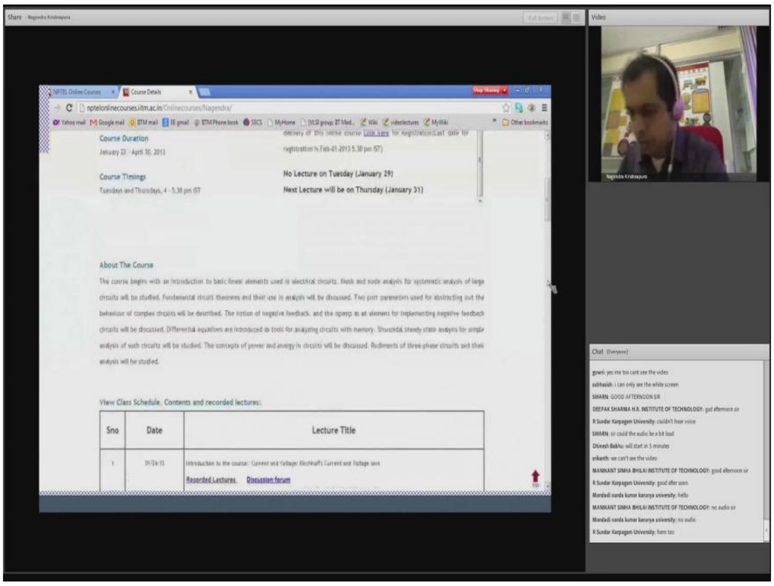
Basic Electrical Circuits

Prof. Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 2
Electrical circuit elements: Voltage and current sources; R, C, L; Voltage sources in series; Example of superposition

(Refer Slide Time: 00:53)



Some announcements during updates to our course website. I think, you see the web browser view shared on your desktop. On the browser, this is the webpage for our course, Basic Electrical Circuits. And then, you see some ((Refer Time: 00:35)) here, which also has the recorded version of the first lecture.

As we go along the recorded lectures, here you can use these to review or ((Refer Time: 00:46)) some lecture, we can also use these things. And here, we also see the class schedule and some tentative list of topics that we are going to cover in every lecture. It is only tentative, it will be revised as we make progress through the course. And again, here also you have links to recorded lectures as well as to the discussion forum.

Now, the discussion forum you can use for asking questions and I will answer them at my convenience. With that let us get started with today’s lecture.

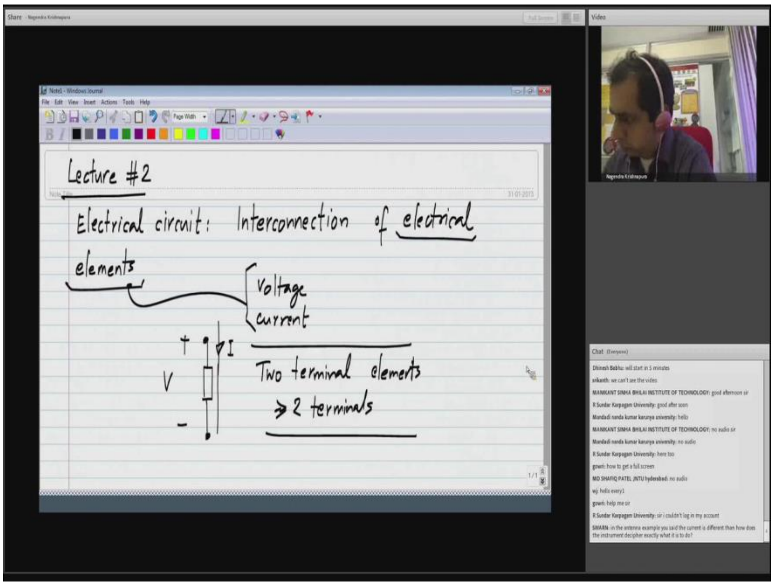
In the previous lecture we looked at ((Refer Time: 02:16)) what current is and related them to the fields. We also described why we can describe everything we want or with only voltages and currents, without going down to the level of details, that we do in magnetics, that is the, with fields and so on. And under certain conditions we saw, that there were two laws that were true for voltages and current from Kirchhoff's voltage law.

It says, that the sum of voltage laws taken with the appropriate polarity around a loop is 0. And another one is Kirchhoff's current law, which says, that a sum of currents entering a node or sum of currents leaving a node is 0. Now, Kirchhoff's voltage law works under the assumption, that the rate of change of magnetic field cutting the loop is negligible and Kirchhoff's current law works under the assumption, that there is no local charge accumulation, ok. Now, both these assumptions turn out to be true for a wide variety of particle circuits. That is why, we are able to use them.

Now, there are conditions under which it is not true, basically during the time period of the signal if the circuit is physically so large, that it takes so much time for, let us say, the electrical signal to go from one side of the circuit to another, then these things will not be true. And a very common example is antenna. Many of you would have seen antennas, which are basically just a wire hanging in the air and, but current is driven into it. Clearly, current does not go off in to the air. What happens is, that the current alternate back and forth within the antenna, you know. And if you look at different parts of antenna, they will have different currents, ok.

KCL is clearly not true in that case, but besides that KCL is true when you are not talking about such large circuits. Large meaning, large compared to the electrical wave length KCL is true and as is KVL. Now, before I go further, if there are some important doubts regarding what we covered in previous lecture, I will go over them, ok. So, it appears that there are no questions. So, we will continue with the lecture.

(Refer Slide Time: 02:08)



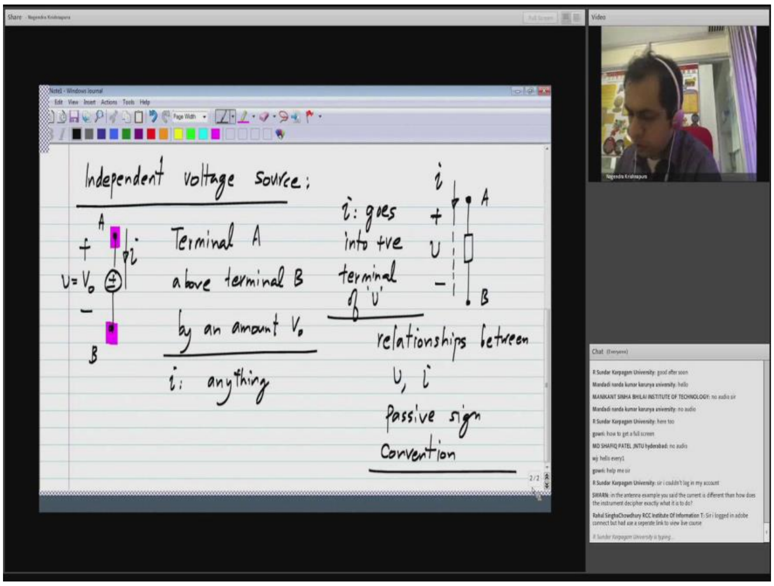
What will do in this lecture is to discuss basic electrical elements, that is, two terminal elements and see their characteristics. So, before that let us see what an electrical circuit is. It is an interconnection of electrical elements. And electrical elements are somethings we will see some examples of that, which take some voltage or current and manipulate them in some way, ok.

So, every electrical element is defined by at least one voltage and a current. So, every electrical element it turns out will have at least two terminals. I will show the terminals here and as I emphasized in the previous lecture, voltage is applied between two terminals and a current is measured into this terminal, and same current will come out of it if it is a two terminal element.

So, electrical circuits will consist of interconnections of elements, which can be either two terminal elements or it can have more than two terminals. The initial examples, that we will see, will all consist of just two terminals. And so, two terminal is a minimum number of terminals we need to have to have a meaningful definition of voltage or even a current, ok. Because the single terminal, we do not know the voltage with respect to what that is they are measuring. And similarly, if you pushing a current into a single terminal, there will be question of where it will go out.

So, we will have two terminal elements or maybe more than two terminals, that is, more than or equal to two terminals. And this course does not worry about how to make these different elements, but we will only take their characteristics in terms of their voltages and currents and use them to make some circuits and analyze the circuits.

(Refer Slide Time: 07:58)



Now, the very first elements that we will take is, what is known as, independent voltage source and this is denoted by a symbol, which has a circle with plus and minus mark. And what is given is the voltage in this polarity and that is equal to, let us say, some v naught. So, this has two terminals. And what this is saying is, that the voltage source will maintain a voltage of v naught between this terminal and that terminal.

Now, one of the things I have to mention before we go further is the convention that we will take for describing the current, currents and voltages of two terminal devices. So, what is done is, to describe relationships between the voltage and the current.

Now, the voltage, let us say, is defined with this polarity, that is, I call this terminal A and terminal B. And I define voltages with A being positive and B being negative, that is, I measure the voltage of A with respect to B. Then, I will also specify the current that is going into the terminal A, ok. Now, of course, it is understood, that the same current will come out of terminal B. Now, we have to have this convention, so that there is no ambiguity.

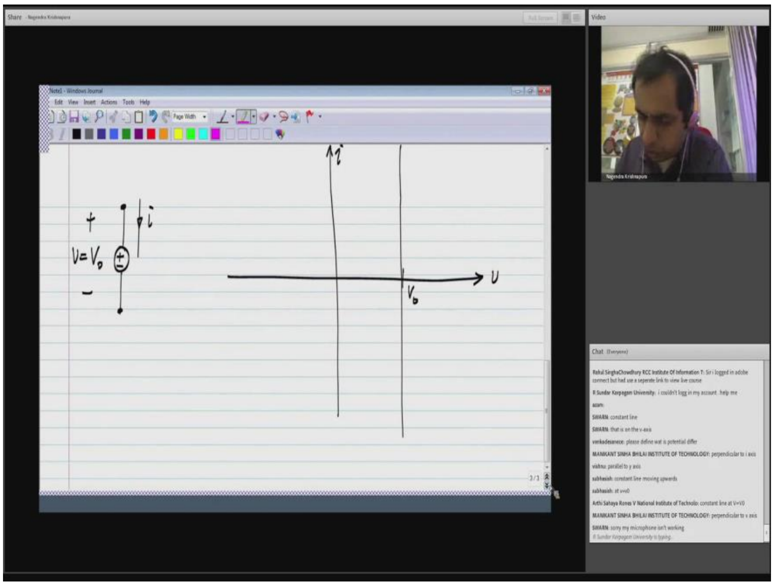
Now, we know, that a voltage can be measured of A with respect to B or B with respect to A. Similarly, current going into A can be measured or current going into B can be measured. Now, what is important is that the definition of voltage and current be consistent. So, if the voltage v is defined with A being positive and B being negative, then the current i is measured with, measured as going into the terminal A and this is known as passive sign convention.

And for every element we are going to use this, that is, the current i goes into positive terminal of v. What is meant is the terminal, which is defined as plus while defining this voltage, ok. So, for the voltage source I have to define the current as going into the positive of this v. Now, this plus and minus sign inside the voltage source only denote the definition. The value of v naught itself could be positive or negative. Again, this is some source of confusion. I do not necessarily mean that because I put plus on top and minus on bottom. This v naught is always positive. I could define it any which way, this, the signs are used, so that I can use the consistent direction for voltages and currents while defining them.

So, what this says is, this terminal A and terminal B, an independent voltage source maintains terminal A above terminal B by an amount v naught. The voltage at terminal A is maintained above the voltage at terminal B by an amount v naught, and the current can be anything. It is not restricted by the voltage source. Is this fine? So, that is like a source of voltage.

And analogy could be made to an infinitely large reservoir, which will maintain its level, ok. Now, you can take voltage out of these reservoir, but it is not going to change its level because there is infinite amount of water in it. Similarly, voltage source is source of infinite amount of current. It can, you can draw any current from it, but it will still maintain one of the terminals to be v naught above the other terminal, ok.

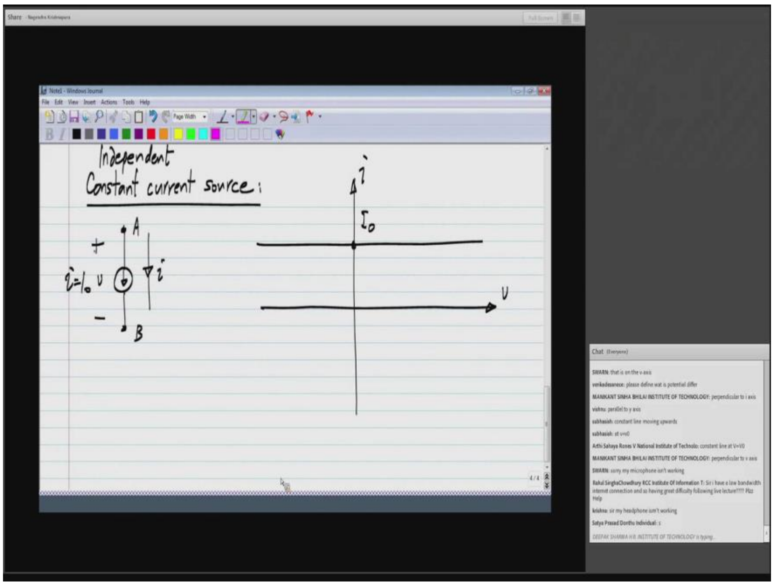
(Refer Slide Time: 12:31)



Now, it is very common to draw a graph of i versus v to obtain a graphical ((Refer Time: 12:49)) of the same characteristic. Now, I would like answers from the participants what would this look like, the i versus v curve for a voltage source? There is a raised hand, with them approved, please ask your question. I think somebody raised their hand in order to ask the questions, please go ahead.

So, a lot of people have answered this. It is pretty obvious, v equals v naught, that is all they will do it and I can do anything. So, this will be a vertical line on the i-v plane at v equals v naught. So, that is the characteristics of its constant voltage source.

(Refer Slide Time: 15:00)



Now, the next element we will look at will be a constant current source. So, this signal means that a current in a node is flowing from A to B. And if we have to measure the voltage across this, we will do it with this polarity. Again, there is question, please ask. We are not able to hear any questions. I mean, somebody raised their hand and I approved the request, ok.

Let us go ahead with the lecture. So, as per our passive sign convention, the voltage will be measured like this and in the current I would be flowing that way. And in the case of ((Refer Time: 16:43)), it will maintain a current of i naught. Please go ahead. ((Refer Time: 17:22))

Sir: hello

no, this an independent current source, so its value is a constant.

Student: Sir, we are talking about constant current source or independent...

What do you mean by constant?

Student: So, we should write it, see last, constant current was, it was a constant current. So, first of all, we talk about independent voltage source…

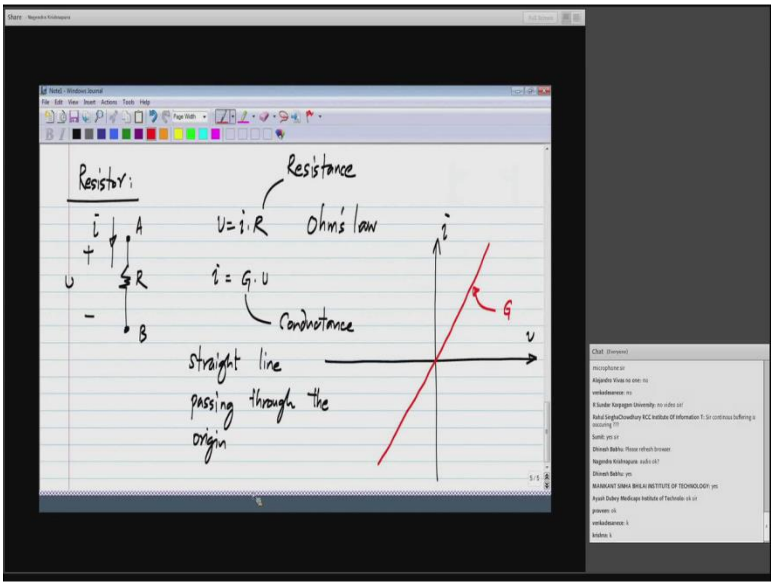
First of all, we have to define what it is independent with respect to independent voltage source. Now, in the initial part of the course we will deal with things that are constant with time, ok. So, that is one definition of what is constant and not. Here, what is meant by independent is, that it is independent of any other quantity in the circuits. So, that is what it is meant. No, no, you can write both. A constant usually refers to constant with time that is what we are talking about. But independent means, independent of other quantities, other electrical quantities in the circuit. And soon, we will come to dependent sources.

So, now, so this is a constant current source, which is of course, independent. And if I plot i versus v, for this it will be a horizontal line with the value i naught. So, what this symbol means is, that it will maintain a current of i naught through it regardless of its voltage. So, the voltage can be anything, but the current value will be i naught. Now, these are useful idealizations of some components that we actually encounter and these are also the sources that will stimulate our circuit.

Now, in case of electromagnetic fields, you know, that charges are the primary source. You place the charge somewhere and it will create an electric field and then, if the charge is moving and actual reading, it will also play at a magnetic field and so on, ok. But in our case we will not go down to the level of fields, but we will deal with the voltage sources and current sources as stimuli for our circuits.

It appears, that some people are having some difficulty with the bandwidths. So, I am going to pause the camera for some time.

(Refer Slide Time: 20:39)



Now, the next basic element that we will consider will be a resistor and as usual, we define the voltage in some way. Now, I have chosen A to be positive and B to be negative, but they could easily do it other way. But what is important is, once you have chosen the sign of v, you should be looking at i going into the positive terminal, ok.

Now, this is the sign convention that is universally followed. So, that is what you must use. And I think, all of you all are already familiar with the relationship between v and i. For this particular element, v would be equal to i times R and this is the famous ohms law, and we can also write i to be G times v. This R is the resistance and this G is the conductance.

Let me plot i versus v for a resistor. What is the kind of plot that we are going to get? I would like to have some answers from participants on what is the plot that we will get. Let me plot i versus v for a resistor. Yes, I can.

Student: Sir, I had a question.

Ok, I am not able to hear the question. First of all, somebody commented, that there is no video because of bandwidth constraints for some users. I am now not sharing my video, I am only sharing the board on which I am writing and many people answered this question as well. The answer is pretty obvious. It is a straight line passing through the origin and the slope of this would be, the slope of this would be G, the conductance G, ok.

Is there a problem with the audio?

Student: Can you hear me Sir? Hello.

Yes, I can hear you.

Student: Hello Sir, question…

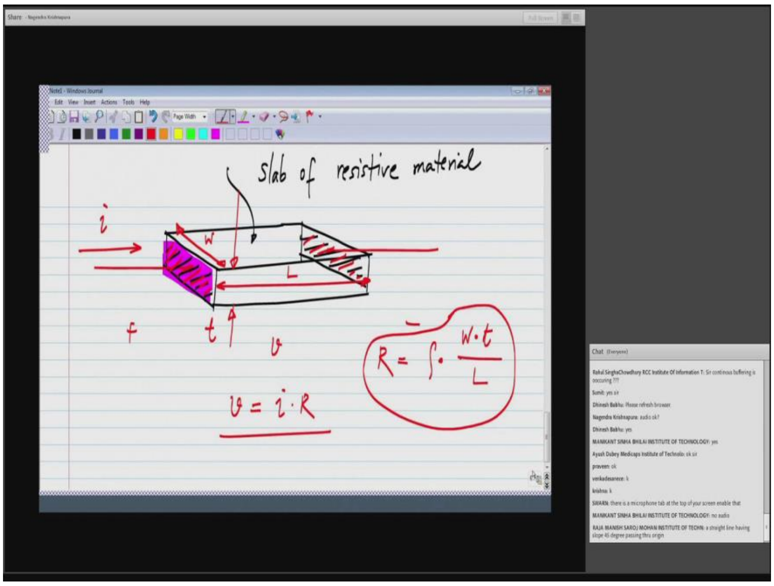
What is the question?

Student: Sir, ((Refer Time: 24:27)), hello…

Let us leave that for later. Some people said, that there is no audio. If this is the case, please type it in the chat window and I would disable the video because some people had very low bandwidth and with the camera on, it was taking a lot of their bandwidth. But if there is no audio, if you are not able to hear me, then please type it in to this. It appears, that the audio is fine.

So, the i versus v characteristics for a resistor would be a straight line passing through the origin and that is quite important. And the slope of that would be g.

(Refer Slide Time: 26:10)



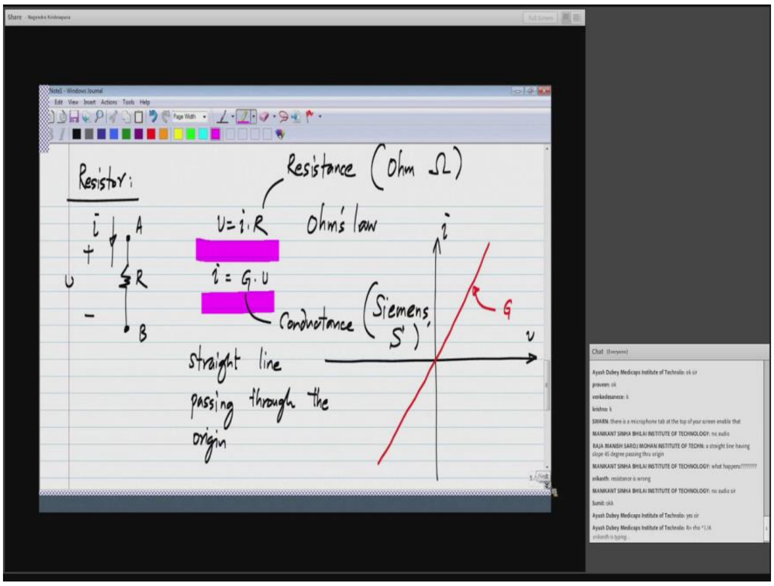
Now, let us quickly look at how a resistor is made. This is not the main focus of the course, but this you can think of as some general knowledge. The easiest way to imagine a resistor that is made, although it may not actually be made, this side is to have a slab of some material and you can have a contact at this end and the other end, ok. And this will serve as the terminals to which you can apply either the voltage or the current. So, you can apply a voltage here and you can measure the current there and which as we said earlier, v would be related to i by this relationship.

So, now, what happens in a resistor is that electrons will be accelerated by the field created by a voltage applied between the two, but of course, it is not moving in a vacuum. So, it will accelerate a little, it will collide with some fixed charges in the lattice of material, then it will start accelerating again and so on. And depending on that material it will acquire certain average speed and that will give you a certain average current, ok. So, you need the knowledge of electromagnetics and materials to calculate the resistance for a given material, but we will not go into those issues. We will take the resistance value for granted and then use that.

Now, for a slab of this type, let us say, this dimension is W and this dimension is L and a thickness of this is t. I think many of you already know, that the resistance R will be the resistivity O, hence the width of the material times thickness, that is, the cross-section area across which the current is flowing, the area of that divided by the length, that is, the length along which the voltage drop is measured, ok.

So, this is the formula for the resistance and it is quite simple for a slab like this, but the résistance itself could be made in any number of ways. You could have resistance, which is coiled into a spiral and so on. In those cases, the formula will be complicated, but what is important for us is the relationship between i and v and it will always be simple like this, ok We will not worry about how to calculate the resistance for a given physical structure, but we will take the resistance as granted and we will use this model.

(Refer Slide Time: 28:59)

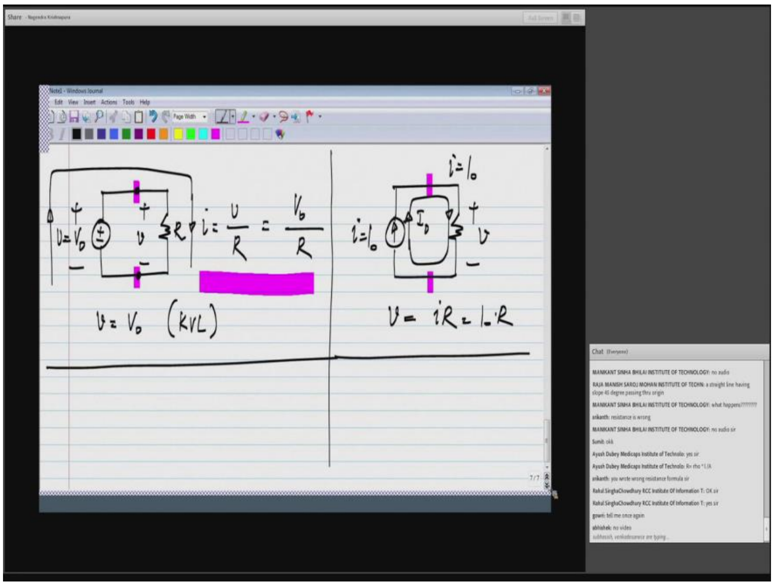


We will use what is given here. v equals to iR or i equals to Gv and sometimes we will use this graph.

Now, there is some comment in the chat window. Somebody said that it is a straight line passing through the origin with 45 degree slope, ok. Now, here you have to understand, that the x-axis and y-axis are different dimensions. So, measuring the slope as 45 degree, it has no meaning. That will be meaningful only if they have same dimensions, then you can say the slope is unity.

In this case, the slope is in conductance, which has some dimensions. So, as we know, the resistance is measured in ohms with this symbol and the conductance is measured in Siemens and this is the symbol S. So, that is the definition of a resistor. That is, once you define the relationship between the voltage across the resistor and current through the resistor, our job is done. We have described this resistor.

(Refer Slide Time: 30:23)



So, what does the resistor do? If I connect the voltage source and the resistance like this, it is pretty obvious, that the voltage across the resistor equals the voltage applied by the voltage source. So, this is v naught and v naught is applied across the resistor.

Actually, this is a very trivial application of Kirchhoff's voltage law. If you sum up the draw from here to there and there to there in the resistor, the sum will be equal to 0. So, under these conditions v will be equal to v naught, this is the KVL. And this is so obvious, that we do not think of it as application of Kirchhoff's voltage law, but that is what we are doing here.

Now, because voltage v naught is applied across the resistor, a current i, which is v by R, which is v naught divided by R, will flow through the resistor and that of course, has to come from the voltage source. So, a voltage source will supply a current to v naught by

R, ok. Now, whether you apply a voltage or apply a current, the resistor bears in the same way. You can think of it as the current generated by an applied voltage or a voltage generated by an applied current. So, I could also do this.

Now, let us say, i equals i naught, that is, what we are saying is, this will maintain a current flowing from top to bottom equal to i naught through the current source. So, that means, that the current flowing here will be equal to i naught, which again is a trivial application of Kirchhoff's current law. The current entering this node here equals the current leaving that, that is, the current supplied by the current source equals the current flowing in the resistor, ok.

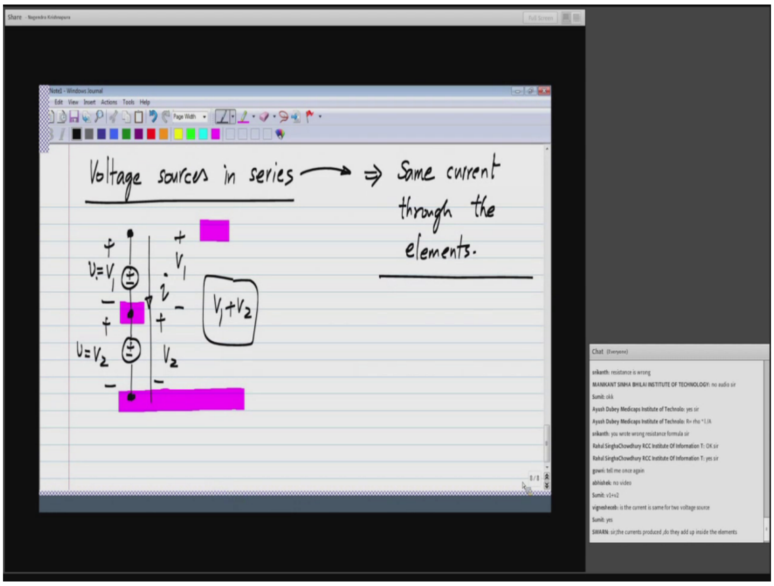
And in this case, the voltage across the resistor, it has to be measured like this because the current is flowing downwards through the resistor and that will be equal to i times R, which is i naught times R, ok.

So, there is a question from Subashish is that… Subashish go ahead, ok, appears that he has dropped out.

Now, so I think all of you would already be familiar with these cases, that is, if the voltage is applied across the resistor, you will have a current v naught by R flowing through it or if current is applied, current sources connected to a resistor, current i naught will flow through it. In this case, you will get a voltage drop of i naught R and in this case, you will get a current of v naught by r, ok.

Now, this comes from very trivial applications of KVL and KCL. I am not going to describe that. If you want to, you let me know, I will do that. But it is pretty obvious, that the same current is flowing in the voltage source and the resistor here and the current source and the resistor on the right hand side, ok.

(Refer Slide Time: 34:28)



Now, one of the properties of the resistor. Now, before we go ahead, let me look at what happens when voltage sources are connected in series. Let us say, I have a voltage source of voltage equal v 1 and another one, whose voltage equals v 2, and they are connected in series.

Now, what does series connection mean? Series connection means, that for two terminal elements, you pretty much connect one on top of the other, that is, every one common terminal. And the important criteria for whether things are connected in series is whether the same current i is flowing through them.

Now, because the bottom terminal of the upper voltage source and the upper terminal of the top terminal of the lower voltage source are connected together, the trivial application of Kirchhoff's current law, the current flowing downwards from the upper voltage source, have to flow into the lower voltage source, the same current is flowing. So, this is what is implied by series connection, ok.

Now, again if you measure with respect to this, the voltage drop here is v 2 and if you measure with respect to this, the voltage loss here is v 1, that is what is given by not in this polarity, sorry, in this polarity. So, again in trivial application of Kirchhoff's voltage law, if I measure from here to there, the voltage will be v 1 plus v 2. So, if you connect voltages in series, the voltages will be sum together.

Please go ahead with the question

Student: Sir, good evening sir.

Yes

Student: Hello

Yes, please go ahead.

Student: Hello Sir, good evening Sir.

Yeah, please go ahead.

Student: You said that in the last ((Refer Time: 36:46)), hello sir.

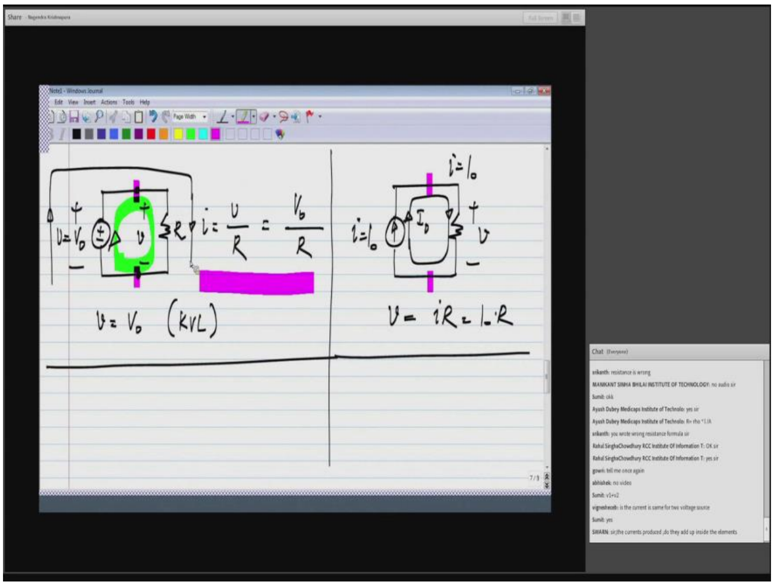
Yeah, please go ahead.

Student: Sir, I was asking, that in the last example that you gave that of using the KVL one voltage source and the resistor, in that the current was flowing from negative side to positive side inside the voltage source.

Yeah, that is correct.

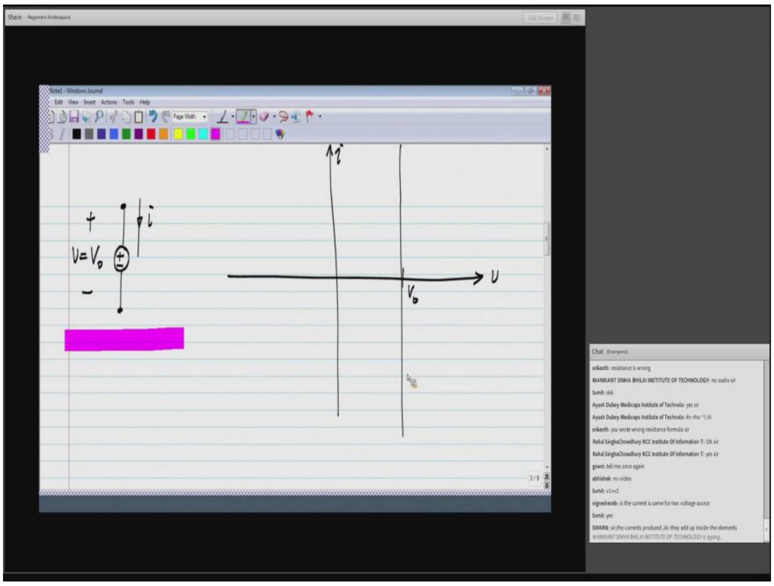
Student: Sir, but in case of independent sources, when you showed the diagram of independent voltage source, you showed, that the current was flowing from positive side to the negative side.

(Refer Slide Time: 37:44)



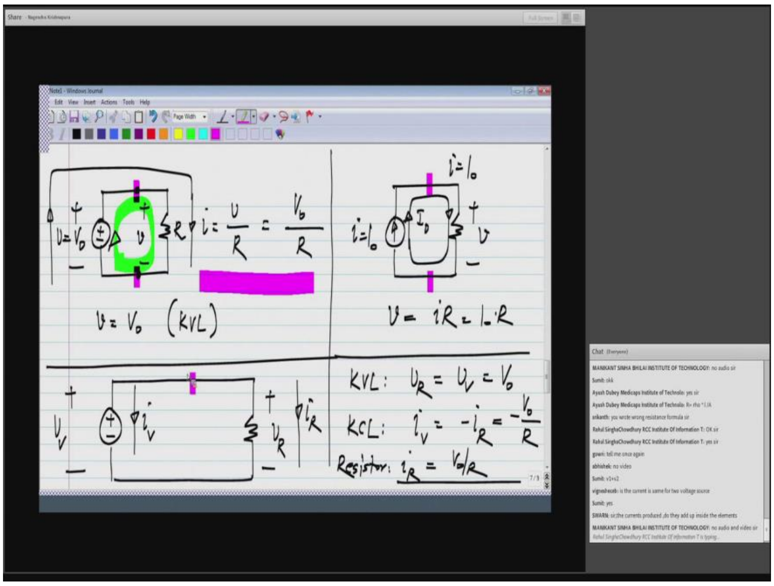
So, again this is a confusion I would like to clear right at the beginning. The question is in this circuit. The current is flowing in this direction, in this direction, that is, it is flowing from bottom to top in the voltage source and then top to bottom in the resistor.

(Refer Slide Time: 38:09)



Now, when I showed the definition of the independent voltage source, I marked i like this. Now, again I will emphasis, then this i does not say that the current is flowing this way. It is only measure to say for the sake of plotting and for the sake of convention.

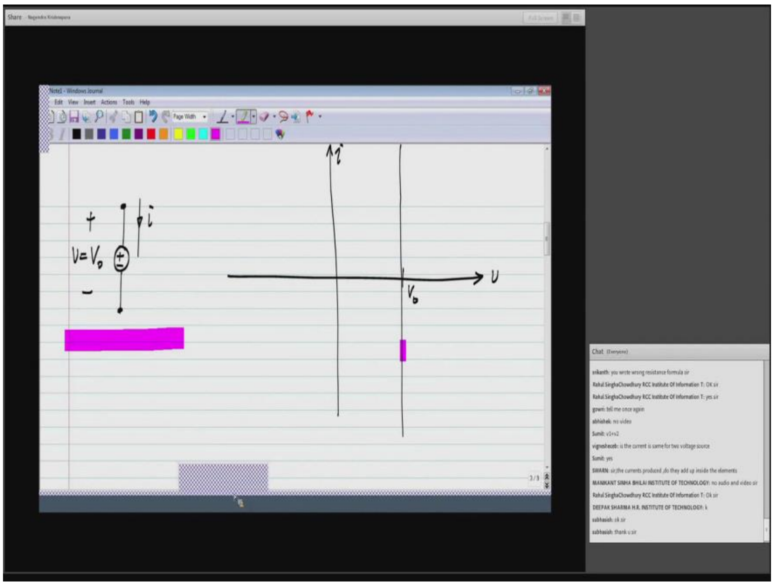
(Refer Slide Time: 38:38)



And if you look at this graph, what it says is, this value of y can be anything, it can be any positive value or any negative value. Now, what it means in this particular circuit, let me show the two parts separately. I will measure the voltage across the voltage source in this direction and I will measure the current through the voltage source in that direction. And for each element, i will follow each convention. Now, I will measure the voltage across the resistor in that direction and the current through the resistor in that direction.

Now, clearly by KVL, V R equals V V, which is equal to the voltage source value V naught. And by KCL applied at this node, i V plus i R will be equal to 0 or i V equals minus i R and i R itself is given by the property of the resistor, which is V naught by R. i R will be V R, i r will be V R divided by R and that happens to be V naught by R and i V happens to be, consequently, minus V naught by R, that is, for this particular circuit.

(Refer Slide Time: 39:55)



And there is no contradiction here because this only shows the direction in which I have measured the current through the voltage source. But this graphs tells you, that it can be either positive or negative, ok. Now, in that particular circuit it happens to be negative. It happens to be equal to minus v naught by R. I hope that clears the doubt. Is that ok?

Let us go ahead with the question. ((Refer Time: 40:28)) I. S, Dubey…

Student: Yes sir.

Yeah.

Student: Sir

Please go ahead with the question.

Student: In the previous explanation, in the previous explanation, in the previous explanation you said, that the voltage is minor [FL] current is i equal to minus V naught by R.

The current as measured in the voltage source as per the convention, that is, minus V naught by R, yes.

Student: Yes sir.

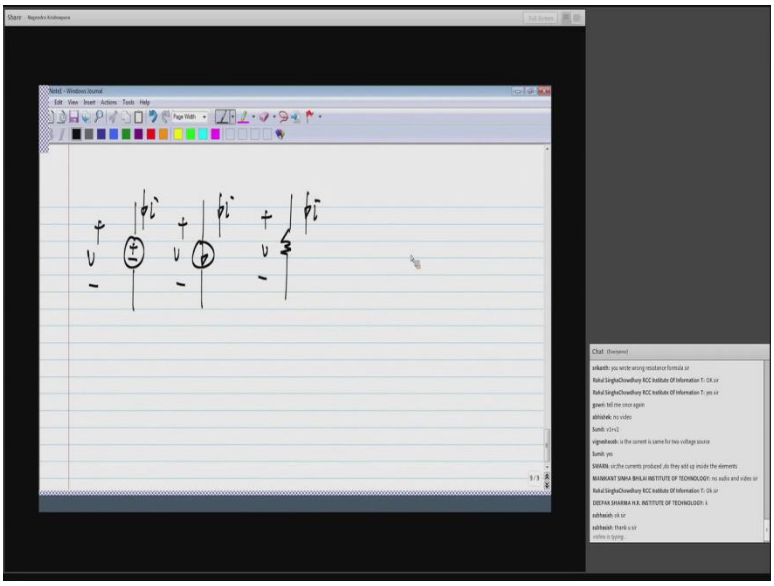
Ok.

Student: So, should not we follow single convention, should not we follow single convention there? Either take it from the positive side or negative, we are following two conventions, for resistors different and for voltage source different.

No, no, I have used exactly the same convention for resistors and voltage sources and I will use it for any two terminal element. So, for each element

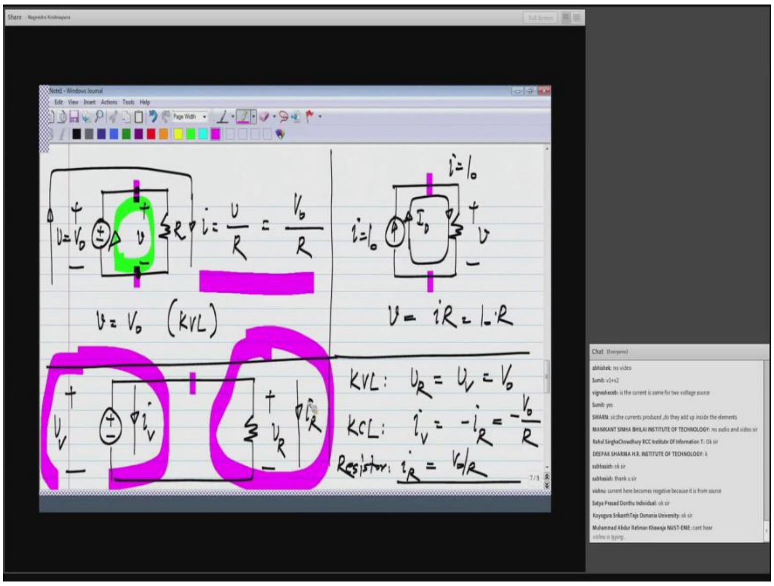
Student: Sir in the last circuit you had…

(Refer Slide Time: 41:35)



For all elements if I define the voltage like this with the upper terminal positive, I will measure currents flowing into the positive terminal of that V, that is, current flowing into the upper terminals.

(Refer Slide Time: 41:57)



And that is exactly what I have used for the current source here, the voltage source here and the resistor here. i R is flowing into this plus sign of V R and i V is flowing into plus sign of V V. So, that is just a convention for denoting the relationship between voltages and currents. It has nothing to do with which way the currents are flowing.

Now, depending on the, depending on the circuit connections and element characteristics, these currents could be positive or negative. In case of a resistor it says, that if you have V R defined like this and i R defined like that, V R will be i R times, i R, V R, i R times R, that is Ohms law.

Similarly, for a voltage source it says, that if I have V V like this and i V defined like that, V V will be some fixed value v naught and i V can be anything, either positive or negative. So, I have used the uniform convention.

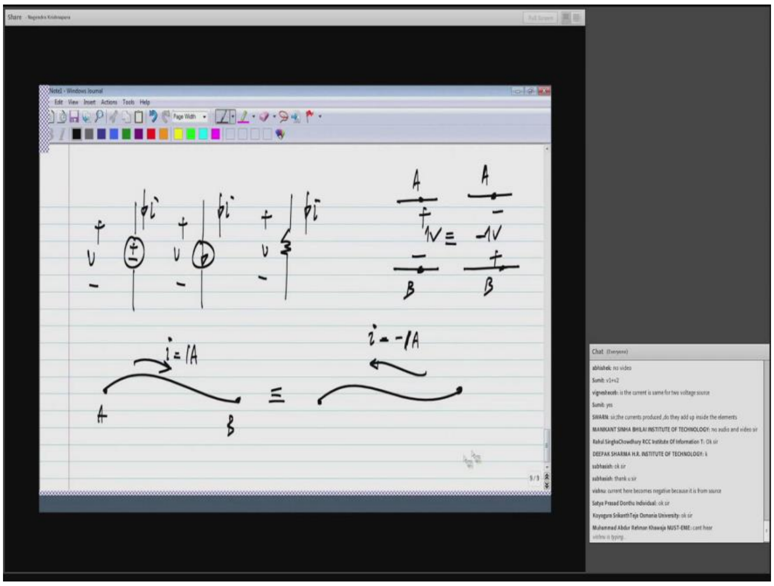
Student: Sir

Now, the voltage, yes…

Student: Sir, if any circuit is given, then we have to independently take the elements and mark them positive negative and we have to go conventionally rather than looking at the circuit as a whole?

No, no, first of all if it is a small circuit, you may not have to do it systematically, but when analyzing last circuit, this is what you will do. So, this is the convention that is used for showing the characteristics of the element, ok.

(Refer Slide Time: 43:29)



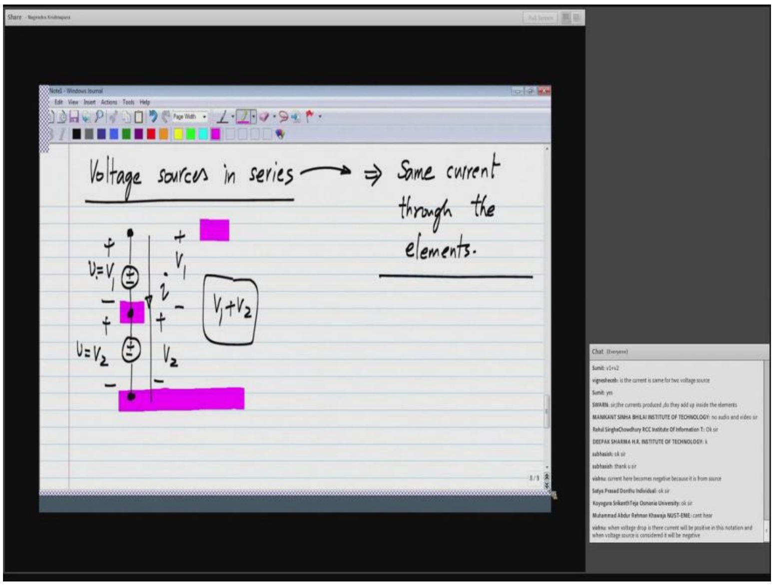
Now, whether the current is positive or not, that depends on the circuit. Like I said in the very first class, I can show a wire with A to B and i mark, i equals 1 Ampere. And similarly, this is exactly the same as saying the current is flowing from B to A, which is minus 1 ampere.

Similarly, if I have two terminals…

Student: Ok, Sir

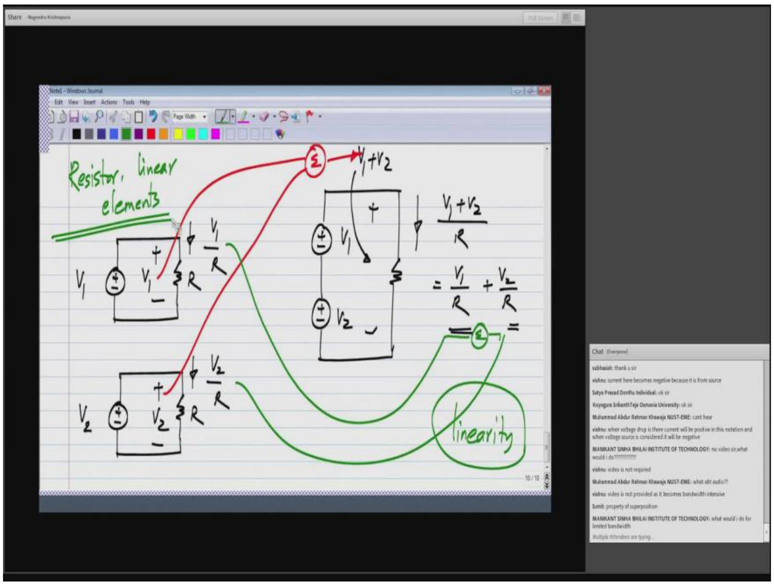
I can say this is 1 volt and that is minus 1 volt. So, these two are exactly the same. So, I can say A is above V by 1 volt or B is above A by minus 1 volt, ok.

(Refer Slide Time: 44:10)



I will go ahead with the lecture. So, now if I have voltage sources in series, the total voltage will be V 1 plus V 2. This is again by some trivial application of Kirchhoff's voltage law.

(Refer Slide Time: 44:30)



Now, let us take three cases. I apply V 1 across the resistor R, the current flowing would be V 1 by R. Similarly, if I have a voltage V 2 applied across the resistor, the current will be V 2 by R. And if I have V 1 and V 2 in series applied across the resistor, the voltage across the resistor will be V 1 plus V 2 and the current through the resistor will be V 1 plus V 2 divided by R, which is V 1 by R plus V 2 by R, ok. The voltage across the resistor here is V 1, voltage across the, voltage here is V 2.

Now, you may think of voltage as cause and current as effect. It could be both ways for a resistor. But if we think of the voltage as cause and current as the effect, we see, that here I have a cause equal to be 1 and as from resulting current here, I have cause equal to V 2, some resulting current and here I have a cause, that is, the sum of these two, that is, the cause. And if you look at the result, that also happens to be the sum of the individual results. That is, if I apply voltages V 1 and V 2 separately to a resistor, i get currents of V 1 by R and V 2 by R respectively. If I apply a voltage V 1 plus V 2, I will get the sum of individual currents, ok.

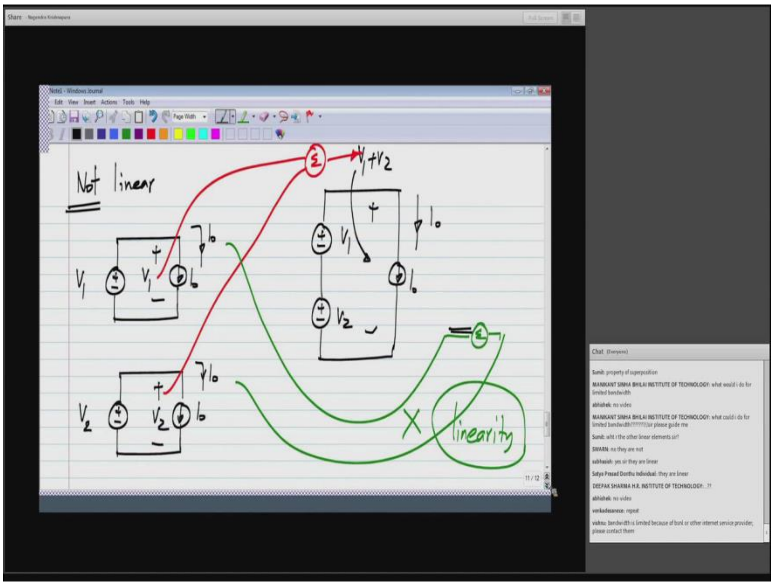
Now, this property is known as linearity. In general, if you have any system, which has certain cause and a certain effect, I can apply different values of cause individually and I can measure the different effects. If I apply all the causes together, all the results, all the effects will be sum together. So, that is the meaning of linearity, that is, if you have stimulus or stimuli, which are combinations of certain values of stimulus and the responses will be sum of individual responses and this is known as linearity. And this will be exploited very heavily in our analysis of circuits, linear circuit form a very large class of useful circuits and mainly in this course. That is what we will be analyzing.

And any element that follows this is known as a linear element. And obviously, a resistor is a linear element because if you apply a cause, that is, the sum of causes, you will get an effect, that is, the sum of effects.

Now, the question I have is, you have discussed three elements, an independent voltage source, an independent current source and a resistor, now please let me know if the voltage source and the current source are linear or not?

Now, I got some responses and some said, they are linear. One person, I think, says they are not linear. Again, the way to evaluate this is to see whether the effect is proportional to the cause or if you have individual causes, will the effect be summed up when you have summed the stimuli.

(Refer Slide Time: 49:31)

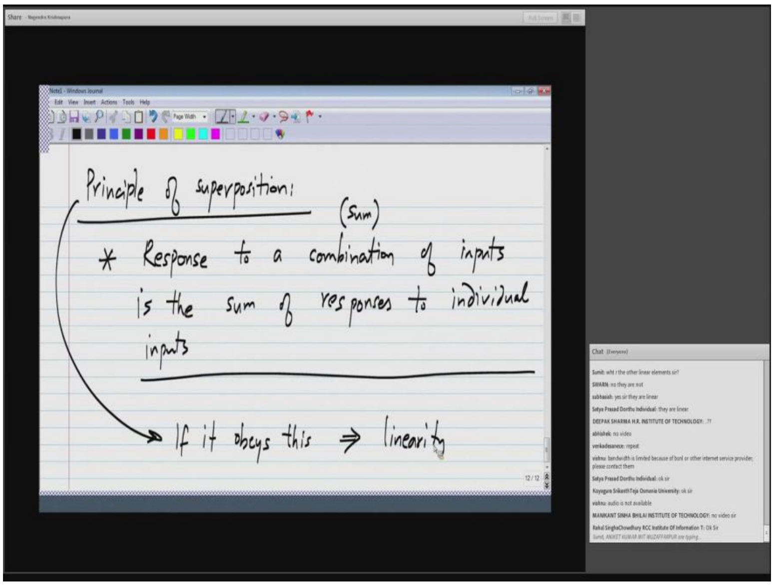


Now, let us consider a voltage source or perhaps let me take a current source because that will be analogous to this example. Now, let me copy over this whole thing. Instead of resistors, let me have current sources, let me have current source y naught and here I have i naught and here I have i naught.

Now, clearly, current in this case is i naught, the current in this case also is i naught and the current in the third case is also i naught, that is, here I apply V 1, the current is any way is independent of what is applied to it. So, it will be i naught, here also it will be i naught and here also it will be i naught. So, although I apply a voltage, that is, the sum of individual voltages here, that is, here I have applied V 1 and here I have applied V 2.

Now, if this principle, that is, if combining the causes will give you a combination of the effect. If that is true, then this current should have been 2 i naught, but it is just i naught, ok. So, in fact, it is not equal to the sum, the resulting current is not equal to the sum of current in individual cases. So, this says, that the current source is not linear in the sense that resistor is linear.

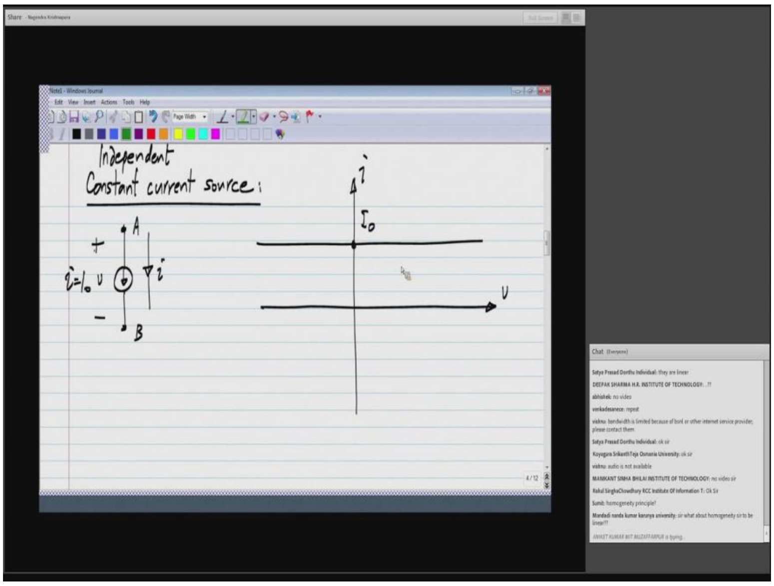
(Refer Slide Time: 51:21)



So, so what says if something is linear or not, if it obeys the principle of superposition? And what is the principle of superposition? The response to a combination or sum of inputs, this is the linear combination that we are talking about, is the sum of responses to individual inputs.

If this is the case, this is ((Refer Time: 52:14)) resistor. Then, it means, that it obeys the principle of superposition. And if it obeys that, it implies linearity where the resistor is linear, the voltage and current source are not linear, ok.

(Refer Slide Time: 52:42)

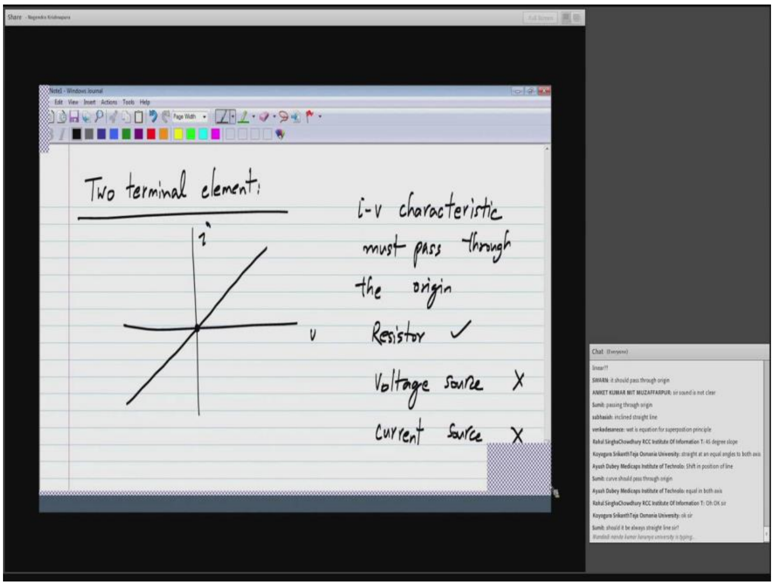


Now, this has nothing to do with the characteristics being a straight line sometimes that is what is called linear. But here for us what linearity means is, whether it applies, whether it obeys superposition or not.

So, now I have another question for the participants. We draw these i-V characteristics and I showed you, that the current source is not linear. And in a similar way, voltage sources is not linear. Also, you can apply it to individual current or voltage source, you will get some voltage. And if you apply the currents together, you will get the same voltage. So, that is also not linear.

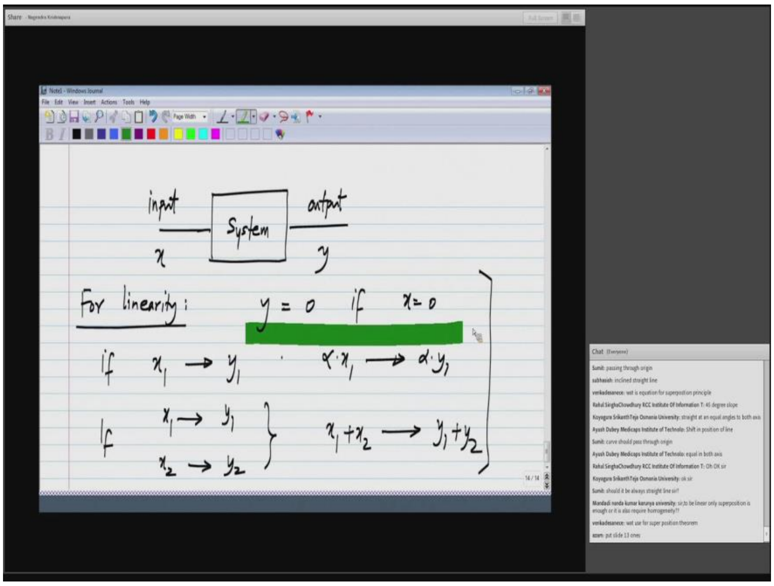
And we have these i-V characteristics that we have drawn. So, what is the, what feature of i-V characteristic guarantees, that the element is linear, that is my question.

(Refer Slide Time: 54:31)



I have a number of responses. Basically, if we have a two terminal element, then if the i-V characteristic passes through the origin, that is linear. And clearly, this is true for resistor, but for a voltage source or a current source it is not true, ok. So, resistor is linear and voltage source and current source are not linear in the ((Refer Time: 55:32)) follow superposition. Now, just some more details of what is superposition.

(Refer Slide Time: 55:39)

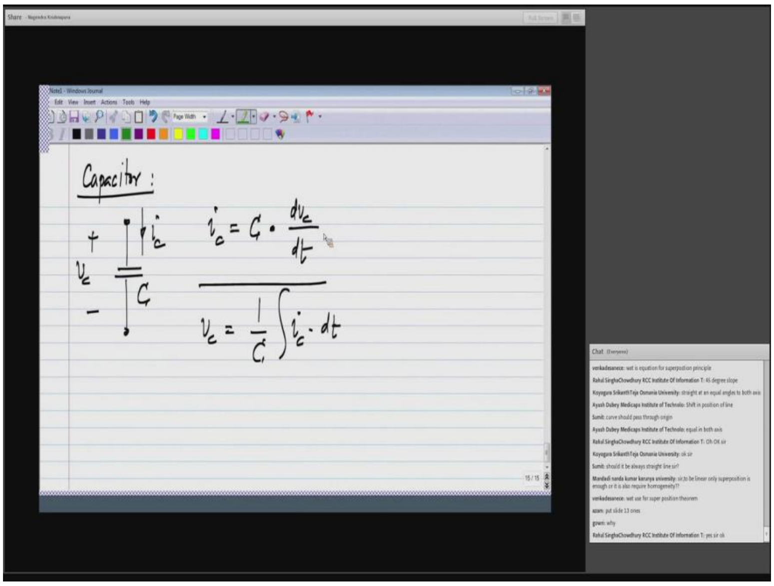


Let us say you have some system and I will show it in an abstract way like this. And here, we are talking about two terminal elements whose stimulus could be a voltage and the response current or vice versa. So, let us say, you have some input and it has some output. I am here talking very generally, I am not saying what system it is and what input output it receives and so on.

So, first of all, for linearity y must be 0 if x is 0, otherwise you can show, that it cannot be linear. Now, if x 1 gives you y 1, then if it is linear, alpha times x 1 will give you alpha times y 1. And similarly, if x 1 results in y 1 and an input x 2 results in an output y 2 and input of x 1 plus x 2 will result in output of y 1 plus y 2, ok.

Now, these are not independent statements that I am making, some of them implies the other. So, these are all necessary conditions. Now, this says, that if you have the i-V characteristics, it has to be passing through the origin.

(Refer Slide Time: 57:24)



Now, let us consider some other simple two terminal elements. This we will just look at the characteristics now and we will analyze circuits with capacitors with inductors later. But ((Refer Time: 57:39)) a capacitor like this and define the voltage V C in this. As usual, I will look at the current in this direction.

So, again it does not mean the current will always be flowing in this way. This is the current that I will measure. Even to measure current i have to choose some direction and this is the direction that I am choosing. And you know, that i C will be some parameter of the capacitor C times the time derivative of the voltage across the capacitor. And if I invert this, I can write the voltage across the capacitor as 1 over C integral i C dt.

Now, one of the things about the capacitors that distinguishes it from resistor is that the value of the voltage across the capacitor depends on the current, not at the present time, but all of the past time, whereas for a resistor the voltage across the resistor at any instant is dependent only on the current at that instant, ok.

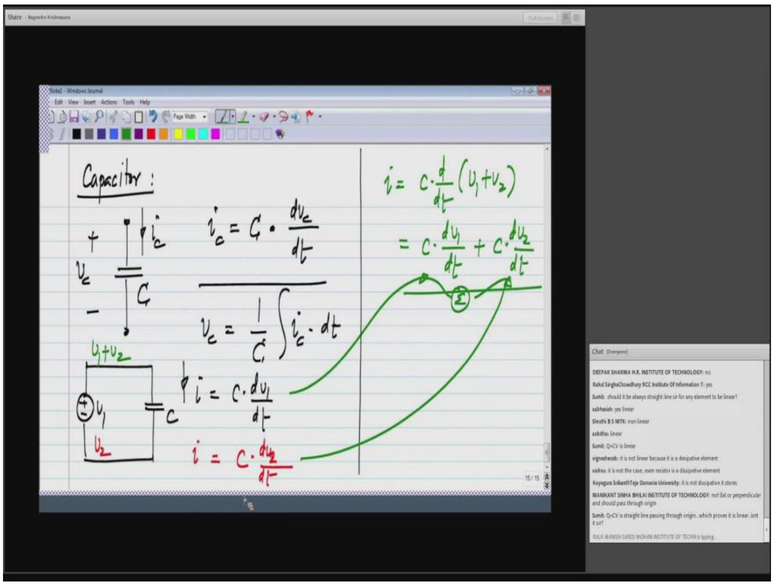
V equal i R is true even if V and i are varying with time. And for each instant, V will be equal to i R, that is, the current at each instant will depend on the voltage of that instant or the voltage will depend on the current at that particular instant, whereas the same is not true for a capacitor. The voltage across the capacitor depends not only the current at the present times, but also the history of the current, ok.

Now, this is the definition of the i-V relationship of the capacitor and because it is related by this derivative, we cannot plot i versus V. We can do that only when the element is memory less, that is, the current and voltages are related only at that instant of time, that is, current in that instant is related only to the voltage of that instance of time, ok. But this gives you the relationship just like we did for the resistor. It is a way, it is a slightly more complicated, it is a derivative.

Now, my first question to the participants, is the capacitor linear or not? We have a number of responses. A majority of them say, that it is not linear or couple of people say, that it is linear, and somebody asked if i vs V was the straight line. Like I said, we cannot plot i versus V for this because if you think about it, that is possible only when the voltage at this, at this instant of time is related to the current, at this instant of time ok.

In this case, that is not true. The voltage at this instant of time depends not only on the current at this instant, but also on the previous instants of time on the history of the current. So, we cannot plot i versus V, but we can still test for linearity. How do we do that?

(Refer Slide Time: 1:01:13)



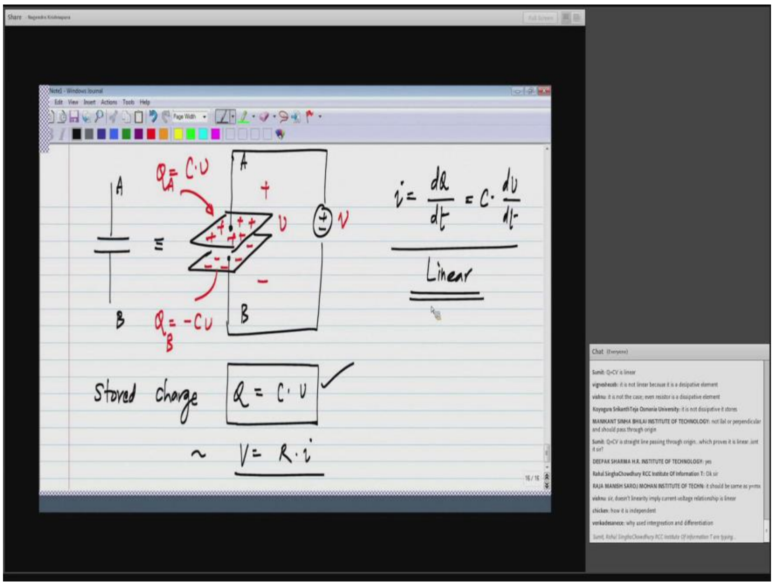
Let us say, I apply V 1 across the capacitor. There will be a certain current i 1, i, which will be C times the time derivative of V 1. Now, first of all, I am assuming, that this voltage is varying with time. Now, it is not varying with time, the formula still applies, but the current will be 0. That is, if you apply a constant voltage across the capacitor, there will be no current flowing through the capacitor. So, the more interesting thing is when the voltage across the capacitor is varying with time. If that is the case, the current will be related to, will be proportional to the time derivative of the voltage.

Now, instead if, if I apply V 2, the current, the new value of the current will be C times d V 2 by dt and finally, I could apply the two together. Exactly the same experiment I did with resistor earlier, V 1 plus V 2. In this case, the new current would be C times d by dt of V 1 plus V 2. But the time derivative of sum of two quantities is the sum of individual time derivative.

Now, clearly the responses are sum of individual responses and linearity is indeed true. So, this is test for linearity. I applied V 1 from the current, I applied V 2 from the current, I apply V 1 plus V 2. What I say is, that the current will be the sum of the individual cases and the capacitor is there for a linear element also. The i-V characteristics maybe more complicated than that of the resistors, but it is still a linear element. I hope that is clear.

Now, there are some comments. Somebody said, that it is not a dissipative element that is why, it is not linear. But it has absolutely nothing to do with whether it is dissipative or not. It only has to do with whether the i-V relationship is linear or not and it is a time derivative and it happens to be linear, ok.

(Refer Slide Time: 1:03:56)



Let us look at a simple construction of a capacitor. The way this is made, again this is just a depiction, the practical capacitor does not have to be made like this. It just has ((Refer Time: 1:04:08)) two conductors, which are separated from each other and two terminals are connected to it, A B, A and B. I described the capacitors by its i-V relationship, but I think, you already know what it is.

If I apply voltage across a capacitor what happens is, so let us say, this is certain V on the upper plate where this V is applied, the positive terminal of V, the plate connected to the positive terminal of V, this is our positive charge. There is a charge, I mean, that whether it is positive or negative depends on the value of V. The charge depends on this voltage V and it is given by C times V. And similarly, on the other plate there will be an equal negative charge and that will be minus C V. I call this terminal A, this is Q A and Q B. So, what a capacitor does? It ((Refer Time: 1:05:09)) charge and the stored charge Q equal C times V, ok.

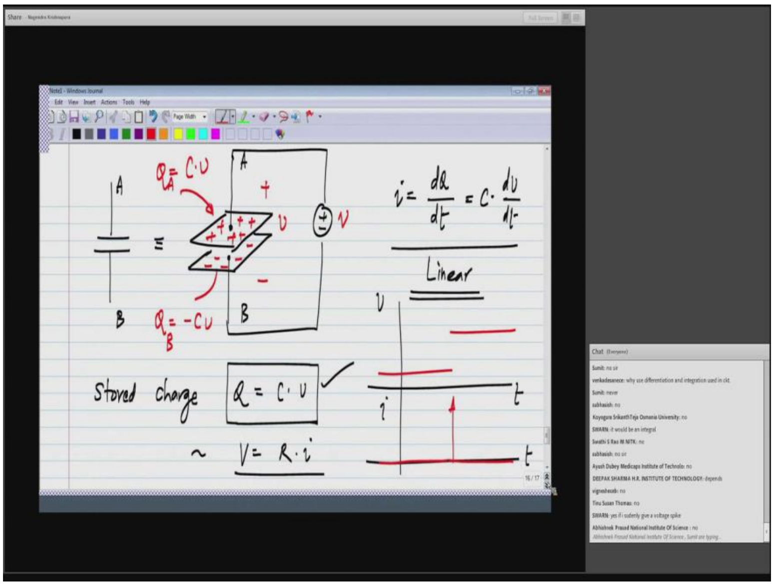
Now, first of all what you have to end up with properly? This stored charge is this means, that on one plate there is plus C times V and on the other plate there is minus C times V. Which is plus and which is minus, that depends on which is the plus terminal of V and minus terminal of V and also this relationship Q equals C V. You see, that this is somewhat similar to V equals R times i and this makes linearity even more obvious, ok.

Now, how do we get the relationship between i and V. You know, that the current is rate of change of charge. What happens is, that as voltage changes, the charge on the upper plate changes. And how will it change? The way it will change is by drawing current from this wire or pushing current into that wire. This charge has to come from somewhere and it has to come from this wire. That means, that the current in this wire will be dependent on change of voltages and the current will be the rate of change of charge, which will be C times the rate of change of voltage. So, that is why it comes about.

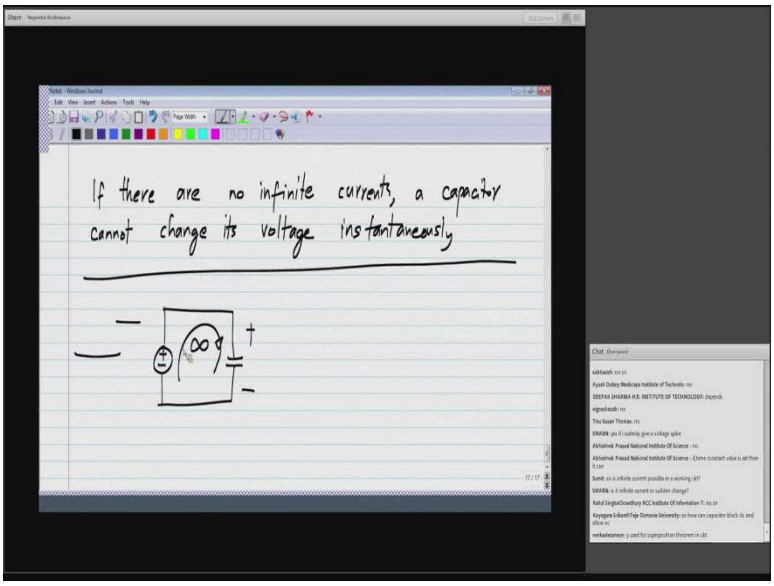
And from this relationship if you plot Q versus V, it will be a straight line passing through the origin and that will be a linear relationship. And the i-V relationship is simply the time derivative of this and time derivative is a linear operator. So, that relationship is also linear capacitor stored charge Q equals C V and that is a linear element.

Any questions about the capacitors?

(Refer Slide Time: 1:08:42)



(Refer Slide Time: 1:09:29)

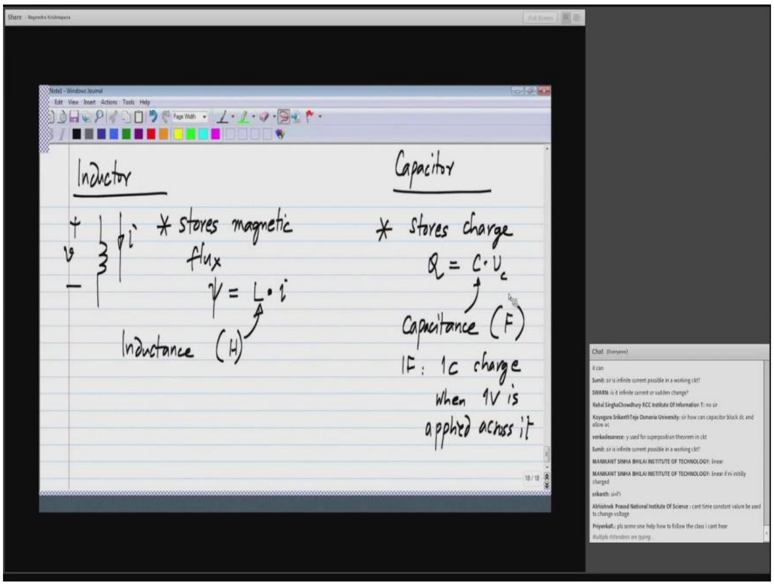


Now, my next question is, can the voltage across the charge change instantaneously? The question is, can the voltage across the capacitor change instantaneously? I think, one person said yes and a number of people said no. My question was, can the voltage across the capacitor change instantaneously, that is, can it be one value and suddenly change to another value?

It is possible. I have to, I mean, normally you hear, that a capacitor holds its voltage and it cannot change instantaneously, but we have to qualify, that let me change this instantaneously. What it means is, that we plot i versus t, it will be 0. And here, it will go to infinity and that is denoted by an impulse and then it can go to 0. So, what we really mean is, that if the currents are finite, the capacitor cannot change its voltage instantaneously. An instantaneous change in the capacitor voltage will demand an infinite current and that is, will demand an infinite current. So, if the currents are restricted to be finite, then it cannot change its voltage instantaneously.

So, if you allow an infinite current, which is not practical, but in theory it is possible, then the voltage can change instantaneously. And one way is, if you have a voltage across the capacitor and the value of the voltage shows changes instantaneously, then the voltage across the capacitor has to be exactly equal to the voltage across the voltage source and it will also change instantaneously, ok. But in this case what happens is, an infinite amount of current will be drawn at the instant of this voltage change.

(Refer Slide Time: 1:10:54)

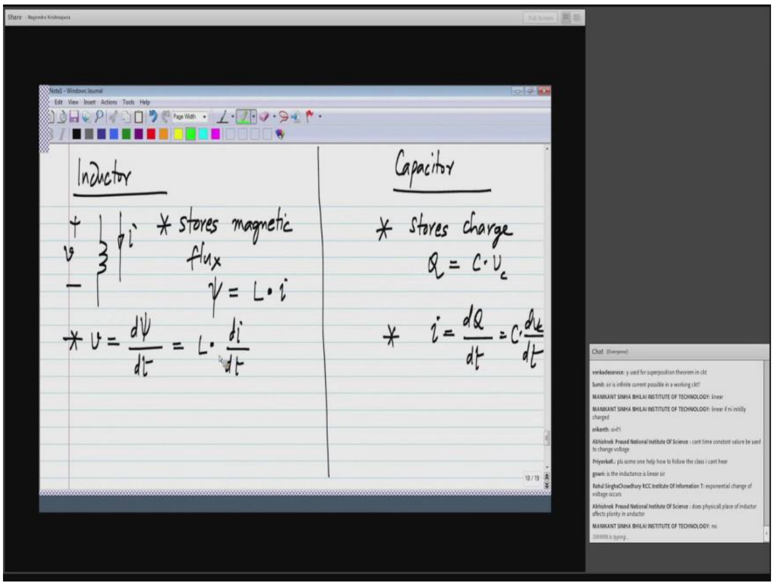


Now, finally, the last of the simple elements that ((Refer Time: 1:10:54)) elements is the inductor and the symbol for an inductor. I think, you are all familiar with it, it is like a coil. Again, I will define V and i in this consistent direction. Now, it turns out, that inductor and capacitor are dual elements of each other. What does the capacitor do, we have already discussed that. It stores charge and the amount of charge is C times the voltage across the capacitor. And in case of an inductor, it also stores something. It stores energy in the form of magnetic field and the capacitor stores energy in the form of the electric field between the plates. This ((Refer Time: 1:11:44)) stores magnetic flux.

We will not worry about the details of construction of the inductor and how much flux it stores and so on. But just like a capacitor stores charge, it stores flux and the amount of flux ((Refer Time: 1:12:01)) it stores is giving by some number L, which is called the inductance of the inductor times the current through the inductor. So, this C, the constant of proportionality is called the capacitance for a capacitor and it is measured in Farads. 1 Farad will store 1 Coulomb charge, 1 Coulomb of charge when 1 volt is applied across it.

And similarly, this is the inductance and it is measured in Henrys. And it turns out, that 1 Henry is the inductance, which will store 1 Webber of flux when 1 ampere of current is applied to the inductor.

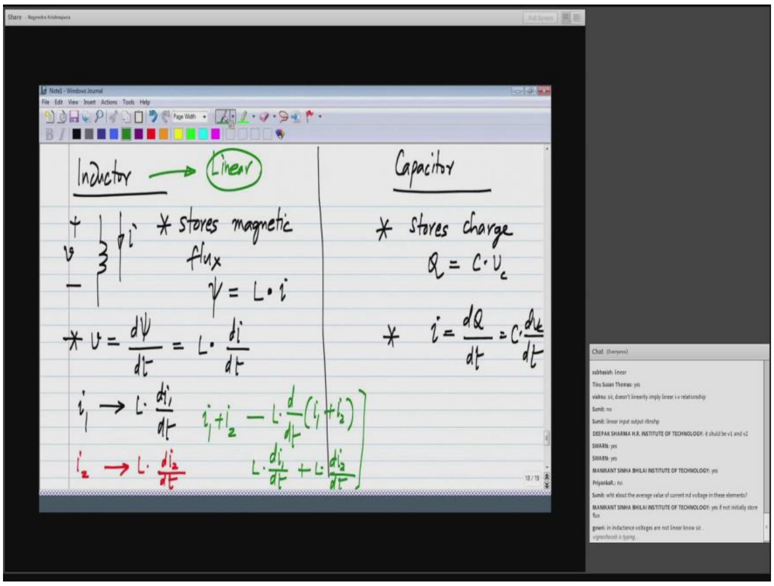
(Refer Slide Time: 1:13:29)



Now, the capacitor had a current, which is basically at the rate of change of charge and which turned out to be C times the rate of change of voltage. And the dual of this volt flow to the inductor voltage across the inductor will be the rate of change of flux linkage through the inductor and that will be L times di by dt, ok.

So, you can see it is an exact counter part of it. There is a question from Priyanka please go ahead. Priyanka.

(Refer Slide Time: 1:15:32)



So, the inductor is the counter part of the capacitor. You can see, that the role of a voltage and current are reversed if you compare it to the capacitor. In a capacitor the current is the time derivative of the voltage and here, the voltage is the time derivative, time derivative of the current.

Now, the question is if the inductor is linear? The question is for all the participants. So, I think, many of you guessed it correctly this time and it is linear because a current i 1 will give a voltage L times di by dt and a current i 2 would give a voltage l times d i 2 by dt. I am assuming, that we are thinking of current source i 1 applied to the inductor current source, i 2 applied to the inductor. And if I apply a current of i 1 plus i 2, I will get L times d by dt i 1 plus i 2 and that will be L d i 1 by dt plus L d i 2 by dt. So, inductor is also a linear element.

If we apply two currents individually you get two voltages and if we apply two currents together, that if we apply the sum of the two currents, then you will have voltage whether it is a sum of the individual voltage. So, this is also a linear.

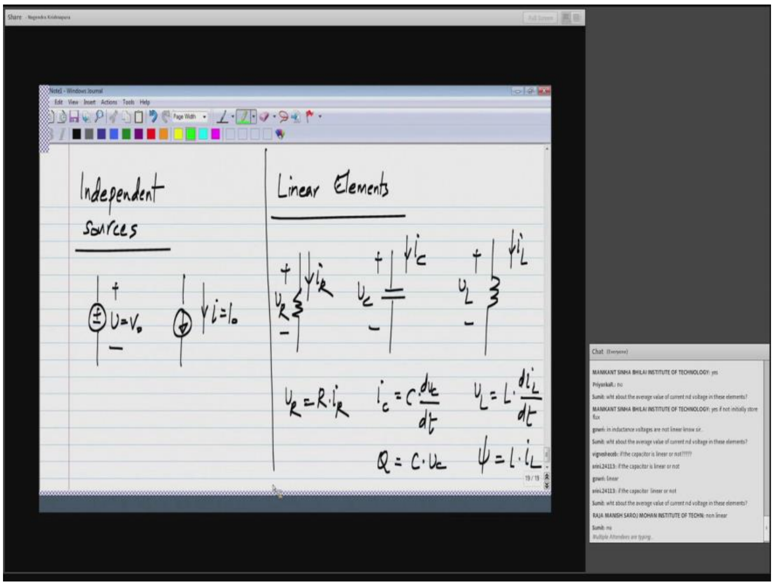
Now, the only one point of confusion that can arrive sometimes with inductors is that in elementary physics textbooks you will see, that the current will oppose the change in flux and it is minus L di by dt. And here we have said, it is L di by dt. Now, there is no contradiction at all here in physics textbooks. You see, they do not show sign conventions. What they mean is, that if there is a current flowing here and it tries to change, then there convention is to measure the voltage like this with plus on the bottom and minus on the top where i is flowing that side. And we will use this convention, that is, the passive sign convention that is consistent for our purposes. So, in this polarity we will minus L di by dt and in this polarity it is plus L di dt.

Also, another thing is, I showed you the simple construction of resistor and that of capacitor, but I will not do that for the inductor because it is rather complicated. But whatever it is, it will consist of some coils of wire and the measurement of inductance depends on geometry of the, geometry of how the coil is wound and what material is used and so on. Again, we do not have to worry about that.

In fact, part of what we learn in this course is, that you can describe the element by the voltage at the terminals of the element and the current through the terminals of the element and not worry about the internal detail, that is, for, that is, for another place and time for us as long as we know the voltage, the relationship between the voltage across the terminals and current through the terminals, you can describe them with some relationship and with that you can do perfect analysis, ok.

Now, this is how the hierarchy of the explanation and this is how we have to build up, right. We cannot go down to the level of fields every time. So, we described all we want with voltages and currents and we can make complicated circuits based on that.

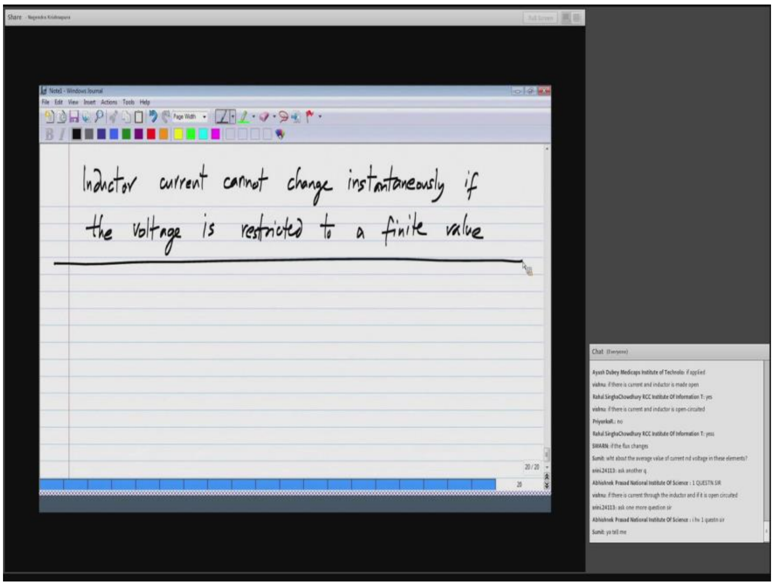
(Refer Slide Time: 1:19:07)



So, we have discussed now independent sources. It is a voltage source and the current source and we have also discussed a certain number of linear elements, a resistor, which follows Ohms law, capacitor ((Refer Time: 1:19:58)) related by… and an inductor whose ((Refer Time: 1:20:08)) related by… and these have some other quantities that has a proportionality relationship.

They are not voltages and currents. In this case, it is charge and in the case of the inductor, it is flux like this and these make it even more obvious, that elements are linear and the derivative operator itself is linear. So, all these are linear elements, ok.

(Refer Slide Time: 81:46)



Now, another question is, can the current in an inductor change instantaneously? Again, many people said, no, it cannot change instantaneously. We have to qualify this statement now. It can change instantaneously if you allow for an infinite voltage across the inductor. If you restrict the voltage to be a finite value, which of course, is a practical test, but in theory it can be infinite. So, if you restrict the voltage finite value, then the current cannot change instantaneously, ok.

So, that is how this brings us to the end of this lecture. Now, in the next lecture we will discuss what is known as a mutual inductor. It turns out, that you can have two coils close to each other. Again, we will not worry about the physical construction of ((Refer Time: 1:22:37)) where the flux induced by one coil can link with the other coil. So, the voltage across one coil depends on current not only through itself, but also on current to another coil. We will discuss that one and a number of other things.

So, today what we did was, first of all we established the, what is known as the passive sign convention and ((Refer Time: 1:22:56)) several two terminal elements, the voltage source, current source and the resistor, capacitor and the inductor. We also looked at ratios of what is meant by a linearity, a resistor, capacitor and inductor are linear. I have got a voltage source, current source or not.

Now, for us what is meant by linearity is, that it is a way superposition makes it a linear element. And what is superposition? Superposition says, that if you have two individual inputs, you will get certain outputs. Now, if you combine the inputs and apply it, the output should be a combination of the individual outputs, that is meant by, that is what is meant by superposition. And on this it will be implied, that if you apply zero input, you should get a zero output. If you do not get, then it cannot be a linear element by that criteria, by that criterion.

The voltage source and current source are not linear, whereas the resistor, capacitor inductor are not linear are linear. So, that you should be able to recognize and also, you should be able to apply the passive sign convention without confusion. It is the only the way, only the polarity with which voltages and currents are measured. It does not say anything about because the actual voltage and current are present, positive or not, ok

So, I take some last questions and end this lecture. Yes Subashish. Ok, it appears, that there are no more questions. I just have some announcements, basically whatever I mentioned earlier, I am going to repeat now. Shrikant, what is the question?

Student: Hello hello

Yes, yes, please go ahead.

Student: I was asking, I was asking that for ((Refer Time: 1:25:24))

Yes, if you want to check for linearity, you have to check for superposition principle. So, the question is, how do you test for linearity, and to test for linearity you have to apply superposition and then see, if it is valid or not, ok.

Now, of course, based on elementary map you will know, I mean, if it is a proportional relationship, if V equals iR that will follow superposition. And similarly, if it is proportional to derivative and so on, it will also ((Refer Time: 1:26:02)) superposition because the derivative is a linear operator. So, after some experience you will be able to simply look at the relationship and see if it is linear or not.

Student: Sir, but in case of capacitor we saw, that ((Refer Time: 1:26:20)).

Yeah, from the iV relationship, the current, the voltages, sorry, the current is C times dV by dt. Now, we cannot say that it is linear, say, we already know the, know, that the derivative, the linear operator and so on.

Now, you are not sure we can test it and see the derivative is linear operator. What it means is, if you take the, differentiate the sum of two functions, you will get the sum of derivative of the individual function, that is linear, ok. With that we come to the end of the lecture.