Basic Electrical Circuits
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Lecture - 6
Circuit Analysis:
Number of KCL and KVL Equations in a circuit
Nodal analysis of a Network with Conductances and Current Sources
Setting up the Equation; Conductance Matrix

Now, if there are no questions, what we will do is to move on to systematic ways of analysing circuits. We have now discussed the number of circuit elements and they can be interconnected in any way and we have to be able to find the solution to the network. That means we have to be able to find the voltage across any branch any component and current through any component, and this process is known as circuit analysis.

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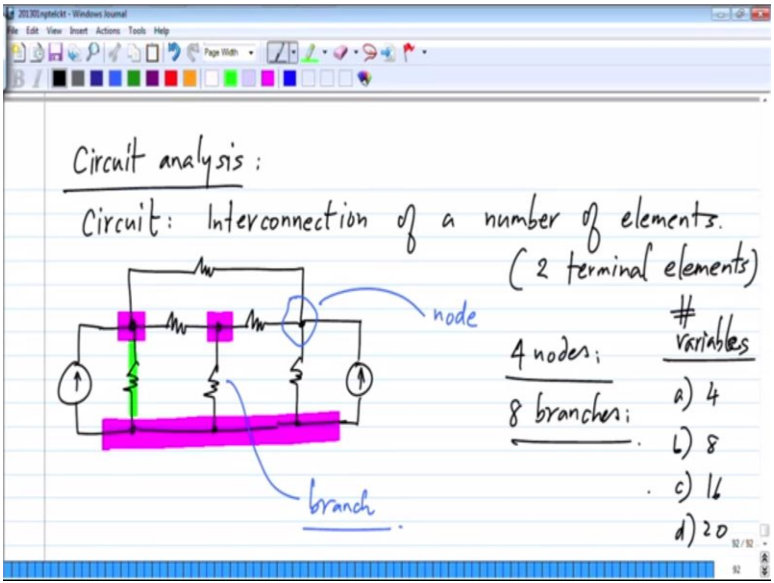

As we know, a circuit is an interconnection of number of elements. Now the elements that we know of are two terminal elements; you can have more than two terminals. But for now let us stick to two terminal elements. So, as an example let me take something, this is just some example that I am going to show.

So, you see a number of elements, we have a number of current sources and resistors connected up like this, and when we are discussing circuits in general and not specifics of each components, we use some terminology that is generally applicable to any element. Now, the point of interconnection of many terminals, this is called a node. Now how many nodes does this circuit have, this is a question for the participants. How many nodes does this circuit have, so I see a number of answers here and many of you said it is 4 and that is correct.

So, we have we have one node here, another node here, another node here and this whole thing is written as an extended line, but of course this is a node. For convenience, we write it like that, but if you just have a wire that is a node, so we have four nodes in the circuit. Now, each element or whatever is connected between two nodes such as this resistance, here this resistance is connected between this node and that node and that constitutes a branch. Now, how many branches do we have in this circuit, each element because we only two terminal element each element corresponds to a branch, so how many branches do we have in this circuit?

So, many of you again said gave the answer correctly that there are eight branches, now sometimes you can combine two parallel elements into a single element and call that a single branch. Now, I will consider each of these things as a branch, each of these elements that is a branch, so that is 5, 6, 7 and 8. So, we have eight branches, now when we say we have to solve a circuit that is we have to do a circuit analysis, what it means is that you have to find the voltage across every branch, every element and the current through every branch or element. So, that is the meaning of the statement that I have solved for this circuit, then you know everything about what each element is doing.

For instance, if you have a voltage source, you can figure out whether it is dissipating power, whether it is consuming power or how much power it is consuming or dissipating and so on, same for a resistor. So, given that how many variables do we have to solve for if we have eight branches, how many variables do we have to solve for? Now, I see a number of answers with numbers like 3, 4, 5 and so on, but my question is the following. We have eight branches in the circuit, now I want to know the current and voltage across every branch and those are the unknowns, so how many variables are there to be solved for?

Perhaps, the question is not clear, maybe I will pose it as multiple choice so that everybody can take another shot at it. I have opened a poll with the choices, please answer on that one, what is the number of variable to be solved for 4, 8, 16 or 20. Some of you have said 4 and some of you have said 16 and 16 is the correct answer, of course because see if you have 8 branches, each branch has a voltage and a current. You have to find out all of it, now some that may be easy, now for instance if you have a current source, you already know the current through that, but that was not the point of my question.

The thing is there are 16 variables to be solved for and somehow or the other we need to find out 16 equations from which we can solve these 16 variables. Now, whether it is easy or difficult it is a different thing, so finally when you say that you have solved for this circuit completely, you have to specify the voltage and current in each branch. Voltage across each branch and current through each branch, so that is what is meant by solving for a circuit.

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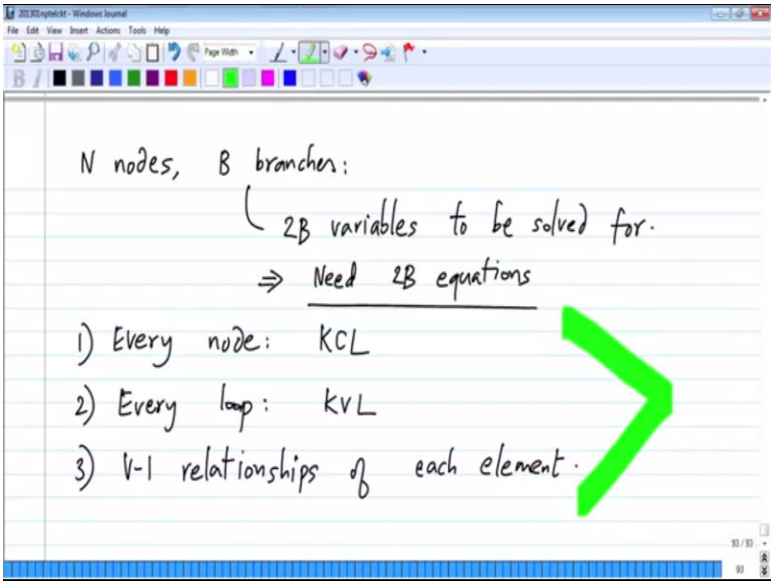

So, in general let us say you have a circuit with N nodes and B branches, you have two times B variables to be solved for. So, in our case B is 8, but whatever the number of branches you have, you have to solve for 2B variables. Now, the question is, what are the 2B equation, so this means that need 2B equations, now what are these, what are the equations that will govern our circuit?

First of all at every node, you can write Kirchhoff's currents law and around every loop, we can write Kirchhoff's voltage law. And finally, we have V I relationships of each element. So, from these things, we have to pick the appropriate set of equations and then solve for the equation, I hope this is clear so far, now let us go through these things one by one.

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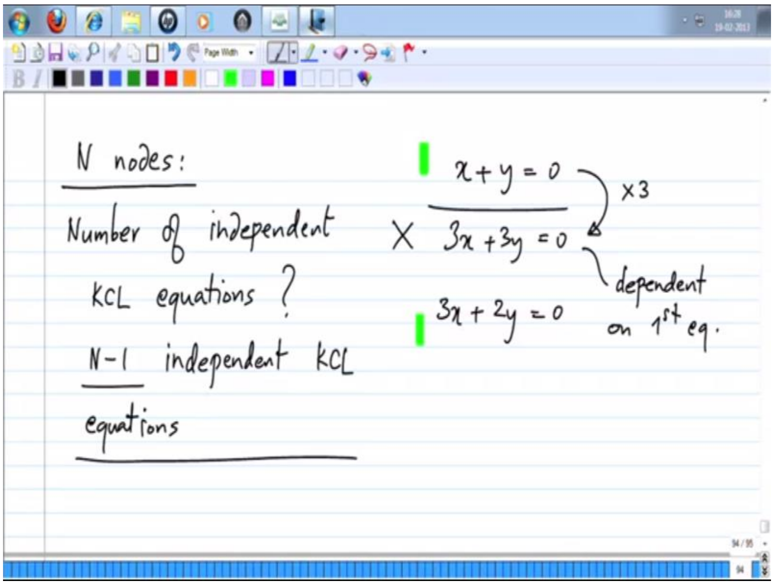

As all of you know to solve for 2B variables, we need to have 2 B independent equations because if you have some dependent equations, they are useless. I think all of you know what dependent and independent equations are, if you have a set of equations, now if some equation can be generated as a linear combination of other equations, then that is a dependent equation. It is not telling you anything that the other equations are not saying, for instance you have one equation over here x plus y equal to 0 and let us say you have another equation, which says 3 x plus 3 y equals 0, clearly you can get this by multiplying the first equation by a factor 3.

Now, if you do that, then that means that this is not telling you anything that the first one did not tell you. So, this is dependent on first equation, so this is not useful, we have a second equation, which was let us say 3 x plus 2 y equal to 0. Clearly, you cannot get this by multiplying this by any number this is an independent equation of this one. So, using these two, you will be able to solve for both variables x and y, now we have N nodes and we know that at every node we have KCL equation. So, my question is how many independent KCL equations we have, if we have N nodes, so if we have N nodes how many independent KCL equations do we have?

At every node we know that the sum of currents flowing out of the node is zero that is always true, but how many independent KCL equations can we generate? So, some of you said N and many of you also said N minus 0 and this is the correct answer, there are N minus 1 independent KCL equations.

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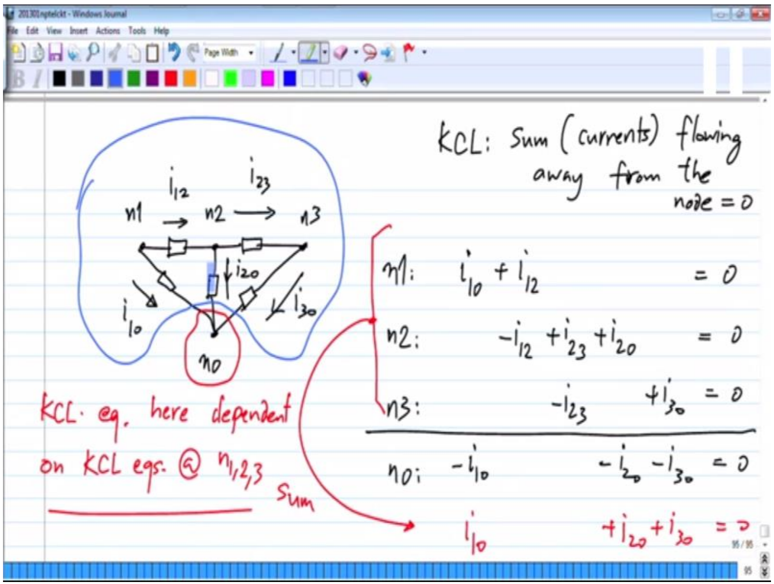

Let me take a simpler circuit than what I had before, these are some elements while writing KCL. I do not care what elements they are because I am only summing up the currents, now let me call this n 0, n 1, n 2, n 3 and let me call these things I 1 0, I 1 2, I 2 3, I 2 0 and I 3 0. Now, what I will do is, I will write KCL equation at every node and my convention is to take the sum of all current flowing away from the node sum of currents flowing away from the node should be equal to 0. Now, what is the equation for n 1, we have 2 Branches I 1 0 and I 1 2 as currents which are flowing away, so I 1 0 plus I 1 2 equals 0 and at node n 2 we have I 1 2 flowing in which is same a minus I 1 2 flowing out.

We have I 2 3 flowing out and I 2 0 flowing out minus I 1 2 plus I 2 3 plus I 2 0 equals 0, finally, at node n 3, not finally at node n 3, we have I 2 3 flowing in which is the same as minus I 2 3 flowing out and I 3 0 also flowing out. So, I have minus I 2 3 plus I 3 0 equal to 0 that is at node n 3 and finally, at node n 0. We will have I 1 0 plus I 2 0 plus I 3 0 equals 0, so at four nodes I have written the four equations, but you see that let me sum the first three. If I sum these three, the equations resulting at resulting at n 1, n 2 and n 3, I sum the three equation what will I get? I 1 0 and I 1 2 will cancel with this minus I 1 2 I 2 3 will cancel with this minus I 2 3, and we will have I 2 0 plus I 3 0 equals 0. So, the node at n 0, the KCL equation at node n 0 is the same as taking all the other KCL equations and summing them together. So, the KCL equation here dependent on the KCL equations at n 1, n 2 and n 3, so we do not have four independent equations, we have only three independent equations.

In general if you have N node KCL at one of the nodes will be dependent on all dependent on the equations at all the other nodes, this is quite easy to see. Now, this we have this circuit what do we mean by writing KCL at node n 0, essentially we take this closed surface and the sum of currents leaving the surface will be equal to 0. By the way I have to change the notation in the last one my usual notation was to take the currents flowing away by this notation at n 0, the currents flowing away are minus I 1 0 minus I 2 0 and minus I 3 0.

So, this last one should have been minus I 1 0 minus I 2 0 and minus I 3 0, the equation I had written was correct, but the only thing was that the signs were reversed, but that does not change the fact that it is dependent on all the other. So, when I write KCL at equation n 0, I take this closed surface and say that sum of all of these currents equals 0 sum of all the currents leaving this surface. Now, instead of looking at looking at the surface surrounding node n 0, let me look at this surrounding the rest of the circuit. So, as it encloses the complete circuit the sum of currents flowing in all these wires that are cutting the surface will also be equal to 0.

Now, you see that the surface enclosing n 0 cuts these three wires and the surface enclosing the rest of the circuit also cuts the same wires. So, what you do what you get by writing KCL equation at n 1, n 2, n 3 and then summing them together is exactly the same as what you get by writing the KCL at n 0. So, in general if you have an n node circuit, there will be n minus 1 independent KCL equations, I hope this is clear, if not please ask your questions, now I think it is quite simple.

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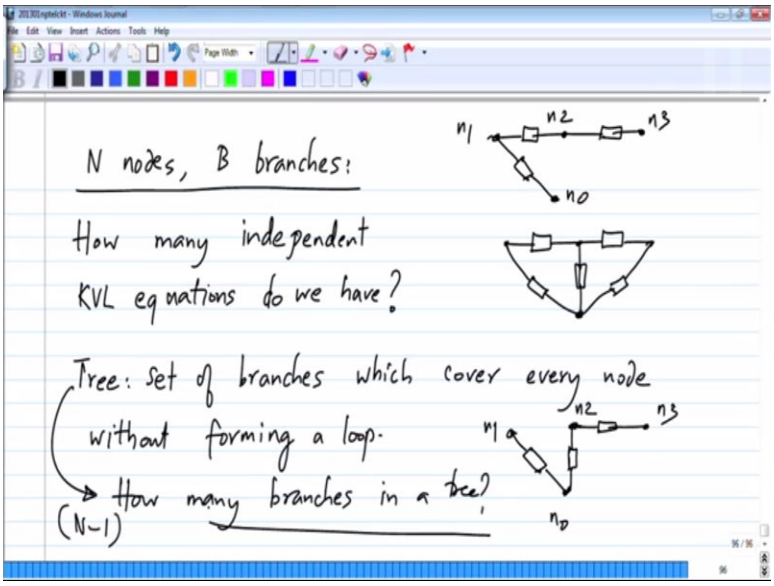

The next thing is I have the same circuit with N nodes and B branches, how many independent KVL Kirchhoff's voltage law equations do we have, please try to answer this one how many how many independent KVL equations do we have. You write KVL around every loop and you have to identify the independent loops, so how many independent KVL equations do we have.

There are lots of loops, but we have to identify independent loops, for instance I will take the same simple circuit I had before. Now, I have a loop here I have a loop here, I have a loop here, so how many would I have some of you have given answers and the answer seems to dependent only on the number of nodes. That is a little strange because you would think that the number of loops will also depend on the number of branches because if you do not have a branch then you cannot form a loop.

Now, I am not talking about this specific circuit if you have a circuit with N nodes and B branches, how many independent KVL equations do we have that is the question. So, it looks like you are not able to identify, so let us go step by step, now how do we know how many independent loops do we have, so to do that I will define what is known as a tree. So, a tree is a set of branches that is out of all the branches we have in the of all the branches we have in the circuit, we will choose a set of branches which cover every node without forming a loop.

What I mean is in this circuit I could choose this, this and this, now as I start from this node go to this one, this one and this one. So if I draw only that part of the circuit n 0, n 1 n 2 n 3, now this will this set of branches covers all the four nodes n 0, n 1, n 2, n 3. There is no loop that is formed that is very obvious from this, now this is not the only choice we can have, some other choice like this, there are many choices. I am not going to list all of them, but for instance, you could have this and this one in the middle and this one that is n 0, n 1, n 2, n 3. So, there are many possible trees, what each what all the trees have in common is that they cover all the nodes without forming a loop, now this should be easier to answer how many branches are there in a tree.

How many branches would be there in a tree, we have a circuit with N nodes and B branches, so how many branches would be there in a tree? Again, some of you are giving numerical answers, I am now interested in general answer for a circuit with N nodes and B branches not for this specific circuit and this most of you are able to get it correctly there will be N minus 1 branches.

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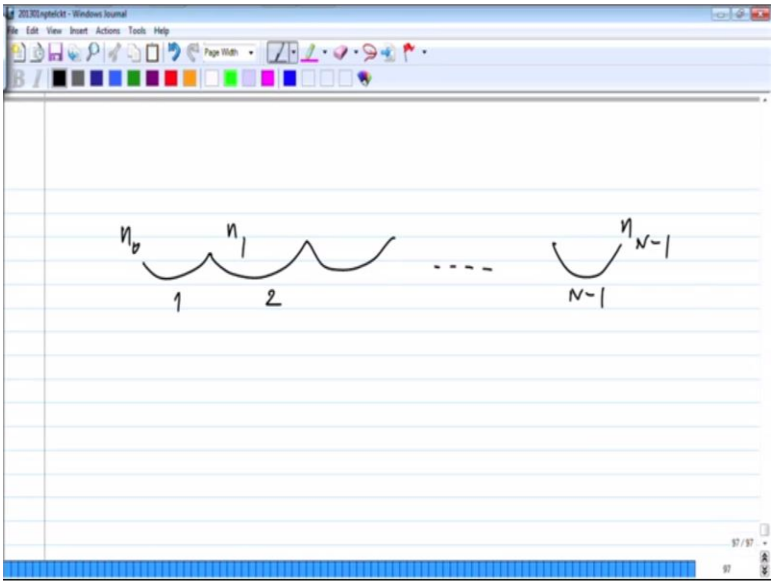

If you have N nodes that is n 0 n 1 upto n N minus 1, so you will have one branch from n 0 to n 1, and another from n 1 to n 2 and so on, and you need n minus 1 branches to form a tree, I think all of you most of you have got this correct. Now, once you have a tree it is very clear that if you had any more branches, if you had one branch, so you have a tree already, so let me redo this by numbering the branches let me redraw the circuit.

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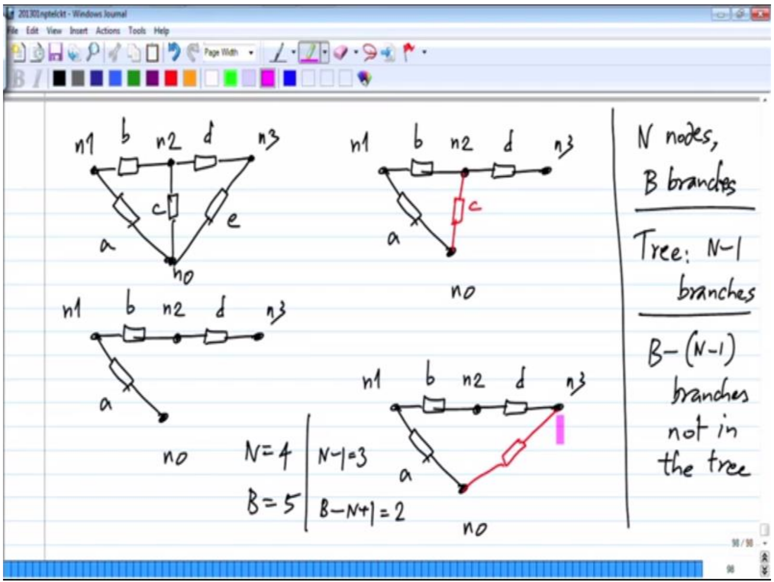

Label the nodes n 0, n 1 n 2 n 3 and I will call the branches a, b, c, d, e, there are five branches in this circuit. Now, I will identify a tree, this is my tree n 0, n 1, n 2, n 3 and this particular tree I have chosen has these branches a b and d. Now, what I will do is, as I said first of all with if you have a circuit with N nodes and B branches any tree, you choose will have N minus 1 branches. So, clearly the remaining branches will not be in the tree that is B minus N minus 1 branches not in the tree, we have a total of B branches and the tree has n minus branches. So, the remaining b minus n minus 1 branches or B minus n plus branches will not be in the tree. Now, what I will do is I will take my tree which is shown here and I will add one of the branches that is not in the tree.

So, in this case I get branch c and then I will add another branch, which is not in the tree, so for this specific circuit by the way number of nodes is 4 and the number of branches is 5. So, in N minus 1 is 3 we have three branches in the tree here and B minus N plus 1 equals 2. So, I have a possibility here and I have another possibility there, now clearly if I add a branch that is not in the tree to the tree, I will form a loop that is very clear because I have gone through all the nodes going without forming a loop. If I add any extra branch, it has to be between the same nodes, I have already cover all the nodes, so it has to cover a loop, so this forms one loop and this forms another loop.

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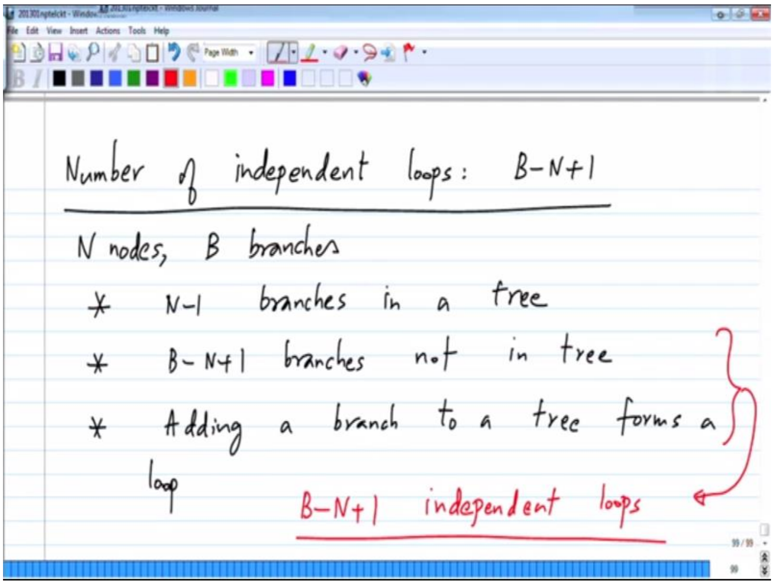

So, that means that the number of independent loops is B minus N plus 1 actually couple of you have already got the answer, Aarti and Sohum. So, we have B minus N plus 1 independent loops, now I will just go through the argument once more for clarity in a circuit with N nodes and B branches, we will have n minus 1 branches in a tree B minus N plus 1 branches not in the tree. Now, adding a branch to a tree forms a loop and how many such possibilities are there, we have B minus N plus 1 branches which are not in the tree. So, we go on add them all one by one to form each loop, so that means that we will have from these two, you can infer that there will be B minus N plus 1 independent loops.

So, this is some involved argument, so if you have any questions or any doubts if something is not clear please ask, now somebody asked what are branches any element is a branch something that is connected between two nodes. Now, we are discussing circuit at a more abstract level later, we will go back to a specific circuit and put a resistor or a current source or voltage source and discuss that, but right now any of those things can be a branch any questions? It appears that no questions on what we discussed this far, so now let us go on some of the other things will become clear as we move along.

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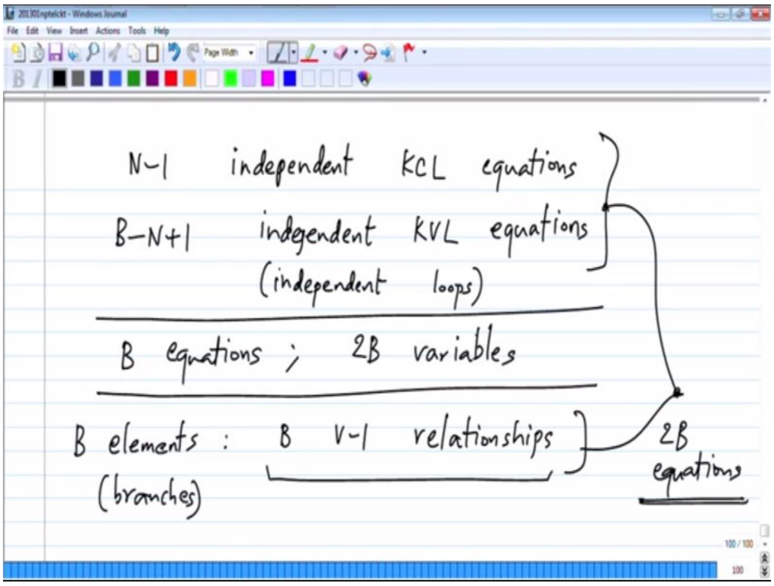

So far, we have N minus 1 independent KCL equations and we have B minus N plus 1 independent KVL equations that is B minus N plus 1 independent loops. So, we have a total of B equations what we had 2 B variables B branch voltages and B branch currents. So, when need B more equations, where will they come from the question is between KCL equations and KVL equations. Together, we have B equations, so where will the remaining B equations come from we have 2 B variables. If we have branches, we have B branch voltages and B currents, where do you thing the rest of the B equations come from because to solve for 2 B variables, finally we need to have 2 B equations.

One of you said, it comes from the VI relationship from each element and that is correct, some of you said Ohm's law, but Ohm's law is very specific to a resistor. So, we have B elements or B branches and each element will have its own VI relationship, so we have B VI relationships and together we have 2 B equations. So, with this, we can solve for 2 B variables which are B branch voltages and B branch currents. So, I hope this part is clear, now we have discussed this analysis only at a high level that is we did not go on analysing any specific circuit, but we have only discussed how to set up the equations.

There are many ways for going about this, first we can look at some simple circuits, see how to analyse that I have chosen to go in the other direction. I will first show you how to analyse any circuit, this is applicable to circuits of any complexity, now of course you will not if you have 100 nodes and so many branches, you will not analyse them by hand, you will not be doing hand calculations. You will set up the equations on a computer and solve it, but this method will be applicable to those things as well. So, given this now we can go and start with specific methods of circuit analysis is this clear, any questions, now there are two methods of circuit analysis.

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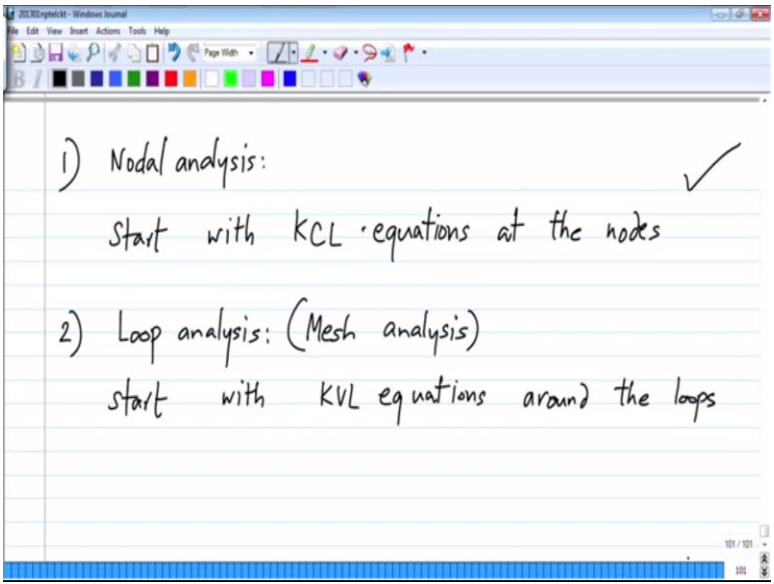

First is called nodal analysis, in this what you do is you start with KCL equations at the nodes. Once you put down the KCL equations, you will solve for a number of things and then you will solve for number of things and then you will solve the complete circuit. So, the first thing you will do is to write down the KCL equations, and if you do that you are basically doing nodal analysis the second method is loop analysis, where you start with KVL equations at around the loops. If you do this, you are doing loop analysis, we will discuss it later and a sub class of this is known as mesh analysis, so we will first start with nodal analysis and see what it is all about.

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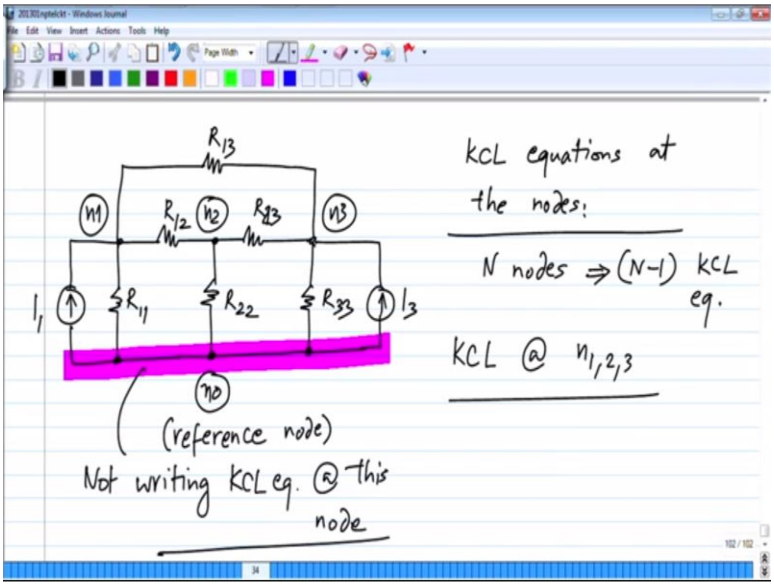

So, let me write down the original circuit, I had this circuit and let me call this I 1 and I 3 I will call this R 1 1, R 1 2, R 1 3, sort R 2 3, R 2 2, R 3 3, R 1 3. The reason for this naming will become very clear and let me call this node n 0, this is n 1, n 2, n 3, now I said that we will start with KCL equations at the nodes. Now, if we have N nodes, it means that we have N minus 1 independent KCL equations, that is if you have N nodes, you will write this KCL at N minus 1 nodes and at one of the nodes you will not be writing the equation. In this particular case, we have four nodes and we will not be writing the equation at one of the nodes, let me choose that to be this bottom node n 0.

Now, such a node the node where you are not writing the KCL equation is called reference node and again the reason for this terminology will become clear. So, what I am going to do is to write KCL equations at n 1, n 2 and n 3. In this case, now I just arbitrarily chose not to write it at n 0, you could have chosen not to write it n 1 and write it at n 0, n 2, n 3 or you can basically out of the N nodes you can choose any one node. Then, say that you are not going to write the equation at that node, so in my case I will write KCL at n 1, 2, and 3.

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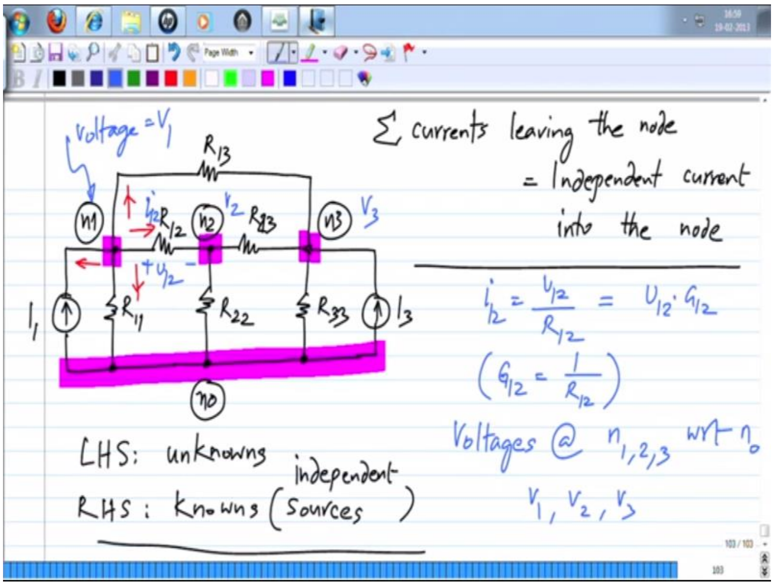

Now, let me write the KCL at node n 1 obviously the current in this branch plus the current in that branch plus the current in that branch plus the current in that branch equals 0. Now, I will also because we are trying to do this systematically for very large circuit, I will use certain convention for writing these equations. What I will do is I will have all the unknowns on the left hand side and all the known on the right hand side and what are the known, what I mean by that are the independent sources, independent sources in the circuit. For instance if I write the KCL at this node the current through R 1 3 is unknown current through R 1 2 is unknown R 1 1 is unknown.

All those things I have group on the left hand side and this current flowing out is due to a current source independent current source. So, that is a known quantity and I will push it to the right hand side, so the way I will write these equations will be summation of currents leaving the node equals any independent current into the into the node. So, that is how I will write the equations and finally, here I have taken a circuit with resistors and current sources. Later, we will also add voltage sources and analyse them, now I have to write the current in each of these resistors and the resistor has a certain VI relationship.

Now, while writing the KCL equation I will use that current voltage relationship for the current through the resistor. So, for instance what is the current through this resistance R 1 2, it will be the voltage across the resistor divided by the resistance value.

So, let me label that for instance let me call this current through R 1 2 as I 1 2 in this direction going from left to right. We know that I 1 2 will be equal to V 1 2 divided by R 1 2, the current through this resistor equals the voltage across this resistor divided by the resistance value. Also, what I will do is, I do not want to keep on writing this thing in the denominator, so instead of this, I will use the conductance form of Ohm's law, which says that it is V 1 2 times G 1 2. So, when I write this it is implicit that G 1 2 is one by r 1 2 that is the conductance of this resistance this resistor. So, I 1 2 is the conductance G 1 2 times the voltage v 1 2, I will write it in this form, because the equation will look neater that is all now it not nothing fundamental about that.

Finally, what is this V 1 2, it is the voltage between voltage across the resistor or voltage at node 1 minus the voltage at node 2, so I have to use this voltages. So, what I will do is, I will use the voltages at n 1 2 and 3, now again while choosing voltages, there are many ways of doing it, I could choose the voltage across r 1 2 as the variable across r 2 3 as the variable and so on. Now, to be systematic and to give a clean structure to the equation what I will do is, I will not use those things, but I will use voltage between n 1 and n 0 between n 2 and n 0 between n 3 and n 0. Now, the definition for n 0 is very clear that is the node at which I am not writing KCL.

All other node voltages, I express as the voltage between that node and the reference node that is between each node and the node, where I am not writing KCL so that I will say voltages at nodes n 1, 2 and 3 with respect to the node n 0. I will call these voltages V 1, V 2 and V 3, now it is very common to say something like the voltage at node n 1 is V 1, what is meant is that the voltage between n 1 and the reference node is V 1. Similarly, when you say the voltage at node n 2 is V 2, it is with respect to reference node never forget that a voltage is always measured between two points, two nodes, so again between n 3 and n 0, it is V 3, so I will say it is V 3, V 2 and V 1.

So, that is about the conventions, so what did I just summarise in this method, which is called node analysis, I will start with KCL equations at n minus node, I will choose some node and call it the reference node. I will not write KCL there, I will write KCL at the remaining nodes and what is KCL it is the sum of all currents flowing out of the node equal to 0. The way I will write it down as the sum of currents basically unknown currents going out of the nodes equal the sum of independent source currents that are coming into the node.

So, if you have a current source connected to the node, I will push it to the right hand side. Now, this is a very common thing to do, you have variables on the left hand side and independent things on the right hand side and you solve for this variable, this becomes very clear as we go along. Finally, the currents in resistors, I will write them in terms of voltages across the resistors times the conductance, currents in each resistive branch equals the voltage across that branch times the conductance.

Now, I have to write the voltages again, I will choose the variables in a systematic way, I will choose the voltage at each node with respect to the reference nodes as the voltage variables. If you have any questions about these definitions, please ask the questions now, so that we can go can go ahead with the circuit analysis in a smooth way, any questions about any part of what I said so far?

Now, that is one question basically asking instead of current sources, if we have voltage sources what do we do? Now, let not take that case, first do it with current sources and resistors later on we will consider the case of independent voltage sources. It looks like things are clear so far, now let me go ahead and write the equations in the manner I described.

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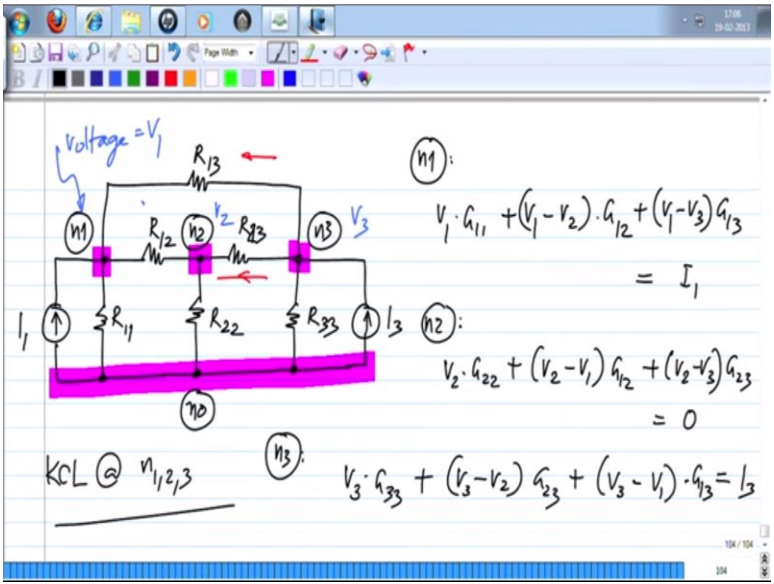

Now, a KCL at node n 1 will be the sum of these currents current through r 1 1 current through r 1 2 current through r 1 3 the sum of those equal the current flowing into this node which is I 1. So, first of all the current through r 1 1 is V 1 divided by r 1 2 or V 1 time G 1 1 and the current through this r 1 2 is the voltage across it, which is V 1 minus V 2 we are always taking current flowing away. So, you take the voltage at this node V 1 minus the voltage at wherever the resistor is connected that is V 2 plus V 1 minus V 2 times G 1 2 plus here we have V 1 minus V 3 as the voltage across r 1 3. Now, voltage across r 1 3 is V 1 minus V 3 and the conductance of that is G 1 3, if I was taking the current going away from this node, I would say minus I 1 because I 1 is glowing in that would be minus I 1.

So, let me first write it like this minus I 1 equal to 0 like I said all the independent sources I will push to the right hand side, if I have to push it to the right hand side, I have to look at the current flowing into the node on the left hand. I have all the currents flowing out of the node and that has to be equal to the current flowing into the node, so as I said, I will put all the independent variables on the right hand side. Now, at node n 2, I will write it similarly, the current through r 2 2 is V 2 times g 2 2 because the voltage across that is V 2 voltage across this and the reference node and the voltage across r 1 2 is V 2 minus V 1.

Now, note that while writing the equation for node one for r 1 2, I took the current from left to right. Now, I will take it from right to left that is because for every node I will draw, I will write the equations in terms of current flowing outwards and the current flowing from right to left is V 2 minus V 1 times G 1 2, I hope that is clear. Finally, if I take this r 2 3, then the voltage across that is V 2 three because i want to take the current V 2 minus V 3 where because I would like to find the current in that direction. So, plus V 2 minus V 3 times G 2 3 equals that should be equal to all the independent current that is coming into this node.

I do not have any independent current source connected to this node, so this will be equal to 0 and finally, at node n 3 I have the current through r 3 3, which is V 3 times G 3 3 plus the current through r 2 3. Again, I will take it flowing away from node n 3, that is I will look at the current in this direction and that would be V 3 minus V 2 times G 2 3.

Finally, the current through r 1 3 that is equal to V 3 minus V 1 times G 1 3 and this has to be equal to total current contributed by independent current sources flowing into the node and that is equal to I 3. So, basically I have written the KCL at the three nodes and I would try to have followed the conventions that I have earlier described all the independent sources on the right hand side and all the variables on the left hand side. All the variables are basically the node voltages with respect to the reference point, I understand that this analysis, it is systematic analysis, so there are many steps like I said earlier in the course also please be very systematic and be very careful about the signs.

When you do these things, especially in the initial parts, when you do not yet have much practice I will stop here and take any questions if any of you is not clear about any of these terms then please ask the question and I will answer them, any questions at all? So, everything is clear there are no questions about how I wrote these things and all the every terms of every equation is clear. So, everything is clear, now although I wrote the equations like this, I will rearrange them slightly, remember the variables here are V 1, V 2 and V 3 the KCL equations are written in terms of the node voltages with respect to the reference node as the primary variables. So, when I say when we solve this equation we will be solving for V 1, V 2, V 3 and from there you can solve for everything else.

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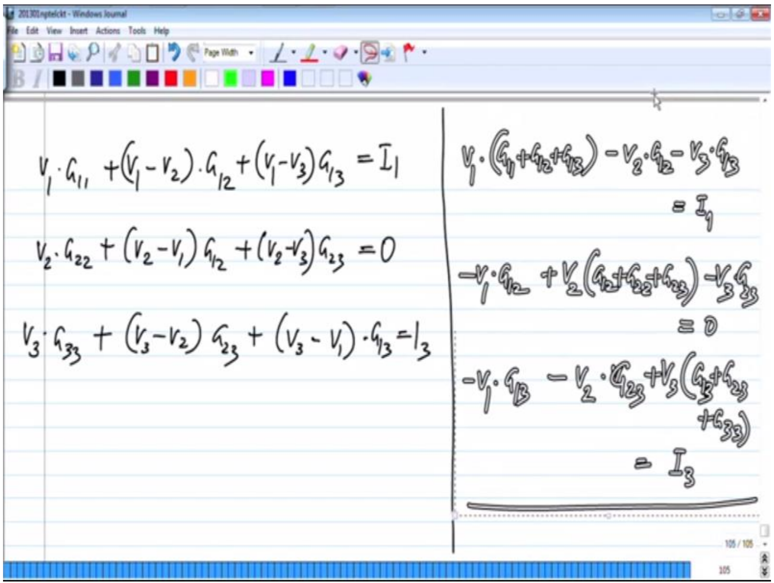

So, this is one of the equations, this is the other equation and finally, this is the third equation. Now, I will rearrange these slightly and what I will do is I will group the variables, that is I have V 1 here, there and there. I do not want to have it like that, so I will write it as V 1 times G 1 1 plus G 1 2 plus G 1 3 minus V 2 times G 1 2 minus V 3 times G 1 3 equals I 1. I grouped all the coefficients of V 1 and V 2 and V 3, then V 2 here I will again write the variables in the same order minus V 1 times G 1 2 plus V 2 G 1 2 plus G 2 2 plus G 2 3 minus V 3 G 2 3 equals 0.

Finally, for the last one minus V 1 G 1 3 minus V 2 G 2 3 plus V 3 G 1 3 plus G 2 3 plus G 3 3 equals I 3, so all I have done is to rearrange the equation. So that I will have the variables together that is each variables have all of its coefficients grouped together. I think this is pretty clear and probably there will be no questions about this.

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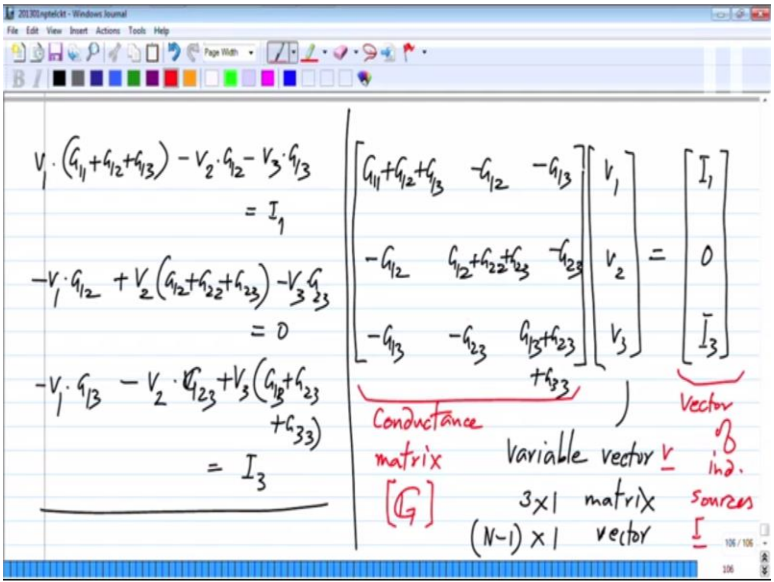

Now, very common way of writing out these equations is in the form of a matrix and this is because it just look neat that is the main reason. Then, on the computer we have methods of solving the matrix solving the inverse of a matrix and that is why we write it like that. Now, the variables V 1, V 2, V 3 I will put it into a vector a vector is nothing but in this case a 3 by 1 matrix or a 3 by 1 vector. In general, if you have N nodes, you will have N minus 1 independent voltages, so N minus 1 times and now I will write this whole thing as matrix times this variable vector equal to the independent source vector.

Now, what is the matrix it is nothing but these coefficient which multiply V 1, V 2 and V 3, so the first element of the first row will be G 1 1 plus G 1 2 plus G 1 3 the second element would be whatever multiplies V 2. You know that when you multiply matrices you multiply, this first element by the first element here second one by second one and third one by third one. So, we will have minus G 1 2 minus G 1 3, similarly for V 2 this minus G 1 2 is the coefficient of V 1 and G 1 2 plus G 2 2 plus G 2 3 is the coefficient of

V 2. Finally, minus G 2 3 is the coefficient of V 3 and here minus G 1 3 is the coefficient of V 1 and minus G 2 3 is the coefficient of V 2 and G 1 3 plus G 2 3 plus G 3 3 is the coefficient of V 3.

It is exactly the same as what I have here each of these rows corresponds to each of these equations and this whole thing will be equal to whatever I have on the right hand side. So, for the first one the right hand side will be I 1 for the second one, it is 0 and for the third one it is I 3. So, this part is called a conductance matrix, obviously it consists of conductance and this part is the vector of node voltages I will call that V, and finally this is the vector of independent current sources I will call that I.

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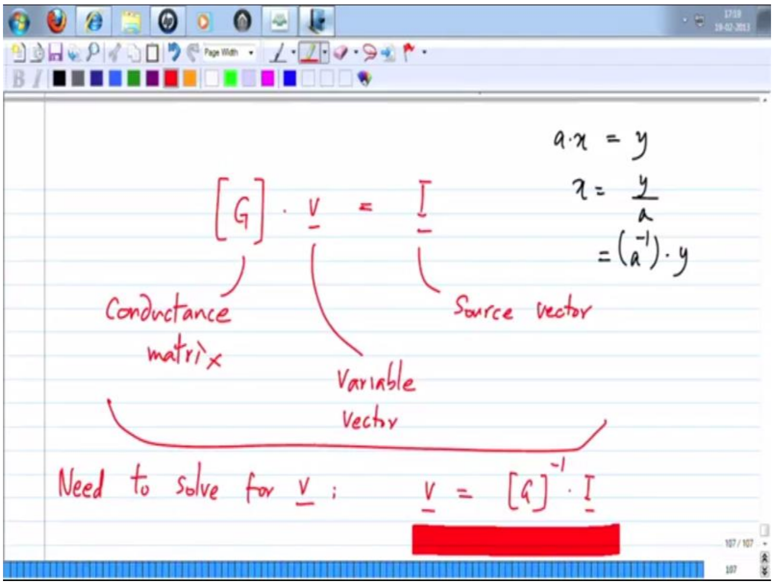

So, when we write this is equations for the circuit based on Kirchhoff's current law and express the currents as voltage time's conductance, we will get this equation of the form conductance matrix g times the variable vector v equals source of independent vectors I. So, conductance matrix times variable vector equals source vector, now you want to solve for V, this means that v equals you know how to do this you have matrix times vector equals another vector this is a square matrix. It will be the inverse of this matrix times I, this is somewhat like if we had scalars, let us say you had a times x equals y, you would say x is y divided by a or a inverse which is 1 by a times y.

Now, we have it in terms of matrices, so this is the solution now, this whole thing it is basically about setting up the equations correctly as I said this is scalable to very large circuit. So, whatever the size of the circuit, you have you will be able to do it like this obviously in exams and in tests and by hand you will not be solving for anything more than circuits with two nodes or at most three nodes. You will not be calculating inverses of 2 by more than 3 by 3 matrix by hand, but right now the point is to understand how to set this up without any error and with confidence for any sized circuit any questions on anything that I have done so far.

What I have done is to set up the KCL equations at n minus nodes and I have chosen the voltage at these N minus 1 nodes with respect to the reference node as the primary variables. It appears that is pretty clear, so right now I do not want to solve this, I am not going to solve this for any particular circuit, but I just want to focus on the structure of this structure of the matrix here, let me copy this over.

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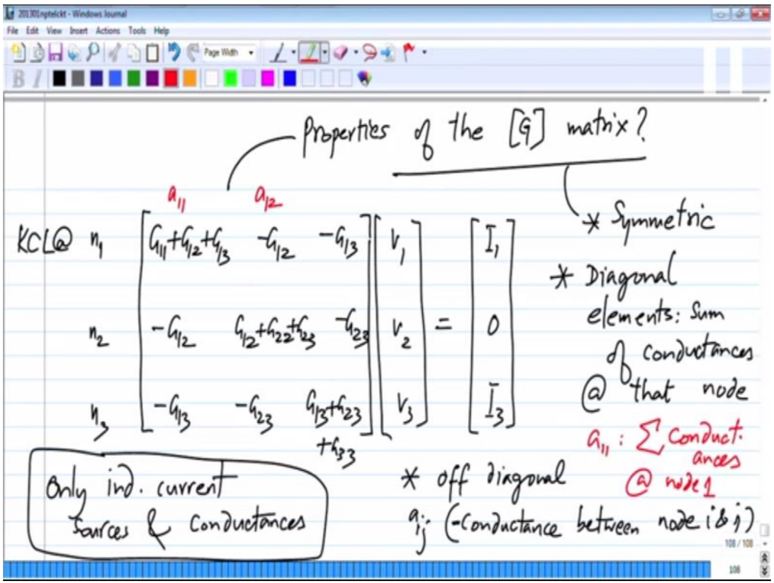

Now, the way I have written it is the first row corresponds to KCL at node 1 and the next is at node 2 and node 3. I have chosen the same order the voltages in the vector because I can write these in any order and I can also re order these and change the order of the entries in the matrix, but the important thing is that I choose to have the KCL equations is some order n 1 first. Then, n 2 then n 3, then I will also have the voltages in the same order V 1, V 2 and V 3 and this is the kind of conductance matrix that I get. Now, my question for you is what do you see about this matrix what properties do you observe in the matrix as it is written.

I think many of you very easily observed that it is a symmetric matrix that is we have minus G 1 2 here minus G 1 2 there minus G 2 3 minus G 2 3 and minus G 1 3 minus G 1 3, so clearly it is a symmetric matrix. Now, let us focus on the diagonal, the diagonal is this, what do you observe about the diagonal elements it is a symmetric matrix what do you observe about the diagonal elements for instance the very first entry of the matrix the element 1 1, what is that?

Clearly, I think this also all of you are able to figure out correctly that the diagonal elements are sum of conductance at that node when I say that node the matrix entry this is the element a 1 1. If I call this a matrix a 1 1 is the sum of conductance at node 1, similarly this a 2 2 is the sum of conductance at node 2 and this is at node 3.

Now, what are the off diagonal entries what is this, let us say I call this element a 1 2, I think you know the matrix notation, you have a 1 1, a 1 2, a 1 3 and so on what is the what is this element a 1 2. Again, I think it is pretty clear the off diagonal elements are if I take the element a i j it is the conductance or the negative of the conductance between the node ii and j. So, that is how we have that is why this entry a 1 2 is minus G 1 2 that is the conductance between conductance connected between node 2 and node 2. It is also why the matrix is symmetric, this is the conductance connected between node 2 and node 1. Obviously that is the same as the conductance between node 1 and node 2, so that is how we have symmetry.

To get this symmetric structure, please note that the ordering of the node equations and the ordering of the vector has to be exactly the same. For instance if I could interchange these two equations, this row will come here, and this row will go there, and similarly, in the right hand side, it will change. Then, the symmetry is lost you will have this symmetric structure only if the ordering of the equations is the same as the ordering of elements in the vector. Now, this can be extended to circuits of any size, the only restrictions we have so far are that we have only independent current sources and conductance.

We have only independent current sources and conductance in the circuit, with that we have all these nice properties that we get, a symmetric matrix with the diagonal matrix being the sum of conductance at that node and off diagonal elements. The conductance between two nodes the negative of that Any questions so far and to solve for this you will have to invert the matrix G and multiply the variable multiply the source vector with the inverse of the matrix G.

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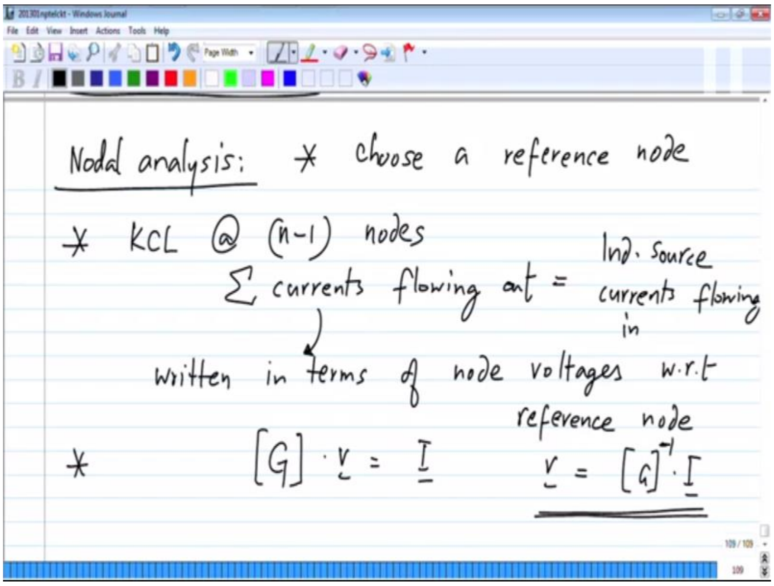

So, in summary nodal analysis consists of first of all writing KCL at N minus 1 nodes first. Before this, let us say choose a reference node write the KCL at N minus 1 nodes in the form of sum of currents flowing out equals independent source currents flowing in and also this current is written in terms of node voltages with respect to the reference node. Now, what this will give you will get an equation of the type g matrix times v vector equals the source vector and solving this will give you the node voltages g inverse times y.

Now, we have not yet solved for the complete circuit like I said earlier, the complete solution consists of solving for every branch voltage and every branch current, but the hard part is done. Now, we have a complicated network and we have found all the node voltages, now if you want to find all the branch voltages that is very easy for instance, the voltage across r 1 1 is nothing but the voltage between node 1 and node 0. So, let me call this I have already called this V 1, that is the voltage between n 0 and n 1 is V 1 n 2 and n 0 is V 2 and n 3 and n 0 is V 3.

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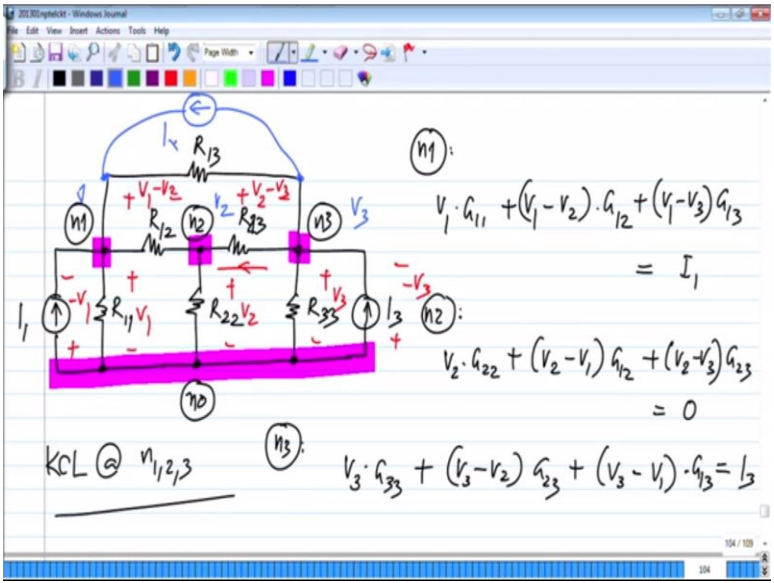

So, the voltage across r 1 1 is V 1, r 2 2 is V 2, r 3 3 is V 3 across r 1 2 it is V 1 minus V 2 in this direction across r 2 3 it is V 2 minus V 3 and across r 1 3 it is V 1 minus V 3. Across this current source, in this direction it is minus V 1 and in this direction it is minus V 3, the reason I wrote down all these things is just to show that all the branch voltages come out of some trivial manipulation of the nodes voltages. So, generally when you say solve for this, we will solve for the node voltages and leave it there and if there are any other specific questions that is questions that are asked like the branch voltage of a specific branch.

You can easily confect it by taking the voltages across that branch that is taking the difference of two voltages or may be in some cases like r 1 1, the node voltage itself is the branch voltage. Similarly, now for branch current you have to go to every branch and look at the current voltage relationships, you know the voltage of every branch, so you will be able to tell the current of any branch for resistors, the current is proportional to the voltage. For the independent current sources, the current is already known, they are independent of the voltages. So, what I want to point out is that after this there are a couple of steps to get all the branch voltages and all the branch currents, but at this point we say the circuit is solved because the rest of the steps are kind of trivial.

You just take the difference between some two nodes voltages to get the branch voltage and you multiply that by conductance to get the resistor currents. So, we have come to the end of this lecture in the next one, we see that our circuit have some limitation we have not analysed the circuit that is universal we only have current sources and resistors. Obviously, we will also have independent voltage sources as well as controlled sources that are dependent sources so that we will take up in the next class onwards.

Now, before we leave a couple of quick questions, so let me first say that to this circuit I will add a current source yet another current source, let me call it I x just for simplicity. Now, please answer the question how will this matrix be changed hope you remember what I wrote, maybe I will copy that over, let me copy over this whole thing what I did was to add this current source in blue.

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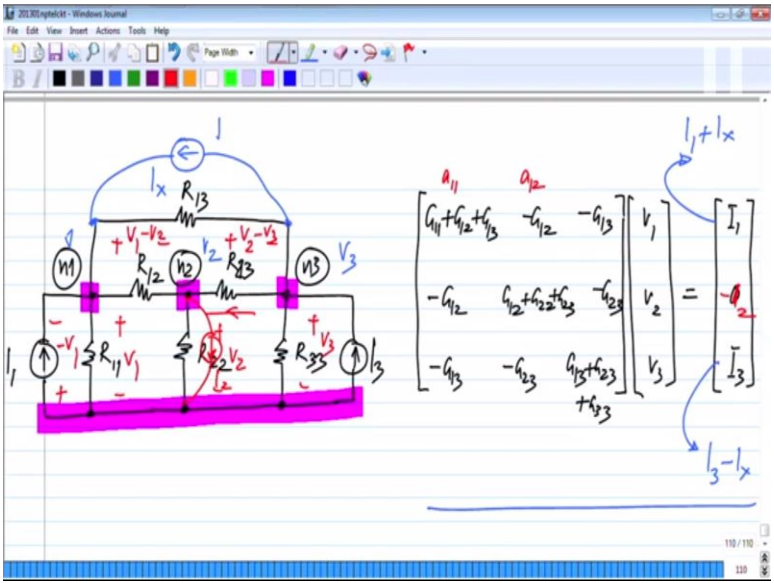

My question is I have set up these equation sin the matrix form how will this be changed that is please answer the question in terms of what will change is it the first of all is it the right hand side that is going to change or the left hand side. I think it is pretty clear because we have added an independent current source that has to appear on the right hand side. So, the right hand side will change this will influence the right hand side, now how will it change which will change which of the entries will change.

So, the right hand side has vector with three elements, so which one will change, again it is pretty clear this is connected to node 1 and node 3, obviously it is only the equation at node 1 and node 3 that will change. Now, what has happened is node 2 we have not changed it at all, so the middle row remains exactly as it is, if you look at node one this current source x is pointing towards node 1. So, it adds to the current flowing into node 1, so instead of this I one it will become this I 1 will become I 1 plus I x and if you look at this node 3, I three is flowing in and this I x is flowing out.

So, this part will become I 3 minus I x and similarly, if I add I two in this direction, this will be minus I 2. So, by now you should have enough confidence to write down this matrix and set it up for any circuit like I said for arbitrary large circuits you are not expected to solve it by hand, but solve it on a computer. You should be able to set up the equation for any size circuit as long as they have only conductance and current sources. So, if you have any questions I will answer them and otherwise we will wind up this session and meet on Thursday, thanks for coming I will see you on Thursday, bye.