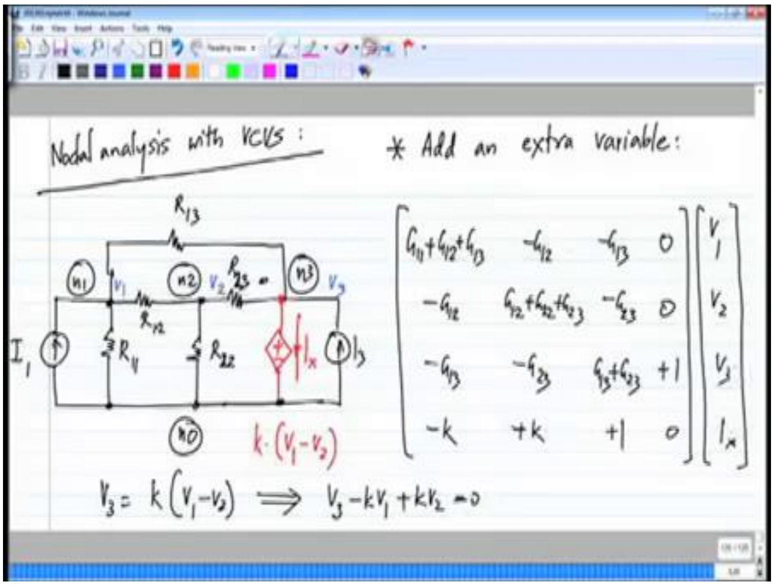

So, let me take this circuit again and what I will do is, instead of R 33 I will have voltage controlled voltage source, and this voltage I say is some k times V 1 minus V 2. So, this controlled sources k times V 1 minus V 2. So, what do we do here? First of all just like voltage source, when we have an independent voltage source, add an extra variable and that is the current through the voltage source, I will still call it I x. So, the equations will be exactly the same as before, with this I x substituting for the current that was flowing in the resistor.

So, if I write down the equations quickly for this one, what happens is at node 1 nothing is changed. So, we have G 11 plus G 12 plus G 13 minus G 12 and minus G 13 and at node 2 also nothing is changed. So, we have minus G 12, G 12 plus G 22 plus G 23 minus G 23. So, this is resistance R 23 by the way, not R 13.

And then, at n 3 we have to change it. So, the equation here would be, at n 3 would be minus G 1 3 times V 1 minus G 2 3 times V 2, and here we will have the sum of conductance's which is G 13 and G 23, but also the current flowing out includes I x. So, my variable vector includes I x and it is flowing away. So, I have a plus 1 over here, and I x does not appear in the node KCL equation for node 1 and 2 so, I have 0 over there.

And finally, I have to write the equation for the dependent source. Now, in case of the independent source, we wrote for instance V 3 minus V 2 equals V 0 and the independent source value was on the right hand side. In case of the dependent source V 3 equals k times V 1 minus V 2, but V 1 minus V 2 themselves are variables. So, what we do is V 3 in this case happens to be k times V 1 minus V 2, but V 3 itself is V 1 and V 2 themselves are variables. So, we take it over to the left hand side, all the variables will be on the left hand side. So, V 3 minus kV 1 plus kV 2 equals 0. So, what do we have minus k plus k and V 3 that is plus 1 and 0, and I do not have the room to write the source vector. So, that I will copy all to the next page and write.

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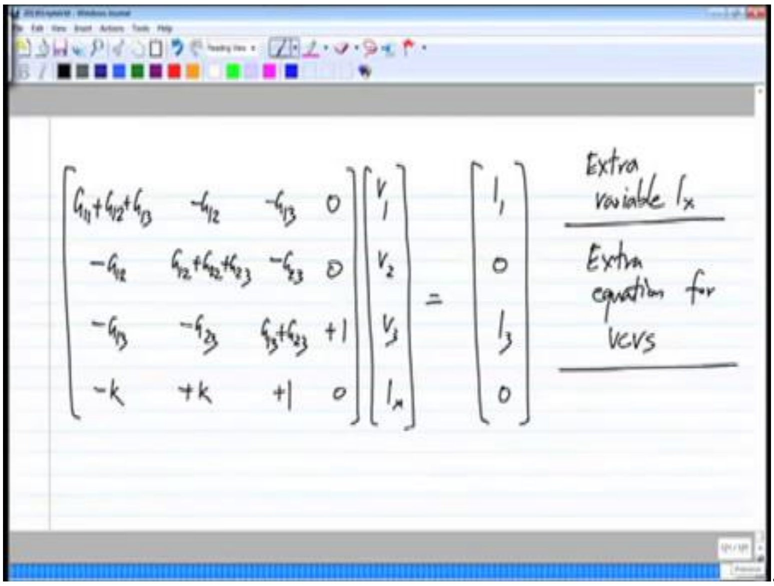

And this equals the total independent current source current flowing into node 1 which is I 1, into node 2 which is 0, into node 3 which is I 3 and the last one, this is simply the equation for the independent, sorry the dependent voltage source. So, this is just 0. So, again we have an extra variable I x, and an extra equation for VCVS. So, the way to handle the dependent voltage source, that is voltage controlled voltage source is exactly the same as handling the independent voltage source. You define an auxiliary variable and go with it, the only thing is that when you write the equation for the voltage controlled voltage source, you group all the variables to the left hand side. So, that is all that is there to it. Any question?

I think one of you had this question, what do you do when you have voltage controlled voltage source? And this is what do you do. Now, the alternative method was to use a super node, I am going to go to that, but before that if there is any questions about this I will take them. It appears there are no questions about this, but there are some questions about what we did before with the voltage controlled current source.

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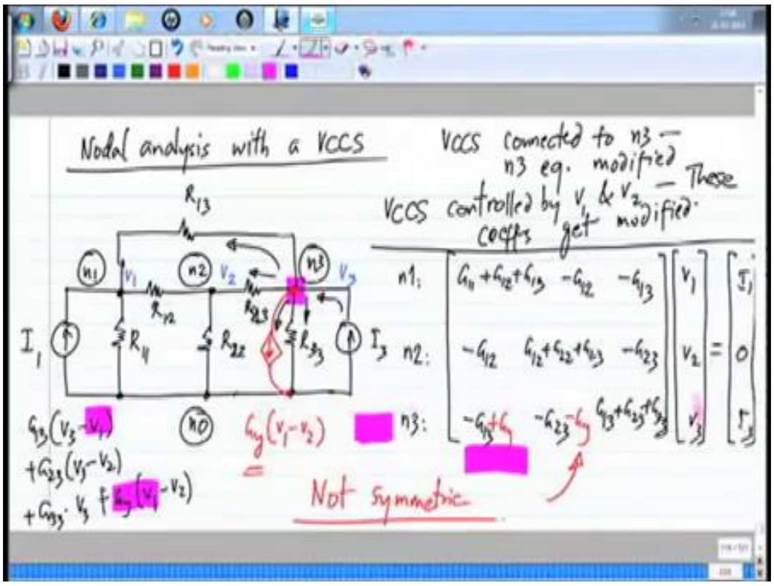

Here, I think the question is, why this entry is minus G 13 plus G y. So, that comes from writing the KCL for this node. Let me write it in a different color here. So, the KCL for this node includes the current there, plus the current ther,e plus the current there, plus the current there, equals the current flowing in from the independent current source. So, the current in R 13 is G 13 times V 3 minus V 1, current in R 23 is G 23 times V 3 minus V 2, and the current in R 33 is G 33 times V 3, and the current in a dependent source is plus G y times V 1 minus V 2. So, if you look at the coefficient of V 1, it is plus G y minus G 13. So, V 1 appears here and here so, it is G y minus G 13 and V 2 it is minus G y minus G 23 So, I hope that is clear.

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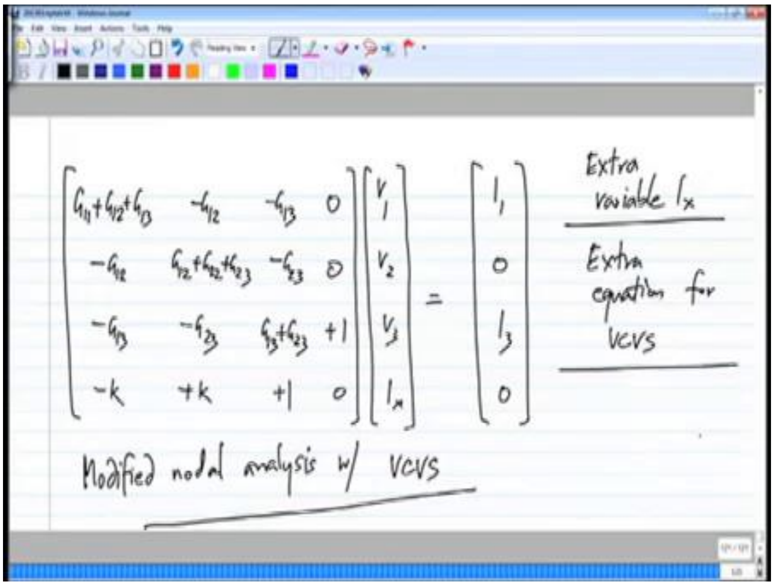

So, this is the, what is known as modified nodal analysis with voltage controlled voltage source. Now, we will try to do the same using the super node.

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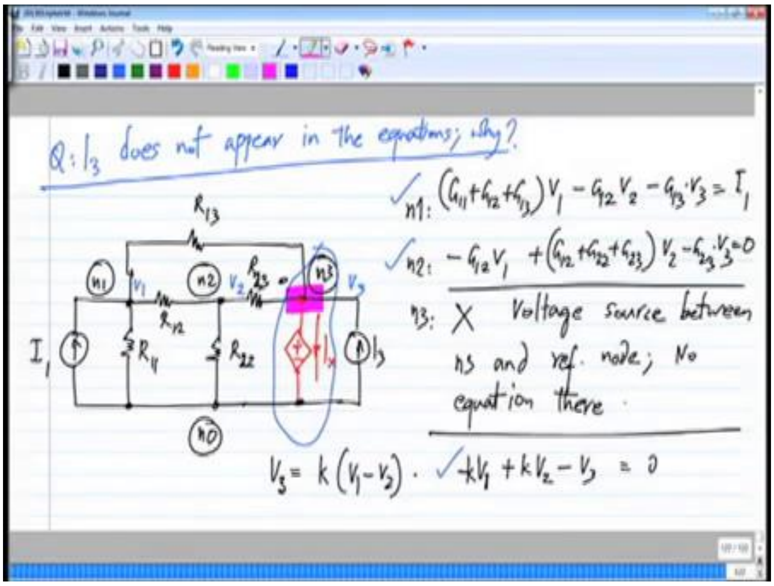

Now, which is the super node that is basically the combination of node n 3 and the reference node, that is where the voltage source is connected. It is a dependent voltage source, but that is where it is. So, we had 3 KCL equations, for n 1, n 2 and n 3. Now, which of these equations will get modified because of the super node? This is a question please try to answer that. Which of these three equations, which of the KCL equations at these 3 nodes will get modified, as a result of making a super node as shown here?

So, I hope all of you are able to hear my question and understand it. My question is we have voltage controlled voltage source, and we would like to solve this using super node. So, I have identified this super node, that is combining the nodes across which the voltage source is connected. Now, which of the node equations will get modified because of this super node? And couple of you answered that it is node n 3, that is correct.

So, what happens to that equation, what should be write, what is the new equation that we should write? Now, because we this voltage source is connected between this node and the reference node, the super node is the combination of this node and the reference node. Now, you do not write any KCL equation at the reference node. So, if the reference node is part of the super node, the node equation simply goes away.

Earlier when we had a voltage source between n 3 and n 2, we combine them into a single equation, that is the total current flowing out of this combined node. When you have something combined with the super node, that equation simply goes away because when you combine it with the reference node, you normally, you do not write any equation for the reference node. So, when any super node when combine with the reference node also goes away.

So, you have two equations, one at n 1 and one at n 2, which are exactly the same as before. And now you also have an additional equation because of this voltage controlled voltage source. And we have this additional constraint that V 3 equals k times V 1 minus V 2. So, as usual we write it with all the variables on left side, which is minus kV 1 plus kV 2 minus V 3 equals 0. And the equations for n 1 and n 2 remain exactly the same as before, that is G 11 plus G 12 plus G 13 times V 1 minus G 12 V 2 minus G 13 times V 3 equals 0. Similarly, minus G 12 V 1 plus G 12 plus G 22 plus G 23 times V 2 minus G 23 V 3 equals 0 sorry, the first equation it is not 0 it is I 1.

So, I hope this part is clear. Now, my question to you is, what happens to this I 3? I 3 does not seem to appear in any of the equations, why is that? So, if you look at these three equations this one, this one, and that one. So, we have three equations and I 3 does not appear in the equations. Why? Yes a couple of you answered that, it is because it is connected to a super node, that is correct, but my question is this super node stuff is some method that we use to analyze circuits.

Now, essentially what this means is that, from these equations forget the fact that we use super node, we got this equations. Now, I 3 does not appear anywhere, but I 3 is very much in the circuit. So, if I 3 does not appear in any of the equations, what it means is, the solution does not depend on I 3 right? So, why does it not depend on I 3? Can you look at the circuit and tell that is my real question.

So, I got a number of responses, some of you have said it is not, it is because there is no parallel resistance and so on. I am not very sure what is meant. What was in your mind when you wrote that, but one of you did answer correctly. See the point is that, this is a current source across the voltage source, now this is a dependent voltage source, but anyway because this is a voltage source, the voltage at this node is set by this voltage source.

So, whatever even if you have current sources here, it is not going to be able to change the voltage here, this voltage source will observe any current that is put into it. So, because you have this current source across this voltage source, it does not appear, it is not it does not matter what the value of I 3 is the solution will remain the same. So, the correct answer is that it appears across the voltage source. So, what we have done is so far is to do…

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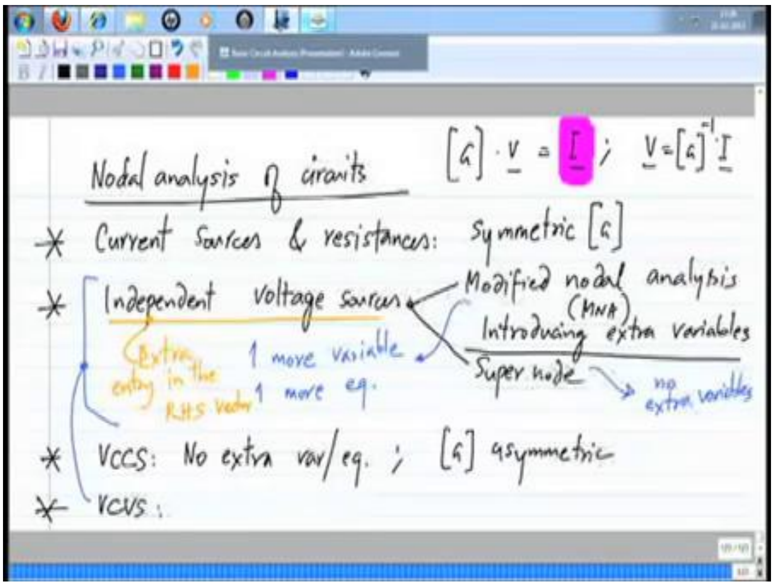

nodal analysis of circuits, and we did it for cases with current sources and resistances. In all cases, the equation can be set up as some matrix at times some variable vector equals the vector of independent sources.

So, we can do this in all cases. Now, when we have only current sources and resistances, this is the cleanest case. We have a symmetric G and in all these cases of course, the vector is obtained as G inverse times I. And when we have independent voltage sources. So, we can do modified nodal analysis, which is MNA or basically we do modified nodal analysis by introducing extra variables. Or alternatively we can treat the two nodes to which the voltage source is connected as a super node and go ahead with the analysis.

Now, modified nodal analysis gives you one more variable and one more equation for every voltage source whereas, the super node stuff, this will not give you no extra variables. And finally, if you have a voltage controlled current source, there are no extra variables or equations, but you will have the G matrix will become a symmetric.

And finally, we have a voltage controlled voltage source, you handle it in the same way, that you did the independent voltage source. Now, when you do the independent voltage source what happens is, you will have an extra entry in the source matrix on the right hand side, in the source vector. Whereas, when you have a voltage controlled voltage source, there is no independent source that is added, whatever is on this side, this whole thing should have only independent sources.

So, that is why you can solve for it correctly that, you have this variable equals the inverse of this matrix which is related to the circuit topology times the source vector. And in case of voltage controlled voltage source, there is no extra addition to the source vector, and you handle it in exactly the same way as independent voltage source by introducing an extra variable and equation or by using a super node in which case you do not have extra variables.

Now, this modified nodal analysis you use it, when you are doing things for the computer, mainly because it is a systematic way of setting up equations and solving it. When you do the super node business it is a little more ad hoc, but it reduces the number of equations. So, when you doing hand analysis, you use the super node. So, any questions on these things?

Now, we have two other types of controlled sources, that is a current controlled voltage source, and a current controlled current source. I will not discuss them in detail here. If you are interested, you can go to the other course that I am offering this semester at IIT madras. I will give you the URL for that, and the details will be there. And you can also try it to do it by yourselves, you add a voltage controlled sorry, current controlled current source or a current controlled voltage source, and see how the equations turn up. If you run into any difficulty please ask me in one of the following classes and I will try to explain, or you can raise it in the forum as well.

So, if you have any questions about anything, we did so far please ask? So, there is a question from Sourav Mahajan asking about why I x is removed? Now, please be more specific, are you talking about why I x is not there when we use the super node, is that the question? So, let me go back to where we had that.

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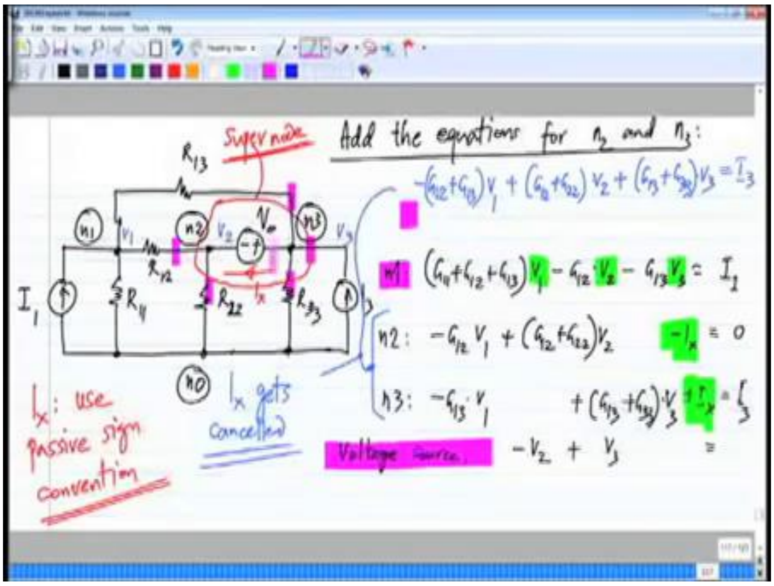

So, first of all super node means this entire thing here so, whatever is inside, it could have components inside, it is the super node. And we know from Kirchhoff’s current law that, all the currents flowing out of a node will sum to 0, but if you have any closed surface of the currents flowing out of that surface will be 0. So, if you look at this, the current flowing out this surface will be through the wires that are cutting this red box that I have drawn, and that is through R 12, R 22, R 33, this current source I 3 and R 13.

The current through the voltage source is completely inside this box and it is not cutting this so, it will not appear in the picture. Similarly, if you want to think about it from the equations, originally we wrote the equation for n 2 and n 3. So, for n 2 you get minus I x and for n 3 you get plus I x. And when you write the KCL for the whole super node, you are essentially adding up the equations for n 2 and n 3. So, minus I x will always cancel with plus I x. Is that fine? Is that the question you wanted to ask?

Now, let me take this particular case. So, this one, I have the equation here, and this refer to a circuit with voltage sources and current sources. We do not have dependent sources, but that is ok. So, I just want to use this, to prove certain things about the kind of circuits we have. I meant this one, where I have the independent sources. So, this is the circuit, let me copy this over to the end, and the circuit itself is not so important. So, I will make it a little small.

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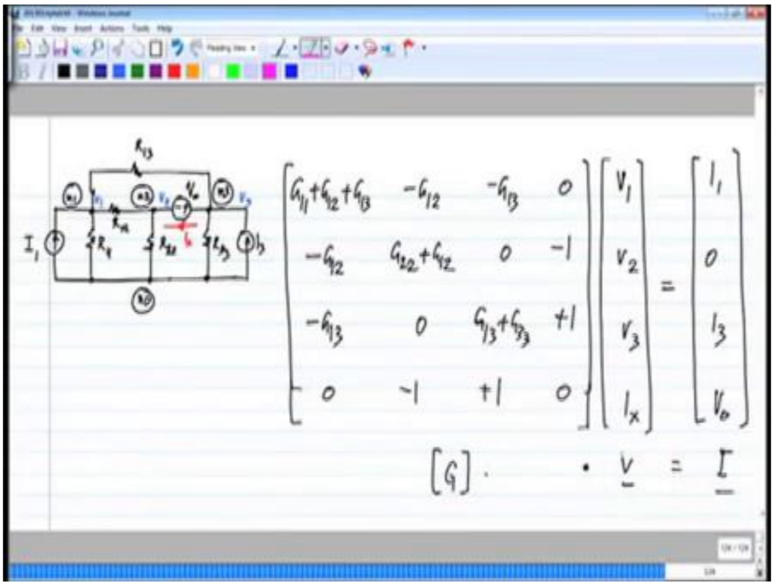

So, this is in the form of some matrix G times the variable vector V, this is the matrix G times the variable vector V, equals the source vector I. Like I pointed out because this has both independent voltage and current sources, this vector V as voltages and currents and this vector I also has voltages and currents, but in order not to change the notation I will continue using this.

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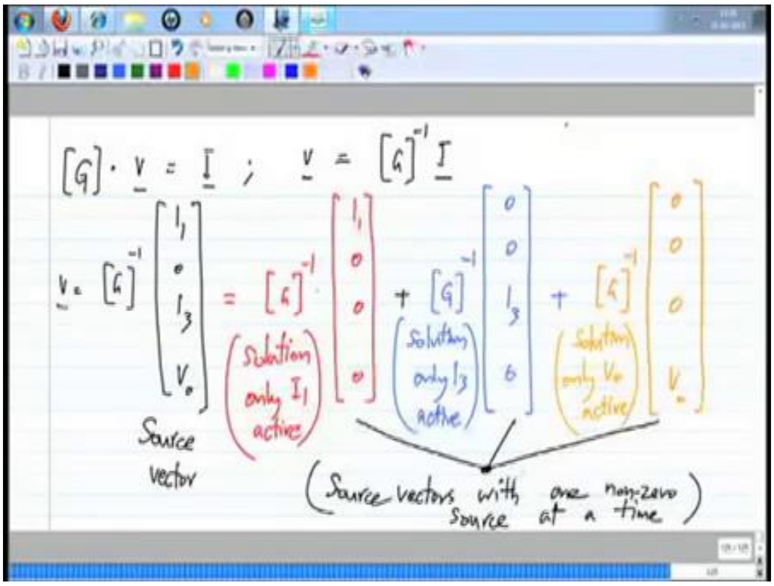

Now, let me expand this out. This G times V equals I. So, obviously the vector V, which is the variable we want to solve for is G inverse times I. Now,, let me light out I completely, V equals G inverse times I 1, 0, I 3, V 0 and this can also be written as G inverse times I 1, 0, 0, 0, that is I take only one element here, plus G inverse times 0, 0, I 3 and 0. In this case, I have taken I 3 alone plus this is the inverse of the matrix, and again G inverse times 0, 0, 0 and V 0. So, this is the source vector, and these are basically source vectors with 1 non 0 source at a time.

So, I can write this vector as I 1 with all 0s and here 0, 0, I 3, 0 and 0, 0, 0, and V 0. And obviously, G inverse times the sum of these three will be G inverse times this, plus G inverse times that, plus G inverse times that. I hope all of you agree with the expansion. So, if there is any doubts about this, please ask me. But what is the point that I am trying to prove here? What does that mean? So, here I have all of them together, and here I have all except one to be 0, here I 1 is non 0, and here it is non 0, and here that 1 is non 0. What is it that? What is the point I am trying to make?

I think many of you got the answer. What I am trying to prove is the property of superposition of these linear circuits. Linear circuits give you this system of linear equations and it follow super position, but this expansion makes it very clear. Now, what is this, this is, this part is solution with maybe it will be clear if I do this.

So, this is solution with only I 1 being non 0. I will say only I 1 active and this is the solution with only I 3 active, and finally, this is the solution with only V 0 active. And we can activate only one source at a time. Find the solution and add up the solutions due to all the sources in the circuit. This is the principle of superposition, and it is use very widely to prove certain things about circuits as well as while doing hand analysis. We will continue from this in the next class. So, for now if you have any questions about this please ask me, and I will clarify them. With that we come to the end of today's class.

Thanks for attending. See you next week.