Solid State Devices
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Lecture - 8
Equilibrium and Carrier Concentration (Contd…)

This is the 8th lecture in this course and the 6th lecture on Equilibrium Carrier Concentration.

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Solid State Devices

Lecture - 8

Equilibrium Carrier Concentration

We have been discussing about carrier concentration of intrinsic semiconductors. First we discussed the qualitative model and in the last two classes we have been discussing the quantitative or the band model. The summary of the model that we discussed so far, the band model is that you have allowed energy levels in a crystal and it allows conduction band.

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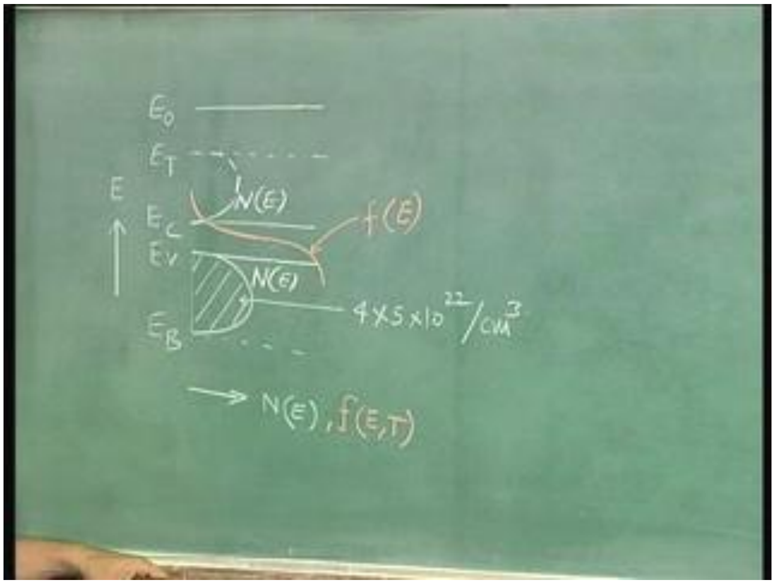

This is the top of the conduction band separated by another band that is the valence band. This is the bottom of the valence band, so this is the conduction band and this is the valence band separated by the energy gap. Then the valence electrons in the crystal occupy the balance band at T is equal to 0, some of these electrons may go up to the conduction band when you raise the temperature; so all the valance electrons at T is equal to 0 occupy the valance band.

Now how are the states for these electrons distributed? We have shown that the distribution of the states is the function N(E) and this area indicates the total number of states in the valence band. That is four valence electrons for each of the 5 into 10^{22} by cm cube silicon atoms. You similarly have a density of states function for the conduction band which we plot only near the band edge because we said that the shape of the function is somewhat more complicated than the density of states function in the valence band. That is why this portion is shown by dotted line.

The third postulate was that, of these available states for the electrons at any temperature only a fraction of the states are occupied and this fraction is given by the Fermi-Dirac function; f (E, T) because it is also a function of T. So this is the fraction of states occupied at any temperature.

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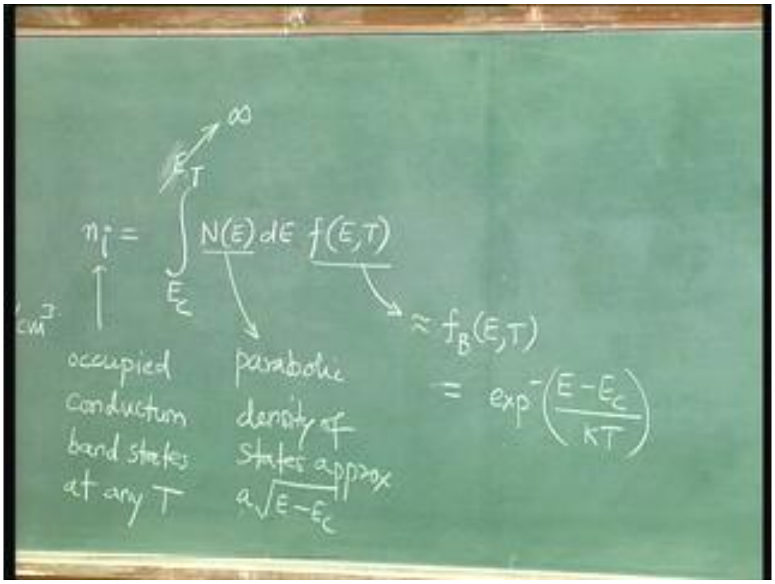

Now adding this information together, we wrote down an expression for the intrinsic concentration and that was n_i is equal to $\int N(E)$ dE, that is the number of states available within an energy interval dE at E, multiplied by the fraction of these states which are occupied at any temperature. That gives the number of electrons within the energy interval dE at energy E. If I were to take the conduction band, that is the interval dE, the shaded area is the N(E) into dE, and this small area here is N(E) dE into the Fermi-Dirac fraction and that small area represents the number of electrons or occupied states in the conduction band. So you integrate from E_c to (E, T) to get the total number of electrons. That is how you get the intrinsic concentration of electrons.

We said that if you make some approximations then the formula for the intrinsic concentration can be simplified. Here in this formula, the left hand side is occupied conduction band states at any temperature T; that is the concentration of electrons. Now we need to make the approximations to get a simple expression from here or replace this by infinity, it makes no difference because here this Fermi-Dirac function rapidly approaches 0 so it does not matter whether you take (E,T) as a limit or infinity as a limit.

The next thing is, for this density of states function use the parabolic approximation; that is parabolic density of states approximation. According to this approximation your state density is given by square root of E minus EC, that is the parabolic density of states approximation. And then for the Fermi-Dirac fraction you will see Boltzmann approximation. So this is approximated as f_B (symbol B is for Boltzmann) is equal to exp to the power minus (E minus E_C by KT). We are writing f (E, T) is approximately equal to f_B (E, T) that is Boltzmann approximation of the Fermi-Dirac fraction and that is given by this formula.

So when you substitute this term, the parabolic density of states approximation here and then the Boltzmann approximation into this and raise the top limit to infinity then we have shown that we have not done the detail derivation but then we said that the formula can be written as: n_i is equal to 2 [2pi m_n kT by h square] whole cube by 2 into exponential to the power minus ((E_c minus E_f) by KT).

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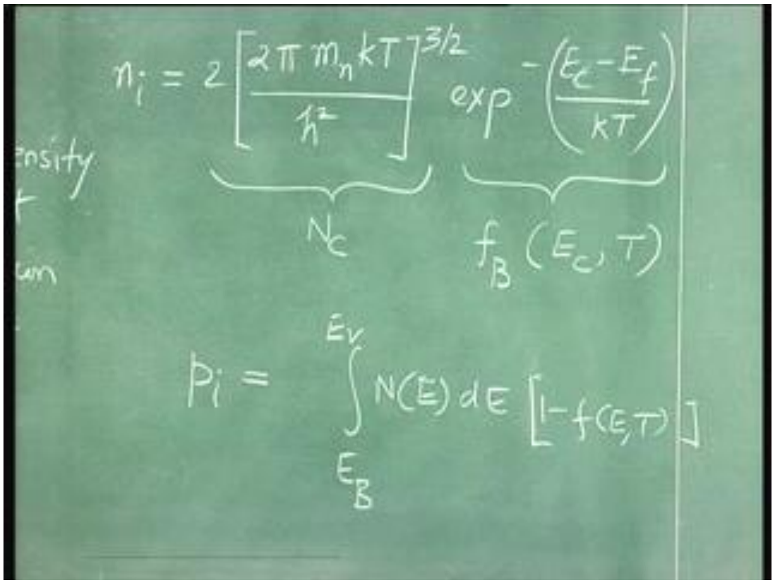

$$n_i = 2\underbrace{\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}}_{N_c} \underbrace{\exp -\left(\frac{E_c - E_f}{kT}\right)}_{f_B(E_c, T)}$$

$$p_i = \int_{E_B}^{E_V} N(E)\, dE \left[1 - f(E,T)\right]$$

It is left to you as a home work to do the derivation. We want to interpret this formula physically, this is where we left in the last class. I have repeated this portion because this is a very important part of the derivation in the energy band approach. How do we interpret this formula? Let us denote this term by N_c, try to interpret this constant N_c. As we have said this n_i represent the occupied conduction band states. Let us look at this function and find out what it represents. Thist represents the Boltzmann approximation to the Fermi-Dirac function at E is equal to E_c and temperature T.

Let me explain that part again, Fermi-Dirac fraction is given by 1 by 1 plus exp minus (E minus E_f by KT). If I want to get the value of this function at the conduction band edge E_c then all that I need is I substitute E_c here instead of E. So this represents the fraction of states occupied within the energy interval dE at E_c. Now if you were to take the Boltzmann approximation to this function assuming E_c minus $E_f \geq 3$ kT; so for E_c minus $E_f \geq 3$kT we will also use $E \geq 3$ kT.

This above function is approximated as this function because one can be neglected. So here if you write $f(E_c)$, T is equal to f_B (E_c, T) and this is the Boltzmann approximation. That means this term represents the fraction of states occupied for energy E_c within an interval dE at any temperature. If this is so and if this is the number of occupied states then it is easy to see that we could interpret N_c as the number of available states at E_c. This is only an effective description; it is not the actual description. It turns out that when you work with the actual description and you come up with the simple formula and if you want to physically interpret that simple formula in terms of some kind of an effective description then since this term represents the fraction of occupied states at E_c at any T under the Boltzmann approximation and n_i represents the number of occupied states, therefore it is evident that this can be taken to represent the number of available states at E_c. So this formula means: Although this is the actual picture where you have the states distributed over energy here, in a simple representation of this we could regard this picture as equivalent to something like this. This is an effective band picture.

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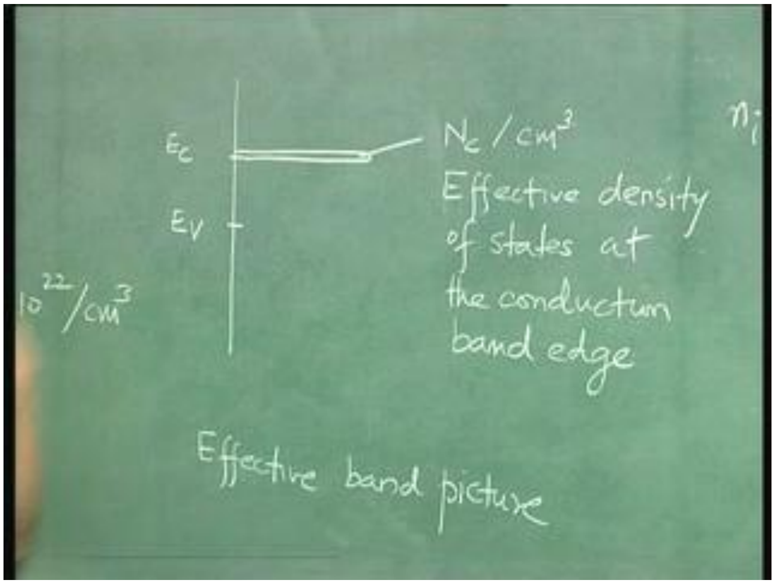

You have E_c here and you have $E_{v;}$ here and you have N_c states all present at E_c. This is the approximation and it is interesting. All the N_c states are as though they are present at E_c. So how do I represent all the N_c states at E_c? I should use the delta function it is like a delta function representation. The area under this function is N_c by cm cube available states at energy E_c so all are available at E_c, this is an effective picture. And then if you want to know what fraction of these electrons are present at E_c then all you have to do is multiply the states by the fraction given by the Boltzmann approximation. That is why N_c is called the effective density of states at the conduction band edge, That is a physical interpretation of that constant N_c.

We will see a similar picture for the valence band edge but before that let us look at one more point. We have said that this formula is derived under the Boltzmann approximation apart from the approximation such as the parabolic density of states approximation. Now how do we know that this approximation is valid? For this approximation to be valid E_f should be below E_c by at least 3(KT). We have said one way of checking the validity is to now derive a formula for E_f and then check whether indeed E_f is below 3 (K T). Before we do that we can also just do a simple logical reasoning to check if this will be true. So can we locate $E_{f\,-}$ the Fermi-level in an intrinsic semiconductor from simple reasoning?

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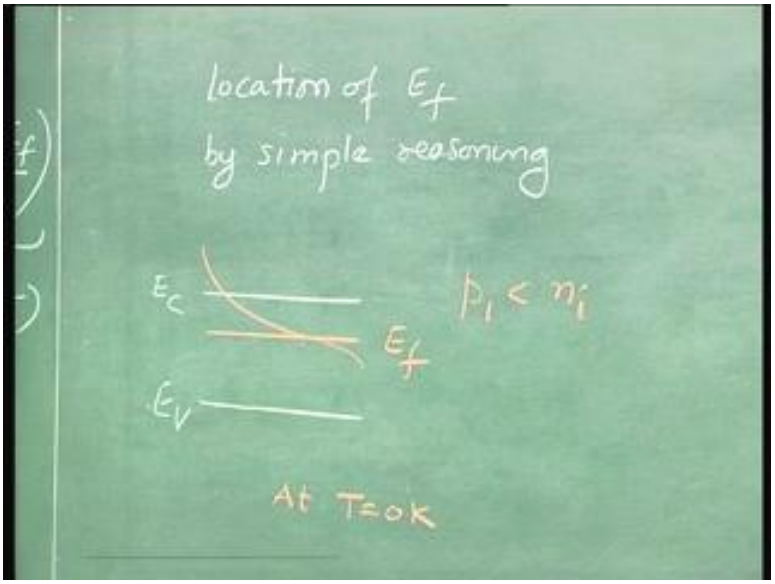

It turns out that we can do it as follows: Location of E_f by simple reasoning: look at the band picture it will show the E_c, E_v, and the energy gap. Let us start with the definition of Fermi-level at T is equal to 0.

The definition of Fermi-level is: it is that level above which no level is occupied at T is equal to 0 and below which all available states are occupied. It is clear that the E_f cannot be in the conduction band because if the E_f was here in the conduction band then we are talking at T is equal to 0 and then we will consider what happens for higher temperatures. At T is equal to 0, supposing E_f was in conduction band, then what it means is that below E_f whatever states are there which are allowed they must be occupied. It means I would have so many occupied states here. All the states which are available in this region above E_c and below E_f there are some states which are available and that is what our band picture says so they must be occupied. This means there must be conduction electrons at T is equal to 0 which is not true.

You know that at T is equal to 0 there are no free electrons therefore E_f cannot be in the conduction band. This means that E_f cannot be more than E_c, this is E_f.. Can E_f be in the valence band? If I assume the E_f to be in the valance band, then again by definition of Fermi-level at T is equal to 0 all states above E_f should be unoccupied which means all the available states which are in this energy range should not be occupied. But we know that at T is equal to 0 the entire valence band is occupied because there is no way the electrons can become free, so they must remain within the valance band. They are all bonded to the atoms in the bond model. So, in the energy band model they are all in the valence band. So again E_f cannot be within the valence band. It is clear that E_f should be somewhere between E_c and E. It has to be within the energy gap. Fermi-level is in the energy gap. This is clear from the simple analysis at T is equal to 0. Now let us go to higher temperatures. Where is the Fermi-level likely to be? Start from T is equal to 0.

Supposing E_f is away from the valence band but close to the conduction band within the energy gap, now at T greater than 0 if I have find out the Fermi-Dirac fraction based on this E_f, it would be something like this. Using this Fermi-Dirac fraction I can find out the number of electrons. Using the formula here I will take this fraction multiplied by available states, and then integrate and so on. This is what we did; I will get some number of electrons. All these electrons have to come from the valence band, which means the number of unoccupied states in the valence band should be equal to the number of occupied states in the conduction band because the electrons which have come to the conduction band for higher temperatures have come from the valence band.

Now it can be easily shown if E_f is close to E_c and away from E_v using the formula for the Fermi-Dirac fraction, the number of unoccupied states that you can calculate in the valence band will be less than the number of occupied states in the conduction band. In other words n_i will turn out be more than p_i.

Unoccupied states in the valence band indicate the whole concentration, we have already said that the whole concentration is equal to electron concentration. This has been translated into number of states, unoccupied states in the valence band are the holes; this should be equal to occupied states in the conduction band in a pure semiconductor at any temperature. If this two turn out to be not equal for a certain choice of E_f then that choice of E_f is not correct. Now how do we show that p_i is lesser than n_i for this condition? We can show that p_i is lesser than n_i for this condition for E_f is very close to the conduction band. The number of holes is given by p_i is equal to $\int N(E)$ dE into 1 minus f(E), integrated over the valence band. This is the formula for p_i from analogy to the formula for n_i. All that you have done is instead Fermi-Dirac fraction indicating the occupied states you are taking 1 minus the Fermi-Dirac fraction because that represents unoccupied states. The other part N(E) dE is the same as in the formula for n_i. This represents the number of available states in energy interval dE at energy E. So, if you integrate this you will get the unoccupied states in the valence band. Now let us look at the Fermi-Dirac function and 1 minus Fermi-Dirac function.

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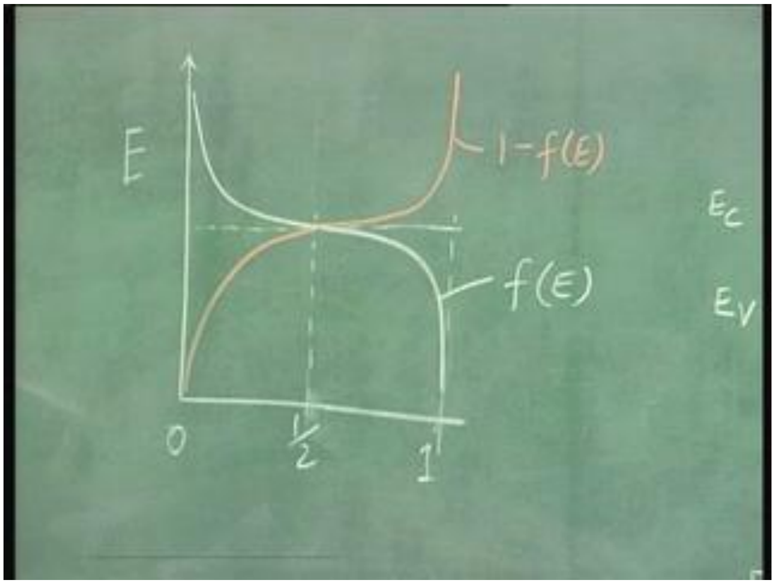

The Fermi-Dirac function is at any temperature T is something like this 0 1 and this is 1 by 2, this is E and this is f(E), 1 minus f(E) is a symmetric function given like this. It is the reflection of this function about this line. Now coming back to this diagram if I were to sketch 1 minus f(E) here then it would be something like this. So this is f(E) and this is 1 minus f(E). You can very clearly see that you put the density of states function, the parabolic density of states approximation, allowed states. Now you multiply f(E) into N(E) and you get this area by multiplying, then integrating, you can do the same exercise here. This is N(E) in a valence band, this is 1 minus $F_{e,}$ and that is the area which is the unoccupied states; that is N(E) dE into 1 minus F_e integrated over the valence band area.

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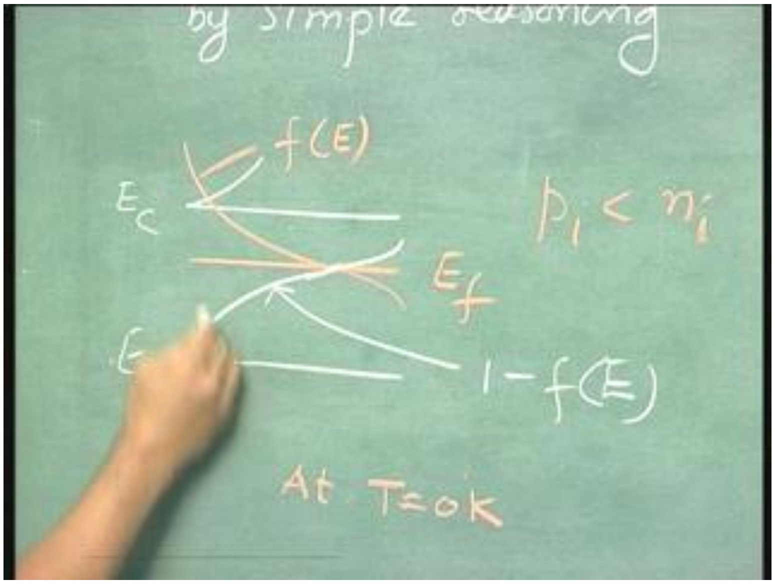

So it is very clear that if your E_f is close to the conduction band edge then this area Ni will be less than this area p_i. In fact if the parabolic density of states function for valence band and conduction band, if the shape of these functions are exactly identical which means in the parabolic density of states approximations the constant a, the parabolic density of states function is given by square root of E minus E_c for the conduction band. For the valance band it is square root of E_v, minus E. So the two constants in the density of states functions for the conduction valence bands are exactly same, then it is clear that this area p_i is equal to $n_{i.}$ if E_f is exactly in the middle of the energy gap.

From this diagram it is very clear that I will get p_i is equal to n_i if E_f is exactly in the middle of the energy gap provided the constants were the same in the parabolic functions. So they are slightly different because the effective masses of electrons and holes are slightly different. From here we can see that E_f will be very close to the middle of the energy gap for T greater than 0 in order that n_i is equal to p_i. If E_f is very close to the middle of the energy gap then what is the distance between E_f and E_c? The energy gap in silicon is 1.1 electron volt. Half of that would be 0.55. The values can be 1.1 depending on from where we have got the data.

Let us say it is 1.1 to first decimal place, then it is 0.55 electron volt; that is E_c minus E_f. Now what is kT at room temperature? It is 0.26 electron volts so 0.26(3kT) is 0.078 electron volts. If you compare 0.55 electron volt and 0.078 electron volts you will always find that E_c minus E_f in a pure semiconductor like silicon and in many other semiconductors will always be more than 3kT at any temperature. Therefore Boltzmann approximation is valid. The effective density of states picture where you assume that there are N_c states available at E_c that is the picture of the conduction band. This shows the effectiveness of the density of states and therefore it is quite valid.

The Boltzmann approximation is valid and this formula n_i is therefore fairly accurate for all practical purposes. We can write down a similar formula for p_i and get an exact expression for E_f. Now we have placed E_f by logical reasoning to be close to middle of the energy gap; it is not exactly at the middle of the energy gap because the effective mass of electrons and holes is not the same. Or the parabolic density of states functions for conduction band and valence band are slightly different their; constants are slightly different.

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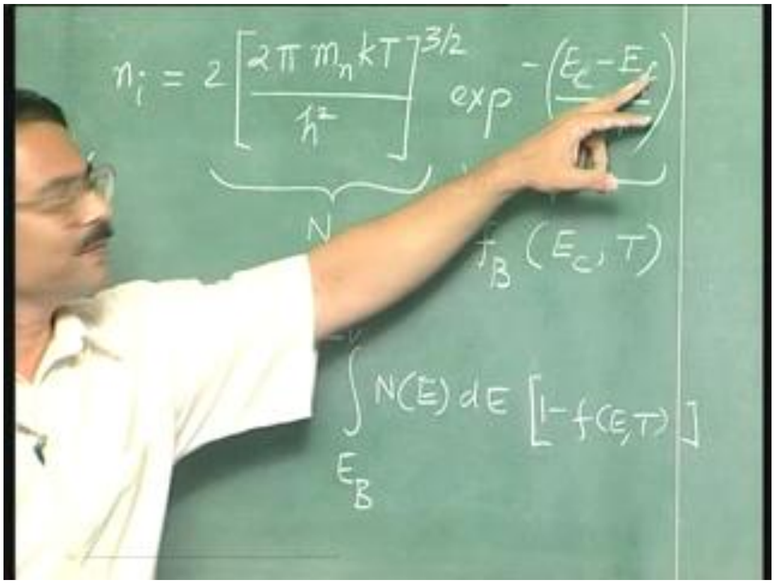

So we need to exactly find out what is E_f. Once you eliminate E_f from this equation then you have a formula for n_i because right now n_i is still a function of E_f and unless you know what is E_f, you cannot know n_i. We want to calculate n_i at any temperature. So, based on whatever discussion we have had now we can write down an expression for p_i. We have p_i written here as equal to integration over the valence band of the function N(E) dE [1 minus f(E,T)]. We use Boltzmann approximation to the Fermi-Dirac function and then use parabolic density of states approximation and you replace the bottom of the valence band by minus infinity. And then your result will be exactly analogous to this. So you can write down the result for p_i simply from the result of n_i.

All you have to do is replace m_n by m_p. Replace E_c by E_v and also you interchange E_f and E_v. So instead of E_v minus E_f it should be E_f minus E_v because this term should be positive. And the Fermi-level is usually below the conduction band edge but is above the valence band edge. So the formula for p_i should be the same as the formula for n_i with the replacements m_n replaced by m_p and E_c minus E_f replaced by E_f minus E_v. Therefore p_i is equal to 2(2pi m_p kT by x square) whole cube by 2 into exponential to the power minus (E_f minus E_v by kT). If you have some difficulty in trying to remember in this formula for n_i E_c minus E_f or E_f minus E_c, you must know the physical interpretation of the formula. That is why we have taken lot of time to interpret this formula. Once you know the physical interpretation this problem is eliminated because you know that this represents a Boltzmann approximation to the Fermi-Dirac fraction. It is a fraction, it should be lesser than 1 there is no doubt. In fact it will be much less than 1. That is why the negative sign is kept outside and the term inside the bracket should be positive.

Once you know this you can remember this formula whether it is E_c minus E_f or E_f minus E_c and similarly for p_i. Now we can eliminate the E_f between these two results by n_i into p_i. When you multiply the E_f will get canceled and we will get the result n_ip_i is equal to

4[2pi kT by h square] whole cube into $(m_n m_p)$ whole cube by 2 into exponential minus $[E_c$ minus E_v by kT].

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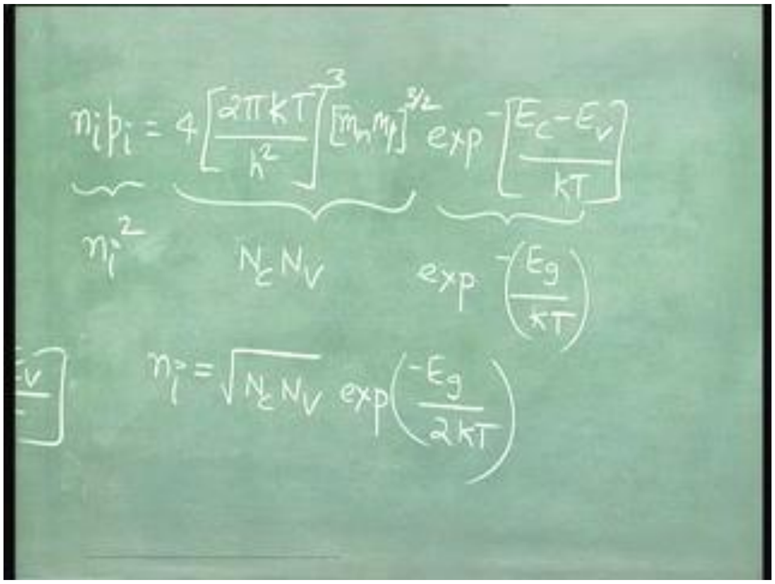

$$\underbrace{n_i p_i}_{n_i^2} = \underbrace{4\left[\frac{2\pi kT}{h^2}\right]^3 [m_n m_p]^{3/2}}_{N_c N_v} \underbrace{\exp -\left[\frac{E_c - E_v}{kT}\right]}_{\exp -\left(\frac{E_g}{kT}\right)}$$

$$n_i = \sqrt{N_c N_v}\,\exp\left(\frac{-E_g}{2kT}\right)$$

The E_f has got eliminated. If we were to write it using the symbol that we have used namely the effective density of states in the conduction band is given by N_c. Now you use an analogous representation there will be an effective density of states for the valance band that will be given by N_v. Why N_c is different from N_v because m_n is different from m_p, that is the only reason. We can write this term as N_c into N_v and recognize the fact that E_c minus E_v is nothing but the energy gap. Therefore this quantity can be written as exponential (minus E_c minus E_v by kT). Further I recognize that this term n_i into p_i since n_i is equal to p_i in a pure semiconductor or intrinsic semiconductor. So I can write this as n_i square or p_i square. The convention is n_i. Now I get the relation n_i square is equal to n_c n_v into exponential (minus E_c minus E_v by kT). Therefore I can write the formula as n_i is equal to square root of $n_c n_v$ into exponential of minus EG (2kT). That is the result for the intrinsic concentration as a function of temperature and the physical parameters such as energy gap and effective mass of electrons and holes. This result is what we have shown earlier.

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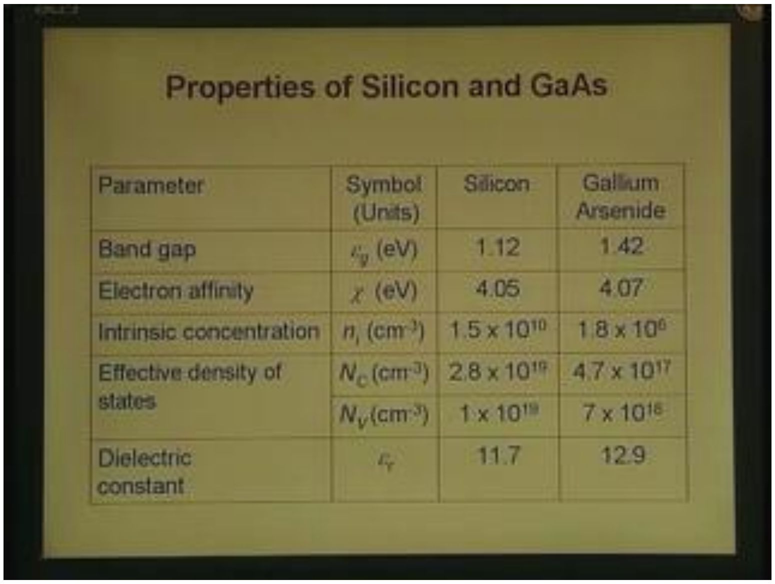

Properties of Silicon and GaAs

Parameter	Symbol (Units)	Silicon	Gallium Arsenide
Band gap	E_g (eV)	1.12	1.42
Electron affinity	χ (eV)	4.05	4.07
Intrinsic concentration	n_i (cm^{-3})	1.5×10^{10}	1.8×10^{6}
Effective density of states	N_C (cm^{-3})	2.8×10^{19}	4.7×10^{17}
	N_V (cm^{-3})	1×10^{19}	7×10^{18}
Dielectric constant	ε_r	11.7	12.9

Before I show the graph let me summarize the quantities present in the energy gap, energy band diagram, or energy band model of silicon. The parameters you need to know is the band gap or energy gap E_g for silicon 1.2 electron volts; for gallium arsenide 1.42 electron volts. The other important parameter is electron affinity which is the difference between the vacuum level and the conduction band edge E_c so E_0 minus E_c that is 4.5 volts for silicon and 4.7 for gallium arsenide so different semiconductors have very close electrons affinity values. Then the intrinsic concentration $n_{i,}$ if you substitute in the formula you will get these values. Now it turns out that if you substitute the effective mass values, the energy gap values and so on you do not get exactly this number. The number given here is the value that is obtained from measurements. So in practice instead of 1.5 you may get a value of 1.1 or 1 etc.

The next slide will give you the effective mass of electrons and holes. You can use those values and you can use this energy gap that is given here and try to estimate the concentration. This kind of problem arises because you are independently measuring these parameters. Energy gap is being measured by some particular approach. The effective mass of electrons and holes are being measured by some other approach. When you want to put it all together some inconsistency will be present.

The normal practice is, whichever parameter you want to use you use its experimental value; but the formula is very important because now what you can do is use this value of n_i for room temperature. These values that are being shown are valid at room temperature, 300k. So if it is not indicated you must take this value as 300k as the temperature. You can use these values of n_i at T is equal to 300k and then you can use the formula to find out the value at any other temperature. That is the way you must calculate the concentration at any other temperature. That means for example, here n_i is given by this, if you know n_i at T is equal to 300k then it means you know N_c and N_v at 300k. You must write an expression for n_i as a function of temperature, you can see what kind of expression you get here.

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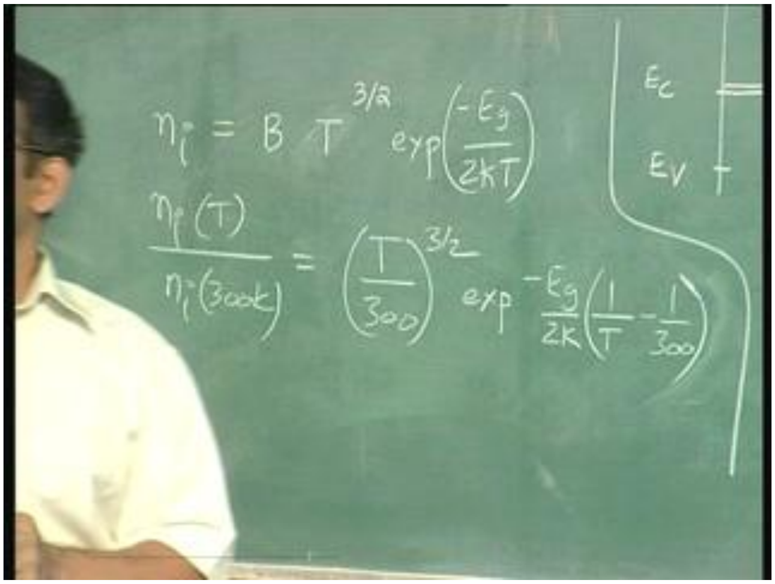

$$n_i = B\,T^{3/2} \exp\left(\frac{-E_g}{2kT}\right)$$

$$\frac{n_i(T)}{n_i(300k)} = \left(\frac{T}{300}\right)^{3/2} \exp \frac{-E_g}{2k}\left(\frac{1}{T} - \frac{1}{300}\right)$$

You will need an expression of the form n_i is given by B into T cube by 2 into exponential of (minus Eg by 2kT) where the constant B accommodates the effective mass of holes, electrons etc. Now you know the value at 300k; so you know n_i 300k, use this value to get the constant B which is independent of temperature, and then use that constant and you can find n_i at any other temperature. So, in terms of n_i is equal to 300k if I want to write down n_i T by n_i 300k will be given by (T by 300) whole cube by 2; this B will cancel because it is independent of temperature into exponential of minus Eg by 2k into (1 by T minus 1 by 300). This is how you can estimate n_i at any other temperature. Therefore one has to follow this approach for results which are close to experimental values.

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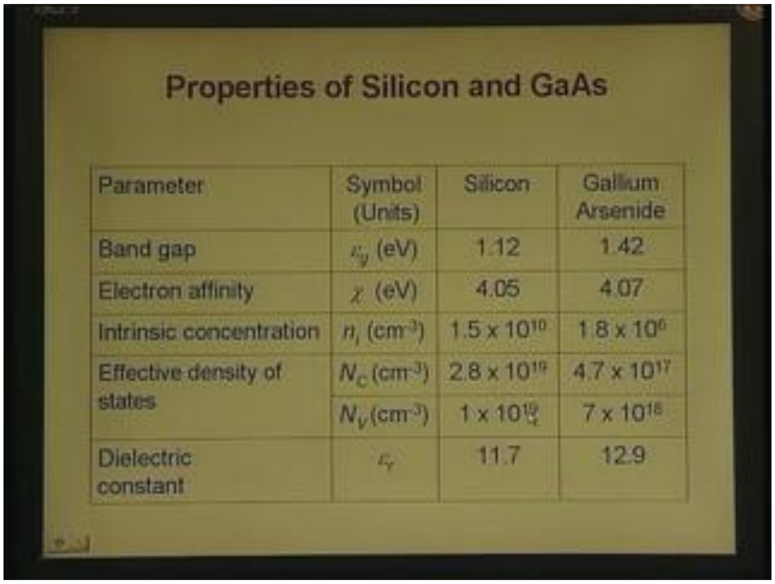

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	N_V (cm^{-3})	1×10^{19}	7×10^{18}
Dielectric constant	ε_r	11.7	12.9

Now let us look at the other parameter such as effective density of states. At the conduction band edge it is N_c these values are given here: for silicon 2.8(10 to the power 19). So order of effective density of states is 10 to the power 19 in silicon. N_v is slightly different which is 1 into 10 to the power 19 by cm cube. It is also important to remember the units. The values for gallium arsenide are given here .So what you find is, the difference between silicon and gallium arsenide is in silicon N_c is greater than N_v but in gallium arsenide N_c is lesser than N_v. All these depend on the effective mass of electrons and holes. So, gallium arsenide has a much lower effective mass of electrons than silicon. Then you have a dielectric constant given here ε_r.

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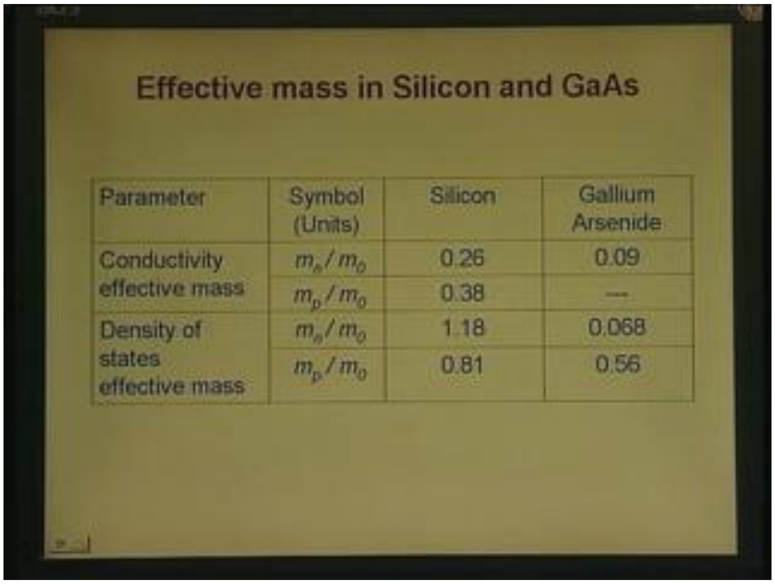

Effective mass in Silicon and GaAs

Parameter	Symbol (Units)	Silicon	Gallium Arsenide
Conductivity effective mass	m_n/m_0	0.26	0.09
	m_p/m_0	0.38	---
Density of states effective mass	m_n/m_0	1.18	0.068
	m_p/m_0	0.81	0.56

This slide now shows you the effective masses. Again these are values at T is equal to 300k. What is important to note here is you have two types of effective masses: conductivity effective mass and density of states effective mass. That is, the effective mass used varies with the physical situation. This is because it is an average value. Now the average is not necessarily the same in different situations. This is not very difficult to understand. If you want to calculate the average of something it depends on the situation. How you average is also an important thing. It is a weighted quantity. It is a weighted average.

Specifically what is important to see here is, if I take the density of states effective mass which is the value that should be used if you want to estimate N_c or N_v effective density of states, that is where this is to be used. Then silicon, the electron is heavier than the hole. Whereas if you take the conductivity effective mass, conductivity situation for estimating the resistivity and so on, you want to find out mobility, later on we will see how this effective mass is useful to find mobility as well.

For example, that is the movement of the carrier in response to an electric field. In that context this effective mass of electron is less than that of holes. So electron is lighter than the hole for conductivity situation. Intuitively this is more satisfying. It is easy to understand because the movement of the hole is actually movement of bound electrons and therefore that movement is going to be slow. So in density of states effective mass you should not invoke the concept of movement. It is some different average used to represent the quantum mechanical situation. The only thing you need to remember is that the effective mass depends on the situation so you must use appropriate effective mass. The values are shown here for gallium arsenide. This value here is not shown because there is no agreement for the exact value. That is all what it means; not that there is no hole effective mass for gallium arsenide under conductivity situation.

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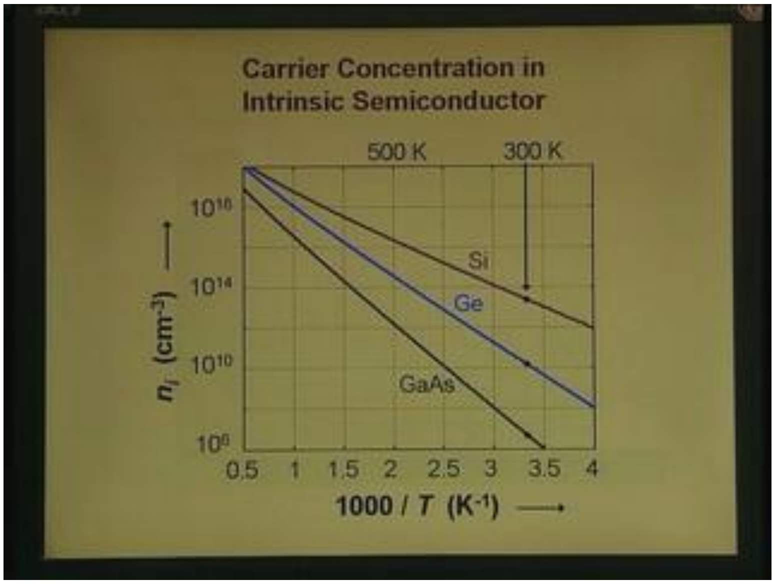

Carrier Concentration in Intrinsic Semiconductor

Next, we come back to the behavior that we want to explain using our formula, n_i versus temperature. So Log of n_i versus reciprocal of temperature, it is a straight line, now this is very clear from our formula. So n_i is equal to square root of N_c N_v into exponential (minus Eg by 2kT). n_i is equal to B into T cube by 2 into exponential (minus Eg by 2kT). So log of n_i is equal to log of B plus 3 by 2 log T minus Eg by 2k into (1 by T). Now this log of T is dependent on temperature but this variation with temperature is very small because it is logarithmic dependence on temperature whereas this particular term which depends on 1 by T varies much more rapidly than this term. And that is why what you have is log of n_i versus, if you plot 1 by T, so log of n_i versus this i by T term would be a straight line. This effect of log of T you will not see here in your line. That is why it almost looks like a straight line. Even though strictly speaking this term is present, its effect is not so much, it does not show up in the variation and this also shows that the slope of the straight line is Eg by 2k. The slope depends on the energy gap and Boltzmann constant k.

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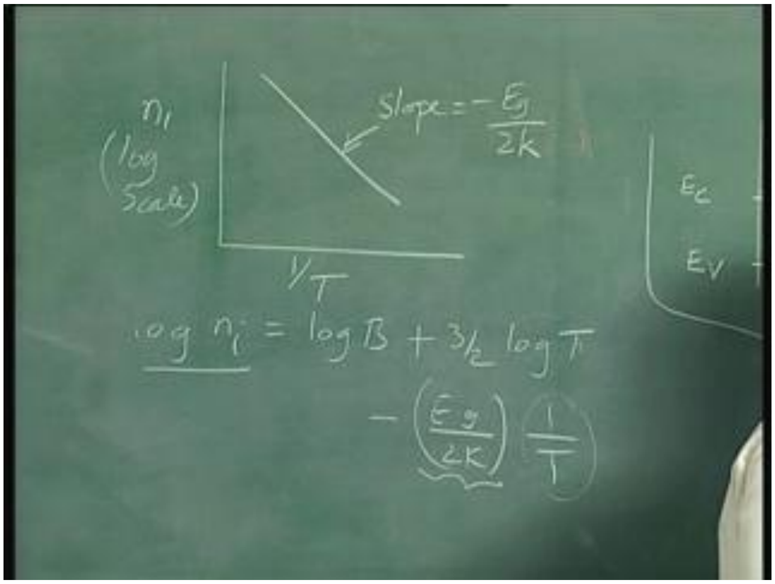

So n_i on a log scale and 1 by T the slope is equal to minus Eg by 2. You can use the slope to get the energy gap of the semiconductor; this is the approach that is used to get the energy gap form the measured concentration. Now, we have got a model for the behavior. The final step is, we will find write down the formula for E_f. So from n_i and p_i I can get E_f. I just have to equate n_i to p_i. I will write down the result is E_c plus E_v by 2, that is middle of the energy gap, minus KT into ln square root of(N_c by N_v) that is the location of E_f. So now you know that N_c and N_v are not very different particularly if you are going to take the log of this quantity, this quantity is not going to be really significant. So it is almost E_c plus E_v by 2. And this quantity here this term depends on temperature. But again this dependence on temperature is negligible because this quantity is again small.

If you take silicon for example the N_c value was 2.8 and N_v is 1. So square root of 2.8 and you take log of that it is really going to be very small. So E_f is approximately equal to E_c

plus E_v by 2. With this we have completed the discussion on intrinsic carrier concentration of the semiconductor. Next we will take up the model of the extrinsic semiconductor or the dope semiconductor and try to explain the carrier concentration versus temperature behavior. So let us see if there are any questions now.

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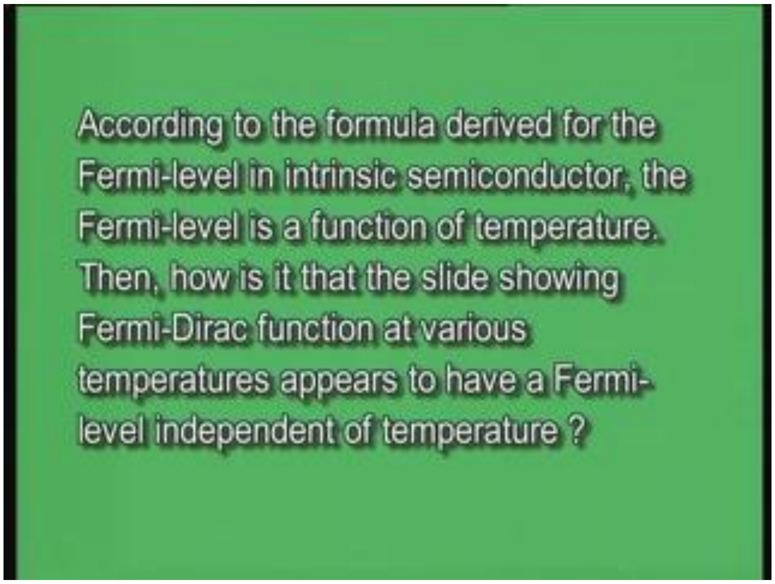

According to the formula derived for the Fermi-level in intrinsic semiconductor, the Fermi-level is a function of temperature. Then, how is it that the slide showing Fermi-Dirac function at various temperatures appears to have a Fermi-level independent of temperature ?

Sir, according to the formula derived for the Fermi-level in intrinsic semiconductor, the Fermi-level is a function of temperature. Then, how is it that the slide showing Fermi-Dirac function at various temperatures appears to have a Fermi-level independent of temperature?

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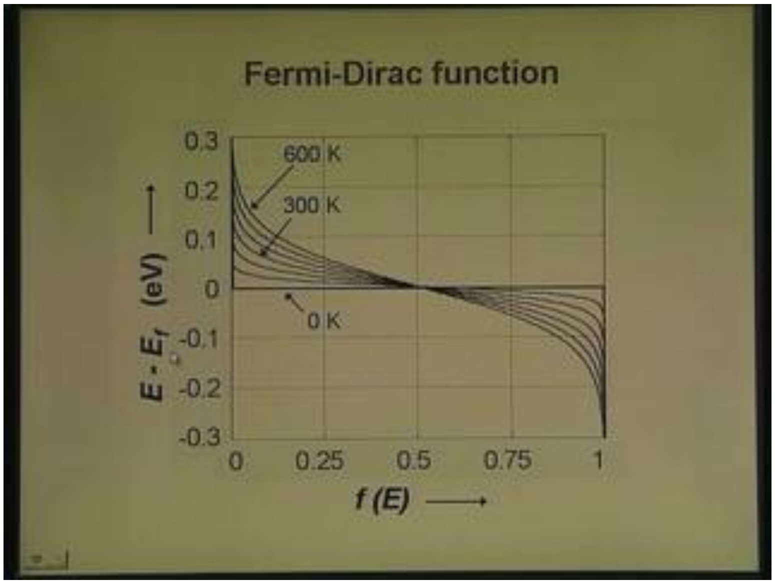

Let us look at the slide. If you note carefully the y axis is E minus E_f and not E. We are plotting the energy as with reference to the Fermi-level and not the absolute energy. So when you take E minus E_f on the y axis then any variations in E_f need not be shown on the graph .If however you plot the function as shown on this diagram where we are plotting the electronic energy on the y axis, and the Fermi-Dirac function as a function of the electronic energy. Here the picture would be something like this variation of Fermi-level with temperature will be seen. Now, this is the line which represents the Fermi-Dirac function for the temperature T_1. For a higher temperature T_2 if I were to show the function it will be shown by this line.

Notice that the Fermi-level for T_2 which is here, this is the line showing the Fermi-level for T_2 is lower than the line shown for Fermi-level corresponding to T_1 which is this one. So E_{f2} is below E_{f1} for T_2 greater than T_1. This is because according to the formula we derived here that is E_f is equal to E_c plus E_v by 2 minus k T into ln of square root of N_c by N_v we can see that as temperature increases because of this term the Fermi-level moves down. So when you plot the Fermi-Dirac function as a function of absolute energy the variation in Fermi-level with temperature will be seen whereas if you plot the Fermi-Dirac function with respect to E minus E_f as shown in this slide, in that case the variation in Fermi-level with temperature will not be seen.

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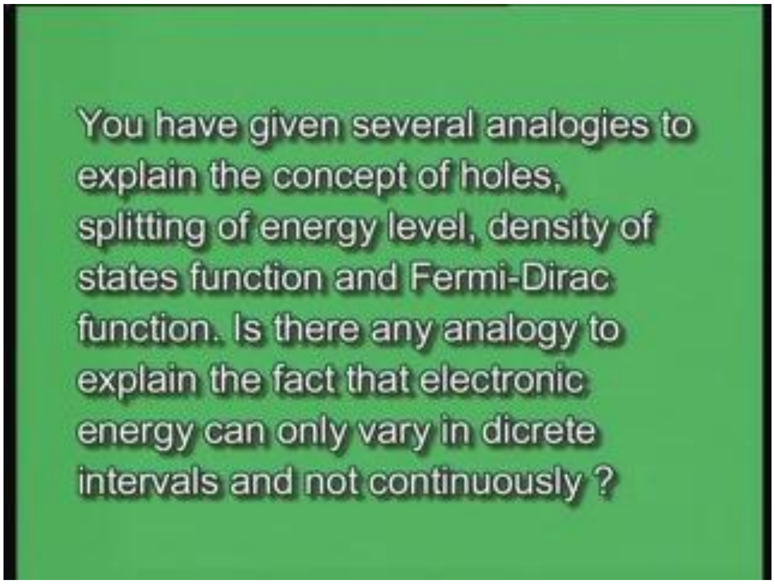

You have given several analogies to explain the concept of holes, splitting of energy level, density of states function and Fermi-Dirac function. Is there any analogy to explain the fact that electronic energy can only vary in dicrete intervals and not continuously ?

Sir, you have given several analogies to explain the concept of holes, splitting of energy levels, density of states function and Fermi-Dirac function. Is there any analogy to explain the fact that electronic energy can only vary in discrete intervals and not continuously? Yes, there are several analogies to explain this fact. A simple analogy is as follows:

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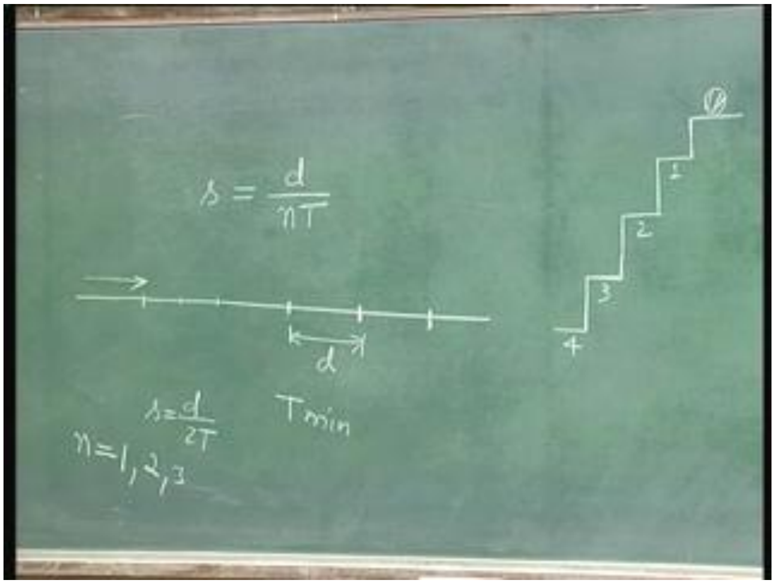

Suppose there is a stair case like this and there is a ball. Now the question is, at what heights can the ball exist on this stair case? So you know that the ball can either exist at this particular stair case that is this region or this or this or this region. It cannot exist, for example, in these vertical portions of the stair case, you cannot have a ball existing somewhere here, it cannot rest there. So, you can see here that the ball can only exist at discrete heights. There is another example which is much more interesting. Suppose there is a street, and this street has signals at regular intervals; let us say the distance between two consecutive signals is d; the signals turn green at interval of T minutes.

Imagine that there is a car which wants to pass through this street uninterrupted even when you have the signals turning red or green. What are the speeds in which the car can move so that it will not be interrupted by the signals? This is the question. Now it can be shown that the speed of the car is given by d by nT where n is an integer. Let us see why this is so? The speed of the car can only vary in discrete steps. So the car is only allowed in these particular speeds. Let us take n is equal to 1. What does it indicate? Now the car is moving that the signal here as turned green, it has passed this signal. After a time T minutes it is right here when this signal is turning green. So it has crossed a distance d in time T. Since the signal has just turned green it can move uninterrupted and go to the next signal and this will continue. So you see the speed of d by T that is, at n is equal to 1 the car moves uninterrupted.

Now another speed that is allowed is n is equal to 2 that is, the speed is equal to d by 2t. Now you see what is happening in this case? The speed is half of the speed for n is equal to 1. That is, the car has passed this signal when it is in green, and next time the signals turns green the car is exactly half way here. When it comes here the signal turns green for the second time, and then again the car moves uninterrupted. This is the way the car can continue moving. It can be shown in a similar manner that for n is equal to 3 also the car will move uninterrupted. So you are allowed speeds given by d by t, d by 2t, d by 3t and so on. So the car’s speed can only change in discrete intervals, discrete steps in this particular situation. Like this there are many situations where the parameters can only assume discrete values. With this we have come to the end of the discussion on intrinsic semiconductors.