Basic Electrical Circuits
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Lecture - 20
General Formulation of Second Order (RLC circuit) Natural Response
Natural Frequency and Damping/Quality Factor; Series/Parallel RLC Circuits
R, L, C in Sinusoidal Steady State

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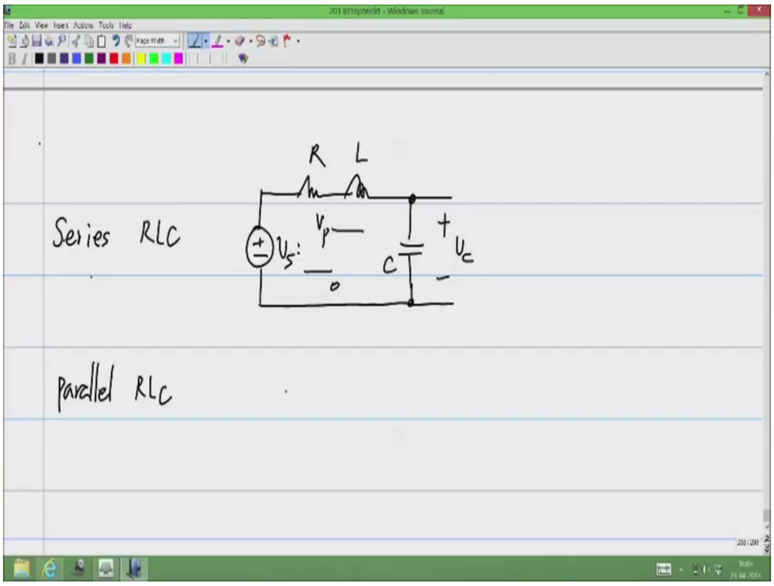

Series R L C circuit, we took to be this and I wrote the differential equation with the capacitor V c as the desired variable and this V s steps from 0 to V p, but in general it could be anything. We are now looking at a constant input and a parallel R L C. We could have this I believe, this is the example I took last week as well. Let me just check, again I will write the differential equation in terms of V C as the variable.

So, we will have in the first case, this is the differential equation we get. Mainly we are now interested in the left hand side of this. The right hand side when equated to 0 you get the homogeneous differential equation. The natural response can be obtained from the homogeneous differential equation. In this case the coefficients will come out to be slightly different in the parallel R L C case. So, this what we end up getting again.

The right hand side is not so important. The left hand side it has the same structure as this one, these 2 look similar to each other. Now, in general differential equation of the second order will be of the form. I am assuming the homogeneous case, that is the right hand side is 0. Now, we know that and a differential equation of this type has solutions of the form exponential p t.

Now, we know this because the time derivative of exponential p t is also exponential p t times some number. So, if you go on differentiating you will go on getting exponential p t and it is quite easy to see that, but could potentially satisfy this equation, for the right value of p. So, how do we find the right value of p. We substitute exponential p t in the differential equation. All the exponential p t will get cancelled out and we will be left with a polynomial in p which is known as the characteristics equation of the differential equation. So, what we get if we do that here.

If substitute exponential p t in this, we will get a 2 p square plus a 1 times p plus 1 equals 0. So, all we have to do is the second derivative is replaced by p square, first derivative I p and this b C itself by 1. So, clearly this will have 2 solutions. There will be 2 possible values of p that will satisfy this. They are given by p 1 p 2 are given by the solutions to this quadratic equation, which is minus a 1 by 2 plus minus, this a 1 square root of a 1 square minus 4 times a 2 divided by 2.

This counts from the very familiar solution to the quadratic equation. Now, again there are 3 possibilities to this. The term on the square could be positive, 0 or negative. Depending on that they will have 2 real roots complex conjugate roots or 2 identical real roots. So, a 1 square greater than 4 a 2 or in other words square root a 2 by 2 a 1 less than 1 or let me put it in another way. Let me copy this over. So, what I have done is, to write the differential equation with some general coefficients a 1 and a 2.So, that it is not tight to some specific circuit.

Previously we got these coefficients in terms of L C and R, we will do that when we go the specific circuit, but this will tell you that it is not specific to R L C circuit. It will be the case for any circuit that has a second order differential equation describing it. We will connect it to the actual coefficients and different circuits. Now, in this case we have 3 kinds of solutions.

One is when even square is more than 4 a 2. This can also be written as, square root of a 2 by a 1 less than half or equivalently a 1 by 2 square root of a 2 greater than 1. So, these are all equivalent statements. I hope that is clear, this inequality says exactly the same thing as this which is exactly the same thing as that. Now, in this case will have 2 distinct real roots. This term under the square root is positive, that means this square root is a real number, that means both this p 1 and p 2 will be real numbers.

This will be different from each other because 1 comes from a plus sign the other comes from a minus sign. Now, what I would like the participants to do is to tell me whether both p 1 and p 2 will be negative or 1 will be positive and 1 will be negative or will both be positive. Please look at this numbers and what I will tell you is a 1 a 2 are both positive this we know for a series R L c. So, a 1 was R C and a 2 was L C because these are real component values which are positive, these numbers will be positive.

So, please tell me if both of these will positive both will be negative or positive and 1 negative. For this particular case, what do you think there is a correction to what I wrote here. This is the roots are I should have a 2 in the denominator, thanks for providing the correction. Now of course, this does not change this inequality in anywhere. So, please let me know if the roots will be both positive both negative or 1 positive and 1 negative.

So, it should be clear that both the roots will be negative because a 1 and a 2 are both positives. So, this number is positive, sorry this number is negative. This the square root of this if we take plus this square root of this we can see that the second 1 will be positive, but smaller than this negative number. So, the overall result is negative even we have minus signs naturally the whole thing will negative also. The answer is that both p 1 and p 2 will both be negative, in this if a 1 and a 2 are positive.

So, we will have distinct real roots and both negative what does it mean for these roots to be negative, it means that exponential p t, if p us negative this means that this response will die out with time exponential p t it decays with time. So, that for that is the implication of p 1 and p 2 being negative, both are negative which means that eventually the natural response will go to 0. This was the case with the R L C circuit and this is also appears with this 1 the R L C circuit.

I hope it is clear why both the roots are negative, when both terms are negative of course, it is obvious when the first term this is negative and we take the plus sign here. This entire term is smaller than this. So, the overall result is still negative. Now, these are all things that we evaluated last time, we did it specifically with values of L C and R. Now, I will do it generally with a 2 and a 1 and show how the two case of series and parallel R L C are different from each other.

This k s 1 a 1 a 1 square equals 4 a 2, what this does is to make the term under the square root 0. So, both p 1 and p 2 will be equal to minus a 1 divided by 2 times a 2. Of course, that will be negative because we have minus a 1 by 2 a 2. And finally, when a 1 square than 4 a 2 in other words square root a 2 by 1 is greater than 1 greater than half or a 1 by 2 square root a 2 is less than 1 we have the term under the square root is negative.

So, the square root itself will be an imaginary number and will have the 2 roots to be minus a 1 by 2 a 2 plus j square root for a 2 minus a 1 square I will reverse the sign inside the square root by 2 a 2. The other 1 will be minus a 1 by 2 a 2 minus j square root 4 a 2 minus a 1 square by 2 a 2. So, more than the exact values of this, what you have to notice is, we have plus j times some number here minus j times, the same number here.

The real parts of the two are the same, the imaginary parts have opposite signs. So, what we have are a complex conjugate pair of roots. So, we have three different cases and there is a reason I wrote the same condition in 3 different ways. This is in terms of coefficients here are defined some parameter less than half and here are defined some other parameter greater than 1. The reason to do that is, that these parameters this 1 and this 1 are some standard parameters that are very widely used.

So, if you write the differential equation like this, let me copy over that. If this is the differential equation a 2 being the coefficient of the second derivative, a 1 the first derivative and 1 is the coefficient of e C. This is important, than the characteristic equation of this is of this form. We have to normalize is this to 1, that is important that is the way we have done it. Then this parameter, square root a 2 by a 1 is known as the quality factor q. It is usually denoted by q, the quality factor and square root a 1 by 2 square root a 2 is known as ((Refer Time: 21:22)) the damping factors.

So, these are constants that are used quite commonly to describe a second order system. So, square root of a 2 by a 1 is known as the quality factor and a 1 by 2 square root of a 2 is known as the damping factor. So, clearly you can write the 3 conditions, these conditions in terms of either the quality factor or the damping factor. So, the same thing is written as q less than half or jaye greater than 1.

Here the conditions will be opposite. Now, we have the series and parallel R L C circuits. Let me copy these things over. So, please tell me what is the quality factor of the series R L C circuit, in terms of the component values in terms of R L and C. Please give me the explanation for the quality factor based on the definition I just gave you. The definition is here, this is the definition corresponding 2 differential equation.

So, what I would like is the quality factor for the series R L C circuit. What is the expression for a quality factor. So, this is a 2, this is a 1 and the coefficient of V C is 1. Like I said, if it is not 1 you have to normalise that by dividing the whole thing by that term. So, the quality factor of the series R L C circuit which is square root a 2 by a 1 is given by, 1 by R square root L by C and the damping factor jaye is given by R by 2 square root C by L. Other couple of you were able to answer this, quite easily.

It is just a substitution of terms. Now, what I would like is the quality factor of the parallel R L C. What is the quality factor of the parallel R L C circuit. I wrote a couple of answers to this as well and it is turns out, the formula is the same as before, it is a square root of a 2 divided by a 1. That would be R square C by L. The damping factor jaye would be 2 by R square root L by C. Also by the way, it is pretty obvious that from these relationships, from these 2 that the damping factor jaye is 1 by 2 times the quality factor and vice versa.

So, the exact damping or quality factor will depend on this coefficients. In both this cases there are L R and c, but you see that the expression for quality factor in fact is the reciprocal of 1 another in the 2 circuits ok. So, that is why I define the quality factor and the damping factor based on the coefficients a 2 a 1 and 1 this term has to be normalised 1 then the second order term is a 2 first order term is a 1, then the quality factor is square root a 2 divided by a 1. Now, depending on the circuit that you have it may not even have R L and C. It can have 2 capacitors or 2 inductors and. So, on you can find the quality factor or the damping factor.

So, especially what I want to emphasis is that, the quality factor expression is different for a series R L C circuit and a parallel R L C circuit. So, do not try to learn any one of these formulas by heart. Depending on whether it is parallel or series the damping factor will be different. Now, one sanity check you can use is the following. The series R L C circuit looks like this and I have nulled the source and quality factor is given by 1 over R square root L by C.

Now, if R tends to 0 the quality factor tends to infinity and if R tends to 0 and the circuit, what will have is only L and C in parallel with each other. Now, if I take it parallel R L C circuit, let me close this and open it again, there is some problem. If I consider a parallel R L C circuit which is nulled, quality factor of this is R square root C by L. If R tends to infinity q tends to infinity and in the circuit will have only L and C in parallel with each other.

So, what I want to emphasis here is that, when quality factor tends to infinity you will have only L and C in a loop. So, the quality factor can tend to infinity if R as 0 or R as infinity. If R as 0 and a series R L C circuit, then you will have a single loop of L and C. The quality factor of that is infinity, if you have R equals infinity in a parallel R L C circuit you will have a single loop of land C and a quality factor of that is infinity.

So, this is another sanity check, that you can use at the circuit level when the quality factor is infinity, that is let us say you get a certain expression for quality factor and adjust the resistance value. So, that quality factor goes off to infinity. Then in the circuit what should be happening is that, you should be left with a single L and a single C in a loop, it should be a lossless loop and q equal to infinity means R equal 0 in this series case and R equals infinity in the parallel case.

So, mainly I am emphasising this over and over because the formulas for quality factor for series and parallel R L C circuits are opposite of each other, but there is nothing contradictory or confusing, there should be nothing confusing about this. It is just that when the quality factor tends to infinity you should be left with pure L and C in a loop. That happens when the parallel R L C is resistance R as infinity or the series R L C is resistance R s 0. I hope that part is clear now. Let us take the k s of quality factor less than half which means the damping factor more than 1. It basically means that a 1 square is more than 4 a 2. So, in this case p 1 and p 2 will both be real and negative. Now, the natural response will be of the form a 1 exponential minus p 1 t plus a 2 exponential minus p 2 t and the constants a 1 and a 2, you can adjust using initial conditions.

Now, qualitatively if you look at this curves, exponential minus p 1 t might be like this and exponential minus p 2 t might be like that. When I plot them was such time. So, the combination a 1 exponential minus p 1 t plus a 2 exponential minus p 2 t. It could be of depending on the initial condition I will assume it starts from the, it could be of some form like that.

It will be just a combination of exponentials or let me take the other case. When the quality factor exactly equals half or damping factor equals 1 and a 1 square equals 4 a 2. In this case it turns out that p 1 and p 2 are real and identical. The natural response it turns out because we have only a single value of p 1, that is p 1 and p 2 are real and identical. The natural response it turns out consists of this form a 1 plus a 2 t times exponential minus p 1 t.

So, this means p 2 equals p 1. So, if you say exponential p 1 t and exponential p 2 t they will be the same as each other. So, the actual natural response will have a 1 plus a 2 times t times exponential p 1 exponential minus, sorry not minus p 1 exponential p 1 t. Then previously also I perhaps wrote this wrongly it is exponential p 1 t and exponential p 2 t not minus because this minus sign is in the value of p 1 itself. In this case when the damping factor is 1 or the quality factor is half, the natural response will be a 1 plus a 2 t times the exponential p 1 t.

Finally, and the damping factor is less than 1 or a 1 square is less than 4 a 2, p 1 and p 2 will be a complex conjugative pair. So, what does that mean the natural response will be of the form. First of all p 1 and p 2 I will write as some real part p plus or minus j times an imaginary part p i. So, the natural response will be of the form, a 1 exponential p 1 t plus a 2 exponential p 2 t, which can be written as a 1 exponential p r t exponential j p i t and a 2 exponential p r t exponential minus j p i t. So, this part is real and these parts are complex.

So, these are complex parts and I will take out the real part. This is for the natural response looks like. Now, the natural response itself is a real number, that is we are taking a voltage here for instance in case of an R L C circuit. The natural response of the voltage V c. So, what I would like from you is the conditions on these coefficients a 1 and a two. So, that the natural response is real. What are the conditions on a 1 and a 2. So, that the natural response is real.

It turns out that because these two are complex conjugates of each other, if a 1 and a 2 are also complex conjugates of each other, then sum will be real number. So, this condition is a 2 being complex conjugates of a 1. So, in that case, the entire expression is real. Now, the whole thing looks very complicated, but mainly what I want to point out is that, a 2 is the when a 2 is the complex conjugate of a 1.

You know that complex numbers can be described by their magnitude and sum phase angle. Then this whole expression becomes exponential of p r times t, the magnitude of a 1, times exponential j phi, exponential j p i t plus exponential minus j phi because here we would have got a 2 which is the complex conjugative of a 1, times exponential j p i t. So, which results in a 1 exponential p r times t exponential j p i t plus phi pus exponential minus j p i t plus phi.

You know that also exponential j x plus exponential minus j x is 2 times cos x. So, this sum is times cos 9 of p i of t plus phi which basically gives us 2 a 1 exponential p r t exponential p i t plus phi. So, what do we have finally, the entire natural response becomes a product of an exponential and a sinusoid. So, that is the qualitative difference I was trying to bring about. There is a question about, how to get the previously recorded lectures. If you go to n p t el website, you will see that all recorded lectures are available.

So, you can go there and then get all of the lectures. Now, let me just summarise the three cases. The first one is when the quality factor is less than half or damping factor is greater than 1, in terms of the differential equation coefficients, a 1 square greater than 4 a 2. The response is of the form a 1 exponential p 1 t plus a 2 exponential p 2 t a 1 and a 2 you choose from initial conditions.

This particular condition where the damping factor is high and that is damping factor is more than 1, is known as the over damped case. If I plot the natural response qualitatively, it will look something like that. Let me plot all of them in the same plot later. Second case when the quality factor equals half and the damping factor equals 1 or in terms of the coefficience a 1 square equals 4 a 2, this is known as the critically damped case. In this case, the natural response again has 2 constants, it is a 1 plus a 2 t exponential p 1 t because p 2 is identical to p 1.

Finally, when the quality factor is more than half or the damping factor is less than 1, in terms of the co efficiencies a 1 squares smaller than 4 a 2. This is known as the under damped case. In this case p 2 on p 1 are complex conjugates of each other and the natural response is turns out of the form some constant exponential of p r t and exponential p i t plus phi.

So, again there are two constants here really, there is the constant a 1 and there is this constant phi and these have to be adjusted from initial conditions. What are p r and p I, basically p 2 and p 1 are this p 1 is p R plus j times p i. It will be a complex number that is where the p R and p i come from. These two constants and these two constants and this, for that all of these are determined from initial conditions. I hope that this part is clear. I will qualitatively show this responses look like and why they are called over damped critically damped and under damped.

I will not go into much more detail about this, but you can try it yourself try to determine a1 and a2 from initial conditions and get the total response. By the way let me also describe what happens for the three cases, in the two kinds of circuits, over damped which means q l s than half or damping factor jaye more than 1. In case of series R L C, this means that basically you have a large resistance.

So, if you work it out based on the formula for quality factor or damping factor, this is what you will see. Parallel R L C value of R s small that is when you have over damped. Critically damped means q equals half or z i equals 1. Under damped is when q is more than half or z has is less than 1 and the value of R is small in a series R L C case or large in the parallel R L C case. So, let me take an example of some series R L C circuit.

So, there will be a certain initial condition on the capacitor voltage. There will be a certain initial condition on the inductor current, which basically can be thought of as initial condition on the derivate of the capacitor voltage. From these you can calculate all the responses, I will assume the k s with V s equal to 0. So, then let me imagine that for some value of R L and C the circuit is over damped. That is the quality factor is less than half or the damping factor is more than half.

So, I will plot V C versus time starting from starting from some initial condition V C of 0. So, what we would see is that, for certain initial condition on V C of 0 and i L of 0 the response may look something like this. This is a typical response of an over damped case. So, this would be over damped and as you go on reducing the value of R. Let say for some value of R it is over damped. As you go on reducing the value of R, at some point it becomes critically damped.

So, for that the response tends to look like this. So, this would be critically damped and finally, when it becomes over damped you see that again I think I made a mistake here. This should be co-sign this should be co-sign p r of t plus phi and because of the presence of this co-sign we will have co-sign sinusoidal response, which has cycles right, which goes alternately between positive and negative values and because of this exponential p r t, the amplitude of the oscillations goes to 0.

So, what we would see, could be something like this. You will see some oscillations and then it can go off to 0. These are not to scale and also the response where is a little bit with the initial conditions, but qualitatively this is what you would see. This happens in a series R L C circuit as R decreases it goes from being over damped to critically damped to under damped, is this fine. Is there any question I will take them now, otherwise we can move on to the next topic.

Then, if you want to get the sub response of a second order system you first calculate this steady state response. This is based on an open circuiting, all capacitors and short circuiting all inductors. Then you calculate the damping factor or the quality factor and from this you arrive the general form of the natural response. The total response is natural response plus steady state response or the forced response.

The constants in the natural response either a 1 and a 2 or a 1 and phi, this have to be adjusted from initial conditions, is this fine. So, this is how the total step response of a second order system would be formed. What we will do next is to find out the response of this circuits, whether it is R C or R L C to other kinds of time varying wave forms. Now, we have analyse them to some extend till now, but we have only considered only constant wave forms or steps which are of course, wearing, but piece wise constants. Now, signals in general can vary in more complicated fashion.

So, what we will look at is the response of this signals to sinusoids, which is a particular kind of time wearing signal. We would not go on to the details, but it turns out that any signal of any shape case be constructed by combination of sinusoids, that is by adding up sinusoids of different frequency and phase. Now, we are dealing with linear circuits. So, if the input is a sum of a different sinusoids, then we know that super position applies, we have to find the response to each of the sinusoids individually.

Then add up the responses to get the final response to the combination of the sinusoids. So, we can analyse the circuit for sinusoidal inputs and with knowing full well that for any other kind of input, if we know it is d composition into sinusoids will be able to tell the response also. That is why you find that most of the time either in analysis or in measurement transits the circuits are characterised using sinusoids.

So, what we will do is, we will look at the response to sinusoidal inputs. This can be done by solving the differential equation, but we will take an easier root and show that what the relationships are. So, what the relationships are for each element when a sinusoid is applied. Then from there what we will be able to do, is directly from the circuit transform it in some way. So, that we can tell what the sinusoidal response is.

Now, there is one cabinet here, if we solve the differential equation, we will see that it will have certain natural response and a certain force response. Now, it turns out that if you have a sinusoidal input the force response is also a sinusoid of the same frequency. It is amplitude and phase can be different. Now, with the method that I am talking about the natural response will not be commuted at all. The method I am going to outline where we transform each element R L and C into something that is appropriate for sinusoidal steady state analysis.

It turns out that essentially will be omitting the natural response completely. So, this will calculate only the steady state or the forced response to sinusoids, but there are still it is still useful enough in practise because there are so many practical situations, where you apply a sinusoid with circuit and you wait for a while for the natural responses to die out and then you look at the output. So, it is still cord useful in practical context and it is much easier when solving the differential equation.

So, that is why we do this thing and this entire business is known as either sinusoidal steady state analysis or fazer analysis. The reasons for all of these things will become clear later. We will talk about fazer later. First I will show the sinusoidal steady state analysis. So, what is this all about. So, first of all let me apply a voltage which is just cos omega t across the resistance R, what is the value of the current i.

Please let me know I apply a sinusoidal voltage cos omega t, across the resistance R what is the value of the current i. So, clearly a current i is V were R which is cos omega t divided by R. Now, let us say apply sin omega t across the same resistance R. This let me call this i 1 i 1 would be V 1 by r. So, V 1 is cos omega t V 2 is sin omega t, this i 2 will again be V 2 by R. We know the real relationship of a resister this is sin omega t by R.

Then, now because this is a linear element super position applies, that is now applied V 1 i got i 1 applied V 2 i got i 2. Now, if I apply let us say alpha 1 V 1 plus alpha 2 V 2 by linearity, I expect the current to be alpha 1 i 1 plus alpha 2 i 2. Now, I will choose particular values of R first which as you will see very easily will become a will make it very convenient to analyse circuits.

I will choose alpha 1 to be 1 and alpha 2 to be j, that is square root of minus 1. If I do that, the applied voltage V which I call let us say V 3 will be cos omega t plus j sin omega t right because it is cos of omega t times 1 plus sin omega t times j. Which of course, you know from basics of complex numbers is exponential j omega t. So, if you apply exponential j omega t the response will of course, be i 3 which is i 1 plus j times i 2 which is exponential j omega t divided by R.

Now, there is nothing surprising here. I could have started off with exponential j omega t and said that, the current is exponential j omega t divided by R because the current is simply voltage divided by the resistance, but the interesting thing comes, when we apply the same to capacitors and inductors right because initially we could always analyse resistance circuit somewhat easily right because V and I were proportional to each other.

We had so many techniques to analyse even very complicated resistance networks like using nodal analysis and so on. Whereas once we had capacitors or inductors we ended up with differential equations. So, it is definitely more difficult than, solving algebraic equations. Now, this history will both adjust illustrates the point. I did it in a convoluted way instead of simply saying I will apply exponential j omega t because then you will not understand the motivation why I did that.

I applied cos omega t sin omega t and by super position I constructed this exponential j omega t. So, let us do the same capacitors and inductors and see what we get. First let me take a capacitor and apply V 1 equals cos omega t to it and what is the current i 1 i 1 is C times the time derivative of V 1 which is basically minus omega C sin omega t.

Similarly, if V 2 is sin omega t that is I apply sin omega t across a capacitor a current i 2 will flow which is C d b 2 by d t which is basically plus omega C cos omega t. Now, if I apply third voltage V 3 which is exponential j omega t and remember I do not have to solve for this separately. I could have solved for this separately, but I will first do it individually.

That is I imagine that, this cos omega t and sin omega t are purposed. I multiply cos omega t by 1 and sin omega t by j to get exponential j omega t. The whole reason I am doing it this way is because I already told you that, we will characterise the circuit with sinusoids. We will use sinusoids to analyse and even in the lab to measure some things about a circuits.

So, cos omega t and sin omega t are sinusoids and somewhat I will construct this some complex number exponential to j omega t. Now that is an abstract thing which is really a mathematical quantity. I cannot get exponential j omega t in the lab, but still it very useful to analyse using that, will see why. Now, I will have this V 3 which is exponential j omega t which is basically V 1 plus j times V 2.

Now, the current i 3 is i 1 plus j times i 2 which turns out to be omega C times minus sin omega t plus j cos omega t, which you can see can also be written as j omega C times cos omega t plus j sin omega t, which is basically j omega C exponential j omega t. I could also have got this directly i 3 which is C time derivative of exponential j omega t which is j omega C exponential j omega t.

Now, why did I do all this, the point is the following. First of all exponential j omega t can be thought of as super position of cos omega t and sin omega t with some multiplying factors. I multiplied cos omega t by 1 and sin omega by j. Then the response to exponential j omega t is also a super position response to cos omega t and sin omega t. Now, what I am trying to emphasise is that, it is that response to cos omega t if I am interested in. I applied cos omega t to some circuit and I want to find what the response is.

Now, in case of a capacitor I can find it quite easily because it is very to differentiate V 1 what is cos omega t, but later we will see the circuits can get quite complicated. Now, what happened I found this super pose response i 3 by superposing this, which is j omega C exponential j omega t or I could even differentiate it directly and I would get the same answer naturally. They have to be consistent. Now, the point is that, if you look at this thing, the applied voltage is exponential j omega t and the current if you see it is proportional to j omega t.

We have some constant, it happens to be an imaginary number in this case, but the point is it is some constant time exponential j omega t. This is not the case when I apply cos omega t, when I apply cos omega t the response is sin omega t. It is not something times cos omega t, it looks more complicated it have to get it by differentiation, but if I apply exponential j omega t as shown using both ways, that the response is some number times exponential j omega t.

It is proportional to the applied voltage and this happens to be just as in the resistors case. In this case we have i 3 to be 1 by R which is the conductance of the register times exponential j omega t. The current is some number times the voltage, the capacitor also has a current voltage relationship in the same form, if the applied voltage is exponential j omega t. So, that is what makes it convenient, but then what I am interested in is, what happens when I apply cos omega t.

So, that is the thing that i will apply in the last. So, that is what I want to know. Now, that is where this view point of super position helps. If you look at this ((Refer Time: 01:24:09)) I will form V 1 plus j times V 2. Let us say I am interested in the response to cos omega t. This is my interest. Now, I have constructed V 3 as super position of V 1 and V 2. So, naturally i 3 the response to V 3 is super position of, this is the response to V 1 plus j times the response to V 2.

Let me in fact rewrite, this write this as minus omega C sin omega t plus j omega C cos omega t. Remember I could get this answer much more easily by simply applying exponential j omega t and doing C divinity of exponential j omega t, but the point is I want to get back this quantity, which is a response to cos omega t. You see that the way this has been conducted V 1 plus j times V 2, the response also will be the response to V 1 plus j times the response to V 2.

Now, V 1 and V 2 are real quantities, their cos omega t and sin omega t and response to V 1 is a real quantity it is 1 real voltage in the circuit response to V 2 is also a real quantity it is a some real voltage in the circuit. Now, this j this multiply are square root of minus 1 helps to keep the two responses separate. You see what I am saying V 1 is cos omega t V 2 is sin omega t, our super position was V 1 plus j times V 2. So, the response also will be response to V 1 plus j times the response to V 2.

So, the response to V 1 will be the real part of the total response, the response to V 2 will be the imaginary part of the total response. So, I can calculate the response to exponential j omega t and take the real part, to get response to cos omega t and if i wanted the response to sin omega t i could take the imaginary part. We do not normally do that, but we just stick with the real part. I will explain what later, I mean it is just an easy thing to do. So, we now have a much easier way of analysing.

I will elaborate on this in the next lecture. The reason it is easier is, with exponential j omega t the relationship for the capacitor becomes somewhat like the resistor that is the current is just proportional to the voltage. Current is the voltage times some number instead of being this differential derivative and so on, but from that we have to calculate the response to cos omega t and that is also easy. The real part of that gives the response to cos omega t.

So, from this we will be able to analyse any circuit consisting of R L and C, when the applied voltage are sinusoids. Is there any questions I will be happy to answer them now, otherwise we will meet on Thursday. I think also there was a lot of lack between what I am saying and what was being received on the net. Hopefully you will be able to what the recorded lecture and fill up any gaps that may be there.

Thank you, I will see you on Thursday.