Electronics and Communication Engineering
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Lecture - 16
RC Circuit Natural Response:
First Order Differential Equation

In the previous lecture, we discussed in detail op amp circuits. We looked at a couple of examples of op amp circuits and also I emphasised that op amp has to be used in negative feedback and I also mentioned how to find the science of the op amp in any circuit such that it is in negative feedback. An op amp is basically a voltage controlled voltage source with a very large gain, because the gain is very large, and it is a negative feedback it forces the input signal of the op amp to be a very small value which can be considered to be 0.

And if the gain of the op amp goes to infinity, that is the idealised version of this which we call the ideal op amp, in that case the input difference voltage of the op amp can be is assumed to be 0. And there is a input current into the op amp. So, based on these things you can analyse any op amp circuit that you have. So, if there are any questions regarding what we did in the last lecture, please go ahead now otherwise we will start with today's topics. So far we have looked at circuits that have resistors and control sources and so on, and all these are known as memory less elements.

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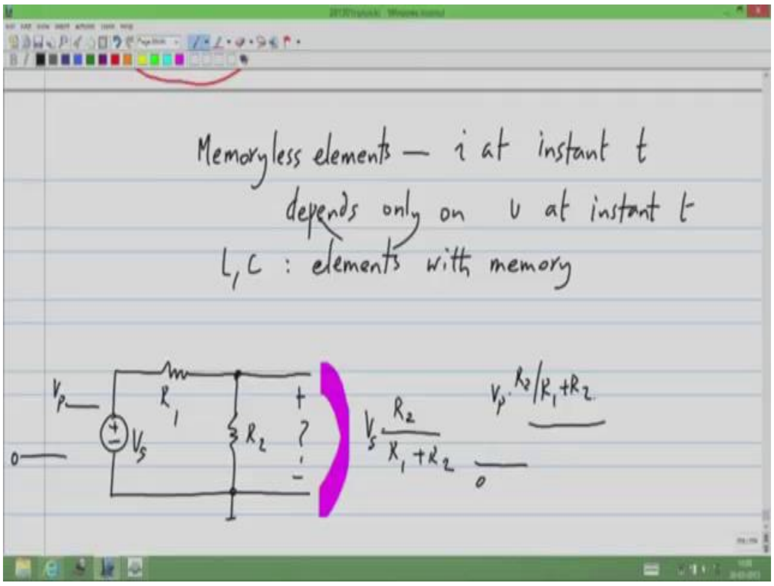

What is meant by this is that the current at instant t depends only on voltage at instant t. That is it does not depend on the voltage at any other instant. Similarly, if you have a voltage to current relationship, the relationship is exactly the same and for all instants and more importantly the voltage at some instant is related to current at that instant only.

Similarly, for controlled sources the controlled quantity is related to the controlling quantity instantaneously only at that instant. Now, if we have capacitors and inductors, we know of this, these properties are not true if you have a capacitor. The voltage across the capacitor depends on not only the current at that instant, but on the history of the current. Similarly, if you have the currents through inductor, it depends not only on the voltage across the inductor, but on the history of the voltage across the inductor.

So, capacitors and inductors are elements with memory, so what we will do in this lecture is to try and analyse circuits which have inductors and capacitors in addition to resistors. Is this okay? Any questions? So, let me take a circuit that we are all familiar with, a resistor, divider. Now, let us say I call this V s and the voltage is 0. What will be the output voltage? Please try to answer this. What is the output voltage? When I say output in this case, I have defined the output to be this particular voltage. What is that voltage going to be? What will be the voltage here, when the input voltage V s is 0? Anybody, What is the voltage going to be?

So, this voltage clearly from the voltage divided formula is V s times R 2 by R 1 plus R 2. And if V s is 0, this voltage is also 0. This is simple, this is a simple resistor divider. Now, let us say it jumps from 0 to some voltage V p. What happens then the output voltage jumps from 0 to V p times R 2 by R 1 plus R 2. So, you can see that the variation of the output voltage with time the horizontal axis here is time. This is time and here also it is time. The variation of the output voltage with time is exactly the same as the variation of the input voltage with time with a scaling factor.

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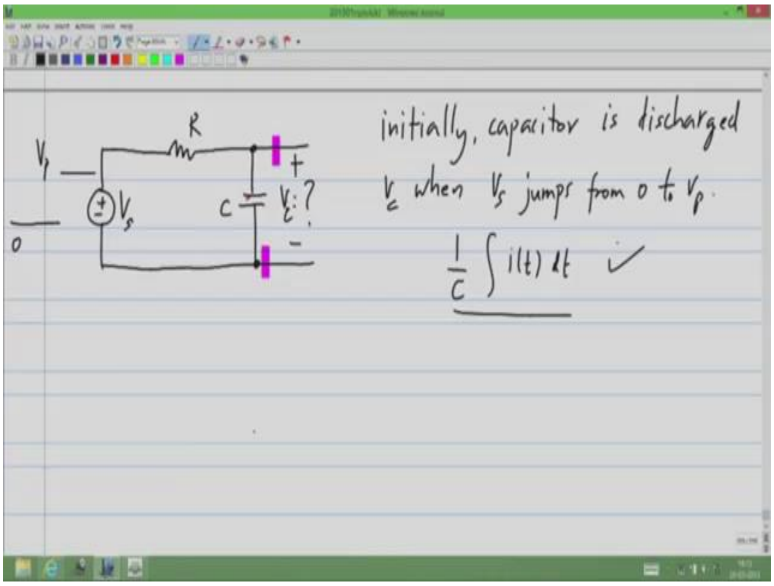

Now, let us say we do something we have V s and I replace R 2 with a capacitor and I have R and C. What happens in this case, let us say, initially the capacitor voltage is at 0 and the input is also 0. Clearly in that case, what happens is that the capacitor voltage is 0 and because V s is 0, no current flows through the resistor which means no current through the capacitor.

So, if there is no current through the capacitor, it is voltage cannot change. So, in this case the voltage remains at 0, if the input is 0. Now, let us say, the input jumps from 0 to V p. What happens to the voltage, please try to answer this, let me call this V c when V s jumps from 0 to V p.

Some of you attempt to answer this. What is the voltage V c going to be when V s jumps from 0 to V p. We have an answer, that says that 1 over c integral i of t d t. This is correct, but the only issue we have now, is that we do not even know what this i of t is. So, that also we have to solve for.

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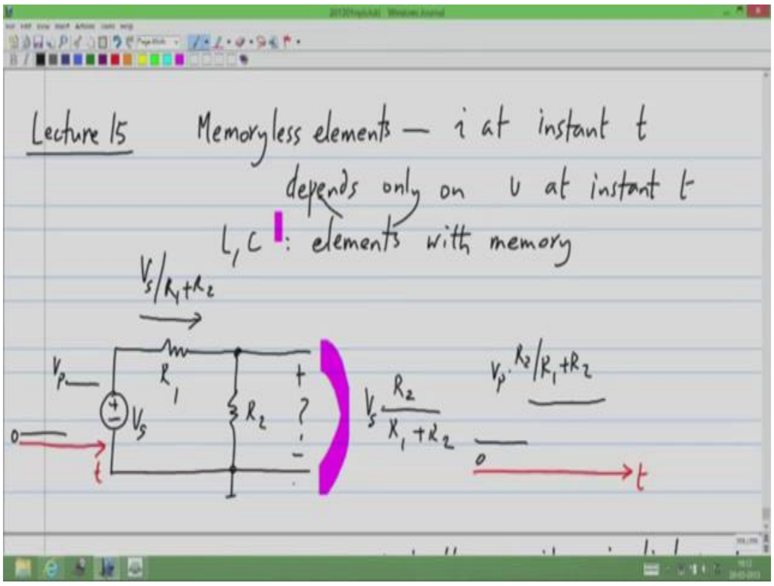

Again, we have done those for the simpler circuit before the current in this is V s by R 1 plus R 2.

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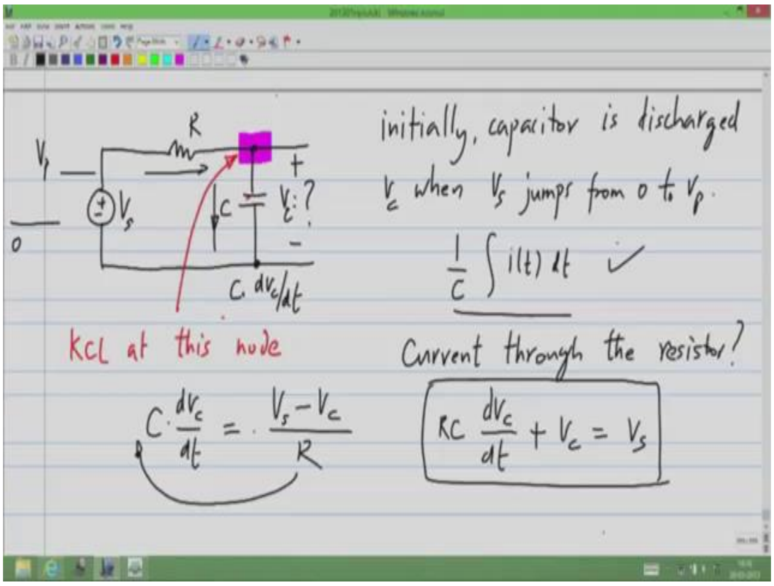

Whereas here we have to find out, and the current itself depends on the voltage of the capacitor. So now, what I would like to do is to write the equation for this circuit so that I can solve it. I will do, what I will do is this. I will write KCL at this node. So, the current through this I will express everything in terms of the capacitor voltage V c. So, the current through this is C times d V c by dt. We know that, that is the current through the capacitor which has voltage V c across it. Now, that has to be equal to the current coming from the resistor.

So, what is the current through the resistor in terms of V c? Please try to answer this. That is correct, so the capacitor current which is C times d V c by dt equals the current in the resistor which is V s minus V c divided by R. What I will do is I will group all of the terms containing V c to the left hand side and I will also multiply both sides with R. So, I will get RC d V c by dt plus V c equals V s.

Now, this is a feature of circuits containing capacitors or inductors or both that the equation governing the circuit will be a differential equation. Whereas previously, they could write a ((Refer Time: 14:07)) called as V c, there is no capacitor here, I will just call that voltage V c. V c equals V s R 2 by R 1 plus R 2. So, it is an algebraic equation. Whereas in this case it is a differential equation, if you have a resistors and capacitors. So, we have to solve the differential equation to find the solution. So, let us try and do that. So, before that we will solve, try and solve a couple of cases intuitively and then go on to finding the solution of the differential equation. Any questions so far?

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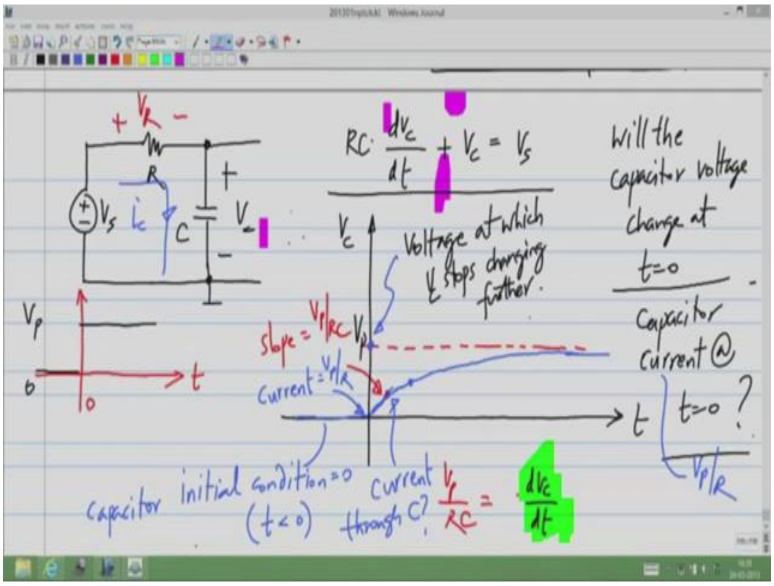

So, let me rewrite the circuit and the equation again. This is the circuit and for arbitrary inputs it is not very easy to solve for these things so we will take specific kinds of input.

So initially in particular, I will take an input that goes from 0 to V p at a particular time instant. So, this is t and let us say this is t equals 0. And we also know that the differential equation governing this is R C d V c by d t plus V c equals V s.

So now, first let us forget the differential equation and try to sketch the output based on what we know about resistor capacitor and the voltage. I already said that initially the voltage across the capacitor is 0. So, if I try to plot V c verses time, before t equal to 0 the voltage will be 0. So that is what I have assumed. So, this is because I say initial condition, but I really mean for t less than 0 on the capacitor.

Now, at the instant that at the instant that the V s steps from 0 to V p, What happens to the capacitor will it change? Will the capacitor voltage change? When I say t at t equal to 0, I mean at this instant will it suddenly change that is the question? What do you think? Yes, so as Aarthi answered the capacitor voltage cannot change suddenly.

Now as I emphasized before, this means that we are already assuming that the currents are finite. So, if the current is infinite the capacitor voltage can change suddenly, but the current is finite and in this case we cannot have infinite current because if we do have infinite current through the capacitor, it means that we also have infinite current through the resistor which in turn means an infinite voltage across the resistor.

So, this capacitor voltage will not change, will not jump at t equal to 0. Now, my question is, what is the current through the capacitor at t equal to 0? What is the capacitor current at t equal to 0? So, clearly it is V s by R or V p by R because value of V s is V p. So, the capacitor current at t equal to 0 this is V p divided by R. So that means, that this current i c at t equal to 0 is V p by R. Now, because the current is flowing through the capacitor its voltage tends to increase, its voltage tends to change and from the direction of the current we know that V c is going to increase.

So, what happens is, it increases so I will show a little bit of increase like this. Now, what is the slope of this increase going to be? What will be the slope? Please try to find out because we know the current through the capacitor, so we should know the slope of the voltage across the capacitor, because the capacitor current is nothing but C times d v by d t, d V c by d t. So, the current through the capacitor is basically related to the slope of the voltage across the capacitor. So, what is the slope of the capacitor voltage verses time? So, when I say slope of the capacitor voltage verses time obviously it should have a dimensions of volts per time, volts per second, something like that, right.

So, we know the capacitor current V p by R. So, we have to equate that, we have to use that, in the equation for the capacitor current and find out the slope of the capacitor voltage. Please try to calculate that. Someone please try to answer this question. So, the current through the capacitor is V p by R, which also in terms of the capacitor voltage is C times d V c by d t.

So, what I was looking for was d V c by d t, the slope of the capacitor voltage, so clearly that is equal to V p by R C. So, the slope at t equal to 0 will be V p by R C. So, it increases a little bit, I have shown it with this blue line over here. Now, let us say after it decreases a little bit, let us say I come to this point what happens then to the current through the capacitor.

Please try to answer this. Now, initially at this point the current equals V p by R, What will it be at the other point? A little bit, I mean for t a little bit more than 0. The capacitor voltage increases, so what happens to the current through the capacitor in this circuit. Somebody please attempt this current through the capacitor here. What happens to the current through the capacitor? So, there is an answer that says slope less than one, but keep in mind that here we are plotting voltage verses time. So, the slope is measured in volts per second.

So, when we have a dimension quantity like that it does not make sense to say it is less than one. So it has to be less than some quantity. So, another answer is that, it is V p by R, but is it true? Why is it V p by R? What is the voltage across the resistor now? The voltage across the resistor at this time instant, what is it going to be? Is it V p?, Is it less than V p?, Is it more than V p. Exactly, so the current through the capacitor is V p minus V c by R. Now, when the capacitor voltage was 0, the current was V p by R, but when the capacitor voltage increases a little bit, the current will decrease.

So, the current here is smaller than the current there. Now, what does that mean? Because the current is proportional to the slope of the capacitor voltage or the other way round, the capacitor voltage, the slope of the capacitor voltage is proportional to the current through it. So, the capacitor voltage will increase a little, but after that the slope will reduce because the current has reduced. Now, if you look at this point the voltage across the capacitor has increased further.

So, that means the current through it is reduced further. If the capacitor voltage increases, the voltage across the resistor decreases because the capacitor voltage plus resistor voltage equals V p. In this time, in this range of time, it equals V p which is a constant so as capacitor voltage increases the resistor voltage decreases. So, the slope reduces further.

So, the slope will go on reducing. Now, in reality it does not reduce in discrete steps like this, continuously it will go on reducing because as soon as the capacitor voltage increases a little bit, the slope will reduce then it increases a little further it will reduce and so on. It turns out that the slope will go on reducing then finally, it will asymptotically reach some value. It will reach some value and after that the voltage does not change much. So, the slope becomes almost 0.

Now, What is this voltage? What is the voltage it almost reaches? What is the voltage at which it stops changing further? Please try to answer this. Clearly, if the capacitor voltage is V p, the voltage here is also V p and the resistance has 0 volts across it. So, that means there is no current flowing through it. So, the capacitor voltage is V p and there is no current through the capacitor so that means that the capacitor voltage will not change at all. So, this would be equal to V p.

So, this intuitive description of how an R C circuit responds to a step is clear. If not please ask your questions and I will clarify it further. What happens is, let us say you start from a discharged capacitor that is a capacitor which has no charge and in this circuit at t equal to 0 there will be no infinite currents. So, the capacitor voltage will not change instantaneously, but at t equal to 0 because the input voltage increased to V p the current will increase. So, the current through the capacitor increases and sorry the current through the capacitor will be a non-zero. It will be V p by R.

So, that means that the voltage across the capacitor increases. As the voltage across the capacitor increases, the voltage across the resistor reduces and that reduces the current and the slope of the capacitor voltage. And this happens continuously, the capacitor voltage will increase little by little and current through it will reduce little by little, reducing the slope of increase. Finally, the slope of the capacitor voltage almost becomes

0 and you can kind of say that the capacitor voltage reaches some value and that value is V p that is because the capacitor voltage will stop changing when the current through this is 0. And if the current through this is 0, current through the resistor is 0. The voltage drop across the resistor is 0, so that means that here also you have V p. So, please think about this and ask me any questions you may have.

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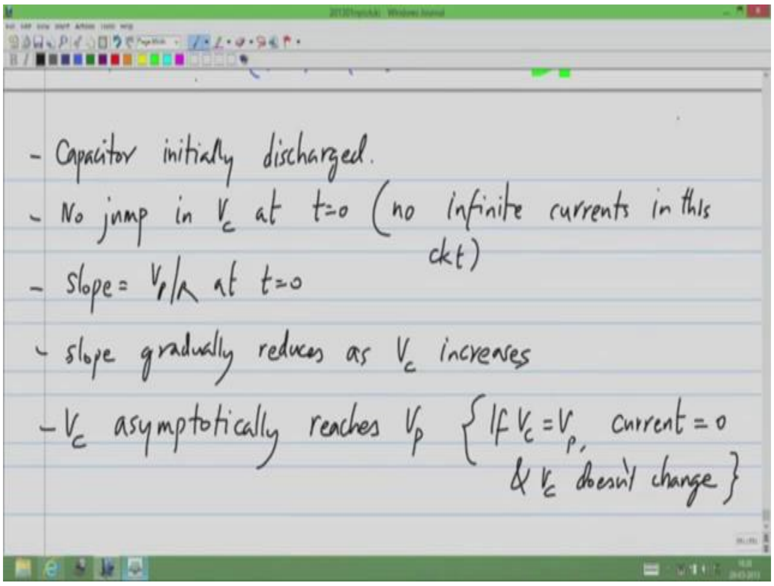

The slope is V p by R at t equal to 0, slope gradually reduces as V c increases and V c asymptotically reaches V p. What is meant by asymptotically is that, it gets very, very close to V p, but does not quite get there, it gets there only at t equal to infinity. So finally, when V c equals V p current through the resistor and the capacitor is 0 and V c does not change anymore. So, any questions about this behaviour of a R C circuit for a step input? This kind of input which jumps from one value to another value is known as a step input. Now, let us go the other way round just for completing the calculation.

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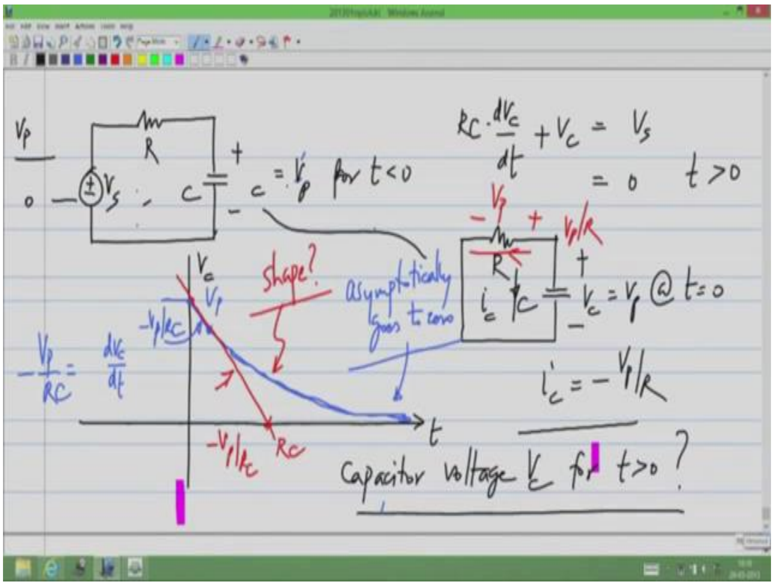

So, let us say initially the voltage V s is at V p and the capacitor voltage V c is also at V p for t less than 0. So, imagine that we take the capacitor and charge it up to V p. So previously, we charged it almost to V p, we charge it up to V p and then after sometime we bring the input V s down to 0. So, this is exactly the opposite of what we had earlier as you are going from 0 to V p, we go from V p to 0. Now, if we look at the differential equation that governs this circuit it is R C d V c by d t plus V c equals V s and this is nothing but 0 for t greater than 0.

So, we have a differential equation with 0 on the right hand side that is the differential equation that governs this particular circuit in this condition. So again, I plot V c verses time, so like I said initially V c is at a value V p and also for convenience I will redraw this circuit with V s equal to 0.

With V s equal to 0, it is a short circuit, I have this C and R and the voltage across this is V p at a equal to 0. Now, what will be the current through the capacitor? So, let us say I call this as i c and because V s equals 0, I have shown it as a short circuit. So, if the capacitor current is flowing like this, I have used the passive sign convention. What is the current i c? So, there is an answer V p by R, but please mind the signs because I used passive sign convention for i c.

So, flowing from top to bottom. That is right, first of all the voltage across the capacitor and the resistor, they are the same in this circuit, they are in parallel. So, this voltage with this polarity is V p at t equal to 0 which means that a current V p by R flows in that direction. So, this i c which is the current going from top to bottom in the capacitor is minus V p by R.

So, what happens to the capacitor voltage after t equal to 0, capacitor voltage V c for t greater than 0. What happens now, will it increase or decrease or remain the same? So, that is right, it is going to decrease . So, it will decrease and the slope is again given by the capacitor current minus V p by R is the capacitor current C d V c by d t. So, d V c by d t is nothing but minus V p by R C. So, the slope is minus V p by R C. So, there was a question here that was whether after charging a capacitor to some voltage can I use it as a voltage source?

Yes, we can in fact it is used like that in some cases, but of course, you have to remember that a capacitor is not a voltage source. As you draw current from it, its voltage is going to change. So, after some point the voltage changes so much that you cannot use it anymore. So, let us say you have, you want a 5 volt source, you can charge a capacitor to 5 volts and use it, but as you draw current from it the voltage will start decreasing. And at some point it may go so below 5 volts, so far below 5 volts that it may be useless.

But in fact, this is used if you want a source for a very short time where you are not drawing so much charge or so much current from the capacitor you can use that. In fact, in many electronic gadgets when you want to change batteries, during the time you take to change the battery there will be no source. So, some of those gadgets may use the very large capacitor. So, in the time that you take to change the battery the capacitor will be acting like a voltage source and supplying power. But of course, it cannot do it indefinitely, so if you leave it without batteries for a long time, it will just discharge and die out.

So, the slope is minus V p by R C. Now, let us say we come to this point. So, What are the ((Refer Time: 42:18)) slope? After that what happens to the current through the capacitor and what happens to the slope of the voltage across the capacitor. The voltage across the capacitor is decreasing, but after it, it decreases a little bit. What happens to the slope of the voltage?

Now clearly, the voltage across the resistor is also V c and as V c reduces, the voltage across the resistor reduces. Which means that the current through it reduces and if the current reduces the slope of the voltage also reduces. So, it does that and then does that, that and that so on.

So finally, it turns out that it will asymptotically reach 0 because the input is 0 for t greater than 0 and if this capacitor voltage goes all the way to 0, then there will be no current through the resistor. And that means that the capacitor voltage cannot change at all, which in turn means that the capacitor has reached its steady state. So, this asymptotically goes to 0. It is clear. Any questions about this?

Now ,we have looked at the behaviour of the capacitor voltage for two cases, when the input jumps from 0 to V p or from V p to 0. Qualitatively it is the same, it starts changing with a certain slope the capacitor voltage and the slope goes on reducing. The magnitude of the slope goes on reducing. So, it starts of steeply and then become less and less steep and finally, asymptotically reaches some value. When you have changed the input from 0 to V p that value is V p, when you change the input from V p to 0 that value is 0. Is this fine?

So, key things to remember are that, are the values of the slopes and so on. So, this slope is minus V p by R C. So, it starts from V p and if it went like a straight line with that slope, it would cut the time axis at t equal to R C. But of course, it does not do that because the moment the voltage reduces a little, the current also reduces. So, the slope reduces and so on. So now, What do you think the shape is? We have done it intuitively and graphically but, we have to also be able to write a formula, an expression for V c as a function of time. So, what is that function? Any guesses, for what that is? So let us see, let me take the second case where the input V s was 0 after t equal to 0.

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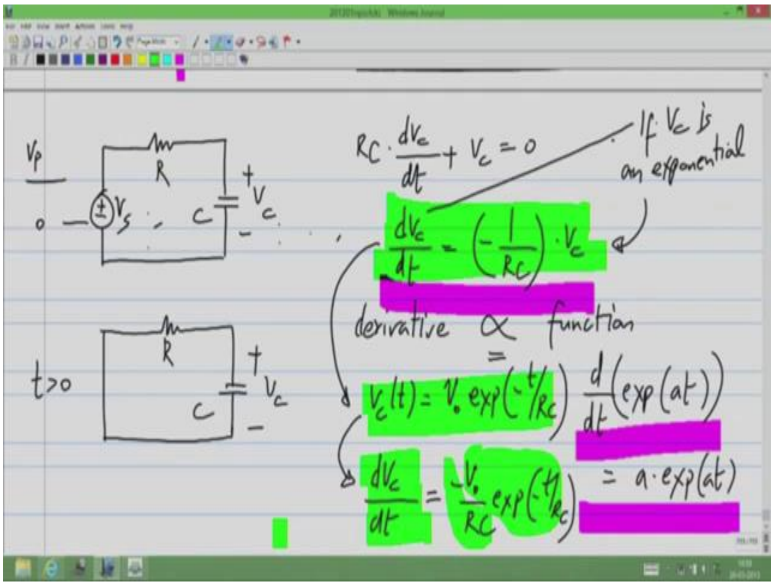

So, for t greater than 0, this is the model with V s set to 0. Now let us see, what shape it is. Somebody answered parabola, it turns out, it is not a parabola. The differential equation governing this is RC d V c by d t plus V c equals 0. Now, I will rewrite this as d V c by d t is minus 1 by R C times V c. So, what this equation is saying is that, the derivative d V c by d t is proportional to the function itself. So, the derivative is proportional to the function. So, what is the function that gives you a derivative which is essentially the same function, it may be a scaled version of that function, but it is the same function.

So, for instance let me do it with some other variables I could have something like this, if I have d x by d t equal to X. So in this case also, the derivative is proportional to the function in fact I have made it exactly equal to the function. Now you know, how to differentiate functions. So, what is the function that you can recall that has the derivative which is the same function. Please try to answer this. Derivative is proportional to the function or may be even equal to the function. What is the function that makes this criterion? So, if you want you can think of the table of derivatives that you would have studied when you studied calculus.

So, think of a function whose derivative is the same function. So clearly, an exponential satisfies this because the derivative of the exponential is also an exponential. Now, let us say I do not want the derivative to be equal to the function, but only proportional to the function. So, I also know that if I had a scaling factor here, exponential a t and I differentiate it. I get a times exponential a t. So clearly in this case, if we see is as an exponential, this function will be satisfied. Also I want a particular scaling factor, right. So, see that this equation is of the same form as this.

So, the derivative of something equals a constant times the same function. So, What is the function that will satisfy this? Satisfy this equation. It is V c of t of the form of exponential minus t by R C. So clearly, now I also can have a scaling factor here. So, let us say V 0 because that will cancel out if V c of t is V naught exponential minus t by R C. Please give me the value of d V c by d t. What is d V c by d t? If V c is V naught exponential minus t by R C, I use the, I use these expressions to find out exactly what goes inside the exponential. If it is exponential a t the multiplying factor here is a. I want the multiplying factor to be minus 1 by R C. So, a has to be minus 1 by R C.

So, V c of t is V naught exponential minus t by R C. So, just differentiate this and tell me what the value is. Clearly, this is minus V naught by R C exponential minus t by R C. So, I can say by inspection that this will satisfy this equation because the derivative is minus 1 by R C. So, this part of it times the rest of it which is the function itself. So, solution to R C d V c by d t plus V c equal to 0. The solution to this is V c of t equals V naught exponential minus t by R C and V naught can be anything, but as far as the differential is concerned.

So, any value of V naught this differential equation will be satisfied, but in our circuit we will need a particular value of V naught. So, how will we find this? How will we find the value of V naught? So, there is another answer which says V naught by R C exponential minus t by R C, I think you omitted the minus sign it should be minus V naught by R C exponential minus t by R C.

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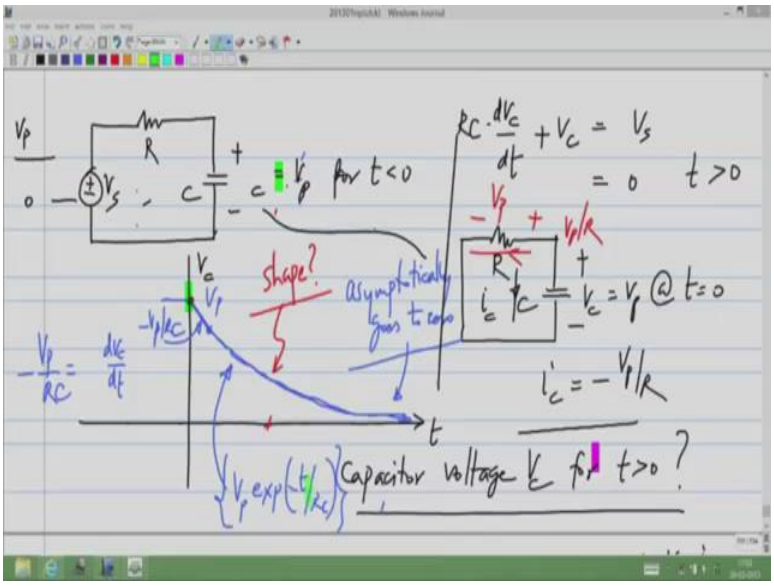

So now, we know that the shape of this blue curve here, the shape of this blue curve is of the form of V naught exponential minus t by R C. So this is, this exponential describes the response of these types of circuits. And also, we know that at t equal to 0, it starts from V p. What is the value of this function at t equal to 0? The value of this function at t equal to 0 is, if I substitute t equal to 0 in this exponential I will get V naught, but I know from the circuit that it is going to be V p. So, for this to correctly describe the blue curve here, this V naught has to be V p. So, this curve is V p exponential minus t by R C. Is this clear?

So, if initially the capacitor had a voltage V p, and the input also had a voltage V p, then the current through the capacitor, current through the resistor would be 0, current through the capacitor would be 0. And it would stay at V p, but if V p jumps, V s jumps from V p to 0 then the output will slowly discharge all the way to 0, towards 0. Now, we have also determined that function, the function is V p exponential minus t by R C. And we are intuitively determined the shape that it has some slope initially and then it goes on decreasing. And that is exactly what the exponential does. Please keep in mind that this exponential is the exponential of t, but with a minus sign inside. So, if we have an exponential with a plus sign, it will lock, it will go on increasing with time. Here exponential of minus t by R C, it will go on decreasing with time. So, any questions about this?

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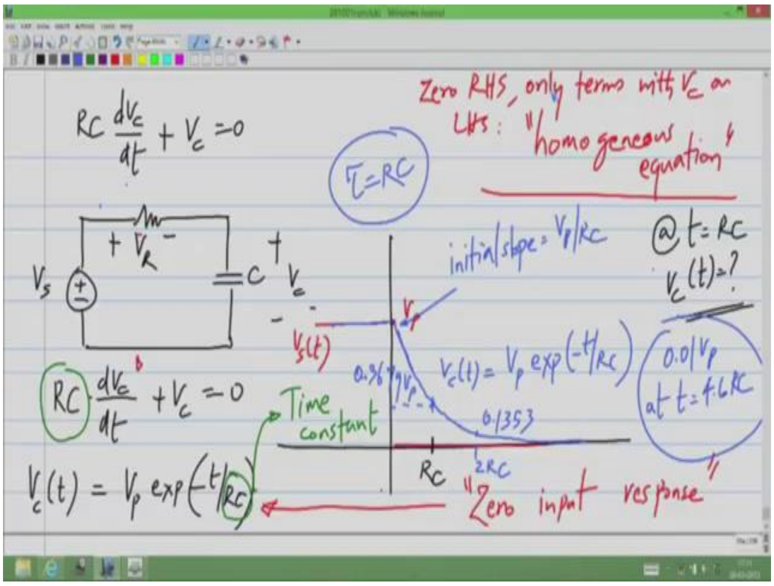

The capacitor voltage across this will fall on exponential. I will redraw the input and output. V s changes from V p to 0, it jumps. So, this is V s of t and V c of t, we will assume initially it was at V p and it slowly goes to 0 like that. And this is V c of t and it equals V p exponential minus t by R C. And the differential equation governing this is R C d V c by d t plus V c equals 0 and the solution is V c of t is V p exponential minus t by R C. So by now, you should be comfortable with these results. If you have any questions about, let me know I will clarify that further. Any questions about this?

Now in this circuit, in this condition the condition here evaluating is with 0 input. So, if you look at this differential equation it has only V c it does not have anything corresponding to the input, the input is 0. So, such a differential equation where the input or the right hand side is 0, I will always try to group the, I always group the variables on the left hand side and the constant inputs on the right hand side. Here, V c is the variable and this kind of an equation which has 0 on the right hand side and only terms with V c on the left hand side, this is known as a homogenous equation.

So, if you have taken some mathematics courses on differential equations you will be familiar with this term, but in this course whatever we need of differential equations we will study them ourselves. And this response because we have 0 input is known as the 0 input response. And this term which appears inside the exponential R C, this is known as the time constant. It also appears here as coefficient of d V c by d t, but we have to be careful. The coefficient of V c has to be 1, in that case the coefficient of d V c by d t will be the time constant. So, it is called time constant because clearly R C has dimensions of time.

We have exponential minus t by something. So this has to have dimensions of time so that we have exponential and inside that some dimensionless constant, dimensionless number. So, this is called a time constant. Any questions about this?

So, like I said before the initial slope is V p by R C. Now, at t equals R C, What is the value of V c? What is the value of V c at t equal to R C? Please try to calculate this. Many of you are able to answer this is V p times exponential of minus 1 or V p divided by exponential of 1 or numerical value is 0.3679 times V p. So, if I mark 1 time constant over here, this is t equals R C. May be it is not to scale. I should redraw this a little better. So, that is what happens and here the value is 0.3679 V p. If I got a 2 time constant it will be 0.3679 square and so on.

So, it will be approximately 0.1353 and it will rapidly reduce. And it turns out that after approximately 5 time constants or 4.6 time constants, it goes down to 0.01 V p. t equals 4.6 R C. We approximate this to 5 R C. The point here is that it will never completely discharge to 0 because if we look at this function V p exponential minus t by R C. It will never become equal to 0, becomes equal to 0 only when t tends to infinity. So, in a finite time it never goes to 0, but it becomes arbitrarily small.

Now, how small you want it to be depends entirely on the context. Sometimes, you may want it to be 0.1 percent of the initial value that is 0.01 V p. Sometimes, you may want it to be 0.1 percent or even smaller, but there is a good number to remember for the voltage to reduce to 1 percent of the initial value. You have to wait for 4.6 time constant. So, many times the time constant is denoted by the letter tau and for this circuit it is equal to R C and in 4.6 time constants it reduces to 0.01 V p. Any questions about this?

So, when the input steps from some value to 0, you know how to solve it. We have the homogenous equation and by solving the homogenous equation, by solving the homogenous equation we get the output to be an exponential. And this is the 0 input response also meaning we have calculated this response for the period when the input is 0. So now, let us get back to the other case where...

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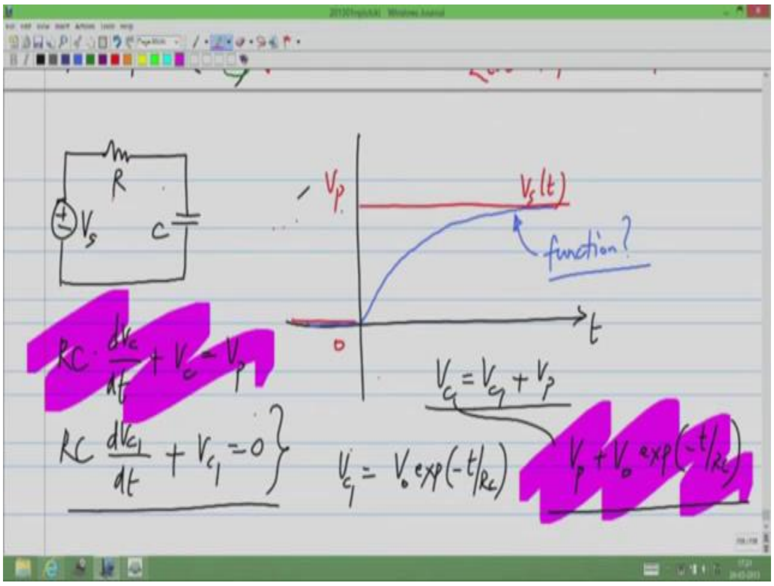

we have R and C. V s and the differential equation governing this is R C d V c by d t plus V c equals V s. The right hand side is no longer 0. So, again I will not take arbitrary functions for V s, but I will take something that jumps from 0 to V p at t equal to 0. So, this is, this red stuff is V s of t.

So, for t greater than 0, the right hand side of this is a constant equal to V p. Now, I already intuitively evaluated that it starts with a slope of V p by R C and it will do that. It will do something like that. Now, we also have to find the function that corresponds to this. So, any idea what this function might be? What might be this function? So, there were a couple of answers. One of the answers is V p 1 minus exponential minus t by R C. That is correct, in fact we will derive it properly. Also there is an answer saying positive exponential. I am not sure what is meant there.

If you mean that the argument of the exponential will be positive that is not correct because if the argument of the exponential is positive you will get a curve that is like this, right. So, it will go on increasing, it will blow at t equal to infinity it will become infinity. That is not the kind of function we have as we have determined from intuition. So, it turns out that we can very easily derive this because we know the result to the previous one when the right hand side is 0 for the homogenous equation. In a very easy way, we can reduce this to the homogenous equation.

So, there is standard tricks and calculus that all of you are familiar with I think. So, let me redefine a V c 1, which is V c minus V p. Now, please tell me, so V p is a constant. So, keep this in mind. So, what will be d V c 1 by d t? What is this going to be? If V c 1 is V c minus V p, What is d V c 1? What is d V c 1 by d t? So, please tell me what is d V c 1 by d t, if V c 1 is V c minus V p. So clearly, if V p is a constant d V c 1 by d t is the same as d V c by d t. So, what I will do is, I will rewrite this equation in terms of the new variable V c 1. So first of all, d V c by d t is the same as d V c 1 by d t.

And if you see this I have V c minus V p equal to 0, if I take V p to the left side. So, that is nothing but V c 1 equal to 0. So here, V c 1, in terms of V c 1 this is a homogenous equation, with the right hand side to be 0. V c 1 is just a new variable I have defined. So, while solving problems you do this all the time right. You define some variable in terms of other variables and substitute. In this case, I have defined V c 1 to be a variable V c minus V p. What I want to calculate for is V c, so instead of that first I will calculate V c 1 and from that I will calculate V c.

So, if I do that and write the differential equation in terms of V c 1, I will get a homogenous equation. So, V c 1 would be of the form V naught exponential minus t by R C. So, V c if I turn around this one, V c is V c 1 plus V p. So, V c will be of the form V p plus V naught exponential minus t by R C. Is this fine? So, this is the general form of the solution to this equation which has V p on the right side. So, this is not a homogenous equation and the solution to this will have V p, it will also have some terms like exponential minus t by R C, which was part of the 0 input response. And it has this constant V naught, which we have to adjust based on the initial conditions.

So, what is V naught and V p? First of all, V p is the size of the step, right. V p the input jumps from 0 to V p. So, that is V p and V naught is just a constant. So, previously here I said V c will be some V naught time exponential minus t by R C, that will satisfy the differential equation. And we calculated the particular value of V naught by looking at the actual solution at t equal to 0. So, V naught happened to be V p in the previous case. In this case, again we have to evaluate the value of V naught, we will do that shortly. So now, we have to evaluate the value of V naught. What we will do is...

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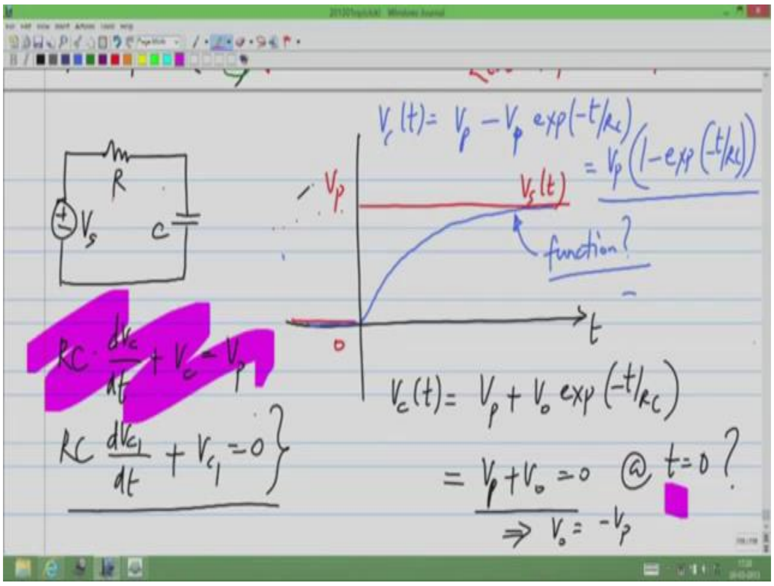

I know that V c of t is V p plus V naught exponential minus t by R C and if I substitute t equal to 0, what will I get? What will I get if I substitute t equal to 0 in this, in this function V c of t? So clearly, exponential of 0 is 1. So, the answer is V p plus V naught. Now, what is the value of the function, actual value of the function at t equal 0? It is 0, right. That we know initially the capacitor has 0 volts and just after the input sets, the capacitor still has the value of 0 volts. So, this should be equal to 0, which means that V naught equals minus V p. It is clear?

So, there will be arbitrary constants in the solution to a differential equation because the differential equation in the general form could be satisfied for any value of this constant, but by looking at the initial condition you will be able to find the particular value, that satisfies it for that case. So in this case, this V naught equals minus V p. So, the actual solution to V c is V p minus V naught exponential minus t by R C or minus V p sorry V p minus V p exponential minus t by R C. Also frequently this is written in this form, V p 1 minus exponential minus t by R C.

So, if you look at the solution here at t equal to 0 both are V p and this cancels off or inside this the exponential will be 1 at t equal to 0 and you have 0. And as t becomes infinity this exponential part becomes 0. So, V c of t will become V p that is what we see it from our intuition. And it starts from V p sorry starts from 0 and then exponentially approaches this V p. It is clear, any questions?

So, we have looked at R C circuit, it is governed by differential equation we have solved for it intuitively we know what it means, that is very important actually. We have of course, taken a special case of only constant inputs. When I say constant, it jumps from one constant value to another constant value. Now, if it does that what happens to the capacitor voltage and we will see later that this happens to every variable in the circuit. Is it undergoes some change with some exponential argument and the argument inside will be minus t by R C or minus t by some time constant. Now, we solved it for 0 input, that is we have some initial condition and the input then jumps to 0. We also solved it for, you have 0, initial condition and it jumps to some value. So, let me just tabulate the results.

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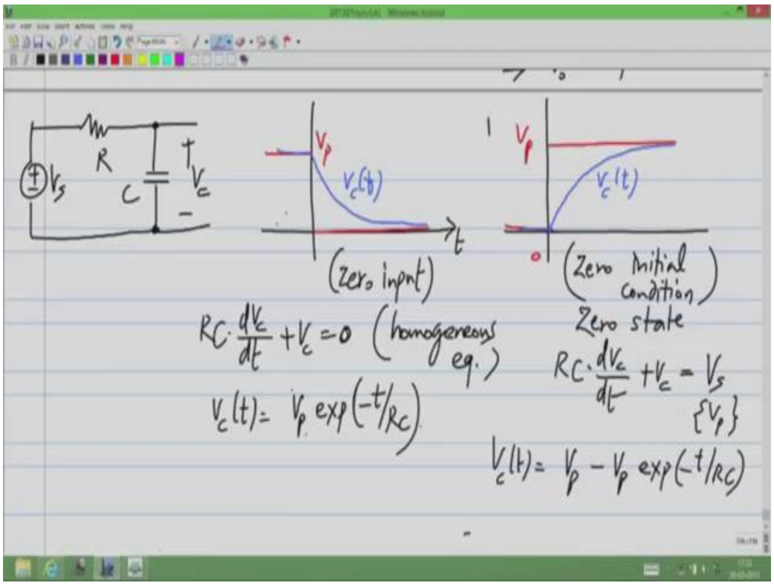

The circuit we are looking at is this. RC and the variable I am trying to calculate is this. I took two kinds of inputs. One, where the input jumps from some V p to 0 and in this case the capacitor voltage V c will do something like that. And the other case was, where the input jump from 0 to V p and the output it turns out, thus something like that. This is V c of t, V c of t.

So, this corresponds to basically a 0 input and in this case we have a 0 initial condition on the capacitor voltage. Now, the capacitor voltage is also known as a state. A capacitor can hold some voltage, so that is known as the state of the capacitor. So, this is also known as 0 initial state or 0 state. So, the differential equation governing the 0 input circuit, is that the important thing here is that it has terms containing V c and its derivatives and right hand side is 0.

So, this is known as the homogenous equation and in the other case the differential equation is R C d V c by d t plus V c equals V s. And we are talking about constant values of V s or V p and the solution to the 0 input case it turned out was V p exponential minus t by R C. And the solution to this case is V p minus V p exponential minus t by R C. So, look at these expressions and classify them into different parts.

It turns out that this part is called the forced response, this is the natural response. The entire thing is the 0 state response in this particular case and this is the 0 input response. We will look at all those things in the next lecture. So, if you have any questions now, I will clarify them. Any questions about any of this R C circuits and differential equations? Then I will see on Thursday.