**Basic Electrical Circuits**
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**Lecture - 11**
**Tellegen's Theorem**

Hello and welcome to lecture eleven, in this lecture we look at few more involved theorems.

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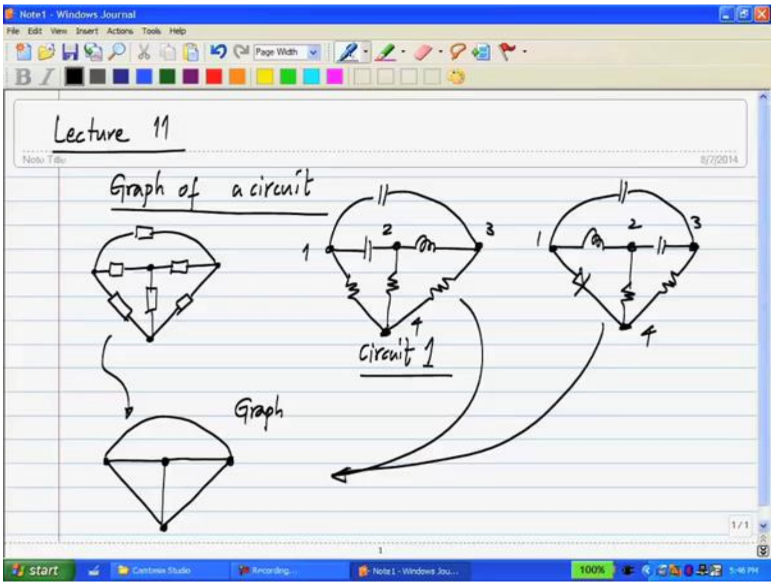


Before I go into the theorem I will introduce the notion of graph of a circuit, so if you have some circuit, each of these could be any elements, we do not make any distinctions between them at these level. A graph of this is basically a picture that contains a node for every node of the circuit. The elements are simply represented as branches this is basically an abstract representation of a circuit and is useful for discussing some very general properties of a circuit, so this is the graph of this circuit. Now, let say we take two different circuits, this is circuit one and I have another circuit, it would have any element.

You see that both circuits have four nodes and both circuits have elements between corresponding nodes, the same corresponding nodes in the first circuit. I have an element between one and two and in the second one also between one and two and so on. It is also clear that both of these circuits have the same graph, so graph representation is not useful for discussing particular properties of some circuit, but for general properties of all the circuits sharing certain graphs, so with this short introduction let us move on.

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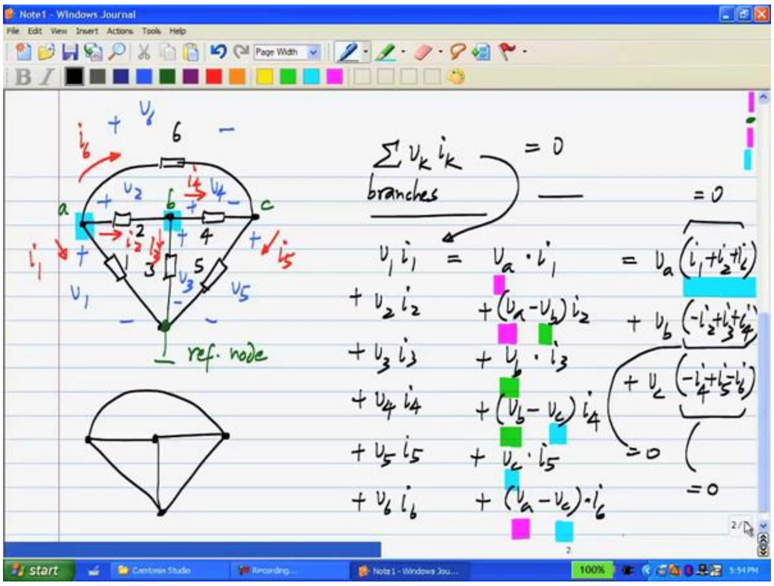


So, first let me take a circuit, in fact I will take the same kind of circuit I had before, I have all these branches and as I mentioned earlier, this circuit has this particular graph and let me number the branches like this 1, 2, 3, 4, 5, 6. For each of them, I will show the voltage across the branch v 1, v 2, v 3, v 4, v 5 and v 6, I will show the currents with the passive sign convention that is I will take the current as entering that terminal of the element whose voltage is defined to be positive.

So, for instance v 6 is like this, so I will take i 6 in that direction i 1 this way i 5, i 2, i 4 and i 3, first what I want to look at is basically some of the products of voltage and current over all branches. I introduced this expression in the previous class, but now we will examine it and see what it comes out to and evaluate it exactly. So, first I will do it for this graph, but I will quickly illustrate that it is not true for this particular graph this will be true for every circuit. So, now what I will first do is I will represent every voltage as difference between node voltages with respect to some reference node.

So, let me take this as the reference node if I call these nodes a, b and c, so clearly I can write first, I will expand these things it is v 1 i 1 plus v 2 i 2 plus v 3 i 3 plus v 4 i 4 plus v 5 i 5 plus v 6 i 6. This whole thing equals and clearly see from Kirchhoff's voltage law that v 1 equals v a minus the reference node voltage which is by definition 0, v 1 equals v a v 2 equals v a minus v b v 3 equals v b and so on. If I complete this for every one of them, I will have v a times i 1 plus v a minus v b times i 2 plus v b times i 3 plus v b minus v c times i 4 plus v c times i 5 plus v a minus v c times i 6.

Finally, first I had this form where I summed v and i over all branches we know that v k i k is the power dissipated in the k branch at that particular instant of time. Now, we do not worry about whether v k and i k are time varying or constant or what kind of elements we have, but at every instant of time the circuit will obey Kirchhoff's current law and Kirchhoff's voltage law. So, we can take these product sum of these products and decompose them like this and turn it into a form like this.

Then, instead of having a summation over all branches I will now group the node voltages together. So, v a i have it here, here and here and how many terms will I have, three because there are three branches connected to node a and what is multiplying v a it is i 1 plus i 2 plus i 6.

Similarly, v b there are three branches connected to node b, so I expect three terms and indeed there are three of them here, here and here and I will have minus i 2 plus i 3 plus i 4. Finally, we see again three branches connected to node c, so I will have one, two and three occurrences of v c. I will have minus i 4 plus i 5 minus i 6 and if I look at each of these terms what is i 1 plus i 2 plus i 6 it is simply the total current leaving the node a and obviously by Kirchoff's current law this whole thing equals 0.

Similarly, minus i 2 plus i 3 plus i 4 is nothing, but total current leaving node b i 2 is entering node b. So, we have minus i 2 and i 3 and i 4 are leaving node b, so we have plus i 3 plus i 4, this sum also equals 0 and similarly, minus i 4 plus i 5 plus i 6, sorry minus i 6 minus i 4 plus i 5 minus i 6 will be 0 because that is the total current leaving node c. So, obviously the entire product will be equal to 0, so though I did it for this particular circuit, I can assure you that it is true for a general circuit.

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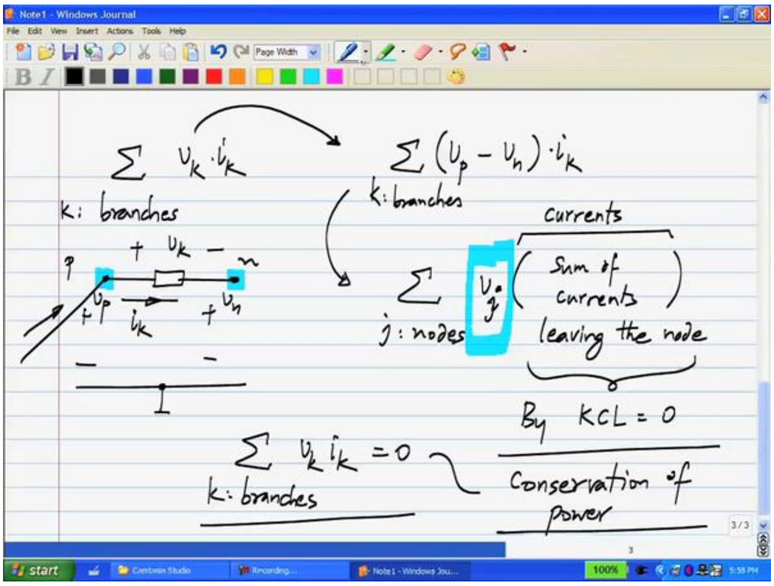

What did I do, I took summation of v k times i k where k basically runs over all the branches of the circuit. Then, I represent each voltage as a difference between two node voltages this is always possible, it could be that the second node voltage could be is 0. If some branch is connected to the reference node, then one of these voltages will simply be the reference node, but obviously any branch voltage equals the difference between the voltage at one terminal and voltage. At the other terminal where v p and v n are measured with respect to the reference node.

Now, if either of these two nodes itself happens to be the reference node, then v p and v n will be equal to 0, but this description holds in any case. So, I can sum over all branches with this modified form then what I will do is I will convert all this into a summation over all nodes, so let me call this j where j runs over all nodes v j. That will be multiplying some terms containing current, now the way we define the voltages if you have a brank k whose voltage is v k, then the current i k goes this way it goes from the plus terminal where as v k is defined to the minus terminal.

We use the passive sign convention, so which ever node has is the positive terminal that is v p it will get multiplied by plus i k and v n will get multiplied by minus i k this i k is simply flowing from this node p to this node n, let me name these nodes p and n. So, if a current is leaving a node, then it will have a positive sign that is we have v p times i k with a positive sign and if the current is entering a node. Then, it will have a negative sign we will have minus v n times i k, so I will have sum of currents multiplying this and basically this will be nothing but sum of currents leaving the node.

When I collect all the terms because I will have current leaving this node with a positive sign and there could be some other branch and currents entering this node that will have a negative sign. So, basically if I change this to summing over all the nodes, then the node voltage v j will be multiplied by something and that something is sum of currents leaving the node which by Kirchoff's current law equals 0.

So, every node voltage gets multiplied by 0, which means that summation of v k i k over all branches equals 0. This is nothing but a statement of conservation of power and with this we will prove more interesting results about circuits and finally, also come to reciprocity theorem, which is quite useful when we have circuits with an input and output port that is two port networks.

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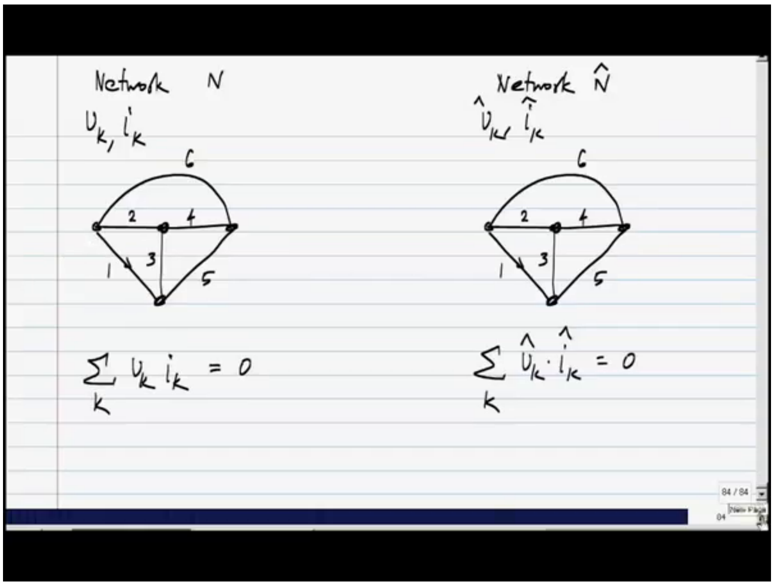


Let us say I have this network n and our network n hat which have exactly the same graphs though the elements may not have anything to do with each other. Of course, as you can see we have not used any linearity or anything, so it could be non-linear also, I could for instance in the network n branch one could be could be a voltage source and the other one could be resister or a diode or whatever it is.

All that is possible and I will denote voltages and currents as v k and i k and in this by v k hat and i k hat clearly from what we just now said sum of v k and i k over all branches i will say or k. That means k running through all the branches is 0 and here what would it be v k hat i k hat would be 0, this is fine.

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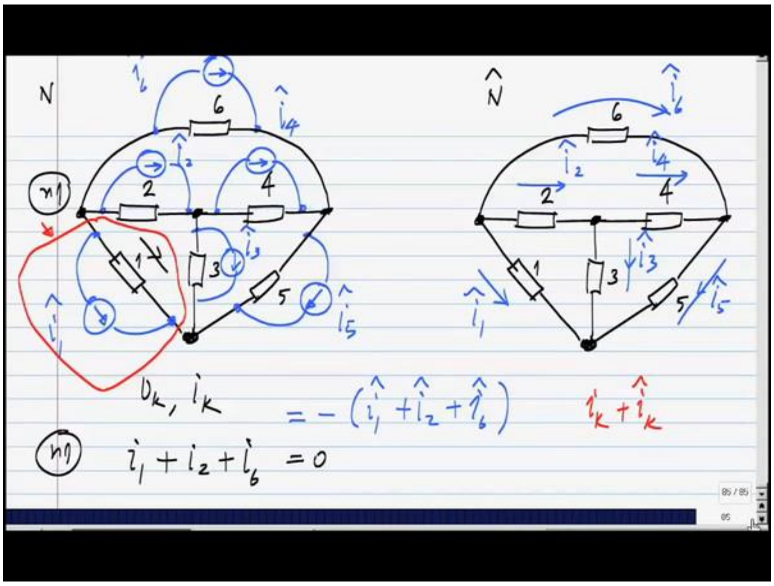

Now, let me show it some bigger some boxes for elements, this is same graph, this is n and n hat and let us say in the network n hat I have currents i 1. Let me show that in a different colour i 1, i 2, i 3, i 4, i 5, i 6, these are the currents that are going into the network and what exactly they are depends on what elements there are in the circuit. Let us not worry about that, what I will do is I will take a current source, whose value is the first branch current whose value is i 1 and connect it across this branch.

Actually, I cannot abuse the notation these are what it is i 1 hat i 2 hat and so on up to i 6 hat. So, what I will do is, I will connect take i 1 hat that is I will look at the network n hat, I will look at whatever current is flowing in branch one make a current source equal to value and connect the source across branch one of network n. Similarly, I will do it for all other branches, now let us say originally the branch voltages in this n circuits are v k and i k that we got by solving for these circuits.

Now, after I connect these current sources, what will be the solutions, I am talking about currents in these black elements voltages here. So, what will be the branch voltages in this circuit in the new circuit superposition understood, so you are saying branch voltages will get added up and currents will get added up why voltages will remain same current will subtract what is superposition. Again, please understand what is super position, superposition is you have your network and you disable some sources in it and you disable some other sources and you add up the solutions individually and that will be the final solution with all the sources active.

But, here that is not what I have done I have changed the network right I originally only had the black elements here to that I have added blue current sources these current sources are not arbitrary. I took another circuit with the same graph, I copied the current values from that into this, so tell me what will be the branch voltages in this v k v k hat remains the same why current remains in parallel things would not remain the same. How would you go about solving for this, first of all lets have this current sources, I had the black network, how would you I go and solve for this one what method.

I would use nodal analysis, now after I add the current source, what will happen to the nodal analysis equation source vector will change and how will it change, by how much it will change what is it no change why? I have not taken arbitrary currents, here it is true to each node I have added some current, but what are those currents the some of those currents added are also 0, because it comes from another circuit, which is valid which follows KCL. For instance, if you think of the original nodal equations I had, let us say original node n 1, you would say these black currents i 1 plus i 2 plus i would be 0.

Now, what else we have would be equal to the currents flowing into this will change to minus i 1 hat plus i 2 hat plus i 6 hat which is also equal to 0. So, if you think of the methodology of solving this circuit by writing down the nodal analysis, you see that at every node the equation does not change at all because the net current added to every node is 0 because I have not added a single current. I have added all the currents by copying it to some other circuit with the same graph. So, at every node if you compute the sum of the blue current sources in the appropriate directions will be 0 that means that the nodal analysis equation remain exactly same as that before.

So, what are the branch voltages, now same as before is that fine, now I can think of this entire thing as a branch that is possible, I mean my two terminal element could consist of my original branch plus this current source in parallel. What will be it, so first of all and what will be the branch current through the blank branches alone, same as before, i k.

Now, with this new branch with the parallel combination of old branch and the current source what is the total current sum of the 2 i k plus i k hat. Please follow the reasoning carefully because this looks very simple, but just follow the steps logically, these branches are have currents i k plus i k hat.

Now, this is a third circuit with the same graph, now each branch will be the original in the black circuit plus the current source in the second circuit, but this is also a graph with that, I mean this is also a circuit with the same graph. If you apply this to the new circuit, what we will get what did we say for every circuit some of the v k sum of product of branch voltages and branch current equals 0. Now, I can do this to new combined circuit, I have to do it for branch voltage here, which is v 1. And the branch current here which is i 1 may be I will call it some i 1 dash or something v 1 dash, so what is the result Sum of v 1 dash and i 1 dash also equals 0.

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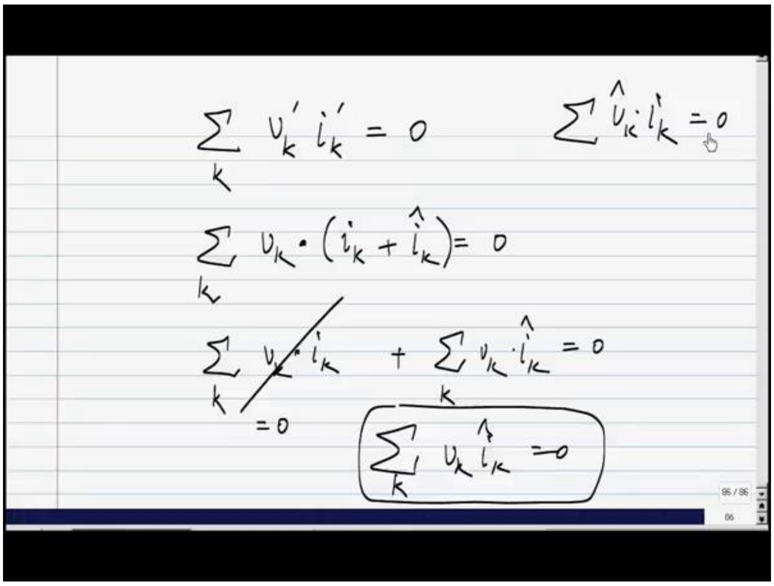


What is sorry sum of v k dash i k dash not v 1 dash i 1 dash and what is v k dash v k the original circuit the original solution to the circuit network n and what is i k dash i k plus i k hat equals 0. So, v k i k plus v k i k hat equals zero v k i k plus v k i k hat equals 0, what next first term we already know is 0 v k i k, this one equal 0, so we have sum of v k i k hat equals 0. Now, this is I mean please understand the significance of this result and we have not used anything rather than to say the circuit obeys the KCL and KVL you have two circuits completely unrelated and except that they have the same graph.

So, we take two voltages branch voltages from the circuit one and corresponding branch currents from circuit two form this current times voltage products and sum. Then, that will also be equal to 0 and this is just a consequence of the circuit having obeyed k c l and KVL. This v k times i k hat that number does not have any meaning for the same in this, I mean if we take v k and i k in the same circuit that will be the power dissipated in that branch.

Otherwise, it has no meaning at all because in fact the elements also could be completely different this here the cape branch could be a voltage source than a current source and so on. But, this theorem is true this is fine what we did was we took two circuits which had the same graph. Then, we formed the circuit, which was circuit number 1 plus current sources copied from the circuit number two and we did that for every branch that is very important.

Otherwise, it would not be true and now you get a third circuit whose branch voltages remains the same branch currents are the sum of the currents in first two circuits and for that also this sum of branch voltage. I mean sum of this branch voltage branch current will be 0, from that we get this very interesting result that you take two circuits from the same graph and you take voltage from this current from that and sum them that will also be equal to 0, any questions?

What do you think of the product sum of v k hat i k 0, how would you go about proving that you could take it take the currents from the first circuit and put it into hat circuit and that is also 0. Now, it is consequence of only KCL and KVL, so it can include elements that we have not considered so far anything that obeys KCL and KVL it will work and it will also work for capacitor and inductors and so on. Now, this network n and n prime need not be two separate circuits, it can be the same circuit at time one and time two.

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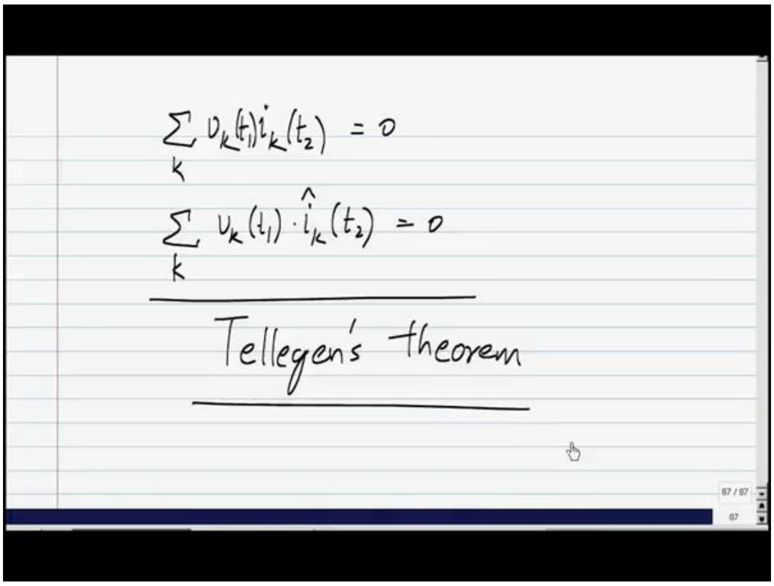


So, it is also true that in the same circuit v k v k at t 1 and i k at some t 2 this will also be 0 you can just think of it as they are two different cases where KCL and KVL are obeyed. Of course, it is true that if you have the voltages from the first network at time t 1 and current from second network at time t 2. This will also be equal to I am saying I can take the voltages from the first circuit v 1 and current from the second circuit at a different time, that is also true. I mean it is yet another case that has the same graph and obeys KCL and KVL, this result is also true for any I mean we only used to circuits, but anything where these laws are obeyed that is sum of flow from these nodes are 0.

The total sum of the total flow from the node will be equal to 0 and also if you go around the loop the sum of branch variables the sum of the across variable will be 0. Here, the across variables is the voltage and the through variable is the current, but it works for many other things also like fluid flow and things like that and this extremely interesting result is known as Tellegen's theorem named after the person who first proved it.

Again, the way we have gone about and proving is very simple, but you have to be convergent enough with KCL and KVL to follow the logic correctly. Now, this theorem itself is rather too general, in fact it is too general to be used directly now we are talking about general properties of the network. In this course, our concern are some specific circuits, so what we will do is we will derive some specific results from this Tellegen's theorem, any questions so far about the proof or statement of Tellegen's theorem?

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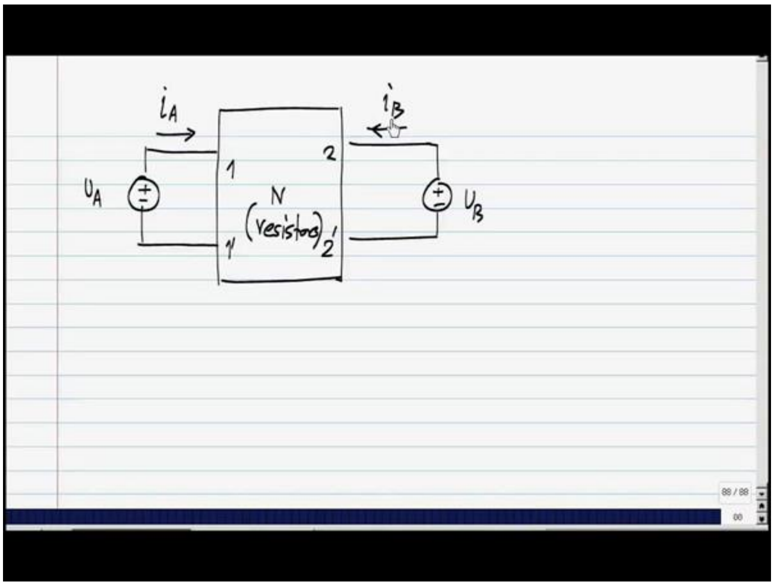


Let me take a network n and this has and this has only resistor and this is only this is more general, I mean less general than before we do not have control sources. We have only resistors in this and let us say we have two pair of terminals to which we can I mean with only resistors if you have only that of course, all the solutions will be 0. All the currents will be 0 and voltages will be 0, we have to connect voltages somewhere and let us say we have two pairs of terminals where we can do. Let us say I have connected v 1 here, actually let me say I have connected v a here and v b the sources could be anything it could be two current sources two voltage sources one voltage and one current source. I am just showing the case of two voltage sources and you these currents i a and i b.

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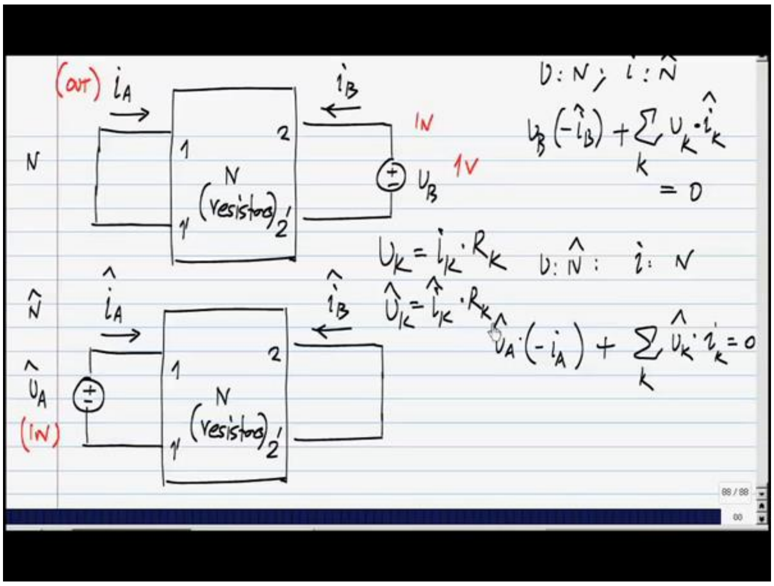


Now, let me first make v a equal to 0, that means I will short circuit this and in the other case i will make v b equal to 0, I will short circuit that one. Now, let me in the first case think of this one as the input and this i a as the result or the output and similarly, in this case I think v a as the input there is only one source and i b as the output. This is a very common view right you have some network you connect something here, you look at the output here. Now, what do you think is the relationship, let us say I connect 1 volt here and I will get some i a if I get if i apply one volt here will i be able to tell what i b is. So, please write down the Tellegen's theorem equation for this and then see what comes out.

Now, you can think of these two circuits right and to avoid confusion, let me put hats on this it is not the same i a and i b that is flowing in there we do not know that. So, I have to put hats on these things, so I now apply Tellegen's theorem to this whole circuit and see what comes out, please do that if i take the voltages from this and the currents from the second circuit and form the products and add them up, that will be 0. Similarly, voltages from the second one and voltages from first one that will also be equal to 0, now for a consistent polarity, I have to multiply v b with minus i b hat because i will direction going in from up to I mean from top to bottom.

So, first I will take if I call this network n and n hat the voltages from n and currents from n hat. So, what is that v b times minus i b hat plus the sum of all voltages inside this resistive network times currents in that hatted currents hatted currents in the corresponding resistive network plus the voltage across this that is 0 that I will ignore this is over k branch is inside the resistive network.

Next, I will take the voltages from n hat and current from n what will I get what is the first term v a hat times minus i a, because of the that is because of the way we have defined the currents plus summation of v k hat i k over all branches equals 0. So far we have not used anything about it being a resistive network so where we will use it?

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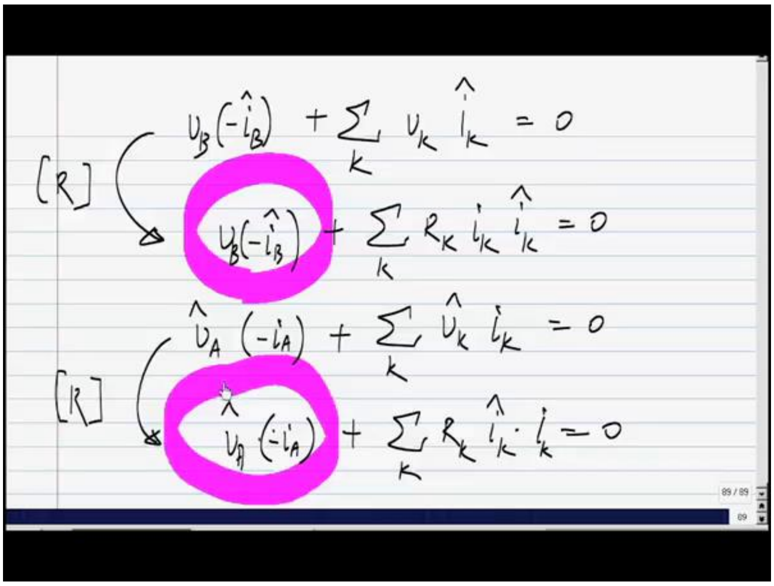

So, v k would be v k is the branch current times the branch resistance, similarly v k hat is i k hat times the branch resistance. So, what we will get out of it and I will use the resistive network part here and similarly, in the other case v a hat minus i a plus v k v k hat i k equal to 0. If I use the fact that it is a resistive network v a hat minus i a plus kr k i k hat i k equals 0, so what you get from these 0, so clearly this part is common to these equation, so this has to be equal to that one.

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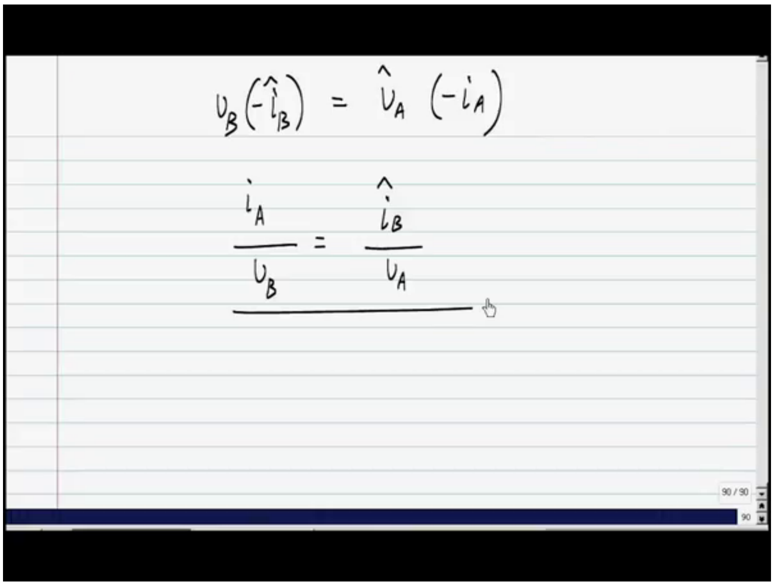

I can of course remove the negative signs now, what is this v a hat this is the hat this is fine, so what is that saying about these cases. So, what is that saying about these two cases?

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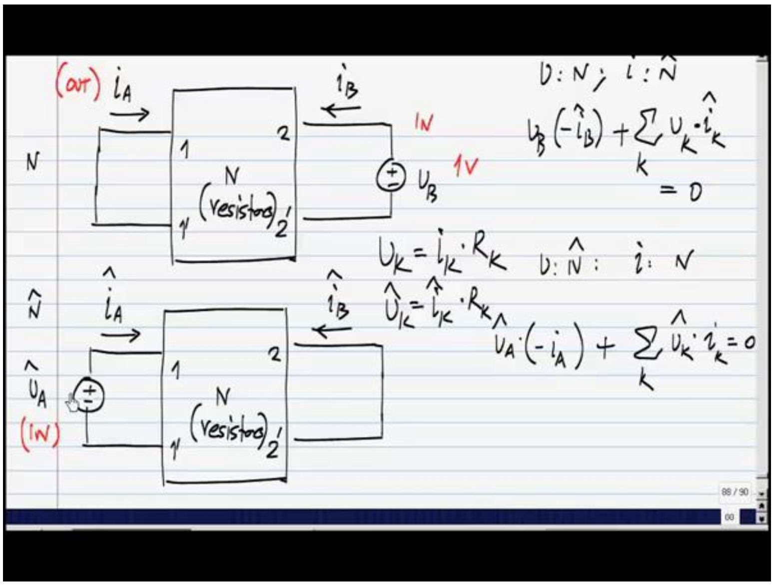


I have a resistive network that is all I know about it could have thousands of resistors, but there are two terminals exposed from this side. On that side, I apply one volt here, short circuit the other side that is I could apply voltage sources to both sides and then I set one of the voltages sources to 0, then I measure the short circuit current here, what will it be? Exactly the same not merely proportional not merely the same it will of course be proportional because it is a resistive network, but that is not what this is saying.

What is the final result we got i a by v b is i b hat by v a hat, it is a very interesting theorem if we have resistors alone this is known as reciprocity theorem the network is reciprocal. If you apply some stimulus here and measure the effect there and we apply the stimulus on the other side and measure the effect here they will exactly the same that is the ratio of the response to stimulus will be the same, so please understand this theorem carefully.

We already proved reciprocity theorem when we had voltages exciting from one side or the other side. Now, there could be other cases that is you could have current sources from either side and voltage from one side and current source from another side. I will not prove reciprocity theorem for all of these cases, but you can follow the exact same steps as I did earlier and prove all of these as well.

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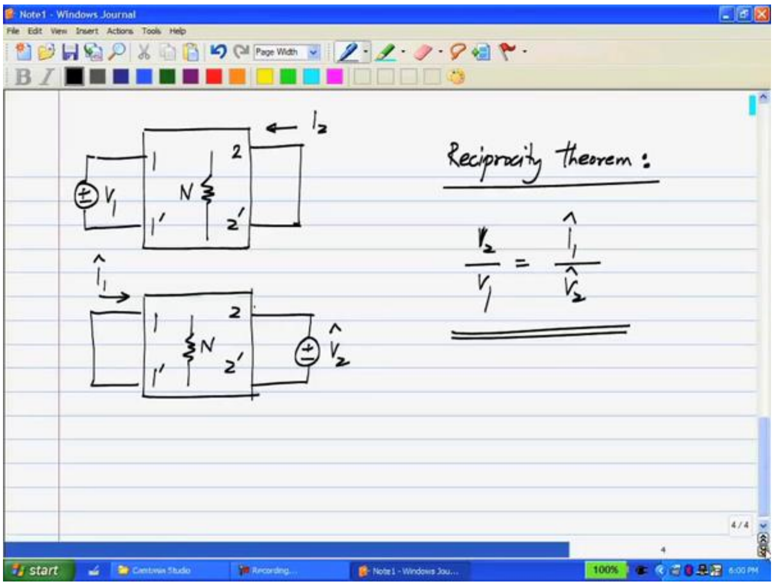


So, just for completeness let me take a resistive network with two ports, 1, 1 prime and 2, 2 prime, I have excited this side with v 1 and short circuited this side and measured the current i 2. In another case, I have the same network n of course that is very important, I excite this side with v 2 hat and short circuit this side and measure i 1 hat. Here, reciprocity theorem says that i 2 by v 1 is exactly equal to 1 hat divided by v 2 hat.

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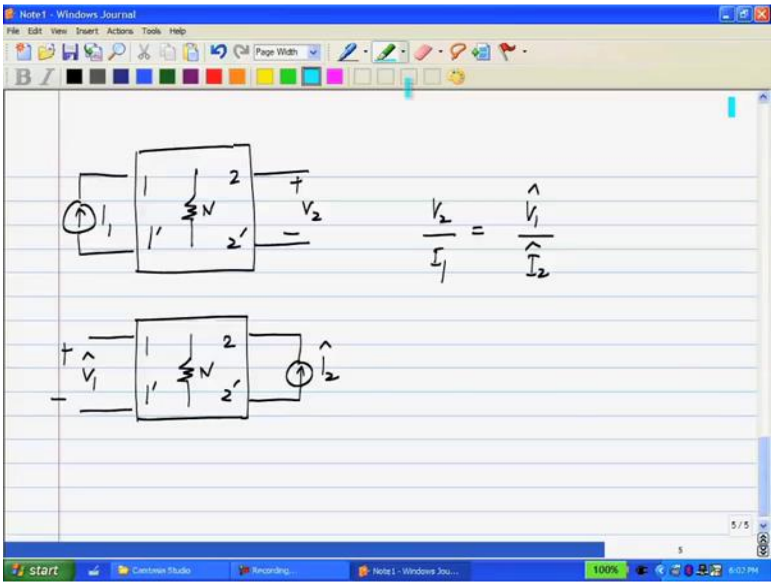


Now, let us look at the other cases again a resistive network excited it to current source and measure the voltage v 2 here in this case excitation of the cause is the current the response or the effect is the voltage. I eject i 2 hat this side and I obtain one hat from the other side and again from reciprocity it turns out that v 2 by i 1 exactly equals i 2 hat by sorry v 1 hat by i 2 hat. That is the ratio of response to excitation when port one is exited is exactly the same as the ratio of response to excitation when port two is excited.

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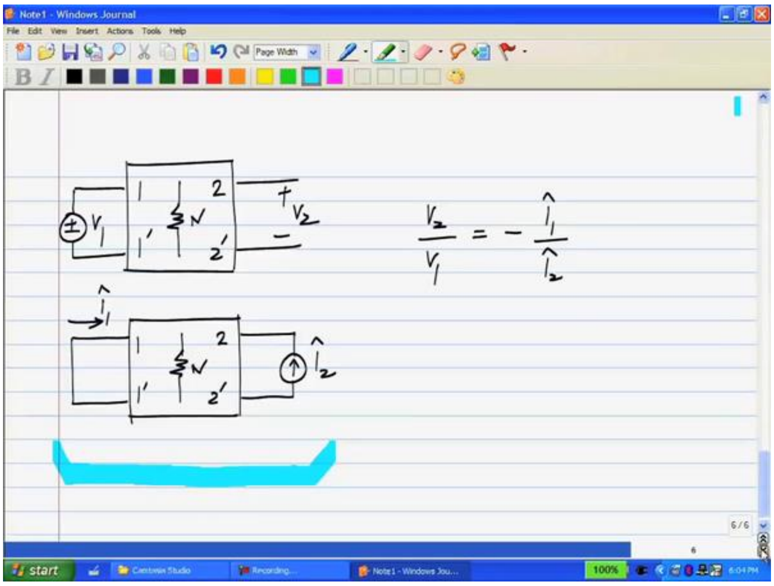

Finally, we have the third case where gain I have a resistive network n and I excite the first side port one with a voltage and measure the voltage that comes out in this side. In the other case, I take the exact same network n I excite second side with a current i 2 hat and measure the current that flows on the first side, please mind the directions of voltages and currents. Otherwise, you will end up with extra minus signs in different places so reciprocity in this case says that v 2 by v 1 which the response by excitation in the upper circuit exactly equals minus i 1 hat by i 2 hat, which is the response by excitation in the lower circuit.

We get this minus sign because of the direction of the current chosen if we had chosen it outwards we would have got a plus sign, but for consistency we have taken all currents as flowing inwards into the resistive port. We have a last case that is left which is trivial that is exciting the left side by a current and right side by a voltage that is simply flipping this picture sideways, so I am not going to discuss that.

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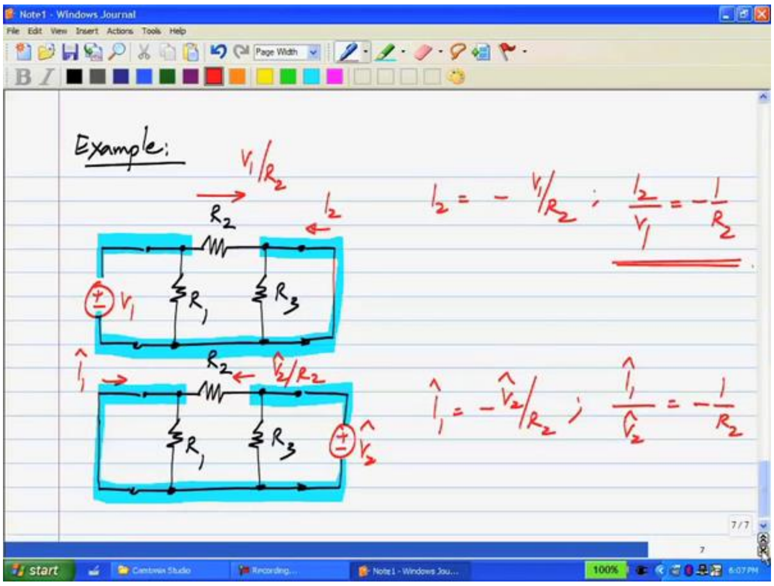


Now, I will show all of these things with an extremely simple circuits as example, my resistive network is just this, as three resistors R 1 R 2 and R 3 and let me connect first a voltage source to this side and I will short circuit to the second side. In the second case, I will connect a voltage source v 2 hat to the right side and short circuit the left side, I will the measure the current i 2 in this case and i 1 hat in that case. Now, it is very clear that due to short circuit this voltage simply appears across R 2.

So, the voltage source is connected across R 2, so the current in R 2 is v 1 divided by R 2 also due to short circuit no current flows in R 3 and this i 2 simply equals minus v 1 by R 2 that is because of the directions chosen v 1 by R 2 is flowing from left to right. Here, i 2 is taken as right to left, so i 2 equals minus v 1 by R 2 or i 2 by v 1 which is response by excitation which equals minus 1 by R 2. If you now look at the lower circuit gain because of the short circuit this voltage v 2 appears across R 2 and therefore the current in R 2 is v 2 hat by R 2.

Again, because of this short circuit no current flows through R 1 and because of the way we have chosen the direction of i 1 hat i 1 hat will be equal to v 2 hat by R 2 which means that i 1 hat by v 2 hat is minus 1 by R 2 minus 1 by R 2. So, this illustrates reciprocity we have already proved for general case, it is always a good idea to work out some real examples and convince yourself that this is indeed the case.

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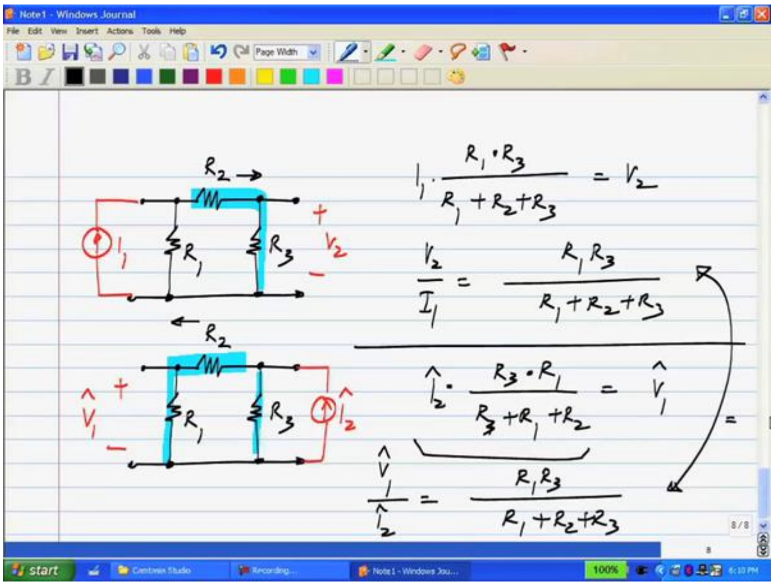

Now let me excite this circuit with currents instead of voltages i will excite it with i 1 on the left side and measure the voltage v 2 that is produced on the right side. Other case, I will take the exact same network excite it with a current on the right side and i w to hat and see the voltage that is produced on the left side. So, now what is the voltage developed here first of all this i 1 divides into two parts, one part goes through R 1 and the other one goes through the series combination of R 2 and R 3. By results on current division, you know that the current that flows here is i 1 times R 1 divided by R 1 plus R 2 plus R 3 and v 2 is nothing but this current times the resistance R 3.

So, this is equal to v 2 the response by excitation v 2 by i 1 will be simply R 1 R 3 by R 1 plus R 2 plus R 3. Now, we can evaluate v 1 hat in exactly the same way this current i 2 hat gets divided into two parts one through R 3 and another one through R 1 plus R 2, so a part of the current that flows through R 2 is given by i 2 hat times R 3 by R 3 plus R 1 plus R 2 this is by current division theorem.

The voltage that is developed across R 1 is nothing but this current, which we have evaluated here times R 1. So, this times R 1 is the voltages is the voltage v 1 hat, so response per excitation v 1 hat by v 2 hat equals R 1 R 3 by R 1 plus R 2 plus R 3 which is exactly the same in this case. So, we have exactly the same response per excitation in the two cases as we expected.

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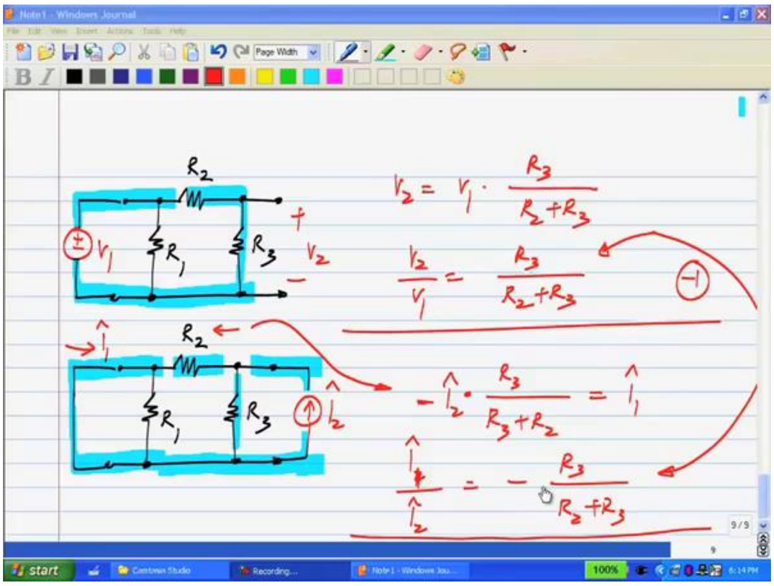

Finally, let me illustrate for a case where you have a voltage on one side and the current on the other side I am also illustrating this the purpose of illustrating this is also that when you apply reciprocity appropriately. So, let me take v 1 on this side and measure v 2 on the right side and in the other case I will apply a current i 2 hat on this and measure the current i 1 hat that flows over there. Now, in the upper circuit this v 1 appears across the series combination of R 2 and R 3 we also have R 1, but that is directly across the voltage source v 1. So, it has no effect, so v 2 is simply v 1 time the resistance divider ratio which is R 3 by R 2 plus R 3 or v 2 v 1 simply R 3 by R 2 plus R 3. In the other case what happens we have a short circuit on this side, which means that no current flows through R 1.

Across that, we have a short circuit, so we just have two resistors R 3 and R 2 in parallel with current source, I hope all of you are able to see that. Otherwise, you can see that the terminals of current sources are here and terminals of R 2 and terminals of R 3 are exactly the same that is they are connected to the same nodes. Now, the current divides between R 3 and R 2 the current flowing through R 2 in this direction is given by i 2 hat times R 3 by R 3 plus R 2. This is from the current division theorem, now all of it flows into the short circuit, but this current i 1 hat is in the opposite direction, so the actual current i 1 hat is minus i 2 hat times R 3 by R 3 plus R 2.

If we calculate the response per excitation i 2 hat, sorry i 1 hat by i 2 hat equals minus R 3 by R 2 plus R 3 and you can see that these two ratios of response per excitation are related by 5, 12 minus one exactly as we expected. It turns out that this reciprocity theorem is very useful in many practical situation and one such situation is where you have a number of sources in a circuit and you want to calculate the response at a given point whether it is current or voltage between two points .

Instead of calculating the response from multiple sources, what you can do is use reciprocity interchange the location of the source and the response and calculate all these responses into a single source, which is definitely easier. By using reciprocity theorem you can get all of the original responses that you wanted in terms it is very useful and it is also used for circuit simulators particularly for noise analysis. Here, you have noises from many parts of the circuit affecting your output you already seen the proof of reciprocity in resistive networks in Tellegen’s theorem, here what I will show is an alternative proof using basic circuit analysis.

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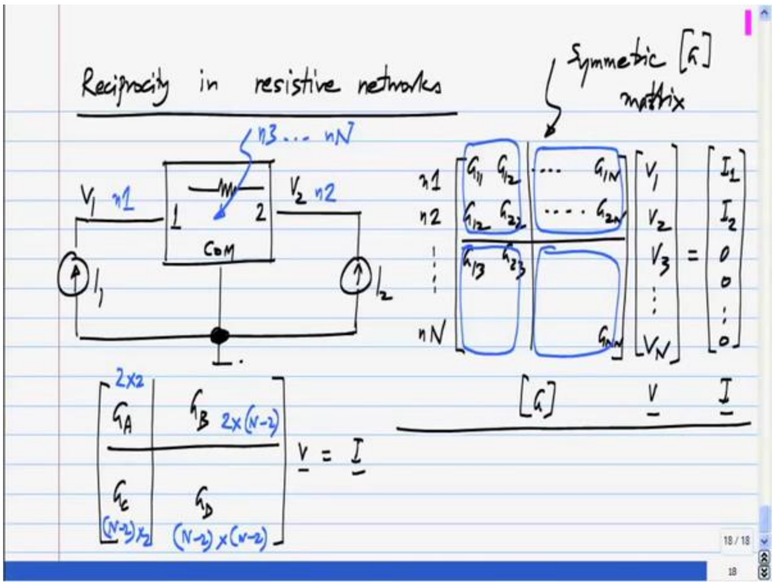


So, what I will do is I will consider a three terminal two port this is port one port two and this the common terminal and the important thing is that this is resistive. Now, let us say just for the sake of it, I will drive it with i 1 on this side and i 2 on that side the reason to do this is pretty obvious I would like to use nodal analysis for the analysis of this. We know that nodal analysis is most convenient when we have only resistors and independent current sources, now what I will do is this network in this is quite arbitrary there can be any number of nodes inside.

So, I will label this as node n 1 and this as node n 2 inside this there could be any more number of nodes and I will call this n 3 2 n capital n there are a total of capital n nodes in the circuit. Now, if you set up the nodal equations for this n 1, n 2, a number of row up to n capital n and remember this is a purely resistive network and the only independent sources in the network are i 1 and i 2.

Now, I will have some g matrix this is the conductance matrix times the variable vector v is v 1 v 2, where this is v 1 with respect to the common terminal and the voltage here is v 2. After that you have v 3 all the way up to v capital n and j times v the unknown vector equals the source vector and the source vector consists only of two non zero elements i 1 which is being injected into node n 1 and i 2 which is being injected into node n 2. The rest of it is 0, of course the network is purely resistive the g matrix is symmetric, so let us i have g 1 1 g 1 2 all the way to g 1 n.

Now, the first element of the second row first column and second row will be also g 1 g 2 etcetera g 2 n and similarly, here we have g 1 3, g 2 3 and so on. So, this will be symmetric because we do not have any control sources in the network the network is purely resistive. Now, for reciprocity basically I need to find the relationship between let us v 1 and i 2 and v 2 and i 1, so what I will do is I will try to eliminate all the remaining variables. So, for that let me sub divide this matrix into four pieces, so there are four sub matrix here as you can clearly see we have one and another one and another one here and yet another one there, so let me write that, let me write those things G A, G B, G C and G D. So, clearly G A is just a 2 by 2 matrix G B has 2 rows and n minus 2 columns it is 2 times n minus 2 and G C is the compliment of that which is n minus 2 rows and 2 columns and G D has n minus rows and n minus 2 columns and this times the variable vector v equals the source vector I.

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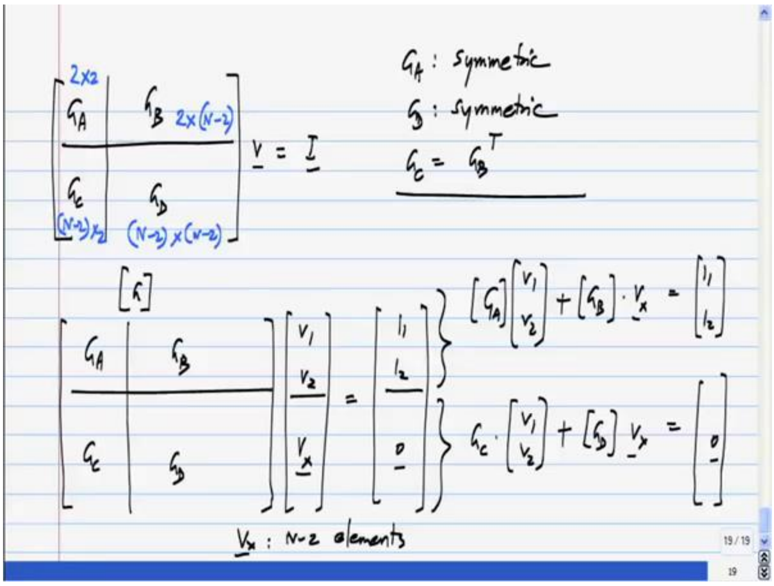


Now, because the g matrix is symmetric we can tell a few things about it, first of all G A will be a symmetric matrix G D will also be a symmetric matrix, and also if we look at G B and G C, G C is nothing, but G B transpose. Now, what I will do is this variable vector v let me write it as v 1 v 2 and then the vector of the rest of them which I simply call v x this itself is a vector v x has n minus 2 elements in it. That will be equal to i 1, i 2 and all 0 there, now I will write this as two separate equations the first part corresponding to these two rows is G A times v 1 v 2 plus G B.

Let me denote it like this to make it clear that these are matrices G B times v x equals i 1, i 2 and for this part I have G C times v 1 v 2 plus G B times v x equals vector all 0's. So, what I will do is basically I eliminate v x from these two equations that is all it is just like eliminating the variable from two linear equation. Except that, we have now vectors and matrices instead of scalar coefficients and scalar variables, but that does not pose any particular complications.

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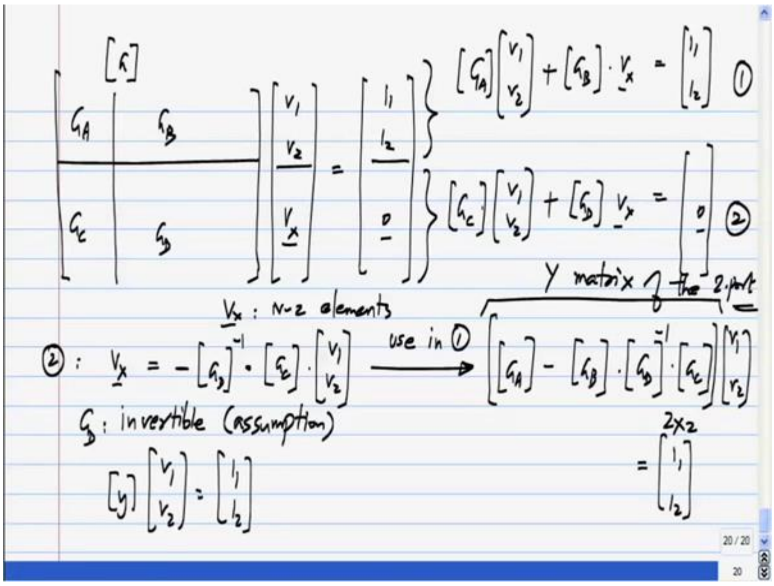


If I recall this 1 and 2, if I solve for v x in the second one it is clear that v x will be G D inverse times minus G C times v 1 v 2. And I will use this n 1 by the way this means that G D has to be invertible and we would not go into complication. We will assume that G D is invertible, then what we have is if we use this n 1 it is pretty straight forward, we will have G A minus G B times G D inverse times G C, this whole thing. Remember, this is a 2 by 2 matrix v 1, v 2 equals i 1 i 2, now from this it is pretty clear that this 2 by 2 matrix times v 1 v 2 is equals i 1 i 2.

So, if you think of this as a two port this entire thing is a y matrix of the two port or the inverse to this would be the z matrix of the two port because what is the y matrix after all y times v 1 v 2 equals i 1 i 2. It is the matrix that relates the port voltages to the port currents, so let us examine the y matrix of this.

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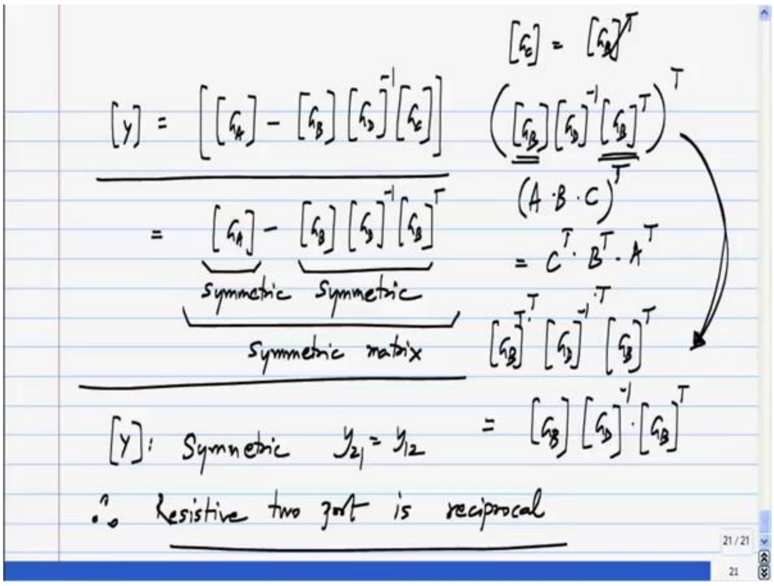


The y matrix of our two port is G A minus G B, G D inverse G C, so first of all we know that G C is G B transpose, so this is equal to G A minus G B G D inverse and G B transpose. So, first of all G A itself is symmetric that we knew earlier we discussed that and if you look at this part we have G B, G D inverse and G B transpose and to check for symmetry. Obviously, you take the transpose of this what will you get if you have three matrices a b c transpose, you know that this is basically c transpose times b transpose times a transpose. So, if you transpose this what you will get is G B transpose that is corresponding to this one times G D inverse and transpose.

Finally, G B transpose and clearly this is equal to G B, G D inverse transpose remember G D itself was symmetric, therefore, G D inverse is symmetric. So, G D inverse transpose is the same as G D inverse and finally, G B transpose, so the transpose of the matrix is the same as the matrix itself, so this is also symmetric this was symmetric we knew this earlier and from these two.

If you subtract one symmetric matrix from another symmetric matrix what this shows is that this y matrix is symmetric which means y 2 is y 1 2, therefore our resistive two port is reciprocal reciprocity means that y 1 2 equals y 2 1. So, instead of starting from Tellegen’s theorem and getting these results, we could also do it by defining a two port and we have some two port parameters. We try to calculate the two port parameters from basic circuit analysis and in this case nodal analysis and you can find that y matrix happens to be symmetrical. Of course, if you find any parameters had to be symmetrical the rest of them follow the same symmetry. Now, Z matrix will also be symmetrical and if you look at G And h parameters g 1 2 will g 2 1 and h 1 2 will be minus h 2 1 and so on, so this is the alternative proof of reciprocity in two port networks.