

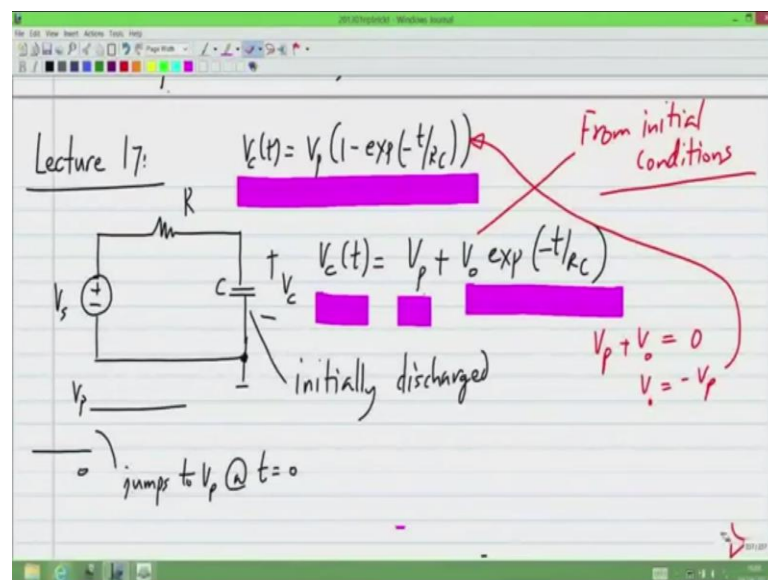
Basic Electrical Circuits
Prof. Nagendra Krishnapura
Department of Electrical Engineering IIT Madras
Indian Institute of Technology, Madras

Lecture - 17
RC (first-order) Circuit, Complete Response with Step Inputs
Transient (natural) and Steady State (forced) Responses
Zero-State and Zero- Input Responses

Hello and welcome to all of you after a long break. In the previous lecture, we were discussing R C circuits, before that we had only considered circuits with resistance which have no memory. So, that means that to evaluate what happens to time varying signal you just have to calculate point by point. All the equations we got while analysing resistor circuits were algebraic equations. Now, when you have a capacitor or an inductor, you end up with have resistor only we will have algebraic equations.

When we have capacitors or inductors we will end up with differential equations. So, that is what we solved and we solved it for a couple of cases had some initial condition and the input was 0. Then I took a case where the capacitor initially had 0 initial condition and we applied step input. Now we will continue from there.

(Refer Slide Time: 01:34)



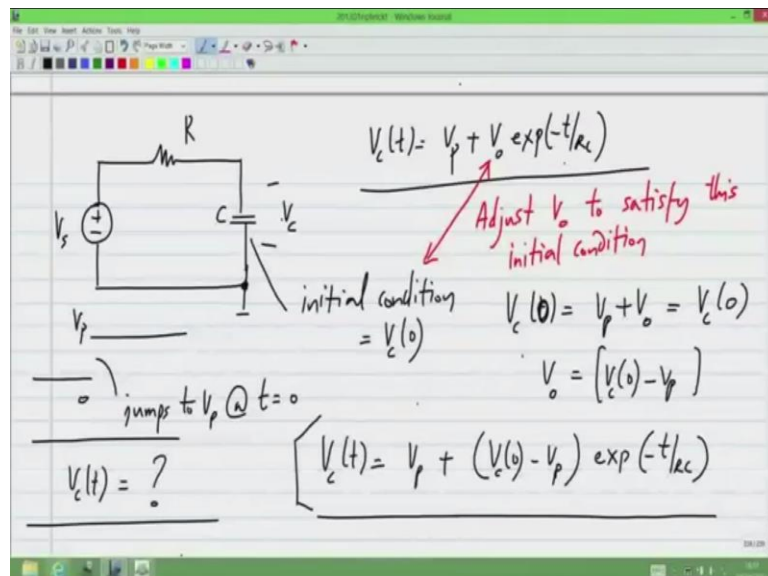
We saw what happens in this case see and the input V_s goes from 0 to b_p . I think I called it, let me check yeah at t equal to 0. Now, in this case we saw that at the capacitor

voltage V_c is given by V_c of the equal V_p times 1 minus exponential minus t by $R C$. So, and the assumption of course, was that the capacitor here is initially, that means for t less than 0 discharged.

So, one of you has raised your hand ((Refer Time: 03:26)) chat window. So, this what we got. So, the general form of the expression turned out to be of this type because of this type where V_c of t would be the V_p . The applied voltage plus some exponential and this coefficient V naught had to be calculated from initial conditions. Now, in this particular case we know that the initial condition of the capacitor is 0 . We will discuss more, the business of initial conditions.

So, at t equal to 0 V_p plus V naught will be the output voltage and that has to be equal to 0 . So, V naught turns out to be minus V_p giving you this expression. So, if you start from a different initial condition of course, you will have different value of V naught. So, is this clear are there any questions on this first for $R C$ circuits how we analysed it, the result that we have got any of that stuff. When we had a 0 input and an initial condition this is what we analyse first the $R C$ and R and the voltage across the capacitor turns out to be the initial condition of the capacitor times exponential minus t by $R C$.

(Refer Slide Time: 06:42)



So, if we combine the 2 cases that is, let me say that I have this circuit. I have this circuit and the initial condition equals V_c of 0 . So, what is the expression of V expression for p in this case please try to answer this. Here I am talking about a case where I had t equal

to 0 at t equal to 0 V_s jumps from 0 to V_p , but the capacitor is not initially discharged at t equal to 0, it has an initial condition of V_c of 0. So, what is the expression for V_c of t please answer this please, try to answer this one. So, all you have to do is take the previous expression that is V_c of t equals V_p plus V_0 exponential minus t by $R C$ and adjust the value of V_0 of for the appropriate initial condition .

So, let me do it here. When V_c of t is V_p and plus this whole expression. So, V_c of 0 would be V_p plus V_{naught} and this should be equal to V_c of 0. So, V_{naught} would be V_p minus V_c of 0. So, the expression for V_c of t V_p plus V_p minus V_c of 0 exponential minus t by $r c$. So, this the expression I think I made a mistake in this. It should be the negative of this V_c of 0 minus V_p and here also it is V_c of 0 minus V_p . Now, this we will spend some time discussing this equation.

This is the general form of equation of the step response that is response to step input whether it be voltage or current for an $R C$ circuit for a first order circuit. $R C$ circuit, we have is an example of a first order circuit, by first order what I mean is like the equation describing this as a first order differential equation. So, let us look at this somewhat carefully.

(Refer Slide Time: 11:46)

$$V_c(t) = V_p - (V_c(0) - V_p) \exp(-t/RC)$$

depends only on the input (forcing function) $\left\{ \begin{array}{l} \text{Forced response} \\ \text{steady state response} \\ \text{particular integral} \end{array} \right\}$

$\left\{ \begin{array}{l} \text{Natural response} \\ \text{Transient response} \\ \text{solution to homogeneous eq.} \end{array} \right\}$

$1/RC = p_1$: characteristic frequency
 time constant

$\exp(p_1 t)$: form a part of the response of any linear 1st order system

Now, this $R C$ by the way is the time constant and this minus 1 by $R C$ equals p_1 . This is the characteristic frequency. Now, you have noticed that for all the cases we calculated they end up with this exponential minus t by $R C$ in the response. Now, there turns out to

be general property of linear systems, that the response will consist of exponential of characteristic frequency times the time. So, exponential $p_1 t$ this will form a part of the response of any linear first order system. In fact all linear systems have this behaviour if you do not have a first order system if you have a higher order system you will end up with multiple exponentials, exponential of $p_1 t$, exponential of $p_2 t$ and so on.

Now, this can be thought of as two parts this part, which is dependent solely on the input this is known as the first response or the steady state response. This part which has this exponential minus t by $R C$, which is a characteristic of the circuit is known as the natural response or the transient response.

So, the natural response of any linear system will be of the form of exponential minus t by $R C$. There are some small modifications, which we will not go into, but this is generally the case. The total response always consists of forced response, which depends solely on the input that is the driving function or the forcing function. The natural response which depends on initial conditions and the forcing function and the natural response of the circuit of the form exponential minus t by $R C$.

Now, let me also write this. So, this force response or steady state response depends only on the input which is the forcing function. This natural response will always have this term exponential minus t by $R C$, which by the way is also known as the natural mode. So, just like you have a string with a characteristic frequency right that you. If you pluck a string it will vibrate at some frequency that is called one of the modes of a string exponential minus t by $R C$ is the mode of the first order $R C$ circuit.

So, all linear system response can be classified into the first response or natural response or equivalently the steady state response and transient response. Another terminology used is here particularly in relation to differential equations and mathematics is this is the solution to homogeneous equation and this is the particular integral. So, these are the same thing and these all mean the same thing. This is general property of linear systems and of course we are now looking at a first order $R C$ circuit.

(Refer Slide Time: 17:30)

$$V_c(t) = V_p \left(1 - \exp\left(-\frac{t}{RC}\right)\right) + V_c(0) \exp\left(-\frac{t}{RC}\right)$$

V_p ——— Zero state response Zero input response } Step response
 ——— 0

Total response ?
 Transient response
 steady state response
 Linear
 What does linearity mean?

Now, we can also rewrite this as $V_p (1 - \exp(-t/RC)) + V_c(0) \exp(-t/RC)$. So, you can see that this part is proportional to the input and it does not have any initial condition term in it. In fact this is known as the 0 state response. So, this is the response you would have got, if the state of the capacitor the initial condition on the capacitor were to be 0. This part which depends only on the initial condition is known as the 0 input response. Now, the reason for this terminology is obvious, the 0 state response is the response you get when the initial state is 0. The 0 input response is the response you get when the input V_p is 0.

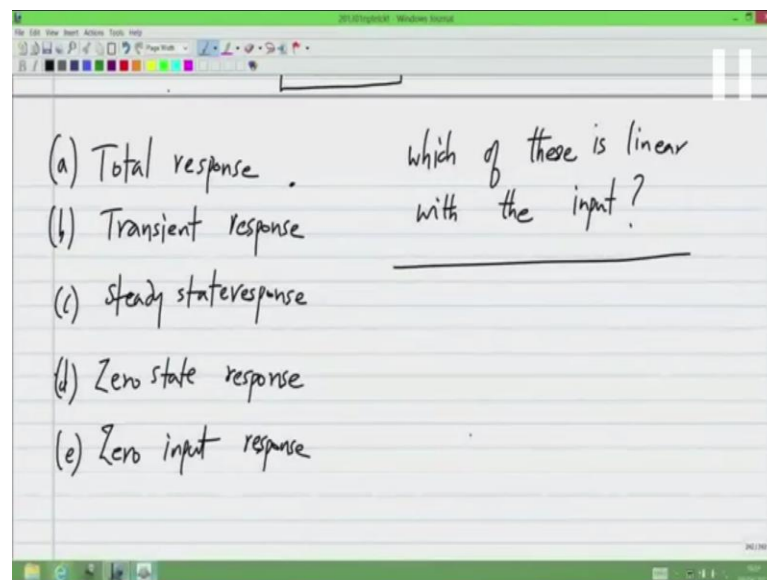
By the way I have to ((Refer Time: 18:58)) once more, that we are only considering a case of a piece wise constant input, what I mean by that, the input is not arbitrarily varying with time it varies from 0 to V_p it jumps from one particular constant value to another particular constant value. What I am particularly warning against here is that, if let us say V_p were to do this V_p were doing something of this sort. You should not substitute V_p of t in this expression that is completely wrong you will get some integral with V_p inside it. You cannot simply substitute V_p of t this for step of V_p .

Now, this is supposed to be a linear system and the response we classified into different parts they have a number of things. First of all they have the total response which is $V_c(t)$ and we have the transient response, we have the steady state response and we also have the 0 state response and 0 input response. All of this is for a system, which we

know is linear. At least we know that it is made of linear components resistor and capacitor.

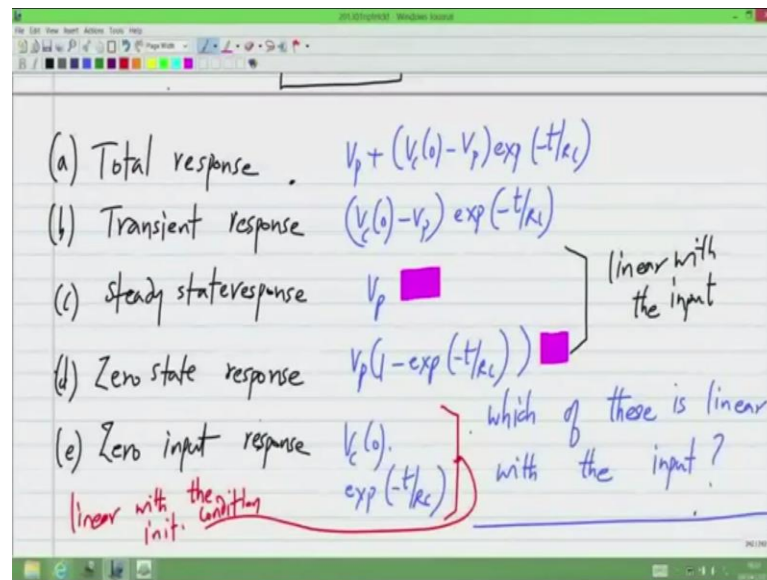
Now, my question is what does linearity means in this case. That is one way to express linearity is to say that some response is proportional to some input or something is proportional to something else. Now, please try to answer this, like is the total response linear with the input is this linear yes or no. You can type your answers into a chat window or I can pose the same question in a different way. We have all this different types of responses or the total response was classified into these different things.

(Refer Slide Time: 22:39)



So, let me put down these things, which of these is linear with the input. You can also indicate your choice on the poll that has appeared. I listed 5 response all either the total response or part of the total response of a first order R C circuit. My question is which of this is linear with this input to the R C circuit. Of course, we are only considering step input at this time please try to answer this. All you have to do is to look at the expressions for these.

(Refer Slide Time: 24:26)



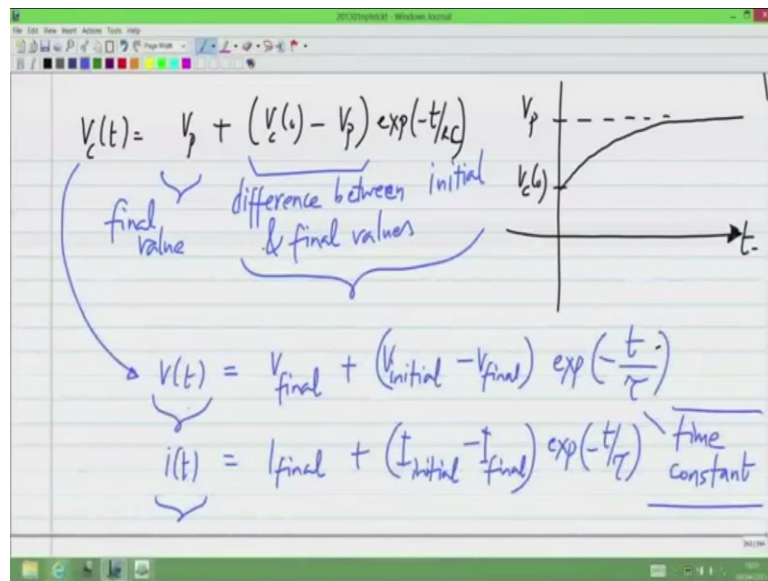
So, the expression for the total response is V_p plus $V_c(0) - V_p$ exponential minus t by RC . The transient response is $V_c(0) - V_p$ minus t by RC . The steady state response is V_p . The zero state response is $V_p(1 - \exp(-t/RC))$. Finally, the zero input response is $V_c(0) \exp(-t/RC)$. Now, from this you should be able to very easily tell which of these is directly proportional to the input.

Now, I got some responses clearly the first one is not proportional to V_p because of this $V_c(0)$. So, this is also not proportional to V_p because of this $V_c(0)$ term. So, steady state response and zero state response these are proportional to V_p . So, these V_p linear with the input. Similarly, this one I did not ask that question, but this part the zero input response will be linear with the initial condition. So, this is what linearity means in this particular context.

The total response will not be linear with either the input or the initial condition. In general if $V_c(0)$ happens to be 0 that is a different matter, that is a coincidence otherwise. The total response or the transient response will not be linear with either the input or the initial condition. The steady state response will be linear with the input because it only depends on the input and the zero state response also depends only on the input. It will be linear with the input and the zero input response will be proportional to linear with the initial condition. So, that is what linearity means in this context.

I mentioned this because we know that in linear system the response is proportional to the input. You can have a resistive circuit, you change the input then the response will change in proportion to that input. I am assuming a circuit with a single inputs. Now, in case of the circuits, which have memory elements like capacitors or inductors, the situation is a little more complicated and it is linear in this particular sense. Now, we can also go ahead and interpret this equation further.

(Refer Slide Time: 29:20)



First of all if I brought this, let us say it starts with some V_c of 0 and reaches a steady state of V_p because t equal to 0. You can see that this V_c of t will be V_c of 0 at t equal to infinity and the exponential has died out it is equal to V_p . So, the response goes as an exponential. Finally, reaches V_p . So, there are different ways to interpret this. If you look at this part of it. The difference between final or initial and final values and this whole term if you look at it says that the difference between the initial and final values decays with time.

So, you have the final value plus any difference was there between the final and initial values will decay with time. So, that is one way to interpret this we do all of this. So, that it turns out for the step like inputs for first order systems after enough practise you will be able simply look into circuit and write the response. Of course, we need a lot of practise to do this initially you should do it systematically, but you should be able to do it after a while and this part of course, is the final value.

So, this V_c of t in general this turns out any V of t in a first order system and by the way it could be I of t . Any current as well, I have demonstrated this with the capacitor voltage in a particular subject, but it turns out that any brands quantity either voltage or current in any first order circuit can be represented in this form for a step input.

So, this is V_{final} plus V_{initial} minus V_{final} times exponential. It is minus t by $R C$, but to make it more general I will write minus t by τ where τ is the time constant. So, this is the general form and like I said exactly the same holds for a current as well I_{final} plus I_{initial} minus I_{final} exponential minus t by τ . So, I hope this is clear in fact they would have seen this type of formula, but it has some basis. Essentially all it says is that it expresses the time by final value plus the difference decaying over time.

(Refer Slide Time: 33:22)

$$V_c(t) = V_p (1 - \exp(-t/RC)) + V_c(0) \exp(-t/RC)$$

$$V(t) = V_{\text{final}} (1 - \exp(-t/\tau)) + V_{\text{initial}} \exp(-t/\tau)$$

Building up to the final value

Initial condition decaying with time

$V_{\text{final}}, V_{\text{initial}}, \text{time constant } \tau$

Now, we also had a expressed the expression in a slightly different way, V_c of t is V_p 1 minus exponential minus t by $R C$ plus V_c of 0 exponential minus t by $R C$. It is exactly the same expressions, but divided differently as 0 input and 0 state responses. Now, again writing it in a more general form V of t would be V_{final} one minus exponential minus t by τ plus V_{initial} exponential minus t by τ . So, compared to the previous expression this gives you a slightly different interpretation.

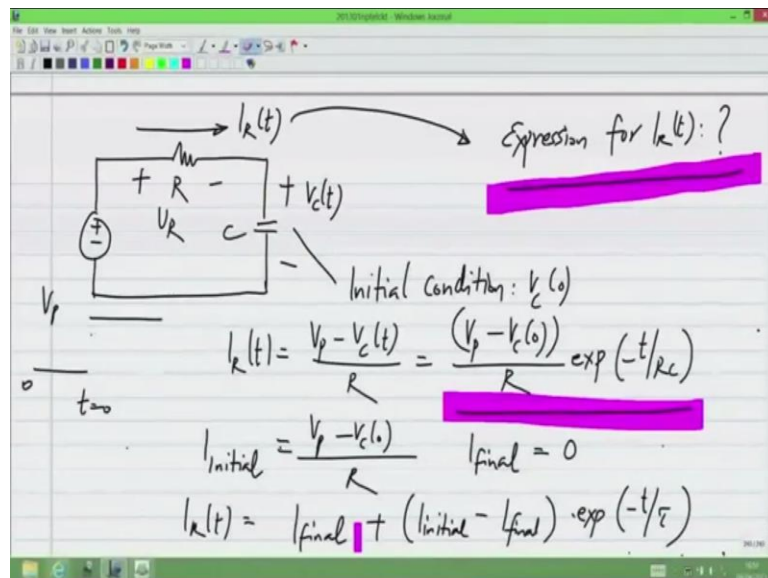
Exactly the same final solution of course, but only interpretation is different. Here you think of this as this term building up to the final value that is if you just look at this part of it, it starts from 0 because 1 minus exponential is 0 and then it builds up to be final.

This you can think of as any initial condition that is there on the capacitor decaying with time. So, t equal to 0 you will only have the initial part and it decays with time. So, this is ((Refer Time: 35:31)) about the interpretation. You can think of total response as 1 part which is building up to the final value and another part which is the initial part decaying with time and like before this also holds for currents.

We will quickly take some examples and see. So, this also holds for currents. So, with this you should be able to write down the step response of any first order circuit. We will also take examples of circuits with inductors and you know that the step response without necessarily having to go through the differential equation. So, as usual in the beginning when you are not very fluent with this I will recommend writing differential equation identifying the term properly and then writing down solution. As you get more and more practise you should be able to do this more conveniently.

Now, to be able to do this we have to identify there are 3 quantities. In this 3 things we have to identify that is V final or the final value V initial or the initial value and also the time constant τ . So, we will see how to do this. Now, the time constant is the same for a given circuit regardless for which quantity you pick.

(Refer Slide Time: 37:21)



For instance let me take the same example R C and let us say that this has an initial condition of V_C of 0. The input steps from 0 to V_p at t equal to 0. Now, we have been looking at V_C of t whatever like you have to do is to give me the expression for I_R of t .

The resistor current, please try to do this please. Calculate the expression for I_r of t the current in the resistor. I got a response that says it is V_c of t by r that is not correct, because the voltage across the resistor is not V_c of t .

So, please try to do this give me the expression for I_r of t . Obviously to one way to find the current is to find the voltage across the resistor and divide it by r . Alternatively you recognize that the current the resistor is the same as the capacitor current. If you know the capacitor voltage you should be able to calculate the capacitor current. You can do it either way I think there are a number of responses, clearly the voltage across the resistor is given by V_p minus V_c of t . So, I_r of t is V_p minus V_c of t divided by r which in turn can be written as V_p minus V_c of 0 divided by r exponential minus t by $R C$.

So, again you see this exponential minus t by $R C$ appearing in the expression. Let us try to relate it to initial and the final condition. First of all if the initial voltage across V_c is V_c of 0 , then the initial voltage across r . So, let me write it like this, V_r is in this direction, so V_r initially would be or let me write the I_r directly. So, I initial is when I say initial it is just after the step is applied it is V_p minus V_c of 0 divided by r . Now, useful special case to consider is when V_c of 0 equals 0 .

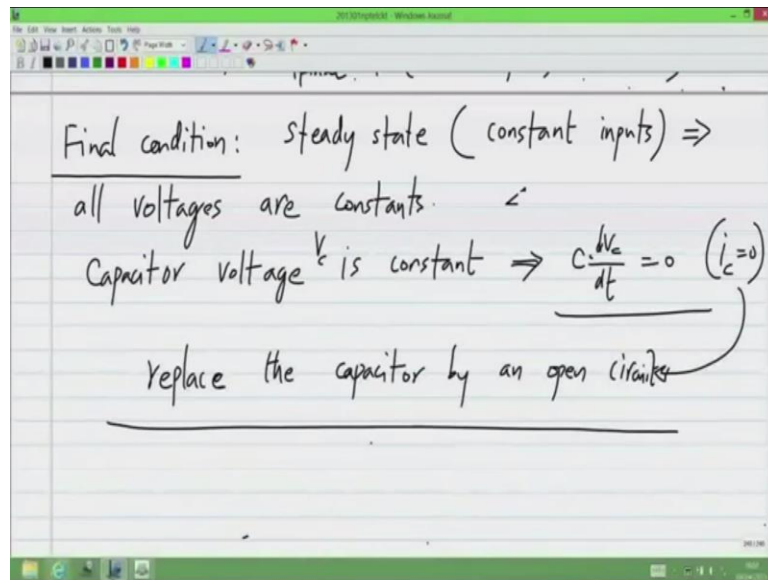
In that case you know that the capacitor voltage is 0 . Initially the input voltage here jumps up the capacitor voltage cannot change. So, the voltage across r will be equal to V_p that is what V_c right. If V_c of 0 is 0 the initial current is V_p by r and that is what will charge the capacitor. We also saw that that will be the slope of V_c that is regulated by the slope of V_c . What is I final, I final is what does it mean to have a steady state in a circuit with capacitors and piece wise constant voltage.

Steady state means that the voltages in the circuit will be constant with time. So, if the capacitor voltage remains constant with time, then there is no current through the capacitor. So, it is like an open circuit. So, clearly the current through the resistor is the same the current through the capacitor. So, I final will be equal to 0 and you again see that whatever we have here this response conforms to the general formula, which is I final plus I initial plus minus I final exponential minus t by τ because I final is 0 . We have only, this part is not there and we have only this part is this.

So, every quantity I mean this is a very simple circuit, but every quantity will correspond to will conform to formula this or this by the way. These two formulas are same the only

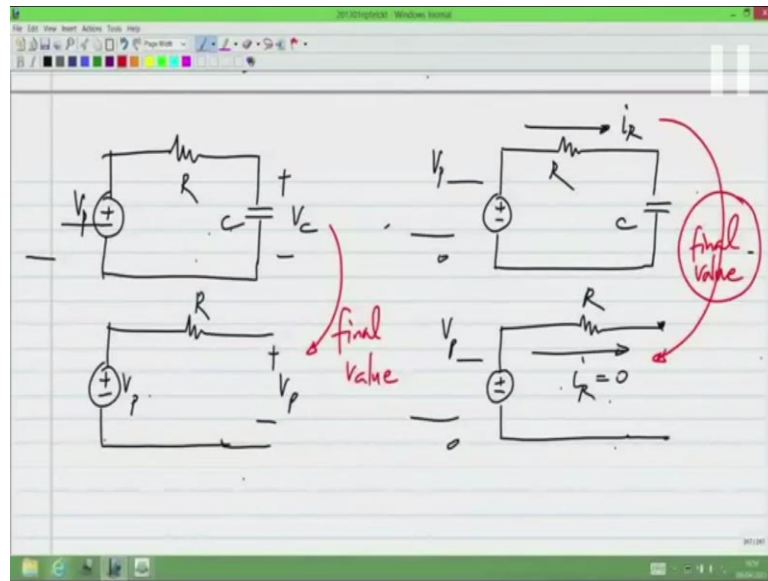
difference is the terms are grouped differently and interpreted differently. Is this fine now, you could also try and see how to evaluate the initial and the final conditions by looking at the circuit.

(Refer Slide Time: 45:39)



Let me first take the final condition. The final condition is after the steady state is reached and steady state because we are looking at constant inputs, means that all voltage is constants. Now, if the capacitor voltage is constant any capacitor voltage. It means that derivative is 0 and that means that the current is 0. The current through the capacitor is 0. Now, if the current through the capacitor is 0 that means that it can be replace by an open circuit.

(Refer Slide Time: 47:05)

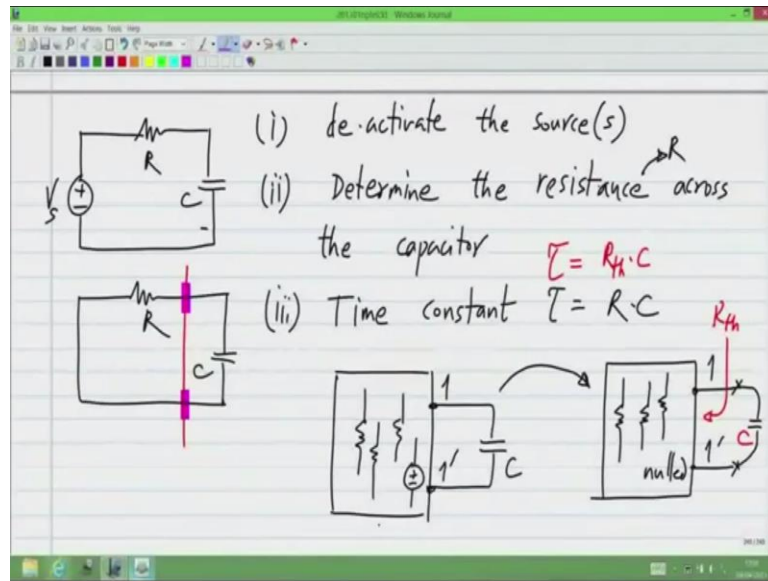


So, I replace the capacitor by an open circuit and find the quantity, which ever quantity is of interest to you. So, this is how you find the value. Let us do it for the case we took this is circuit R C and let us say I am interested in the capacitor voltage V_c . Now, to get the final value what I will do is to open circuit the capacitor that is the capacitor is not there. So, clearly the voltage that appears here equals V_p and this would be the final value.

Now, this circuit is very simple, but the same principle applies to all the circuits. Similarly, let us say I am interested in the resistor current. Then I can take a case of inputs stepping from 0 to V_p to calculate the final value, I open circuit the capacitor and clearly this current would be 0 and that indeed is the final value of I_r .

We will take a slightly more complicated example later. So, is this thought clear any questions about what we discussed? So far what we did was to extend discussion from last class there we had mathematically evaluated the solution to the differential equation. Now, here what we have done is to identify certain characteristics of the solution to the first order system with a step input. We can use those characteristics directly to write down the solution with little bit of experience. So, if you have any questions I will take them now. It appears that the things are clear.

(Refer Slide Time: 50:57)



Finally we have to identify the time constants. So, there is a question, but it looks like it is incomplete. I am not able to understand the question here from one g industries we also have to identify the time constants that I will come to later. In case of our circuit which is only R and C it is very easy. So, let me call this V s. So, first of all you deactivate the source then determine the resistance across the capacitor. Finally, let us say the resistance is R in this case. Finally, the time constant tau will be R times C.

Now, this is very simple and silly for this case because we have only R and it is very obvious what the resistance across the capacitor is when I say what across capacitor what I mean is between these 2 terminals. Now, this principle applies even to very complicated circuits as long as you have only one capacitor.

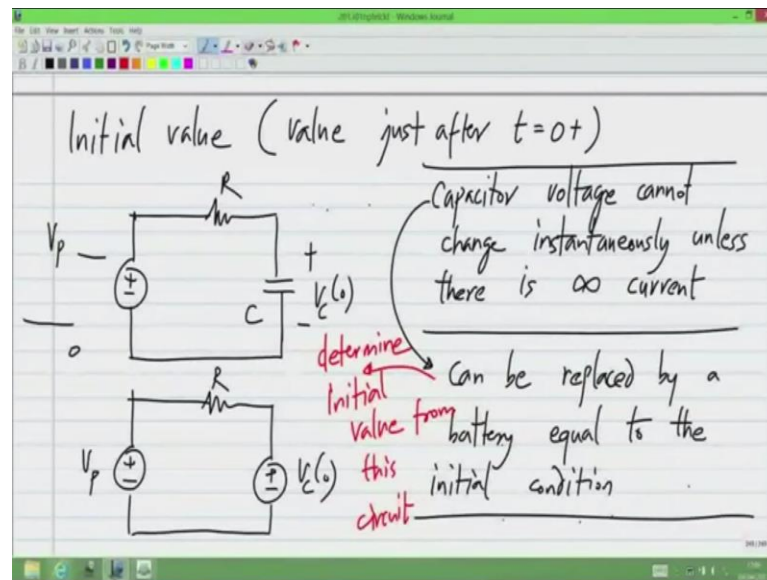
Let us say I have a circuit with a number of resistors and I show only the capacitor like this connected to terminals one prime. I also have a number of sources. I will show one example here. Now, what I have to do to find the time constant is, first of all I numb this circuit that means that any independent source is set to 0. If it is current source it is an open circuit, if it is voltage source it is a short circuit and I have these 2 terminals 1, 1 prime across which the capacitor is connected. So, what I have to do is to determine the resistance of the circuit looking into 1 1 prime. Now, this we have done many times right. It is basically the ((Refer Time: 54:32)) equivalent resistance looking into the terminal 1 1 prime.

So, this R_{th} is what we have to determine and once you do that the capacitor value is c the time constant τ will be given by R_{th} times C . So, you can very easily verify that it is true for all very simple circuit. Looking between these 2 terminals to the left side we have only R , but even if we had a very complicated circuit, it would be true you would have to find the equivalent resistance looking this and time constant would be R_{th} times c is this clear.

So, we know how to find the final value. You know how to find the time constant, we also have to find the initial value. When I say the initial value, it will be just after the step is applied that is what we will proceed to do. Any questions please interrupt. I am not getting many questions today. So, that means either you are understanding everything or probably understanding nothing. So, if you are understanding everything good, but if you have difficulty understanding something please ask the questions on the chat window on I will answer one.

So, R_{th} is the notation I use for the thermal resistance looking into $1-1'$. So, if you have a circuit with 2 terminals $1-1'$, then when you deactivate the circuit and find the resistance that is the thermal resistance across those 2 terminals. So, that is R_{th} , now we have to find initial condition. Again I am talking about any variable not just the capacitor voltage. There is another question can we use r_l network for this. It is not clear to me, are you talking about circuit with resistors and inductors. The same principle applies I will come to those examples a little later. Exactly the same principle applies because with a single inductor it is also a first order circuit.

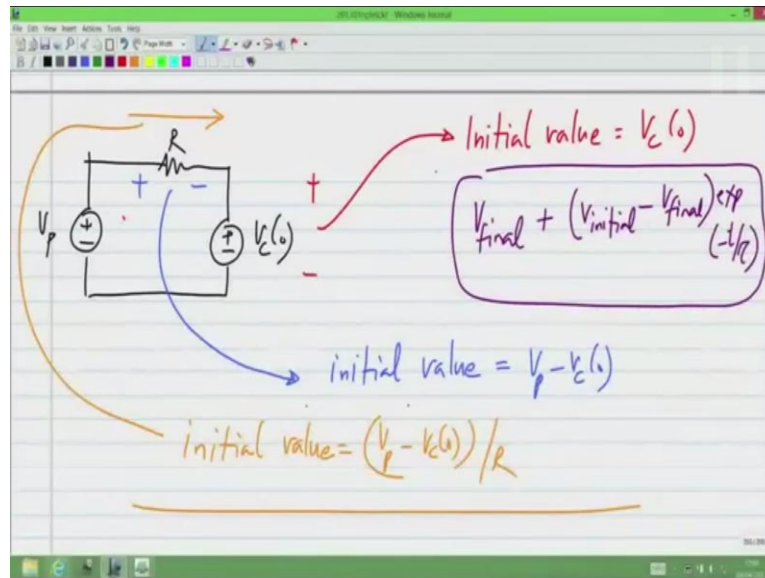
(Refer Slide Time: 57:49)



Now, let me look at how to calculate the initial condition or the initial value. When I say initial value, it is the value just after t equals 0 plus. So, again let me take my simple circuit example. The input jumping from 0 to V_p I have resistance R and I have a capacitance c initially charged to V_c of 0 . So, the principle we use to find initial value is the following that a capacitor voltage cannot change instantaneously unless there is infinite current.

In fact we will come to cases where there is indeed infinite current, but initially we will assume there is no infinite current. So, the capacitor voltage does not change at that instant. So, for that particular instant capacitor can be replaced by a battery whose value equals the initial condition. So, that means that instead of this V_c of 0 . What I do is I have a battery or may be let me use the usual symbol for the voltage source V_c of 0 . This is V_p just after t equal to 0 . Now, clearly we see that any quantity we take right. So, this is the network from which we calculate the initial condition. So, we replace the capacitor by a battery and then you do the circuit analysis.

(Refer Slide Time: 1:00:45)



So, let me write it down first. You will determine initial value from this circuit. So, first let me say that I am interested in this voltage that was the capacitor voltage. Clearly the initial value equals the equals V_c of 0. Again it will become more clear if we take a slightly more complicated example. If I am interested in the resistor voltage the initial value of that equals V_p minus V_c of 0 because this is now, when you remove the capacitor this is just d c circuit analysis right circuit analysis with d c sources. So, it is easy that is why we choose to do this.

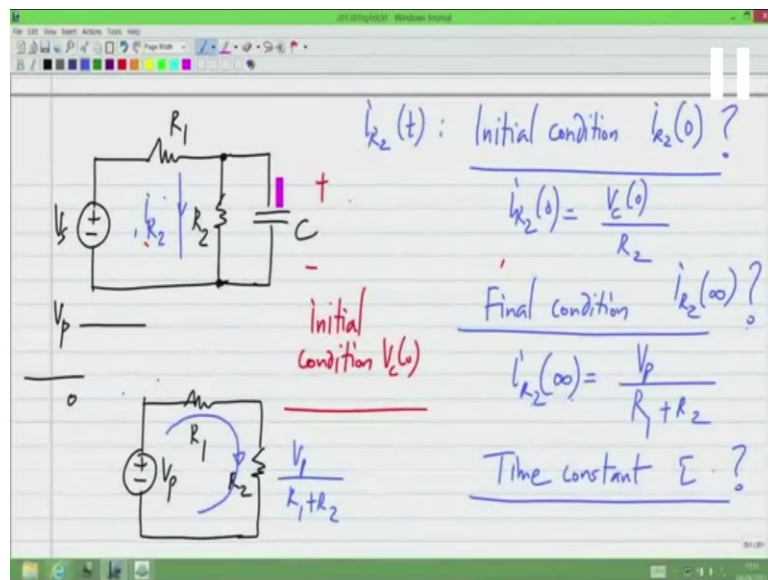
Let us say I was interested in the inductor current and the initial value for this sorry the resistor current not the inductor current. The initial value for this is V_p minus V_c of 0 divided by r . Now, we know how to calculate all 3 parameters that appear in the general equation for the solution to first order system with a step input. What were the 3 things the initial value the final value and the time constant. The initial value is obtained by replacing the capacitor with a battery equal to the initial condition 1.

If the initial condition is 0 then you replace the capacitor by a short circuit. So, then find whatever quantity you want it could be a current it could be a voltage whatever quantity for which you are trying to write the expression, that is that you find from this new circuit and that is the initial value. How do you find the final value, we are looking at constant inputs. So, in steady state all the voltages and currents will be constant. If capacitor voltage is constant its current is 0. So, they can simply open circuit or remove

all the capacitors from the circuit and from this new circuit we can analyse the final value and finally, a time constant.

How do we find this we look at the terminals across which the capacitor is connected. We find the equivalent resistance looking into it after nullify all the independent sources. So, that gives you the time constant. So, we have all three. So, you should be able to simply then write down the expression, whether it is voltage or current $V_{\text{final}} + V_{\text{initial}} \exp(-t/\tau)$. So, is this part clear, any questions about this. Now, let us take an example which is only very slightly more complicated than what we had. Of course, I would like you to participate and answer the questions that way you will get practise in doing things like this.

(Refer Slide Time: 1:05:36)



So, let us we will again take the same kind of input starting from 0 to V_p . Now I could calculate any quantity I want. So, first I will try to let us try to calculate these 2 things. This is V_c and another 1 is I_{R_2} . First, let us calculate V_c . Let us say this is what we are interested in. So, we need the 3 things, the initial value the final value and the time constant. So, we will give you the initial value of t , or rather let me this is rather because we have already calculated the capacitor voltage. Let us do it for I_{R_2} that current through resistor r_2 of t . Let us say the capacitor has an initial voltage initial condition V_c of 0. Now, please give me the initial condition for I_{R_2} of t . So, I have given the algorithm, how to do this and the initial condition means just after the step is applied.

I told you how to do this you replace the capacitor with an appropriate battery and do this. So, please give me the expression for $I R^2$ of t the initial value of that. What is the initial condition, what is what is the initial condition of $I R^2$ of 0. So, that is correct, it is response from Arti it is clearly $I R^2$ of 0 is V_c of 0 divided by r^2 . In fact in this case you do not even have to replace it by a battery and analyse because if you have V_c of 0 here that appears directly across r^2 . So, initially the current here would V_c of 0 by r^2 .

Now let us get to the final condition, which is also typically $I R^2$ of infinity. What would this be. So, again I have to give an algorithm all you have to do is follow that ok. Of course it is not a good idea to blindly follow the algorithm without understanding it. So, if you have difficulty with the logic. So, get back to me and I will explain it again. So, again there is a response and for the initial condition we replace this with a battery and that voltage appeared across r^2 for a final condition. We have to open circuit the capacitor. So, we will be left with this V_p which is the value just after t equal to 0.

We have to open circuit the capacitor to do this. Once we open circuit the capacitor the value here the current here would be V_p divided by r_1 plus r^2 . So, the final condition is V_p divided by r_1 plus r^2 because once the capacitor is open circuited I mean no current flowing through that it is V_p by r_1 by r^2 . Finally, we need the time constant τ what is this going to be. Again I outlined algorithm for this, you have to look at the resistance that appears across the capacitor if all the independent sources being void. So, please do that and let me know what will be the time constant τ .

(Refer Slide Time: 1:12:34)

$$i_{R_2}(t) = i_{R_2}(\infty) + (i_{R_2}(0) - i_{R_2}(\infty)) \exp(-t/\tau)$$

$$= \frac{V_s}{R_1 + R_2} + \left(\frac{V_c(0)}{R_2} - \frac{V_s}{R_1 + R_2} \right) \exp\left(-\frac{t}{(R_1 || R_2)C}\right)$$

$$\tau = R_{th} \cdot C = (R_1 || R_2) C = \frac{R_1 R_2}{R_1 + R_2} C$$

So, what we need to do is we have this circuit here. So, first I identify the terminals across which the capacitor is connected that is this much. Also I said V has to be 0, when V has become the 0 when the voltage source is 0 it is a short circuit. So, I will end up with this r_1 r_2 and I have to find the resistance looking into 1 1 prime and clearly that resistance is r_1 parallel r_2 because you see that r_2 r_1 appears in parallel r_2 .

So, the resistance here is r_1 parallel r_2 and the time constant is nothing but the product of the resistance and the capacitor c time constant τ is r_1 plus r_2 times c . With this we will be able to construct the expression for I_{R_2} of t . So, I_{R_2} of t is the final value plus the initial value minus the final value which decays over time, which is basically V_s by r_1 plus r_2 plus. This should be I_{R_2} of infinity I_{R_2} of 0 is V_c of 0 by r_2 minus V_s by r_1 plus r_2 exponential minus t by r_1 parallel r_2 . By the way this notation for r_1 parallel r_2 , I think all of you are familiar with r_1 parallel r_2 times c .

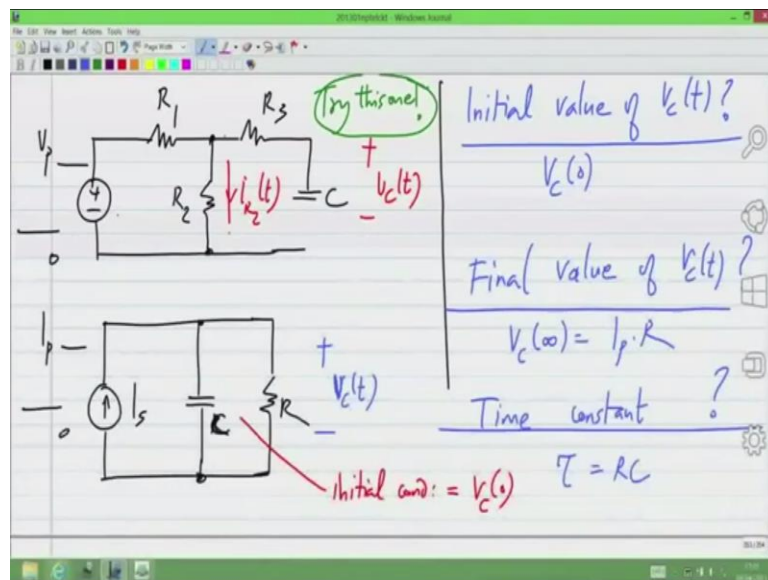
So, this is the expression for I_{R_2} of t inside you can write down the differential equation and make sure that this satisfies this. I encourage you to do that especially in the initial parts when you do not have enough practise. So, this is by identifying general pattern in the solution to the first order differential equation. For a step input we can write down any expression. We wrote it for I_{R_2} , I could write it for the current through r_1 or the current through the capacitor or any quantity in the circuit, are there any questions at this point. So, one question is why we are taking τ as r_1 parallel r_2 τ is

not taken as r_1 parallel r_2 . The time constant τ is the thermal resistance looking into r_1 times c or $\tau = r_1 \times c$, which is as I said thermal resistance. Looking into this is r_1 parallel r_2 times this c .

I do not know if this is a confusion with notation this is r_1 parallel r_2 times c which is the same as $r_1 \parallel r_2$ by $r_1 + r_2$ times c . Now, another question is how to find I_{R1} of t and if it is the same, no clearly it will not be same as I_{R2} because I_{R1} will be I_{R2} plus the current through the capacitor. So, it will have a different expression and you can evaluate it you can find the initial condition of I_{R1} the final condition of I_{R1} . The time constant will be the same because you see that the procedure to evaluate time constant does not depend on what input is there because we null the input and what output we are taking. That is that variable we are considering.

The time constant is a property of the circuit. So, the time constant will be the same for that. So, you can evaluate I_{R1} by yourself by following this algorithm. Any other questions. So, you can try it for any other circuit as well. So, if you want some examples you can put instance.

(Refer Slide Time: 1:20:57)



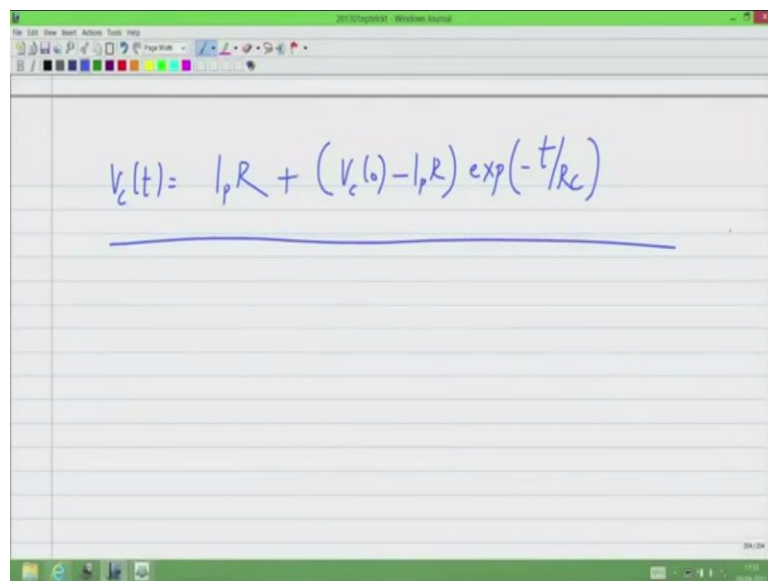
You can try this and you can calculate any value. For instance you could calculate V_c of t or you could calculate I_{R2} of t , but again assume that input jumps from 0 to V_p . Now, also the input does not have to be a voltage it can be a current. Let us say I have some I_s and I have a capacitor and resistor in parallel. Let me take this very simple case and let

me say that the initial condition here on the capacitor is V_c of 0. The current starts from 0 to V_p . So, let us try to do those very quickly for this case that is for this problem I encourage you to try the upper problem by yourself and this problem.

So, let me say what I am interested in as this particular voltage, which is the voltage across the capacitor V_c of t . So, what is the initial value of V_c of t . Please give me some response. I know that to construct the total response I need the final value of V_c of t and also the time constant. So, first please give me the initial value of V_c of t . This is very, very easy in this case. So, clearly we are looking at the voltage across the capacitor. So, this V_c of t at t equal to 0 is V_c of 0 itself. So, this is V_c of 0 and what is the final value of V_c of t .

So, if we open circuit this capacitor for the final value all of this I_s will flow into the resistor. So, the voltage across this is current times is resistor. So, the final value of V_c is simply I_p times r and what is the time constant. Again it is very simple, in this case you have to null the independent source that is you have to null current source. So, if you null the current source that is you open circuit it. So, this part goes away completely and across the capacitor you just have the resistor R . So, the time constant is the resistor you have across the capacitor time is the resistance value. So, it is $R C$.

(Refer Slide Time: 1:26:10)


$$v_c(t) = I_p R + (V_c(0) - I_p R) \exp(-t/RC)$$

So, the expression for V_c of t is final value plus the initial value minus the final value times exponential minus t by $R C$. Please take care of these things, if you have any

doubts or something needs to be clarified, we can discuss them in next lecture. So, what we have done is to detect a general pattern in the response of a first order system. In the next lecture, we will extend it to RL circuits. It is a very simple extension because we have already done RC we have spent. So, much time on it is quite easy to do that and then move on to second order circuits.

Thank you, I will see you on Thursday.