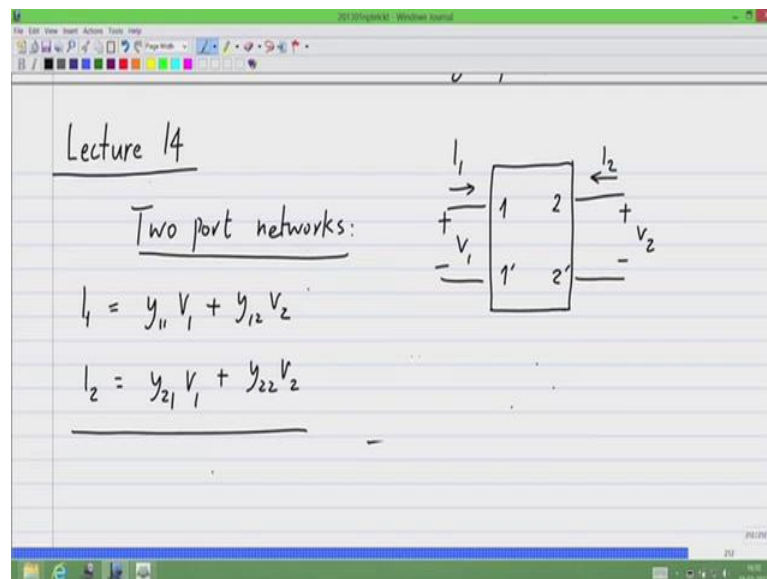


Basic Electrical Circuits
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Lecture - 14
2 Port Parameters (Z, h, and g); Reciprocal 2 Ports

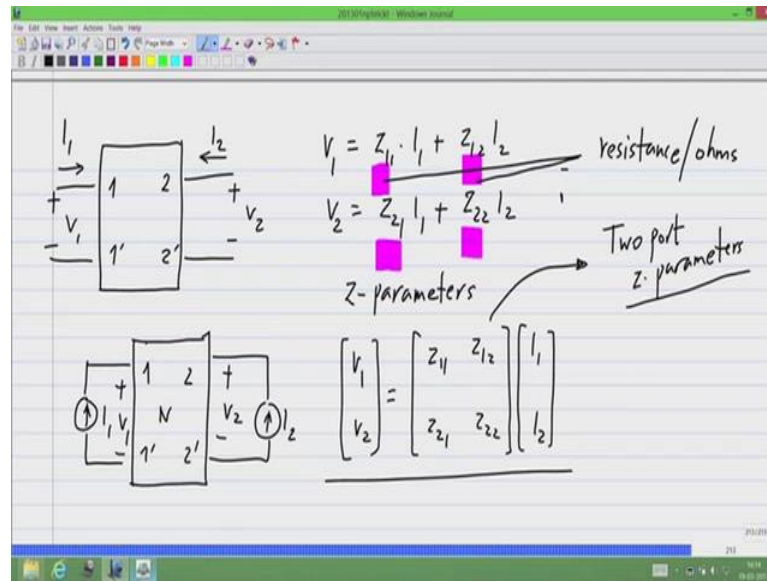
Hello everyone, welcome to another lecture of basic electrical circuits. In the previous lecture, we were looking at two port networks and we considered y parameters and g parameters. y parameters are whether you think of voltage cells being applied or voltage cells as independent variables, and the currents into the two ports as dependent variables. Now, when you write the currents as a function of voltages, a linear combination of voltages, you have this proportionality constants which are y parameters. Now, you can also have current cells independent variables and voltages dependent variables, and so on. So, we will discuss those parameters today. I believe now we are set to go.

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So, we have two ports, that is two terminal space, where we can apply voltages and currents. So, previously we wrote the currents as a function of currents as functions of voltages. Now, this is not the only way to do it, you can also write the voltages as functions of currents.

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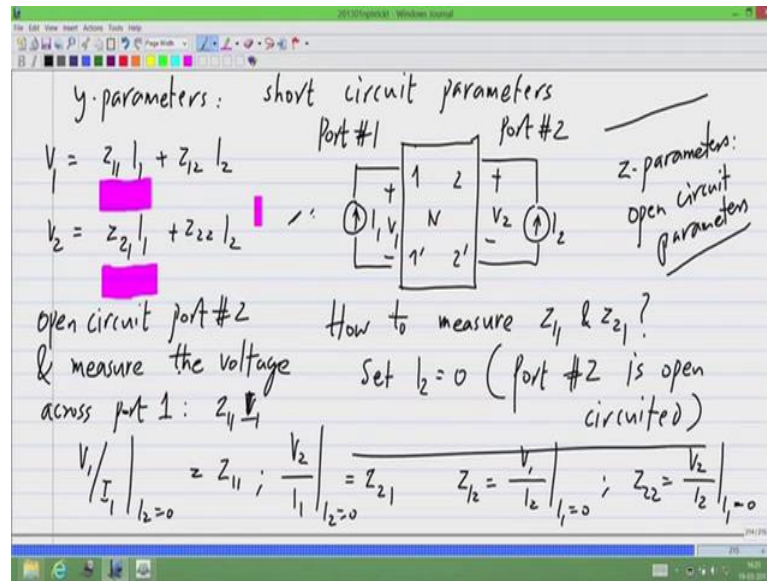


If we do that, we get expressions of this type, but V_1 and V_2 are linear combinations of I_1 and I_2 . Now, clearly these constants which give you voltage from currents, they have dimensions of resistance or units of ohms. Now, this set of parameters is known as Z parameters. Now, what this means is, we think of a picture like this I_1 , I_2 , that is current supplied to the 2 ports and we are measuring the 2 voltages, that appear as a result. Now, the same thing can be written in matrix form, times I_1 and I_2 .

So, are there any questions about the definitions of the Z parameters? Any questions about the 2 ports Z parameters. The use of Z parameters is similar to the use of other parameters, when you have a 2 port, that is when you have a circuit with some input and some output, you can treat these 2 as 2 ports. You have a complicated circuit inside, you do not want to worry about the circuit, you just want to describe the input voltage and current and the output voltage and current. This can be done in many ways and Z parameters is 1 of the way of doing that.

In this case all Z values are resistance or impedances, we have been talking only about the resistive circuits, so they are real numbers and they will be resistances. So, now we interpreted the y parameters as short circuit parameters.

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This we discussed in quite some detail in the previous lecture. That is to measure y parameters, you have to short 1 port and measure the applied voltage, and measure the current at 1 port or other port. So, those are y parameters or short circuit parameters. Now, there is a question which is better y parameters or Z parameters? There is no such thing as better, this is a macro convenience, right,

Sometimes you calculate using conductances, sometimes you calculate using resistances. Now, which is better depends on the context, it is more a question of convenience. So, y parameters and Z parameters are equivalent description of the same network. Now, sometimes some are easier to use and we will later see some of them may not exist for certain types of circuits and only some of them will exist. Now, please tell me just like in y parameter we short circuited 1 port in order to measure the y parameter. So, in this case I have.

So now my 2 port picture is over here. So, let us say I want to measure Z_{11} and Z_{21} . Please let me know what you will do, let us say you are given this 2 port in the lab and asked to measure it in anyway. Please let me know what you will do to measure Z_{11} and Z_{21} . I think this question is repeated, I already answered this question, which is better y or Z. It is a matter of convenience. How would we go about measuring Z_{11} and Z_{21} in this case. Please try to answer this question, how will I measure Z_{11} and Z_{21} .

Previously, we short circuited the output. So, why should we short circuit the output? I think I made a mistake in writing this, this is I should have written I_1 and I_2 .

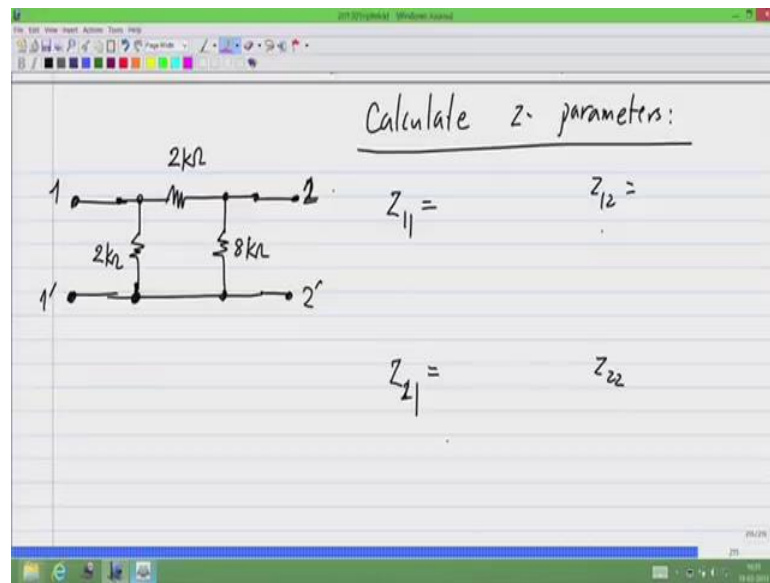
So, these are I_1 and I_2 . So, 1 of the answers is the output port 2 is short circuited. Why is that the case, how will I measure Z_{11} in this circuit? If I want to determine the value of Z_{11} , what should I do? What should I apply to port 1? What should I apply to port 2? How should I measure it? So, please try to answer this, how will I measure Z_{11} and Z_{21} . So, a couple of you were able to answer it correctly, clearly Z_{11} . Z_{11} appears only in this term. So, if I make this term 0, it would be most convenient. So, what I will do is, set I_2 equal to 0, which means that port 2 is open circuited.

So, there are some who have responded saying that we have to short circuit the terminals. No, we have to set I_2 equal to 0, which means that we have to open circuit port number 2, in that case we will get only these terms $Z_{11} I_1$ and $Z_{21} I_1$ for V_1 and V_2 . So, open circuit port number 2, which sets I_2 equal to 0 and measure the voltage across port 1 that will be $Z_{11} I_1$. So, V_1 by I_1 with I_2 equal to 0 is Z_{11} . Similarly, V_2 by I_1 with I_2 equal to 0 is Z_{21} . How much voltage appears across port 2 with when a current I_1 is applied to port 1 that is Z_{21} .

Similarly, we can easily see that Z_{12} is V_1 by I_2 with I_1 equal to 0 that is port number 1 open circuited and similarly, Z_{22} is V_2 by I_2 with I_1 being 0. So, because we measure the, measure the Z parameters by open circuiting 1 port or the other port, that is to measure Z_{11} and Z_{21} . we open circuit port 2 to measure Z_{22} and Z_{12} . We open circuit port 1. These Z parameters also known as open circuit parameters.

Any questions about this definition of Z parameters and how to do we measure it? We have to open circuit other port, open circuit 1 of the ports and measure two Z parameters, and open circuit the other port and measure two more Z parameters. Is this clear? There is a question, what do you mean by symmetry? I am not clear about the context of this question. I mean what circuit does this refer to or anything like that. Now, let us quickly go through an exercise, it is probably best if we take the same circuit that we took last time.

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Previously, in the previous class we calculated y parameter of this network. So, this time let us calculate Z parameters of the same network, that will also let you easily appreciate the relationship between y and Z parameters. So, please calculate the Z parameters. First calculate Z_{11} and Z_{21} , Z_{12} and Z_{22} . So, please calculate this and give me the values. First let us start with Z_{11} and then go to Z_{21} . So, you can use the expressions I gave earlier Z_{11} is when you set I_2 equal to 0, V_1 by I_1 , Z_{21} is when you set I_2 equal to 0 V_2 by I_1 and so on.

So, what will be the value of Z_{11} here. I hope all of you are able to start solving this because it is just a simple calculation of circuit with 3 resistors. Please be careful with the units, I mean it is a 1.66 ohms. I am not sure how that came out, all our resistances of 2 kilo ohms, 4 kilo ohms and 8 kilo ohms. I got 1 answer for Z_{11} . What about Z_{12} ? What is Z_{12} ? Please calculate Z_{12} or Z_{21} , whichever you find is easier and similarly, Z_{22} .

So, the question is to calculate Z parameter of the network the resistor values are given and I have also explained how to calculate Z_{11} , Z_{21} , Z_{12} , Z_{22} , etcetera. To calculate Z_{11} and Z_{21} , you have to open circuit port 2 and to calculate Z_{12} and Z_{22} , you have to open circuit port 1. So, there is a question asking for more explanation, but please be more specific which part do you want an explanation for. Will anyone calculate the value for Z_{21} or Z_{12} ?

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$V_1 = z_{11} I_1 + z_{12} I_2$
 $V_2 = z_{21} I_1 + z_{22} I_2$

y-parameters: short circuit parameters
 z-parameters: open circuit parameters

Port #1 Port #2
 How to measure z_{11} & z_{21} ?
 Set $I_2 = 0$ (port #2 is open circuited)

open circuit port #2 & measure the voltage across port 1: $z_{11} I_1$

$\left. \frac{V_1}{I_1} \right|_{I_2=0} = z_{11}$; $\left. \frac{V_2}{I_1} \right|_{I_2=0} = z_{21}$

$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$; $z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$

So, as I mentioned earlier to calculate Z_{11} , you set I_2 equal to 0 and take V_1 by I_1 and this is not some mysterious formula I came up with. If you look at this V_1 is $Z_{11} I_1$ plus $Z_{12} I_2$. If you set I_2 equal to 0, this part goes away, that is if you open circuit port 2, the second part goes away and V_1 will be $Z_{11} I_1$ so V_1 by I_1 will be Z_{11} , and V_1 by I_1 is nothing but the resistance looking into port 1.

So, V_1 is here and I_1 is flowing there, so V_1 by I_1 is the resistance flowing into port 1 sorry, resistance seen in port 1. And similarly, if you set I_2 equal to 0 here V_2 is $Z_{21} I_1$. So, if you apply I_1 , measure the value of V_2 , take the ratio that will be Z_{21} , that is if you apply a current here, how much voltage develops in port 2 that will give you Z_{21} .

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Calculate z-parameters:

$$Z_{11} = \frac{5}{3} \text{ k}\Omega = 1.67 \text{ k}\Omega$$

$$Z_{12} =$$

$$Z_{21} = \frac{4}{3} \text{ k}\Omega = 1.33 \text{ k}\Omega$$

$$Z_{22} =$$

$$Z_{11} = 2 \text{ k}\Omega \parallel 10 \text{ k}\Omega = \frac{2 \text{ k}\Omega \cdot 10 \text{ k}\Omega}{2 \text{ k}\Omega + 10 \text{ k}\Omega}$$

$$= \frac{20}{12} \text{ k}\Omega = \frac{5}{3} \text{ k}\Omega$$

$$i = \frac{2 \text{ k}\Omega}{2 \text{ k}\Omega + 10 \text{ k}\Omega} \cdot I_1 = \frac{1}{6} I_1$$

So, let us know that from this case, first of all to calculate Z_{11} and Z_{21} , we have to open circuit port 2, that means that basically you do not connect anything here. This is open circuit and you apply V_1 . So, now you apply here sorry, I made a mistake in this, you apply I_1 and you measure the value of V_1 .

So now as I said V_1 by I_1 will be simply the resistance between these 2 terminals. I hope that part is clear to everybody. If you have 2 terminals, and if you measure the voltage in this way, and the current going into the plus terminal with the plus sign, then V by I is nothing but the resistance looking into those 2 terminals, by definition that is the resistance. So, here what do we have? We have the resistance looking into here would be 2 kilo ohm in parallel with this whole thing and this is open circuited.

So, we have 2 kilo ohm plus 8 kilo ohm, that is 10 kilo ohm. And 10 kilo ohm in parallel with 2 kilo ohms. So, Z_{11} will be 2 kilo ohm parallel with 10 kilo ohm, which is basically 2 kilo ohm times 10 kilo ohm divided by 2 kilo ohm plus 10 kilo ohms, which is 20 by 12 kilo ohms or you can also write this as 5 by 3 kilo ohms, which is basically 1.67 kilo ohms.

So, essentially you have to do circuit analysis with a single source I_1 applied here, you have to find V_1 , so V_1 by I_1 will be the resistance looking between these 2 terminals. Also if you apply I_1 here, we have to find the value for V_2 , that is the voltage that appears between these 2. To do that, first you calculate how much current flows here. If I

I_1 is applied here, how much will be the value of I that you can get from current division. So, this I_1 will produce some I , it will produce some current in here and some current in here.

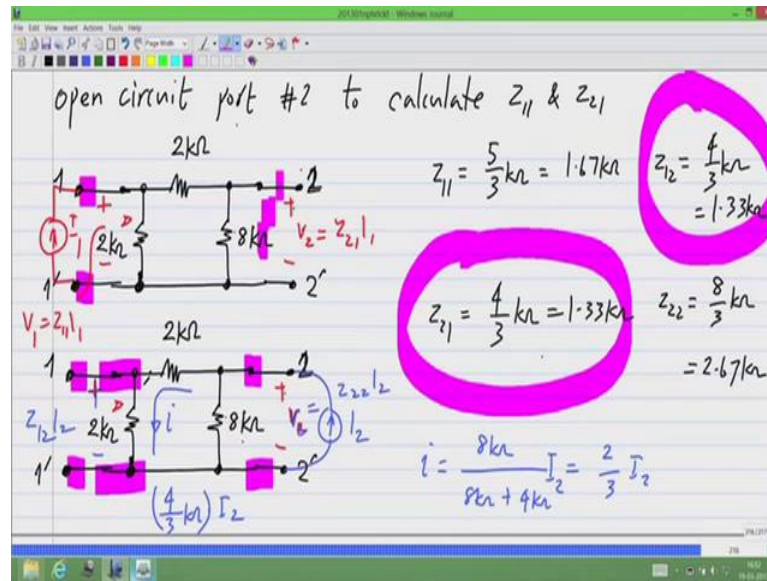
So, the value of I will be, how much is I ? Please try to answer this as a fraction of I_1 , I_1 will be, I will be proportional to I_1 , but how much it will be, what will be the fraction. If you have I_1 applied here, how much of it will flow through this 2 kilo ohm and 8 kilo ohm series combination.

So, I will be from the current theorem, you have current division formula, you have 2 kilo ohm by 2 kilo ohm plus 10 kilo ohm times I_1 , which is basically sorry, 2 kilo ohm plus 8 kilo ohm, which is, this is indeed 10 kilo ohm, this is the series combination of this 2 kilo ohms and 8 kilo ohms this I_1 by 6 th of I_1 . So, if you apply I_1 here, 1 by 6 will flow here and 5 by 6 will flow there. And what is across 8 kilo ohm, which is V_2 is 1 by 6 times I_1 times 8 kilo ohm.

So, this proportionality constant is nothing but Z_{21} which is 8 by 6 or 4 by 3 kilo ohms, which is 1.33 kilo ohms. So, I have already given here, the value of Z_{11} is 1.67 kilo ohms. So, I have calculated Z_{11} which is the resistance looking into 11 prime with port 2 open circuited, that is 1.67 kilo ohms, and Z_{21} that is you apply current I_1 between 11 prime, measure the voltage V_2 that appears between 22 prime with port 2 open circuited it is Z_{21} , that is 1.33 kilo ohms.

So, somehow the class is not very responsive today. Please try to solve this problem. And now I have solved half the problem and calculated Z_{11} and Z_{21} . Please try to calculate the values of Z_{12} and Z_{22} . Please try to do that I will be back in a minute, I am back please give me the answers Z_{12} and Z_{22} .

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So, I think a number of you have been able to get the answers. Now I will redraw the picture open circuit port number 2 to calculate Z_{11} and Z_{21} . Let me copy over the network circuit instead of redrawing it. So, if I apply a current like this here I get Z_{11} I_1 and here I get Z_{21} I_1 . And for calculating the other two parameters I apply a current, let me use a different color to port 2, so like I said in the first case here this V_1 will be Z_{11} I_1 and v_2 will be Z_{21} I_1 . If I open circuit port 1 and apply I_2 then this V_2 will be Z_{22} I_2 and V_1 will be Z_{12} I_2 . So, Z_{22} is nothing but the resistance between these two terminals, resistance looking into port 2 with port 1 open circuited.

Now, I have all the values, Z_{11} is 5 by 3 kilo ohms or 1.67 kilo ohms Z_{21} also we already calculated it is 4 by 3 kilo ohms or 1.33 kilo ohms and Z_{12} , that is let me first calculate Z_{22} , Z_{22} is nothing but the resistance looking into 2 and 2 prime and that is this parallel combination is 4 kilo ohm an 8 kilo ohm. Z_{22} is 4 kilo parallel 8 kilo which is 4 kilo ohm times 8 kilo ohm divided by 4 kilo ohm plus 8 kilo ohms. This is 8 by 3 kilo ohms or 2.67 kilo ohms.

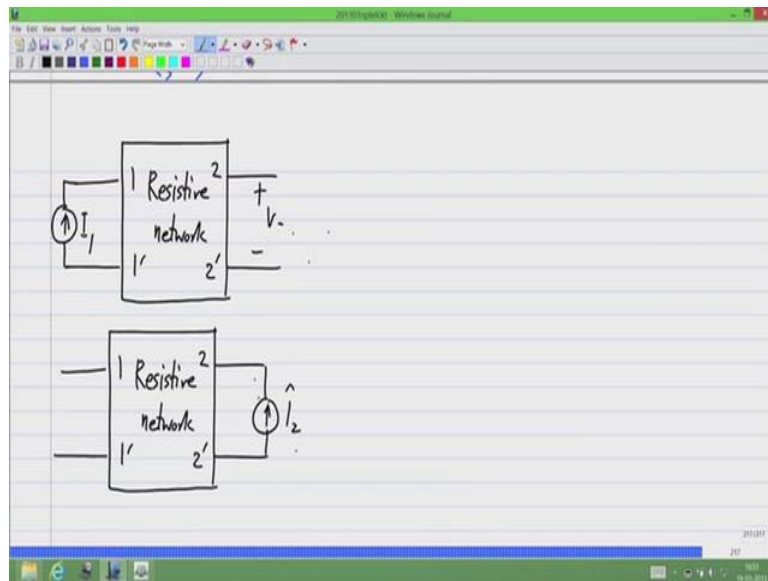
And similarly, this Z_{12} you can calculate by first finding out how much current flows into this, if you apply I_2 a certain fraction of the current I will flow into that one. So, I will be from the current divided formula 8 kilo ohm divided by 8 kilo ohm plus 4 kilo ohms times I_2 or basically two-thirds of I_2 . And the voltage across the 2 kilo ohm is

nothing but this $I_1 \times 2 \text{ kilo ohm}$, so this $Z_{12} = I_2$ which is the voltage across this 2 kilo ohm resistor is nothing but $4 \text{ by } 3 \text{ kilo ohms times } I_2$.

So, this is Z_{12} , 1.33 kilo ohms or $4 \text{ by } 3 \text{ kilo ohms}$. So, we were able to calculate all the Z parameters, the circuit is very simple, but even if you have an arbitrary connection of resistors, you should be able to calculate the Z parameters. So, any questions about the calculations? Any questions about the calculations? Now, we also notice that Z_{21} and Z_{12} have exactly the same value.

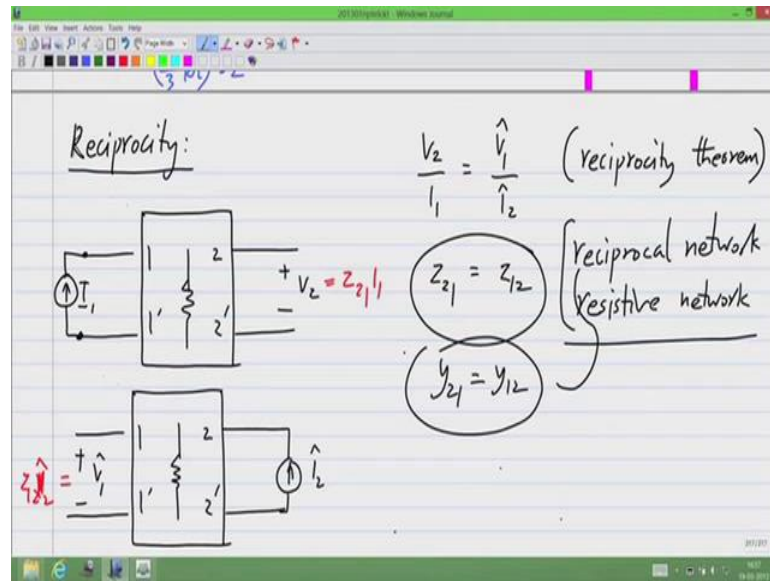
Z_{21} and Z_{12} have exactly the same value, so it could be a coincidence for this particular circuit or it could be a general rule, so what do you think it is? So, you think it is a coincidence or a general rule? As couple of you immediately guessed it is because of reciprocity we had discussed reciprocity theorem with different kinds of sources connected to the two ports.

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And by the way this apply is only if the circuit has only resistors, no control sources and so on. In that case, in that case we had seen that if this is I_1 and here you open circuited that is $I_2 = 0$, then we had this other version of this circuit, where we had I_2 at here. So, in this case we measure V_2 . Sorry, about that windows general crashed. So, basically my white board disappeared, I am going to repeat whatever I said just now. So, reciprocity we had discussed earlier.

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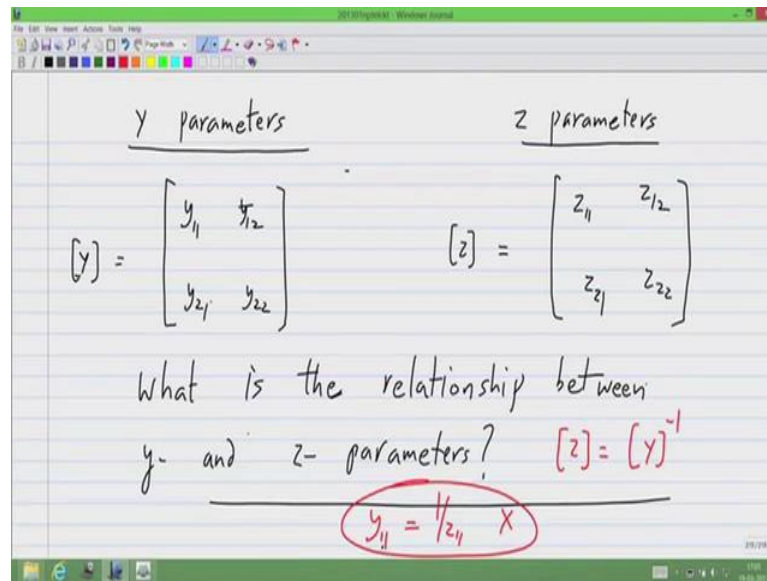


Now, we discussed the case where we apply current sources on either side you apply I_1 and measure V_2 this is 1 case, the other case is you apply \hat{I}_2 and measure \hat{V}_1 and reciprocity said that V_2 by I_1 equals \hat{V}_1 by \hat{I}_2 . By the way like I said this is if the circuit consist only resistors. Now, someone answered coincidence, this is not as I am just explaining.

So, V_2 by I_1 is \hat{V}_1 by \hat{I}_2 , now this V_2 by I_1 is nothing but $Z_{21} I_1$ right, Z_{21} is basically the voltage that appears at port 2, when port 2 is open circuited and I_1 is applied to port 1. Similarly, in this case if port 1 is open circuited you apply \hat{I}_2 to port 2, \hat{V}_1 that appears is $Z_{12} \hat{I}_2$. So, from this reciprocity V_2 by I_1 which is Z_{21} is the same as \hat{V}_1 by \hat{I}_2 which is Z_{12} .

So, this is true for any reciprocal network and if you have only the resistors in the network, it is also true. Is this clear? So, this is because of reciprocity, earlier we had seen that y_{21} equals y_{12} also because of reciprocity. So, reciprocity implies these two. Any question about this?

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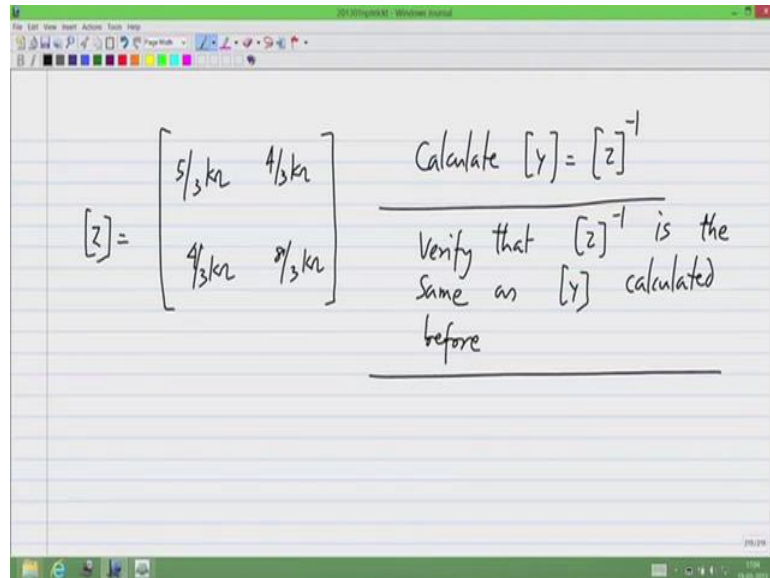
Now, we have these two sets of parameters we have discussed so far. So, there is a question what is reciprocal network, please go back and view one of the previous lectures, where we started of from diligence theorem, and derived reciprocity relations for 2 port network and reciprocity relations for a 2 port network. When the network consists only of resistors, then you will be able to understand. We derived it for a few conditions, that is when both sides are excited by voltage sources both by current sources or 1 by current 1 by voltage source.

So, now we have this set of y parameters y_{11} , y_{12} , y_{21} , y_{22} and Z_{11} , Z_{12} , Z_{21} , Z_{22} . Now, what do you think is the relationship between these two parameters? What is the relationship between y and Z parameters, that is if I give you y_{11} , y_{12} , y_{21} , y_{22} , will you be able to get Z parameters? There is an answer saying 1 by Z or Z inverse. What inverse are we talking about here? I think you are able to answer this, the matrix, the matrices are inverses of each other, that is this is the y matrix and this is the Z matrix, and the relationship is that Z equals y inverse or vice versa, and y will be equal to Z inverse and so on.

So, it is very important to not make this mistake of y_{11} equals 1 by Z_{11} , this is not correct. The individual parameters are not reciprocals of each other y_{11} is not 1 by Z_{11} , y_{22} is not equal to 1 by Z_{22} , and so on. So, the entire matrix is the inverse, the entire Z matrix is inverse of y matrix. So, I hope this is clear. So, you can verify this for yourself.

Today we calculated the Z matrix for this network. There is a reason I took the same network as we discussed last time.

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The image shows a digital whiteboard with handwritten mathematical content. On the left, a 2x2 matrix $[Z]$ is defined as:

$$[Z] = \begin{bmatrix} 5/3 \text{ k}\Omega & 4/3 \text{ k}\Omega \\ 4/3 \text{ k}\Omega & 8/3 \text{ k}\Omega \end{bmatrix}$$

To the right of the matrix, the text reads: "Calculate $[Y] = [Z]^{-1}$ ". Below this, a horizontal line separates the instruction from the verification step: "Verify that $[Z]^{-1}$ is the same as $[Y]$ calculated before".

Z matrix is 5 by 3 kilo ohms, 4 by 3 kilo ohms, 4 by 3 kilo ohms and 8 by 3 kilo ohms. Now, you can calculate, please try this at home. calculate the inverse of this matrix and in the previous lecture we calculated the y parameters for this matrix, so please verify that. Z inverse is the same as y calculated before. Is this fine? So, please try this yourself.

So, if there are any questions on y and Z parameters I will take them otherwise other parameter sets. So, I would not spend as much time on this other parameter as I did on y and Z, because by now the theme is very clear and it will become very repetitive if I go on and on about this, so let us look at the other two cases.

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Y: $\{I_1, I_2\}$ in terms of $\{V_1, V_2\}$; Z: $\{V_1, V_2\}$ in terms of $\{I_1, I_2\}$

H: $\{V_1, I_2\}$ in terms of $\{I_1, V_2\}$

$V_1 = h_{11} I_1 + h_{12} V_2$ (dimensionless)
 $I_2 = h_{21} I_1 + h_{22} V_2$ (units/dimension)

hybrid parameters

The diagram shows a two-port network with ports 1, 2 and 1', 2'. An independent current source I_1 is applied to port 1, and an independent voltage source V_2 is applied to port 2. The voltage across port 1 is V_1 and the current entering port 2 is I_2 .

So, for y parameters we express I_1, I_2 in terms of V_1, V_2 and for Z parameters we express V_1, V_2 in terms of I_1, I_2 . Alternatively we can express V_1 and I_2 in terms of I_1 and V_2 and this is known as h parameters, that is we think of let us say of linear network, we think of applying a current to port 1. So, I_1 is applied here and voltage to port 2 if voltage is applied over there, and we measure V_1 and I_2 . Now, because it is a linear network both V_1 and I_2 will be linear combination of I_1 and V_2 .

So, V_1 will be $h_{11} I_1$ plus $h_{12} V_2$ and I_2 will be $h_{21} I_1$ plus $h_{22} V_2$. So, in this case the V_1 and I_2 are dependent parameters and I_1 and V_2 are independent parameters, independent quantities. So, firstly this is just linear combination, so we can always re-arrange any of this relationships in terms of the other. So, now please tell me the units or dimensions of h_{11} . What kind of quantity is this h_{11} ? What are the units? So, someone answered no units, but please see that h_{11} is multiplying I_1 to give V_1 , so h_{11} times I_1 has to be voltage.

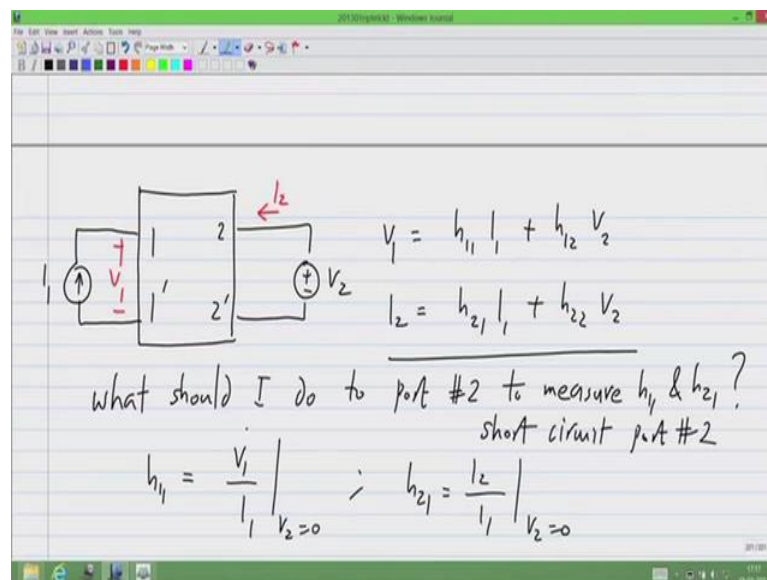
So, what are the dimensions of h_{11} ? There are two questions one is, can we use these parameters for n port networks? Yes you can. So, if you have n ports, you will have n square parameters and you will, you can use any of these parameters. So, you can use either y or Z and let us say you have 3 ports, for y parameters you will express I_1, I_2, I_3 in terms of V_1, V_2, V_3 . There is another question, which says I did not understand h parameters. I am not clear which part of this is not understood?

So, as couple of you answered, the units are ohms or dimensions are that of resistance. This h_{11} multiplies a current to give you voltage, so this has to be a resistance. So, I will just write ohms here now, what are the units or dimensions of h_{12} .

What are the units or dimensions of h_{12} ? Remember h_{12} is multiplying a voltage V_2 to give you a voltage V_1 . So, what are the units or dimensions of h_{12} ? So, clearly it is dimensionless or unit less? Similarly, what is it for h_{21} ? Units and dimensions of h_{21} ? Someone answered mho or Siemens for h_{12} , that is not correct. So, units and dimensions of h_{21} ? So, again h_{21} multiplies a current I_1 to give you a current I_2 , so this is also dimensionless.

And finally, what are the dimensions of h_{22} ? So, h_{22} multiplies with voltage to give a current. This clearly has dimensions of conductance or units of Siemens. So, the point is in h parameter, the four different parameters have different units, that is why these are known as hybrid parameters. So, the next question is let us say I want to measure h_{11} and h_{21} , h_{11} and h_{21} . So, what should I do to port 2 in order to measure h_{11} and h_{21} .

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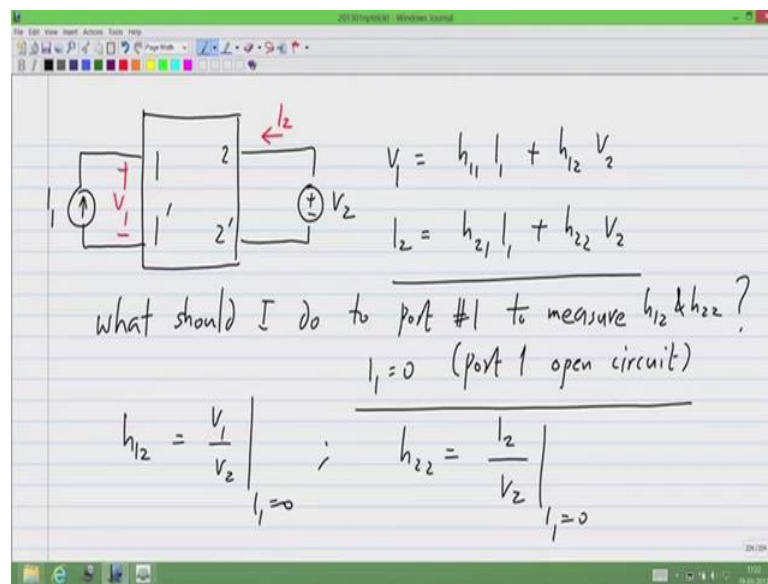
A couple of you answered that port 2 is open circuited. Please explain why it should be open circuited? And there is another question, will h_{12} equals h_{21} ? That is actually a very interesting question, we will take it up shortly. So, clearly when you are measuring h_{11} and h_{21} in general parameter 11 and parameter 21, you have to set an independent

source on port number 2 to 0. So, in this case we have to set V_2 is 0. Now, what happens if V_2 is 0? What does it mean, is it a open circuited or short circuited?

So, clearly it is short circuited. If you set your voltage to be 0, so that means that it is short circuited. Actually there is some confusion here, somebody responded saying an open circuited voltage will be 0. That is not correct, if you have an open circuit between 2 points, there can be any voltage between those 2 points, but there can be no current. So, a 0 voltage means a short circuit 0 voltage source, and a 0 current source means an open circuit. So, clearly to measure h_{11} and h_{21} we have to short circuit port number 2.

So, h_{11} is V_1 by I_1 with V_2 equal to 0 and h_{21} is I_2 by I_1 with V_2 equal to 0. So, h_{11} is nothing but the resistance looking into port 1 with port 2 short circuited, and h_{21} is nothing but the current gain, the ration of current in port 2 to current in port 1 with port 2 short circuited. Similarly, let me copy this over.

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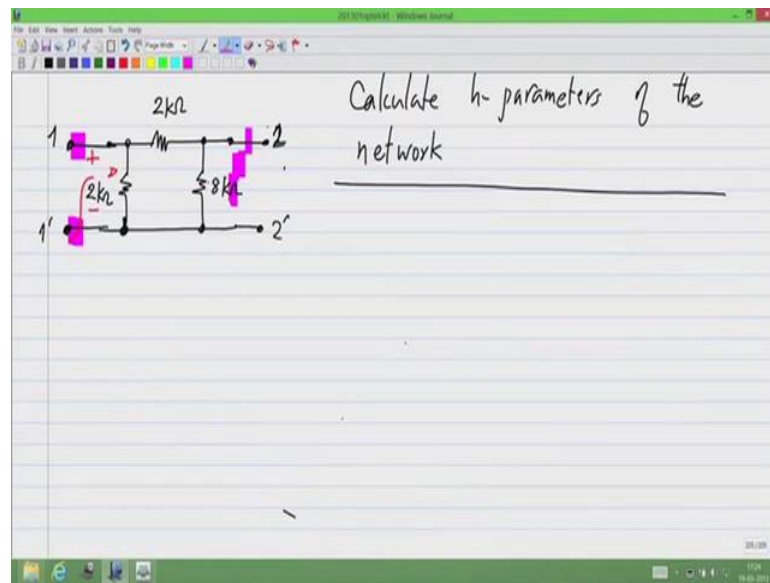
So, the next thing is the obvious extension of it, what should I do to port number 1 to measure h_{12} and h_{22} ? What should I do? To measure h_{12} h_{22} I should do something to port number 1. What is it that I should do? Clearly I_1 should be set to 0, now what does that mean? Port 1 is open circuited, so h_{12} is V_1 by V_2 with I_1 equal to 0 or port 1 open circuited and h_{22} is I_2 by V_2 with port 1 open circuited.

So, these are the definitions, you can see that the general scheme is the same for all parameter sets, the units the quantities that we calculate will be different. In y parameters you are always calculating the ratio of currents to voltages. In Z parameters you are always calculating ratios of voltages to currents. And in this hybrid parameters sometimes it is current by voltage, sometimes its voltage by current, it is also current by current and voltage by voltage. So, everything is possible. Is this fine? Any questions about h parameter definitions?

So, again there is a question on application of this parameter. So, application for any parameter is the same. It is to give a compact representation of a complicated circuit, you could have hundreds of components in this, but let us say only 2 ports are exposed, that is 2 terminal phase are exposed to you. You can describe the behavior of this entire complicated circuit with this 4 parameters and you could choose y or Z or h parameters or we will see another one, which is known as g parameter. Like I said a few times before, which one you choose depends on convenience.

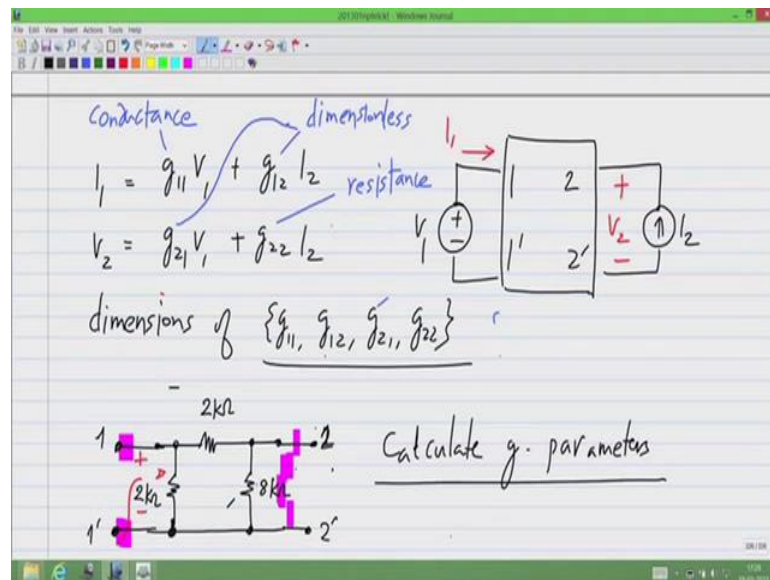
Now, traditionally there are components known as bipolar junction transistors which are use to make amplifiers, so those are described with h parameter. And there is a question of it is hybrid? See, if you look at y parameters, all the y parameters have conductances, they have dimensions of conductances and Z parameters, all of them have dimensions of resistance. Then these h parameters their dimensions are all mixed up, one of them is a resistance, one of them is a conductance and two of them have no dimensions, so that is why it is called hybrid. And this last question is, is there any circuit element? I am not sure of what this question means, where are the circuit elements?

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So again I would encourage you to calculate h parameters of this network. It is the same network I had earlier, so I am not going to go through this, please try to do this as an exercise and if you run into any difficulty, please let me know in the next lecture. Now, finally we can also express here we have expressed V_1 and I_2 in terms of I_1 and V_2 .

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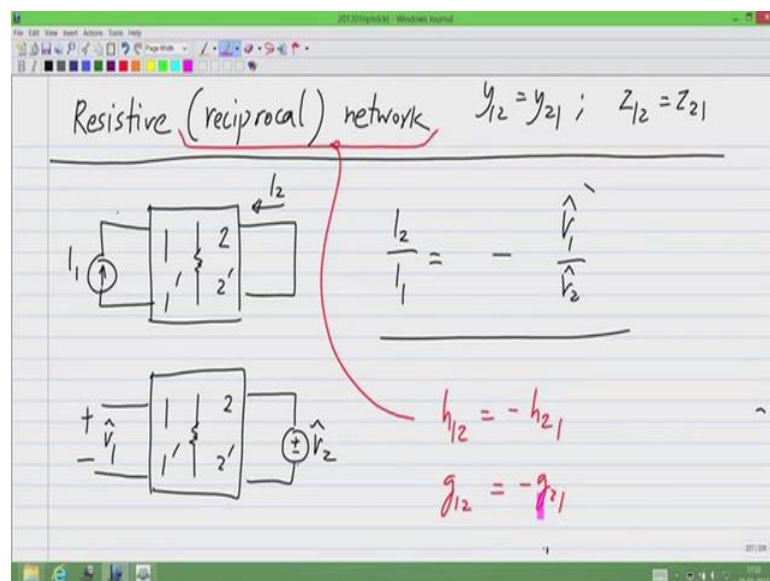
We can also express I_1 and V_2 in terms of V_1 and I_2 . And in this case, we think of a voltage being applied here and a current being applied to port 2, and we measure current I_1 and voltage V_2 .

So, very quickly please give me the dimensions of g_{11} , g_{12} , g_{21} and g_{22} . What are the dimensions of g_{11} ? Clearly g_{11} multiplies a voltage to give current, so it has dimensions of a conductance or units of Siemens. What about g_{12} ? g_{12} is clearly dimensionless, it multiplies a current to give current. And g_{21} ? It is also unit less, it multiplies a voltage to give a voltage. And finally g_{22} ? g_{22} multiplies a voltage to I think I made a typo while writing, this is I_2 . g_{22} multiplies a current to give voltage, so this has dimensions of resistance. So, this also hybrid, you have conductance resistance and dimensionless parameters in fact they are exact opposite of h parameters.

So, this is another set of hybrid parameters. So, I will not go through this again, so you can figure out how to measure g_{11} and g_{21} , how to measure g_{12} and g_{22} , what you should do to port number 2 and port number 1 and so on. So, I think you will be able to this for yourself, and also as before just for to get some practice calculate the g parameters of this circuit. Somebody answered that g_{22} will not have units, that is not correct. g_{22} will have units of resistance, dimensions of resistance.

So, why is the name g given? I have no idea, it is some historical reason. There was h and there is g , so I really do not know why g is given. It is, it can be a little confusing because we use g for conductance as well, but this g in 2 port stands for some hybrid parameter. Now, finally we know that for a resistive network which is reciprocal.

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We saw that y_{12} and y_{21} are the same and Z_{12} and Z_{21} are the same. Now, is there any such relationship for h parameters or g parameter? So, just to refresh your memory we discussed reciprocity for this case, for I_1 with this side short circuited and we measured I_2 . And in the other case, we had V_2 , we had this side open circuited, V_2 hat and V_1 hat. And there was some relationship between I_2 by I_1 and V_1 hat by V_2 hat. What was the relationship for, what was the relationship for a reciprocal network between I_2 by I_1 and V_1 hat by V_2 hat?

So, we derived this by putting a current in, current on one side and a voltage on the other side. What is the relationship between I_2 by I_1 and V_1 hat by V_2 hat for a reciprocal network, for a resistive network? Relationship between these 2 ratios for reciprocal networks, there is an answer which says both are equal, there was a slight twist I_2 by I_1 equals minus V_1 hat by V_2 hat. If you apply reciprocity theorem and calculate the reciprocity and relationship, this is what you will get.

So, this means that for the hybrid parameter for a reciprocal network h_{12} will be minus h_{21} and g_{12} will be minus g_{21} . So, they are related h_{12} is the effect of port 2 on port 1, h_{21} is the effect of port 1 on port 2. So, in general parameter 12 shows the effect of port 2 on port 1, parameter 21 shows the effect of port 1 on port 2, and in a reciprocal network these 2 will be related in somewhere. For y parameter the relationship is that they are equal, and for h and g parameter they are equal and opposite of each other. Any questions about this?

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$h_{11} = Z_{11}$

(a) Yes

(i) No

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$h_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}$$
$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}$$

So, finally someone on the chat window commented that h_{11} equals Z_{11} . Is this correct? What do participants think? Is h_{11} is the same as Z_{11} ? So, the answer is no. It is not the same because if you think about it, V_1 is $h_{11} I_1$ plus $h_{12} V_2$ in terms of h parameters, and V_1 is also $Z_{11} I_1$ plus $Z_{12} I_2$. But if you look at the value of h_{11} , it is V_1 by I_1 with V_2 equal to 0, that is port 2 short circuited whereas Z_{11} is V_1 by I_1 , but under different conditions with I_2 equal to 0 with port 2 open circuited. So, h_{11} is not the same as Z_{11} . In fact in all these parameter sets, the parameters will be different from each other.

Although one can be derived from the other, they are not same as each other. I hope that is clear, there were number of questions asking about the details on reciprocity and so on. So, we went through a long discussion on that in one of the lectures. I suggest that you please go through that lecture, if you have any doubts we can discuss that. And then there is some question asking for books on networks and network theorems. Books like are good, but I will go through the books again and then give suggestions next week.

Thank you, I will see you next week.