Basic Electrical Circuits

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Lecture - 21
Sinusoidal Steady State Response of RC and RLC circuits

Hello everyone, welcome to another lecture of basic electrical circuits. In the previous lectures we looked at second order systems and are transient response and so on. What we just started to do at the end of the lecture was to determine sinusoidal steady state response of linear circuits. Essentially, what it means it means is that we want to determine the steady state solution to an input like cos omega t. Of course, that can be a phase shift it could be cos omega t plus phi.

Now, it is possible to substitute that in the differential equation and substitute the known form of the solution, which is some combination of cos and sin and work out the answers, but it turns out there is a lot easier way. That is what I was describing, I first showed it with the case of a resister which change another trivial, but it importance becomes very clear when we use that analysis for circuits containing capacitors and inductors. So, that is what we are going to do in this lecture.

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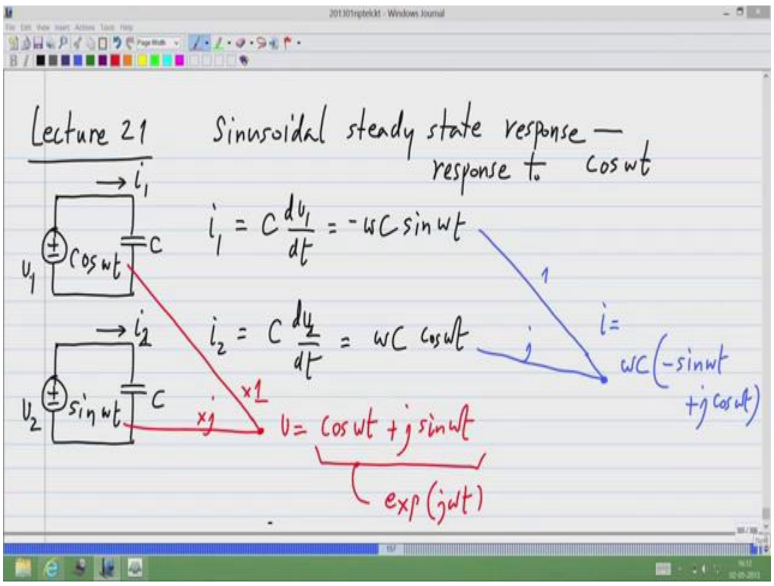

If you have any questions about previous lecture please go ahead and ask. I will discuss that before going on with the topics of this lecture ok. So, let me start with our lecture today. What we mean by steady state response is that, the transient response has already died out. Now, that is a topic for another set of lectures in itself, but you have to also verify whether the transient response dies out.

Now, in this course what we will assume is that, for all the circuits that we consider with real R L and C s the transient response indeed will died on if you wait long enough. So, after that what you evaluate will be the steady state response. So, that is why it is also useful to calculate this because the total response always consist of the transient and steady state response. After a certain period of time for a class of circuits known as stable circuits, the transient response will die out. So, the total response will be equal to the steady state response.

That is why it is useful to calculate this if the transient response was always dominate there is not much point calculating the steady state response, but it turns out that the steady state response will dominates in a lot of practical cases, after a long time. How long is long depends on the time constants of the circuit. So, now let me go back to the case with the capacitor. So, what I mean by this is, response to some stimulus of the form cos omega t. Now, I will assume that just like I did at the end of the previous lecture. That the voltage across the capacitor is cos omega t. So, let me call this V 1 i 1 then will be minus omega C sin omega t. Now, let me apply sin omega t, I will get a current i 2 equals C times the derivative of V 2 which is omega C cos omega t. Now, clearly these are linear components the capacitors, here is the same capacitor. Now, if I apply some super position of V 1 and V 2, I will get the same super position of i 1 and i 2. I hope this part is clear that I take some super position of V 1 and V 2.

What do I mean by that I will take some alpha 1 times V 1 plus alpha 2 times V 2, then the total response will be alpha 1 times i 1 plus alpha 2 times i 2. So, let me do that and the particular linear combination I will take will be this multiplied by 1 and this multiplied by j. So, V will be cos omega t plus j sign omega t. The response to that will be the same super position, I have to multiply this by 1 and multiply this by j. So, I would get omega C times minus sin omega t plus j cos omega t.

This will be the total current I, this is ok. So far, I have not done anything profound I just for a single capacitor given a voltage I can easily find the current by differentiating the voltage. Now, I also take some particular super position of voltage is sin 2 cases and the current sin 2 cases. Now, the important thing is that, from basics of complex numbers we know what this is, this is equal to exponential j omega t.

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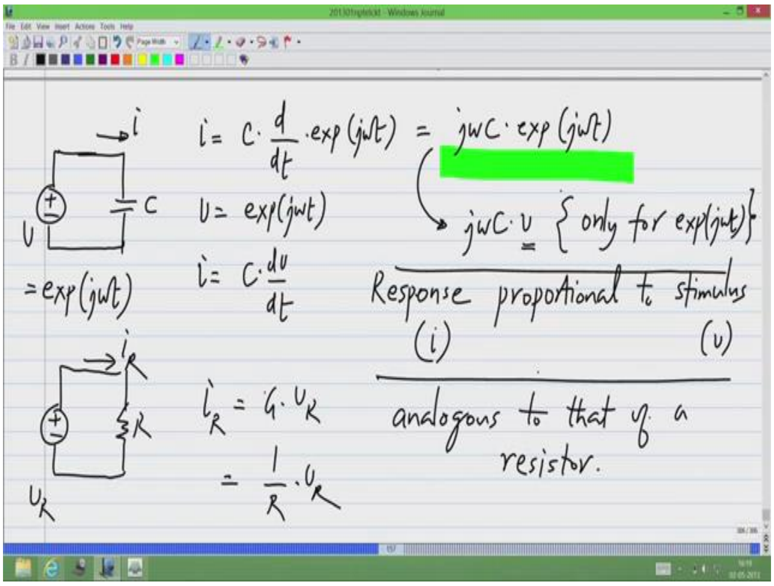

So, essentially this super position is like taking a capacitor C and applying a voltage V equals exponential j omega t to it. Previously I first applied cos omega t and then sin omega t and considered this super position. Now, what I will do is I will take this exponential j omega t and try to calculate the current directly. Now, this also is trivial, in case of a capacitor. It is C times the time derivative of exponential of j omega t, which is basically j omega C exponential j omega t.

It does not look like, if have done anything significant here we calculated the response to exponential j omega t directly by differentiating and also by taking super position of cos and sin omega t. Now, but the most important thing which I pointed out last week as well is that, in this case remember V was exponential j omega t. So, the current for this particular value of V turns out to be j omega C times V.

Now, of course, this is not general rule, the general rule is that i a C times the derivative of the voltage, but if the voltage is an exponential, exponential j omega t then you clearly see that the current is proportional to voltage. So, this is a very important thing and in fact there is a whole body of work a linear circuit analysis based on this. Now, we will not go into the depths of this, but I will only point out that, this exponential j omega t, if you apply to any linear system, the steady state response will consist of exponential j omega t times something.

So, the output will be proportional to the input in this particular case. This is true even if you have a differential equation, like you have capacitors and inductors in a circuit. So, you will have what the circuit described by a differential equation not an algebraic equation. Even then the output will be proportional to input as long as the input is exponential j omega t. Now, the point is that, this the relationship looks similar to that of a resistor right, for a resistor if you apply any V let us say V r, the current row the resistor is always proportional to V r, I r is G times V r or 1 over r times V r.

Now, for a capacitor also if the voltage is exponential j omega t, the similar type of relationship holds. This j omega C is analogous to conductance of a resistor. So, the point is now we have earlier had complicated circuits using resistors and we were able to analyse them. We have use node analysis, mesh analysis etcetera the set of the equations. We can invest some matrix and get the solution. Whereas, it was a little more complicated for circuits with capacitors and inductors, we ended up with differential equations, but now what I am saying is as long as the excitation is exponential j omega t the response will also be exponential j omega t times something.

So, the relationship looks similar to that of a resister and every technique that we have used for calculating quantities in a resistive circuit can be used here. Any trick we have all the circuit theorems we had for resistive circuits is also useful for these circuits. Is this clear or there any questions about this. This is the most important thing the algebra showed so far is rather trivial, but the important thing to realise is that for exponential j omega t, the response is exponential j omega t times something.

In other words the response is simply proportional to the stimulus. Any questions about this. Now, this exponential j omega t is some fictitious input I created by super posing cos and sin. My actual input is cos omega t. Now, I have to find the response to that, it turns out that, if you calculate the response to exponential j omega t finding the response to cos omega t is very easy.

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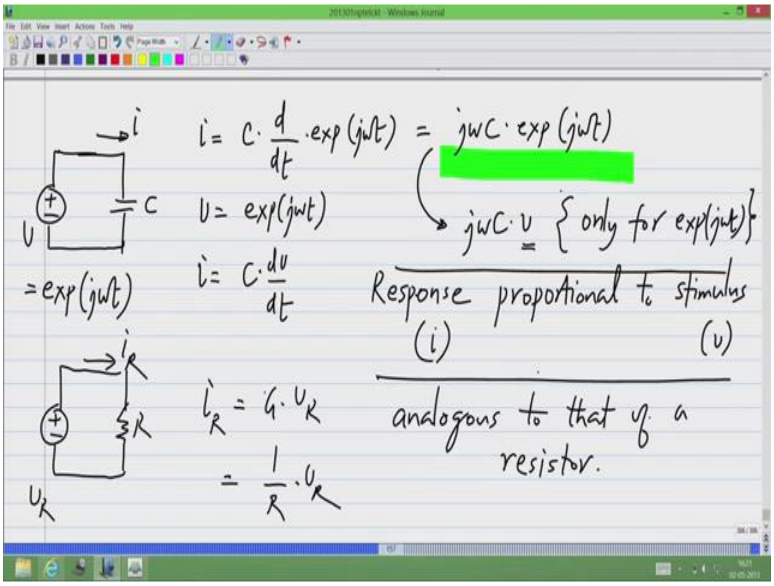

So, we generate this exponential by taking cos plus j times sin and the response also is the response to cos omega t plus j times the response to sin omega t. Now, in this particular set of calculations and this is true of any other calculation involving real circuits and real inputs all the coefficients everything will be real. The only place where this j or square root of minus 1 appears is in this when you take the super position. So, each of the responses will be real numbers and the total response to exponential j omega t is response to cos plus j times response to sin. So, the response to cos omega t can be simply obtained by taking the real part of the total response.

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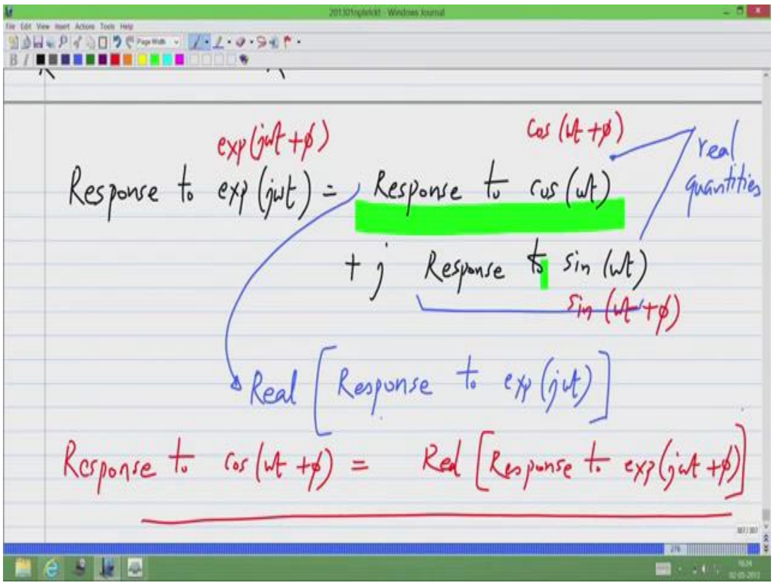

This is what we are interested in. Now, this it-self is some real quantity that means that it has 0 imaginary part and this part also is a real quantities. So, the response to cos omega t can be just obtained as real part of response to exponential j omega t. Now, that is the convenient part of it, the response exponential j omega t is easy to calculate because you get this proportionality between V and i for every component including capacitor s and inductors.

I showed it for the capacitor, I will quickly show it for the inductor also, the analysis of any circuit is like analysing a circuit with resistors. You do that any the calculate response to cos omega cos omega t is simply calculate real part of the response to exponential j omega t. This is much easier than applying cos omega t to your circuit and finding the response.

So, that is why this is extremely widely used and this analysis is known as sinusoidal steady state analysis and this will also lead to fazer analysis, any questions. Now, of course, you do not always want to calculate response to cos omega t, what I mean is the stimulus may not always be just cos omega t. It maybe cos omega plus phi or it could be sin omega t which can also be expressed as cos omega t minus phi by 2 radiance.

So, in general you may have a phase shift, but that is very easy to accommodate here because if you have exponential j omega t plus phi, that will be the response to cos omega t plus phi and j times response to sin omega t plus phi. So, the response to cos omega t plus phi would simply be the real part of response to exponential j omega t plus phi. So, we will see examples of this. Any questions about it the principle of the basic idea of sinusoidal steady state analysis. So, we will see examples of this, any questions about it the principle of the basic idea of sinusoidal steady state analysis.

So, there is a question about how do we visualise a complex resistance. The term is impedance, it is not called resistance anymore I will come to that. Now, what is the visualization for it, what it really means is that, in a resistor if you apply cos omega t as a voltage, the current will be cos omega t. There will be no phase shift, I mean a capacitor or an inductor or in general a circuit containing capacitors and inductors if the stimulus is cos omega t the response will be cos omega t plus some phase shift.

So, the complex impedance essentially has two numbers the magnitude and phase. So, the magnitude tells u how much the amplitude of the sine wave is altered and the phase tells you how much the phase is altered. So, you need two numbers where as in case of resistive circuits you only need to tell how much the amplitude is ((Refer Time: 22:13)) because the phase is not going to be changed at all, I hope that is clear. So, let me quickly do it for an inductor. If I have i 1 plus omega t plus phi I will take cos omega t phi directly in this case forced into an inductor.

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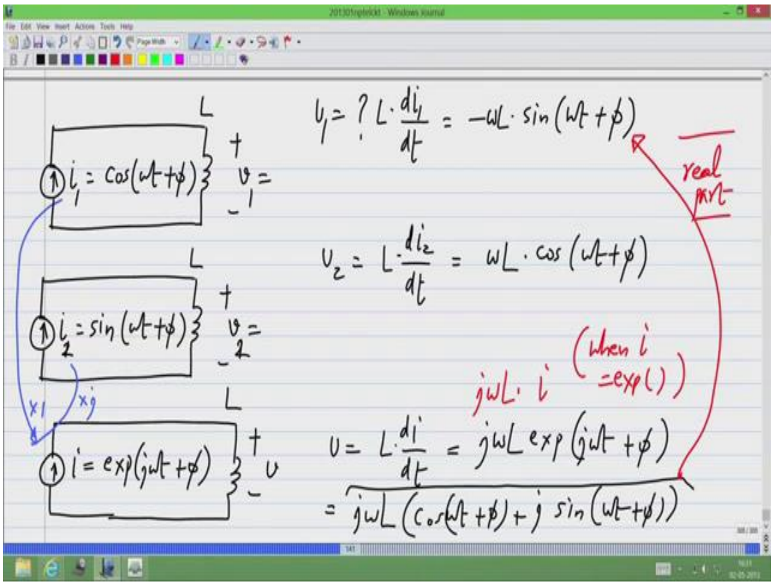

What will be the voltage across this the inductor value is l, what is the value of V 1. Please let me know it would be L time derivative of i 1 which is minus omega L sin omega t plus phi that is correct. Similarly, if I have sin omega t plus phi the voltage V 2 would be L time derivative of i 2 which will be omega L cos omega t plus phi. So, again if I form a new input which is exponential j omega t plus phi.

This you can think of is being formed by i 1 times 1 plus i 2 times j voltage V will be L time derivative of i, which is j omega L exponential j omega t plus phi. Now, I can quickly verify whether I said earlier is true, this V is nothing but j omega L times cos omega t plus phi plus j sin omega t plus phi and you can see the real part of this. The real part of it would come from the product of j omega L and j sin omega t plus phi and it is nothing but minus omega L sin omega t plus phi. You should take the real part that is what we will get. Also like I said the voltage will be j omega L times i when i is an exponential.

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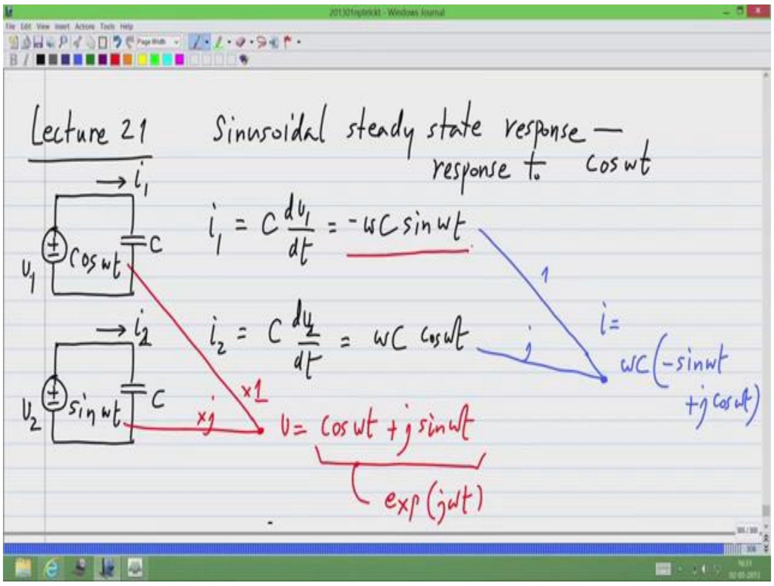

So, this works for inductors and capacitors, and also here like I said this response to cos omega t is minus omega sin omega t and that is exactly the same as the real part of this one.

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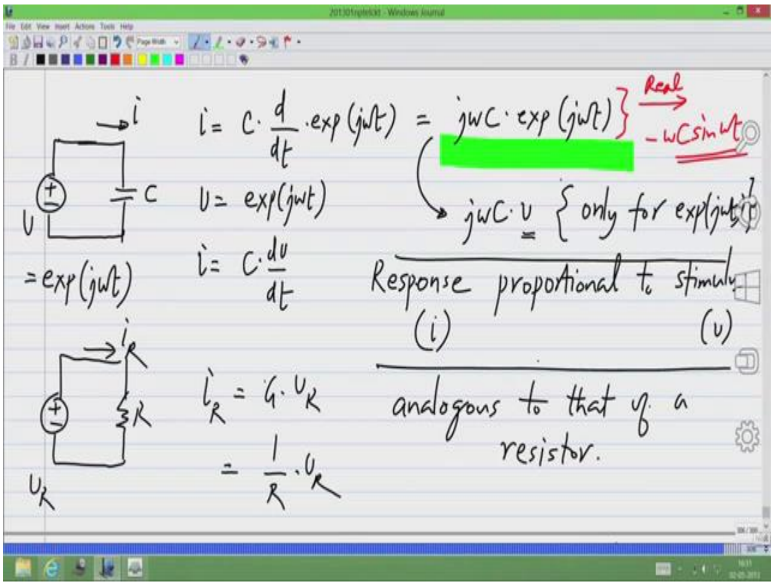

So, if you take the real part you will get minus omega C sin omega t. So, in general if you want the response to cos omega t plus phi.

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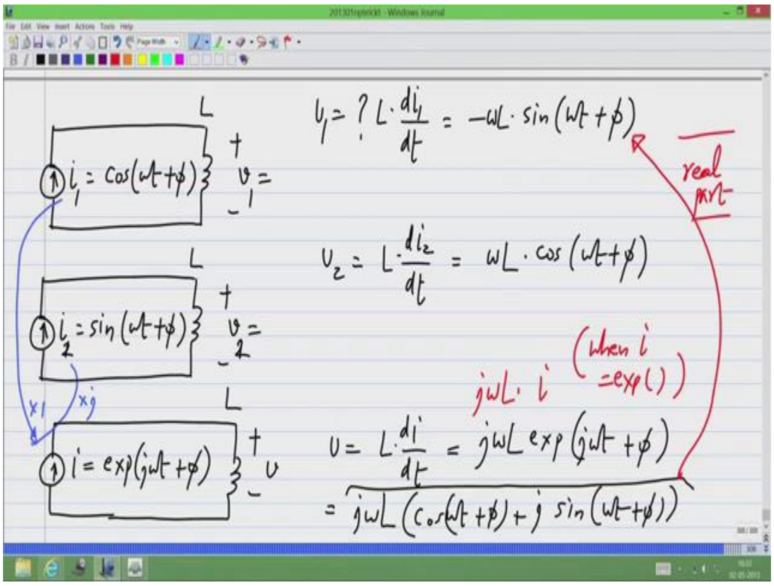

You find the response to exponential j omega t plus phi and take the real part. The reason we do this seemingly round about thing is that, the response to exponential j omega t plus phi is lot easier to calculate then for sin omega t or cos omega t. It does not involve any differentiation or integration or finding the solution to a differential equation. Now, here when I did it I showed it by differentiation, but you see that the voltage is just proportional to the current. So, by finding the proportionality constant will be able to solve it exactly as we did for resistors. So, I hope that part is clear.

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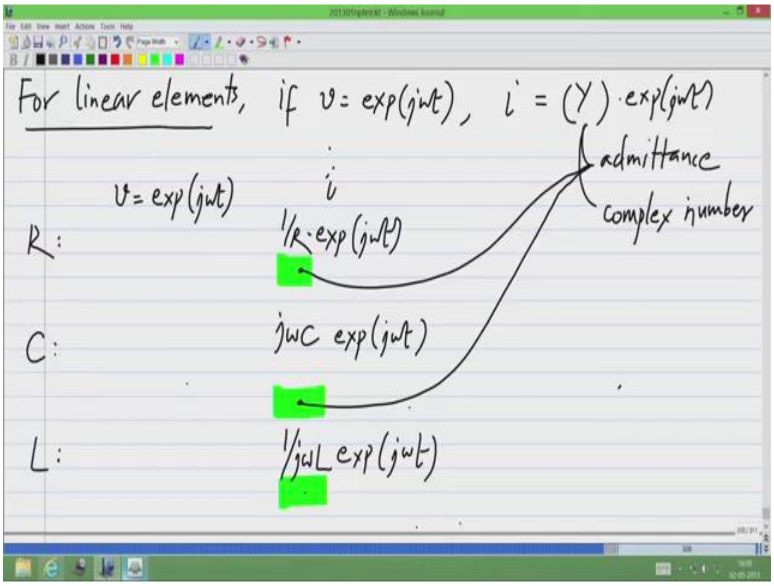

So, if for linear elements which is very important, everything here uses linearity otherwise we could not have use super position etcetera. If I is exponential j omega t, we will be proportional to exponential j omega t and is proportionality constant is in general a complex number it is known as the impedance.

It is in general a complex number. So, if you take a resistance, if i is exponential j omega t, then we would be r times exponential j omega t, for C if i is exponential j omega t we would be 1 over j omega C exponential j omega t. This is the inverse of what we calculated there we calculated i from V. Now, we are calculating V from i as long as they are proportional I just becomes a reciprocal and for L have j omega L exponential j omega t.

Now, the resister of course, was always proportional power. So, for a resistor V and i were always proportional and the proportionality constant was the resistance R. Now, it is also proportional for C and L. So, that is what I said and these numbers are nothing but the impedances of these components. The impendence of a resistor is nothing but the resistance itself.

The impedance of a capacitor is 1 over j omega C and the impendence of an inductor is j omega l. So, instead of going from i to V we can also go from V to i. So, let us say if V is exponential j omega t for a linear elements the current. I will be some number times exponential j omega t and that number naturally is also a complex number. It is denoted by y and that is known as the admittance and it is analogue is to conductance of a resistor.

If we evaluate for these three components, V is exponential j omega t, then I will be quite easy to calculate i 1 by r exponential j omega t j omega C exponential j omega t and 1 by j omega L exponential j omega t. These numbers 1 by r j omega C and 1 by j omega L are the admittances of these components, is this clear. Now, because of this proportionality, relationship between V and i the analysis of any circuit reduces to that of resistive circuits, any questions about this. Let us take a very simple example.

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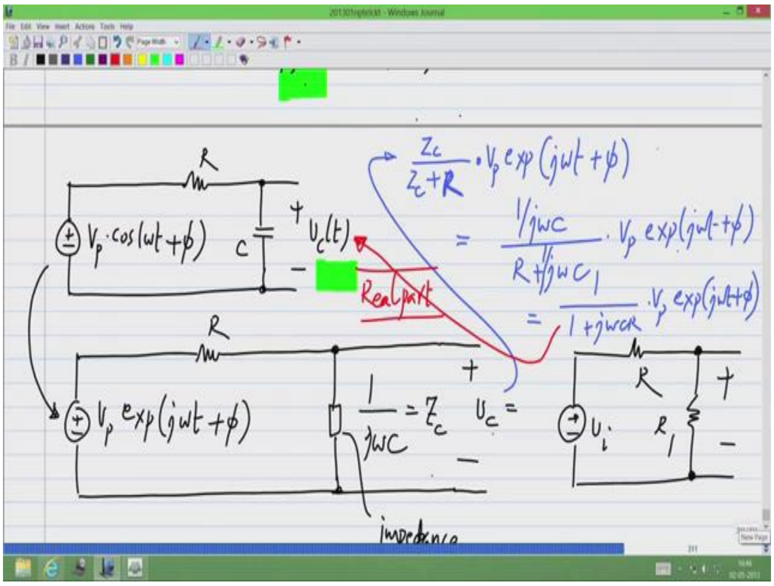

What I want to calculate, is the response of this circuit are C to V p cos omega t. Now, I could go and solve the differential equation, but the easier way, which is what I have been advertising all alone is instead of this. So, in this case I will get some V C of t, this is what I want to calculate. I will go about it in a somewhat roundabout way, but which is actually easier.

So, let me make it to be with general cos omega t plus phi and here it will be exponential j omega t plus phi. Now, the point is that because the excitation is exponential t omega plus phi. Now, voltage is in currents everywhere will be of the form exponential j omega phi times some number. So, I showed this for capacitors and inductors, but it is true of any linear circuit. So, then what I do is, because I have exponentials everywhere, I can use the proportionality relationship between V and i.

In case of resistors the relationship is the same as what it always was, V equals I r and in case of capacitors it will denote the capacitor by this box which stands for some impedance the impendence of the capacitors 1 by j omega C. So, this is the impedance of resister is just R. Now, because everything is like a resister, in the sense that the voltages and currents are proportional to each other.

We analyse this circuit just like we analyse resistive circuits, the only difference is that when we had purely resistive circuits all the constants were real. Now, we also have complex numbers as constants and we will interpret that properly. So, let me call this Z

C the impedance of the capacitor. So, what will be the voltage here V c, what is this going to be. What is the V C going to be in this circuit. When I have exponential j omega t plus phi like I said you can analyse this exactly as you would analyse resistive circuits and that is because V and i are proportionate to each other.

I am sure you would be it would be very easy for you to analyse. Some circuit like this where I had R and then some R 1, I asked you for the voltage here. This circuit is exactly except instead of R 1 we have this impedance X c. So, please tell me the value of V c, what would be the value of V c.

So, clearly you can use the voltage divider formula. This V C is going to be Z C by the G plus r times the applied voltage which is V p exponential j omega t plus phi and which can be further written as 1 by j omega C R plus 1 by j omega C V p exponential j omega t plus phi, which can be further simplified R plus 1 by j omega c. So, 1 by 1 plus j omega C r V p exponential j omega t plus phi is this. Now, if I take the real part of this, I will get V C of t. I will leave that to you to calculate it will be some sinusoid, but if you take the real part you will get the response to V C is this part clear, fine it is not very difficult to do. So, let me do it and show it to you.

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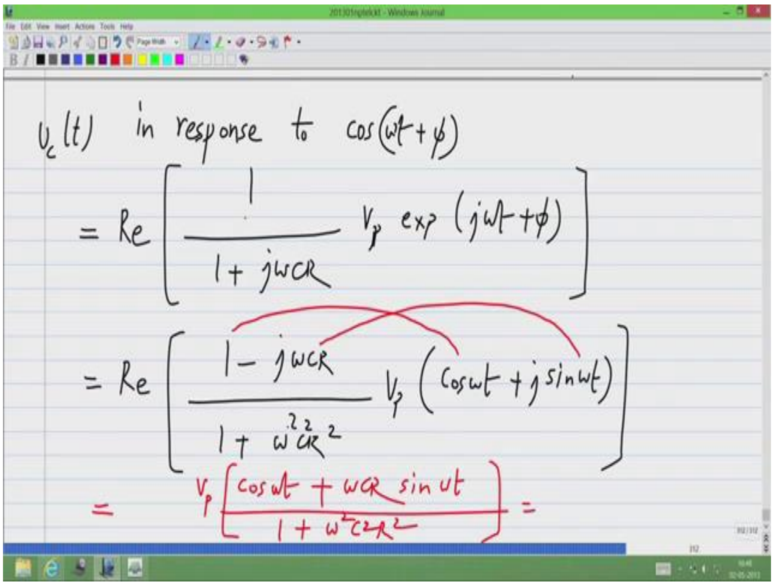

So, V C of t in response to cos omega t plus phi is real part of. So, in general for all this analysis, you need to brush up your manipulation of complex numbers as it is rather simple, but you still need to know to use them fluently. So, we will have we will first rationalise this part.

So, when you take the real part, you will get one term from the multiplication of these 2 and another 1 from multiplication of those 2. I will write out the answer right away which is that V p times cos omega t plus omega C R sin omega t divided by 1 plus omega square C square r square. So, this is what it is and you can always reduce this to single sinusoid with some phase shift is this ok. Any questions about any of this in fact we will find a simpler way to write down V C of t just in a moment.

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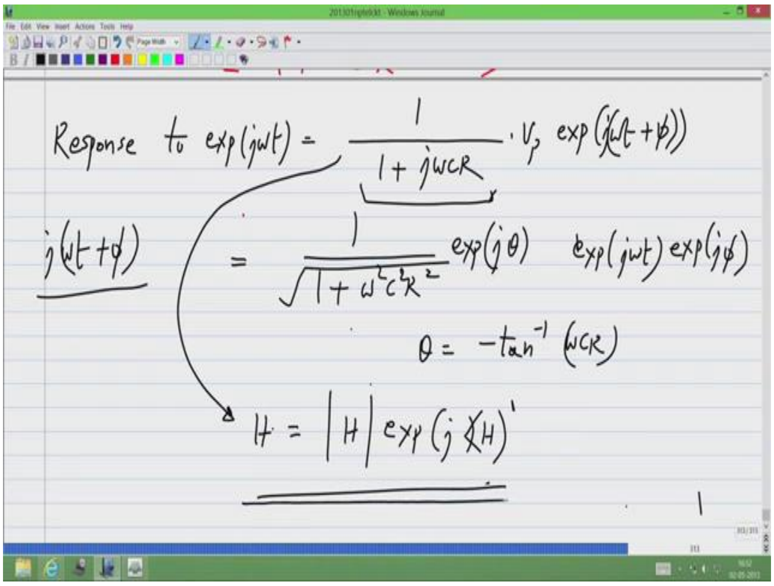

So, the easier way to do that is response to exponential j omega t. We saw was 1 by 1 plus j omega C R, V p exponential j omega t plus phi. In general, it will be some number times exponential j omega t. So, in this case I also have exponential j phi, by the way when I say j omega t plus phi, if j is multiplying phi as well. So, everywhere j omega t plus phi without brackets, but what it meant was that, it is j times omega t plus phi.

The real part of that is cos of omega t plus phi, that is what I had meant. Now, this there will be some complex number multiplying this exponential j omega t, which also it is easy if you write it in polar form. That is let us say this number is rather, let me do it only for this part of it can be written in the magnitude in phase form. The magnitude of this is 1 over 1 plus omega C R square root of that and the phase of that is exponential some j theta where theta is minus ten inverse omega C R.

So, in general whatever complex numbers, you have you write it in polar form. So, if I call this complex number as H it can be written as the absolute value H or the magnitude of H times exponential j angle of H.

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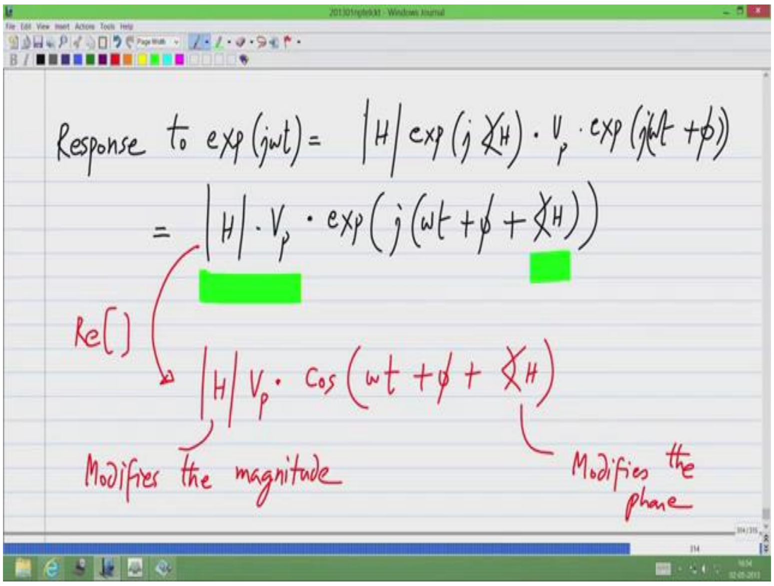

So, once you have that the response becomes magnitude of H exponential j times angle of H ties V p times exponential j omega t plus phi, which of course, can be written as magnitude of H times V p and exponential j omega t plus phi plus angle of H. So, you can clearly see the role of the magnitude and phase, earlier there was a question from Sharmila, about how do you interpret a complex impedance.

Now, in this case it is not an impedance, but any complex number multiplying exponential j omega t should be interpreted like this. So, the magnitude scales the amplitude of the sinusoid and the phase shifts the phase of the sinusoid because if I write it in this form taking the real part is quite trivial right. If I take the real part what do I get I will get a sinusoid whose amplitude is absolute value of H or magnitude of H times V p times co sin of omega t plus phi plus angle of H.

So, this modifies the magnitude and this modifies the phase. So, this is true for a single elements. Also if you have let us say the voltage across an inductor being j omega L times the current through the inductor. There is a modification of magnitude by omega land modification of phase by 90 degrees.

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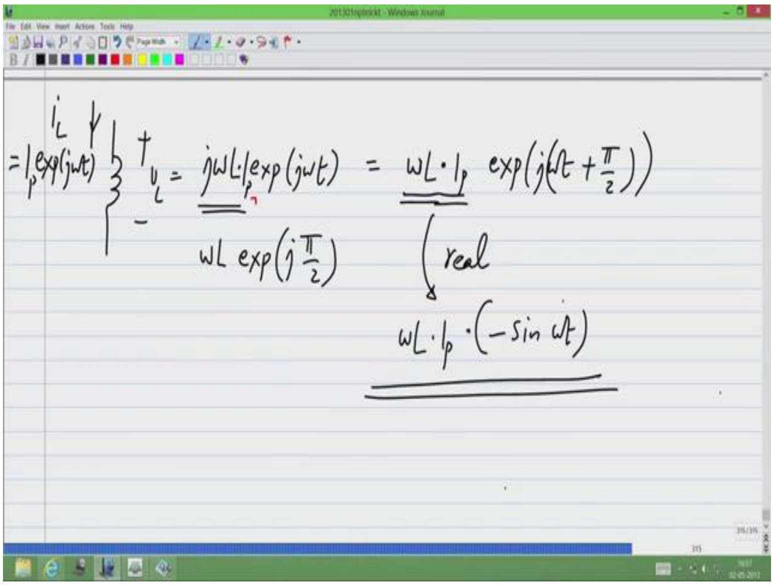

So, you can see that, this can be written as omega L exponential j phi by 2. So, this V L becomes omega L times exponential j omega t plus pie by 2. Let me just for clarity say that the peak value of the current is i p then we will have i p here also and here. So, you can see that the magnitude of the impedance omega L modifies the magnitude of the voltage of amplitude of the voltage.

So, and then the phase of the impedance which is pi by 2 modifies the phase of the sinusoid. So, if I take the real part of this I would get omega L i p which is the amplitude of the sinusoid of the voltage times exponential j omega t plus pie by 2 which is minus sin omega t. So, I hope this is clear, if not you just do a calculation with a couple of inductors and capacitors and it will become very clear. So, if there are any questions on these i will take them now.

Now, you see that everything becomes as easy as analysing a resister, resistive circuit only thing is you have to handle complex numbers that is all and that is not very difficult, but what you should not lose out in all this algebra is the connection to reality, which is that you are trying to find response to cos omega t plus phi. That you do by finding the response to exponential j omega t plus phi and taking the real part. If you take the real part of that you will get some sinusoidal wave forms which is the response to cos omega t plus phi, any questions. So, just to summarise

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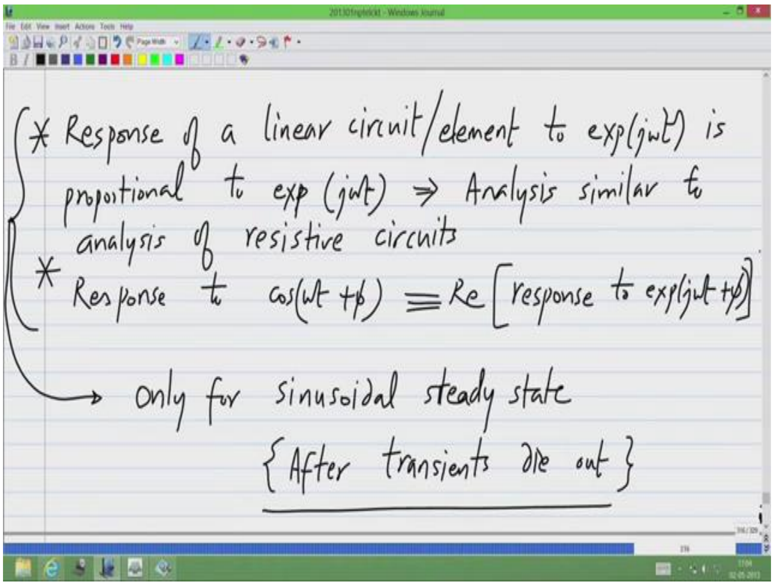

This implies analysis similar to analysis of resistive circuits and response to cos omega t plus phi basically can be obtained by taking the real part of response to exponential j omega t plus phi. Finally, we should not forget why we did all of this, where this is applicable and this is only for sinusoidal steady state. That means that I apply sinusoids which is omega t plus phi and then you find the steady state, which is after the transients die out. I hope this part is clear.

Now, from here we it turns out we do not even have to write exponential j omega t everywhere because it appears everywhere in all the expressions. So, we will omit that and the resulting stuff is what is known as a fazer analysis. So, let me take this R C circuit again and if I have I will just use exponential j omega t, but whatever I say will also apply when I have plus phi over there.

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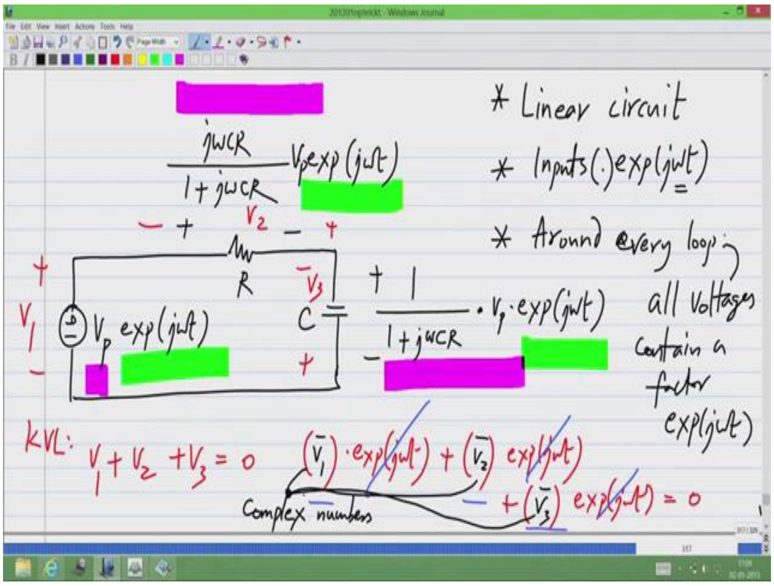

I will just write out the solutions, you can verify these things yourselves. Here we saw the voltage was 1 over 1 plus j omega C R V p exponential j omega t and here it turns out, it will be j omega C R by 1 plus j omega C R exponential j omega t. Now, do not worry about the details, what I want to point out is that if you have a loop, where the input to the circuit. So, I am assuming that we have a linear circuit and the inputs are of the form all inputs must be of the form exponential j omega t, that is at the same frequency same omega. If you have multiple inputs its, but they should all have the same frequency.

So, if you have multiple sinusoidal inputs of the same frequency the inputs all will be of this form. Here may be some multiplying constant, but they will all be of this form. So, then if you look at any loop every voltage across elements in a loop will have exponential j omega t right. Here I have illustrated this with exponential j omega t here, there and there. Now, this should be predict or because I said any response in linear circuit will be proportional to exponential j omega t if the input is exponential j omega t.

So, you take a loop and you only apply exponential j omega t. So, every branch voltage would consist only of exponential j omega t times something. So, everything will be like this. So, the point of this is that what k V L says Kirchhoff's voltage law. It says that sum of voltage is around the loop equals 0. So, let me call ah these voltage drops let us say V 1 V 2 in that polarity and V 3 V 1 plus V 2 plus V 3 is 0 in this loop.

This is the statement of k V l, I know that V 1 is something times exponential j omega t and V 2 is something else times exponential j omega t and V 3 is yet another things times exponential j omega t and the sum is 0. Now, exponential j omega t is a common factor to all of these I can omit them and do any calculations only with these numbers.

That is I do not need exponential j omega t at all, what I need are these numbers this, this and this in fact I omitted V p here V p should be here as well. So, in general these numbers multiplying exponential j omega t will be some complex numbers. I will denote that by V 1 bar V 2 bar and V 3 bar V 1 V 2 V 3 V bar will be some complex numbers.

I can omit this multiplying factor exponential j omega t and right k V L only in terms of these complex multipliers of exponential j omega t. Now, it is very clear that exactly the same holds for k C L as well because all currents will have exponential j omega t and k C L says that the sum of all currents entering an odd or leaving an odd is 0. Now, because all currents contain exponential j omega t, I can simply rewrite the currents in terms of the sum of the complex multiplying factors.

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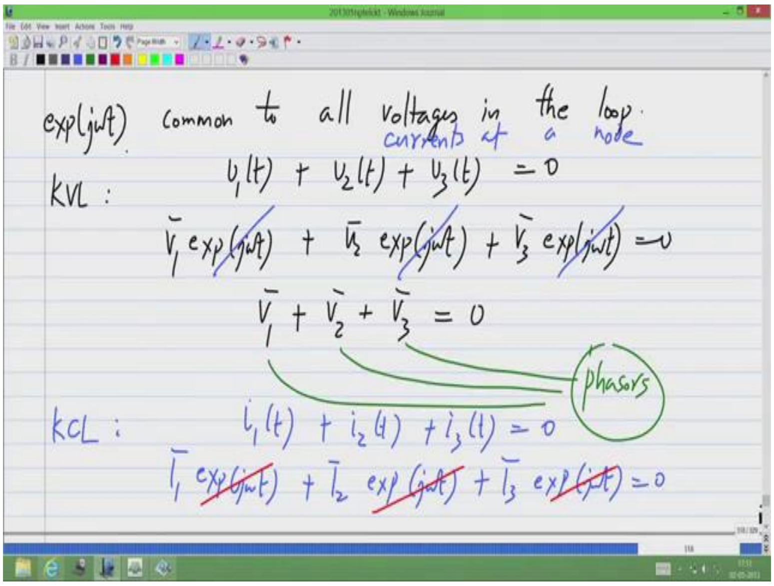

So, k V L which says V 1 of t plus V 2 of t plus I will take the 3 voltages equals 0 will reduce to some complex number V 1 exponential j omega t plus some complex number V 2 exponential j omega t plus V 3 exponential j omega t equals 0. Now, this is common. So, i get rid of that. I can simply write k V L in terms of the complex multipliers.

Remember V 1 is not the voltage V 1 times exponential j omega t is the voltage, but that appears as common everywhere. So, I will ignore that all together and I will write V 1 plus V 2 plus V 3 to be 0. Again the algebra is quite simple is and I think you can learn it very easily, but it is also important to figure out what is the meaning of this. What do this complex numbers mean and it turns out that this is a very widely used analysis. These complex number which are multiplying exponential j omega t are known as fazer.

Similarly, just like this exponential j omega t is common to all voltages in the loop. It is also common to all currents at a nod. The statement of k C L which is i 1 of t plus i 2 of t plus i 3 of t equal to 0 becomes i 1 exponential j omega t plus i 2 exponential j omega t plus i 3 exponential j omega t equals 0, from which I will again remove the factor exponential j omega t. So, i 1 plus i 2 plus i 3 which are complex numbers 0 and these i 1 bar i 2 bar i 3 bar are the fazer of currents.

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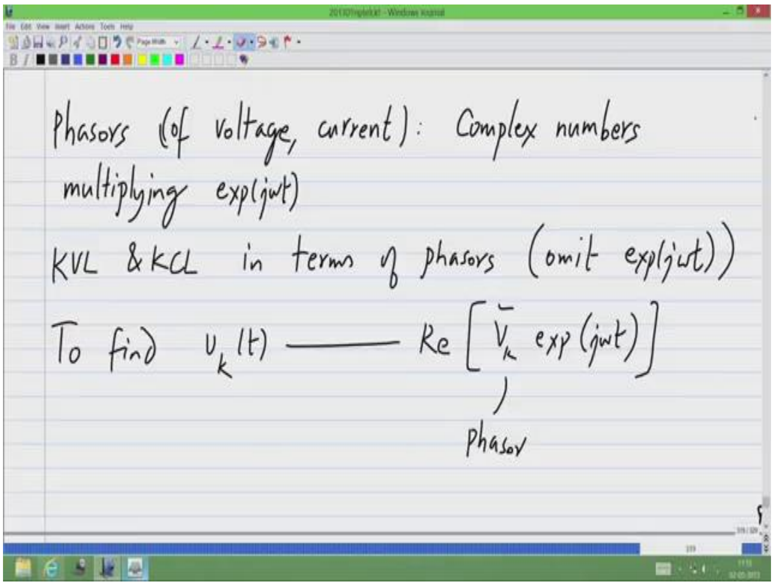

Now, what happens is that everything just looks like DC circuit analysis containing DC analysis of a circuit containing only resistors, but once you see let us say fazer i 1 to figure out what the voltage is you should take real part of i 1 times exponential j omega t. So, we can write k C L and k V L in terms of fazers basically omit exponential j omega t, which mean that to find some voltage V k of t you should take real part of the fazer of V k timers exponential j omega t. This omega is something that you need to know.

The very important thing is that, even if you have multiple sources all of them should be at the same omega, that is when you will have all the voltages and currents being proportional to just exponential j omega t. If you have omega 1 and omega 2, you will have exponential j omega 1 t and exponential j omega 2 t in different proportions and you will not be able to cancel them.

If at all you find the situation where you do have multiple frequencies let us say omega 1 and omega 2 because these are linear circuits what you can do is, first you disable all the sources of omega 2 and do the analysis of only the sources at the frequency omega 1. Then you disable all the sources at omega 1 and analyse the circuit for frequency omega 2. Then you add up the results separately after finding out the real part of fazer times exponential j omega t, is this fine.

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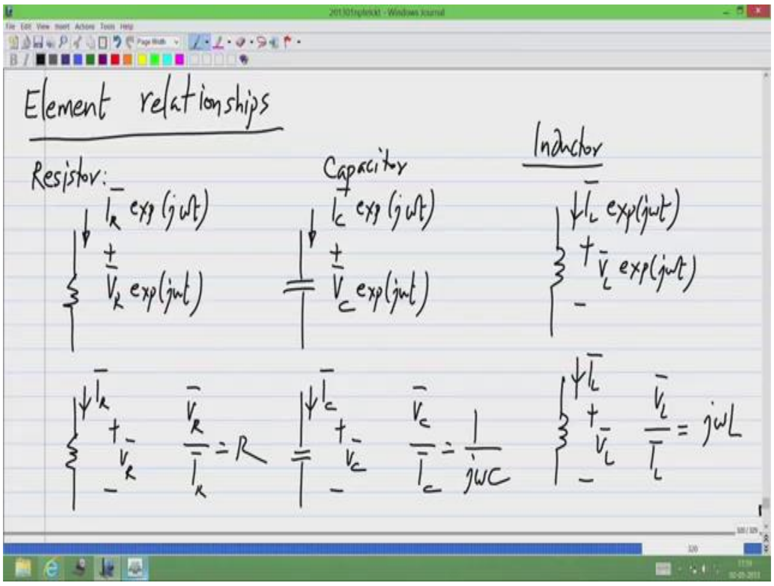

So, again I will show this with very simple example, first of all the element relationships these we have already worked it, but I will show it just for the sake of clarity. The resistor if it has some voltage V r exponential j omega t are the current I r exponential sorry, V r bar exponential j omega t and I r bar exponential j omega t, we omit exponential j omega t and simply write I r and V r. The relationship of course, is the familiar relationship of the resistor V r by I r equals R.

If, we have a capacitor and we will have fazer size C and V C the fazers will be related by 1 over j omega C. I will have the inductor current and the inductor voltage, which are fazers times exponential j omega t. The relationship between these two with only fazers would be V L by I L equals the impedance of the inductor which is j omega l. So, we can omit this exponential j omega t from everywhere. Do our analysis and only when you need the expression for the signal in the time domain as a function of time, you need to take this fazer times exponential j omega t and take the real part. Any questions about this? So, let us take a simple example.

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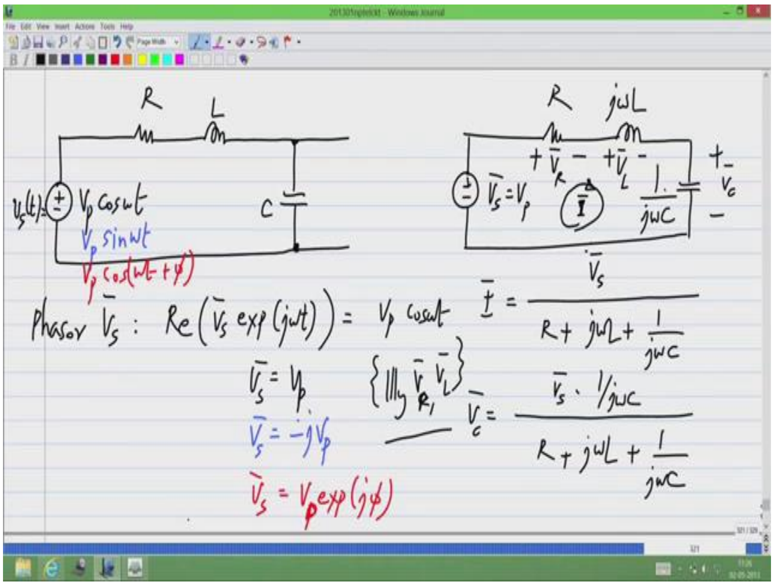

So, let me take the series R L C circuit and I want to find the response to V p cos omega t in steady state. Now, if I want V p cos omega t, what is the fazer corresponding to this the fazer is such that. So, let me call this V s of t lower case V s and the fazer of V s of t is such that real part p of V s exponential j omega t is V p cos omega t.

It is pretty clear what V s is, V s must be equal to V p. So, if you have a source cos omega t V p cos omega t, the fazer corresponding to that is V p. Alternatively if you had V p sin omega t the fazer would, what would be the fazer corresponding to V p is sin omega t, what is the fazer. The fazer multiplied by exponential j omega t should give you the voltage. I work it out for V p cos omega t, if this has to be V p sin omega what should be the fazer V s V s bar. Please try and answer this question.

Clearly this will be minus j V p I mean general if you need the fazer of V p cos omega t plus phi the fazer corresponding to that would be V p exponential j phi. So, this is what we will use as the source. So, I will take this V p cos omega t. So, the source itself would be represented by a fazer V p and the resistance will remain as a resistance. The inductance basically is an impedance j omega L and the capacitance is an impedance minus it is 1 by j omega C, which can also be thought of as minus by omega c.

Now, each of these has some voltages across it, which is represented by fazers V r V L and V C and there will be a there are single loop and there is a certain current i flowing through the loop. When I say current i this, i is the fazer and the actual current is exponential this i bar times, sorry real part of i bar times exponential j omega t. So, i would be V s by r plus j omega L plus 1 over j omega C. It is just V s divided by the total impedance and V C would be V s times 1 over j omega C r plus j omega L plus 1 over j omega C and soon.

So, you will be able to find all of these things very easily. You see that this circuit is just as complicated to analyse as a single loop of three components, three resistors in series. So, that is all that is there to it, this is how you do fazer analysis, but please do not forget the connection to reality, which is that if you want the voltage across the capacitor you take real part of the fazer V C times exponential j omega t. Any questions, these also can be formed there is a reason I took this second order circuit. We will take up exactly what this things do to, the input signal in later lectures. Just for completeness I will give you another example.

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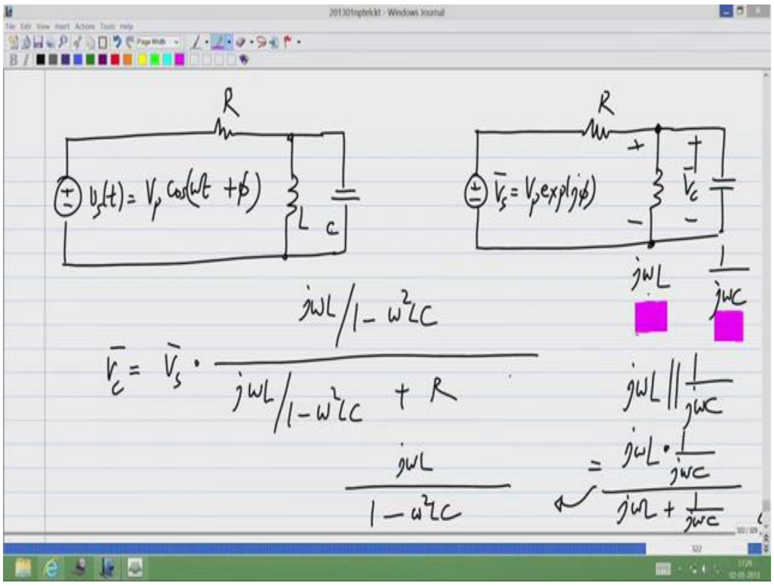

In this case, let me take V s of t to be V p cos omega t plus phi. This time I will take a parallel R L C circuit as I mentioned earlier if you null the input source if L C r are in parallel it is a parallel R L C circuit. So, in terms of fazers I would have to replace V s by it is fazer, which is V p exponential j phi R will remain as an impedance r L will be j omega L and C will be 1 by j omega c.

Now, similarly, everything can calculated. So, for instance if we want the voltage across the inductor which is also the same as the voltage across the capacitor V C, what do we have it is V s times, the parallel combination of these two impendences is j omega L parallel 1 over j omega C which is j omega L times 1 over j omega C divided by j omega L plus 1 by j omega c.

Which can also written as 1 minus omega square L C by taking j omega C to the top. So, I will have j omega L 1 minus omega square L C j omega L 1 minus omega square L C plus r. It is just an impedance divider instead of a resistive divider. We have parallel combination of these 2 impedances and that divided the total impedance which is R plus the parallel combination. So, let me just complete this before ending the lecture, which I will take R to the numerator and we will get something like.

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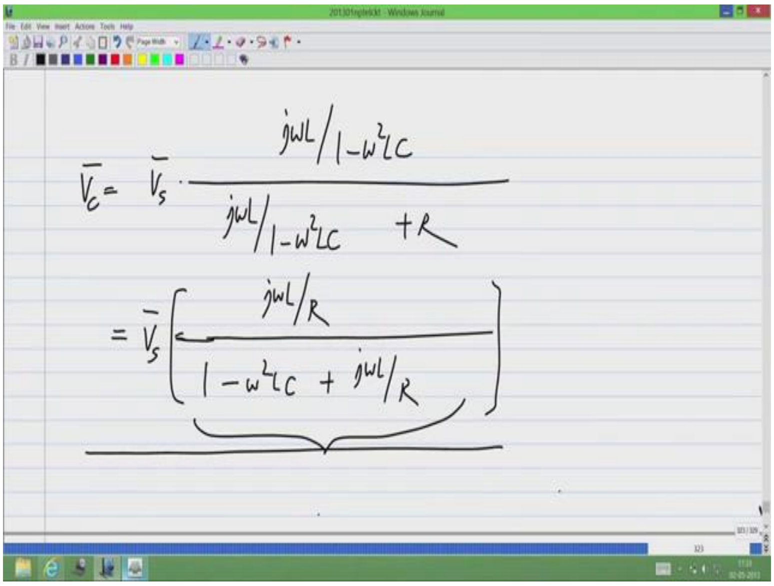

Now, again what does this mean, there is a complex number multiplying V s which will modify the amplitude and phase of the sinusoid. If V s is a sinusoid V C will be a sinusoid at the same frequency and it is amplitude and phase will be modified how much they will be modified by that will depend on the amplitude and phase of this number the magnitude and phase of this complex number.

So, I hope that part is clear that brings us to the end of this lecture. So, the summaries as follows. If you apply exponential j omega t to a linear circuit every voltage and current will also be of the form exponential j omega t. This gives us a lot of simplification and analysis exponential j omega t itself is first of all a fictitious input. In the lab we would apply sin cos omega t etcetera, but the response to cos omega can be obtained from the response to exponential j omega t by taking the real part.

This is because all the co efficient all the component values everywhere are real and assured by super position, that you can think of exponential j omega t as combination of cos omega t and sin omega t. So, that is number 1, secondly because for every element voltage and current will have exponential j omega t, voltage becomes proportional to current, just like that for a resistor. So, even though you have inductors and capacitors those circuits can be analysed, in a way that we use for resistive circuits, which you agree was very because we did not have any differential equations.

The only ((Refer Time: 01:27:22)) here is that this applies only when the input is of the form exponential j omega t, that is for the sinusoidal inputs and in steady state because the transient response has to die out. And that we cannot calculate using this method, but all the usual circuits that we consider in this course, will have transient response that die out over time.

Finally, because this exponential j omega t is common to everything, all voltages and currents we start writing Kirchhoff's current laws and voltage laws by omitting the exponential j omega t all together and we will consider only the complex multipliers for each of these voltages and currents and those are known as fazers. This analysis using fazers is knows as known as fazer analysis and it is very convenient to use whenever you have sinusoidal inputs to a linear system.

So, any questions about this I will answer. Now, there is a question that says that fazers have a relationship to Laplace or Fourier transform. Now, the Laplace transform analysis of systems is derived from a similar reasoning that, if you have exponential s t given as an input to a linear system, the response will also be of the form exponential s t times something. This is true for any linear differential equation, if you have a linear differential equation with a input of exponential s t the solution will also be exponential s t times something, the steady state solution.

So, that is the relationship. Now, what is the exact relationship. If you take Laplace transforms, if you do Laplace transform analysis of a circuit and substitute s equal to j omega you will this essentially this fazer analysis. Now, the Laplace transform and also Fourier transform analysis can also be used for signal which are not sinusoids. The way we have presented it is only for sinusoids. So, if you reduce, if you use the Laplace transform for analysis of sinusoidal signals, essentially what you get is the fazer analysis.

So, you can think of this subset of Laplace or Fourier transform. Now, Laplace transform is used for circuit analysis. The Fourier transform in particular is mainly used for signal analysis there is another thing, which we not touched at all. Now, we have showed that you can analyse circuits with sinusoidal inputs using fazer analysis. There is also the Fourier transform, which is that any signal can be thought of as a combination of sinusoids. So, like if we square over a triangular wave can be thought of as a combination of sinusoids. So, what they Laplace transform on Fourier transform analysis do is to, combine these two ideas, that the response exponential j omega t is exponential j omega t and any signal can be decomposed as some combination of exponential j omega t. With this basically you can analyse any circuit, any linear circuit with any input using Laplace transform and also may be less conveniently using Fourier transform.

So, this fazer analysis is a sort of sub case of a Laplace or Fourier transforms, when you restrict the input to be single frequency sinusoids. I hope that answers the question. So, then break. Now, it turns out that for all the institution, which were subscribed to the online lecture the exams have already started. So, this will be the last online lecture, but we will continue to post the lectures, but recording it offline and we will also have the assignments and you will have about three weeks or a month to complete them. So, please watch the web page for these details there will be up shortly.