Solid State Devices
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Lecture - 40
MOS Field Effect Transistor (Contd…)

We have been discussing the characteristics of MOS Field Effect Transistor.

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In the previous lecture we started deriving the current voltage relations in terms of the charge conditions in the device. Let us summarize what we achieved in the previous class.

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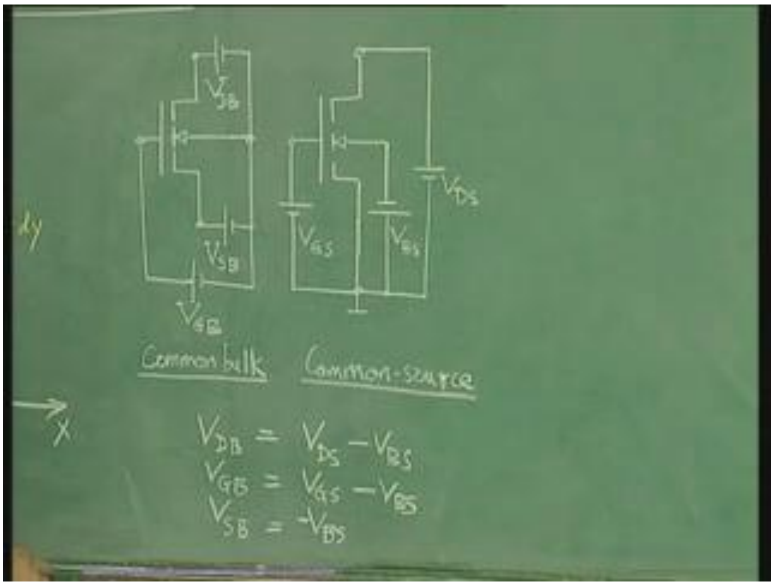

What we showed is that it is easier to start the analysis of the MOSFET in the common bulk condition where the voltages are applied to the source to the drain and to the gate with respect to the bulk. In such a case if you look at the device structure shown here the charge conditions in the device can be readily expressed using the MOS capacitor equations that we derived in one of the earlier classes.

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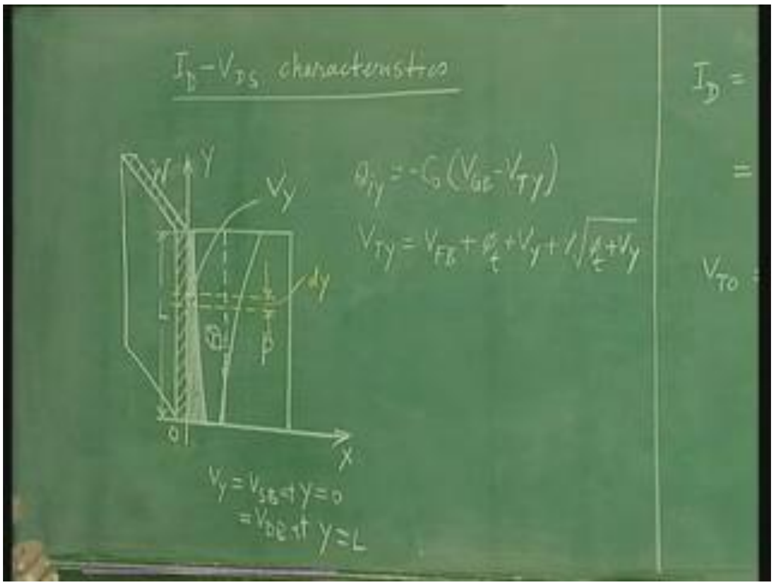

So the analysis is very simple when you look at it from the common bulk point of view. However, in practise the device is used in the common source mode, the analysis in the common bulk mode can be readily converted into an analysis in the common source mode as follows: This is the common source configuration where all the voltages have been shown applied with respect to the source. The conversion from common bulk to common source mode is achieved by substituting for the various voltages of the common bulk mode by the voltages of the common source mode as follows: So we will obtain the current in the common bulk mode as a function of drain to bulk voltage gate to bulk voltage and source to bulk voltage and then we will substitute the right hand side of these equations in these voltage terms.

So we will replace V_{DB} by V_{DS} minus V_{BS} V_{GB} by V_{GS} minus V_{BS} and V_{SB} by minus V_{BS}. When we do that you will get the equations for characteristics in the common source point. Further we also explained the approximations to the five basic equations which will help us to derive these characteristics. The result of the approximations was that, in an n-channel device the current is carried predominantly by electrons and then above threshold which is when we are considering the device the current is carried mostly by drift. That is as far as the transport equation is concerned. The continuity equation on the other hand says that the current through the channel inversion layer is constant as a function of y.

If you see the diagram here the current here along this channel, the current is flowing from drain to source that is in downward direction and that current will be constant even though the channel charge is going on changing. That is the statement of the continuity equation. Then we said, as far as Gauss's law is concerned the two dimensional picture of the electric field distribution is approximated by a one dimensional distribution by neglecting the variation of the y directed electric field in the y direction. So change in y directed electric field is neglected when you are trying to find out the amount of charge controlled by the gate voltage.

You cannot neglect the effect of E_y for determining the current because this E_y that is contributing to the current that is drain to source voltage. The drain to source directed electric field is contributing to the current so that field has to be taken into account in current calculation. But what we are saying is that when you want to find the amount of charge that is carrying the current and since this charge is predominantly controlled by the gate to bulk voltage that is the field in the x direction therefore we can find out the charge based on the information about the x directed electric field neglecting the y directed electric field. So the y directed electric field effect is neglected only while calculating the charge that is contributing to the current.

By using this charge and the y directed electric field we will calculate the current. We have said that this approach is called the gradual channel approximation because it means that the charge in the channel is changing very gradually from source to drain. The charge or the conditions in the channel are changing very gradually from source to drain. So, the y directed variations are therefore neglected while estimating the charge.

Now let us proceed from this point and show how we can derive the current voltage relation. The first step is to separate this device into small elements. Each small element is of length dy in the L direction or the y direction and then let us look at the charge at this y. So we will separate out this element and analyze it. So let us draw this small element in an expanded fashion.

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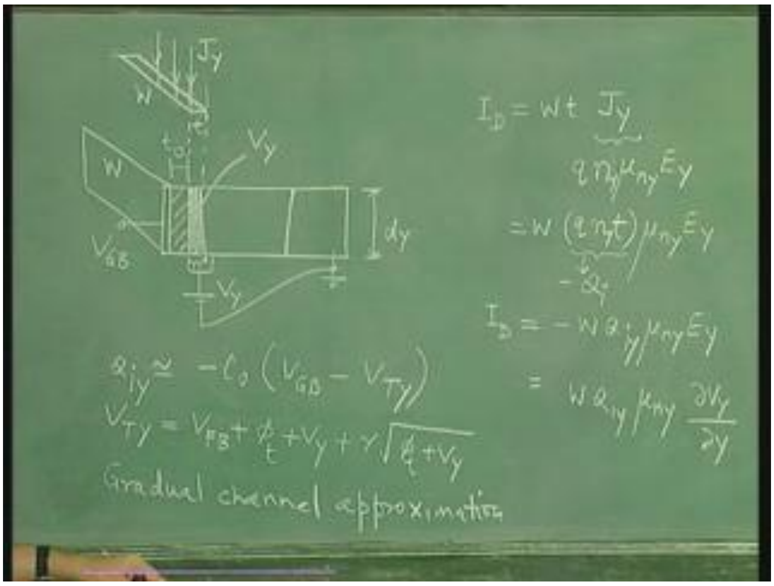

This is the inversion charge, this is dy and this is the depletion layer. Please note that the variation in depletion layer over dy length is very small in the y direction. So we can assume that both the charges namely the inversion charge as well as the depletion charge is uniform over the width dy. This is gate oxide and here you have the gate. Now the voltage in the channel here is V_y. Since we are applying all voltages with respect to the bulk the bulk is considered to be the common or the ground and the voltage here is V gate bulk.

Now the voltage here is V_y means that this particular element is equivalent to a MOS capacitor with a source here reverse biased by the value V_y with respect to the bulk. That is the meaning of this element having a voltage V_y. So, to know the amount of inversion charge here in this region we can invoke the analysis of the MOS capacitor in the presence of the body effect. So we can write down an expression for the inversion charge as follows: Q_i is equal to C_0 which is the capacitance associated with the oxide layer of thickness t_0 into V_{GB} that is this voltage minus the threshold voltage with a negative sign because inversion charge is made of electrons and it is negative. So the equation is Q_i is equal to minus $C_0(V_{GB}$ minus $V_T)$

Now please note this threshold voltage incorporates the effect of this V_Y. And since this V_Y is changing along the channel from the source to the drain as shown here the threshold voltage is changing from source to drain. That is why we will put a suffix y to the threshold voltage term. Now we can write the expression for V_{TY} by recalling our results from the MOS capacitor with body effect. So V_{TY} in an Ideal MOS capacitor including body effect is given by phi_t plus V_Y which is the potential drop in the depletion layer here that is the potential drop in the silicon plus the potential drop in the oxide that is gamma into square root of phi_t plus V_Y. Then we include the effect of the flat band voltage in a real capacitor and we obtain this V_{TY}. Therefore the equation goes like this: V_{TY} is equal to V_{FB} plus phi_t plus V_Y plus gamma into square root of phi_t plus V_Y. This is what we must substitute here to get the charge Q_i.

Now you see that Q_i is available in terms of the gate to bulk voltage and the voltage V_Y in the channel at y. Now once this is known we can the expression for the current. How do we write the expression for the current? We know that the current following here in the channel from drain to source in this element dy is the current I_D because by continuity equation the terminal current is the same as the current throughout the channel.

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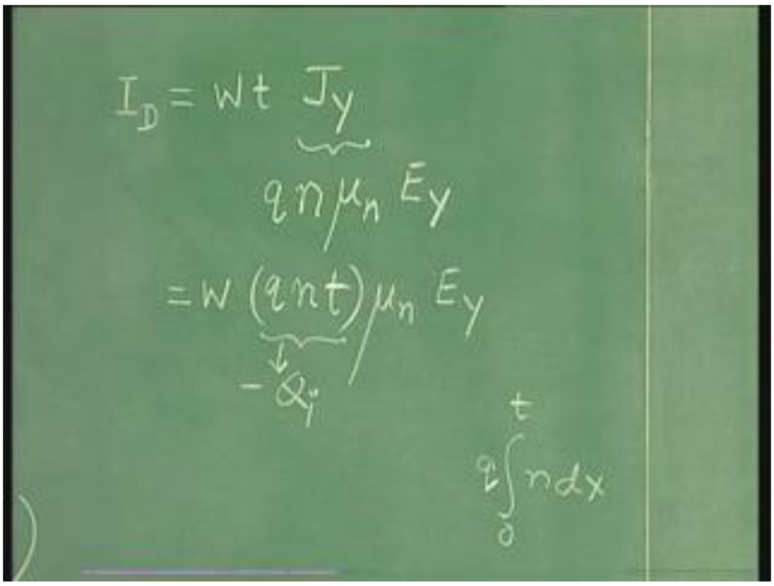

So we can write I_D which is the current as the drift current due to the charge here. That we can write as the area of cross section of this channel multiplied by the current density. Now, what is the area of cross section of this channel? The area of cross section means the thickness of this particular inversion charge layer multiplied by the width W which is in this direction. That is the area of cross section of this channel.

Let us draw it separately to show this, it is something like this where this is W and this is the thickness of the inversion layer that we will denote as t. Please note that this t is different from this t_0 which is the thickness of the oxide. Ultimately we will not need this t, we will show how we can avoid using this t. But this is the area of cross section of the channel and this current is flowing through this area and this current density is J_Y.

For simplicity we will assume that the inversion layer charge is uniformly distributed over its thickness. Actually this is not the case the inversion charge varies rapidly with distance. For example, if you see here inversion charge will be maximum here it will go on decaying. In fact by the end of this layer it will go to almost 0. But then to derive a formula we will start with a simple picture and we will show that you can extend the same simple picture to take into account the variation of inversion charge. So if you assume the charge to be uniformly distributed over the thickness t then we can write the expression for current as follows. So I_D is given by the area of cross section of flow that is W into t that is this cross sectional area multiplied by J_Y which is the current density y.

Now this J_Y we can write as q into n which is the inversion layer charge concentration per unit volume, the electron concentration per unit volume in this inversion layer. We are assuming it to be uniform for simplicity. And then it is multiplied by the mobility of inversion layer electrons into the electric field E_Y here. So the electric field is in this y direction. So the expression is: I_D is equal to W_t J_Y which is qn mu_{ny} E_Y. Now what we will do is we will club some terms as follows: We will write this as W into q n t so this t and this n will club into mu_{ny} into E_Y this is the current. So this is as follows: is equal to W(q n t) mu_{ny} E_Y.

Now notice this particular term q n t what is the dimension of this term? You can see that this is charge concentration per unit area. What is a unit area? Unit area is the area of this gate that is this area, this is the gate. This is nothing but the inversion charge with a negative sign because inversion charge is negative and this quantity is positive so you are putting a negative sign. So this quantity is Q_i the inversion layer concentration of electrons multiplied by the thickness of the inversion layer is actually Q_i. So this is the sheet concentration of the charge.

Now, if n is not uniform over t, that is, if the inversion layer concentration is not uniform from the interface inside all that happens is that this n into t here is replaced by an integral n into dx over the thickness t as something like this: integral n dx from 0 to t in x direction. And still this quantity multiplied by q will become the inversion layer charge Q_i.

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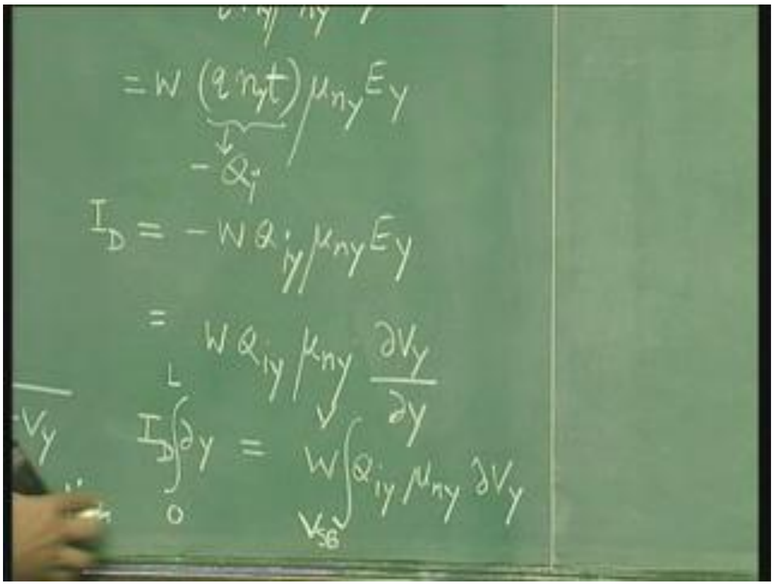

Now that makes our job very easy so we are writing I_D is equal to minus W Q_i mu_{ny} E_Y. Now here we should note that the inversion layer charge is also a function of y because if you look at this picture the inversion layer charge changes with y. Also, the electric field both in the x direction and in the y direction is a function of y. Therefore the mobility which is the function of the electric field both in the x direction and in the y direction is also a function of y. Therefore in this expression here we must put a suffix y both in the inversion layer charge as well as to the mobility.

So this n is actually ny. This is our expression where this Q_{iY} has been written in terms of the various voltages in this formula here. Here we will again put a suffix y to indicate that Q_i is a function of y. Now please note that this expression which has been written assumes a one dimension electric field only in x direction and neglects the y directed electric field variation, this is the result of the Gauss's law. So this is the result of the gradual channel approximation. Therefore strictly speaking we must put the approximate sign here.

Now, for this we have written down this particular result I_D is equal to minus W Q_i mu_{ny} E_Y and it contains the transport equation which says that the current is mainly because of drift as said here qn mu_{ny} E_Y and continuity equation because this current I_D is equal to W_t J_Y which is qn mu_{ny} E_Y is constant as a function of y in the channel and therefore is equal to the terminal current. This is how this equation I_D is equal to minus W Q_i mu_{ny} E_Y together with the equation here V_{TY} is equal to V_{FB} plus phi_t plus V_Y plus gamma into square root of phi_t plus V_Y we have used all the five transport equations and there approximations. Now, how do we proceed from here and find out the current? Please note, E_y, Q_{iY} and everything else is changing with y. So what we do is, we separate the variables but before that we replace E_Y as gradient of the voltage V_Y. So the same thing we write as; W Q_{iY} mu_{ny} E_Y is equal to dowV_Y by dowY with a negative sign and that negative sign and this negative sign we will cancel and put it as a positive sign here.

Now we note carefully that Q_{iY} is a function of V_Y. Therefore in this equation we can now separate the variables, we can shift dy this side because Q_{iY} is known as a function of V_Y then we can do the integration. Now what will we do with mu_{ny}? We will see that shortly. So now separating the variables we write I_D dowY is equal to W Q_{iY} mu_{ny} dow V_Y. Now we can integrate both of these left hand side and right hand side. The integration limit will be from y is equal to 0 to y is equal to L. Let us look at this in the context of our picture of the device. y is equal to 0 is this and y is equal to L is this, this is the limit we are integrating from here to here. When we do that the channel voltage V_Y this V_Y at this point it is equal to V_{SB} so V_Y is equal to V_{SB} and y is equal to 0 and it is equal to V_{DB} at y is equal to L. So we substitute this here so when y is equal to 0 it is V_{SB} and when y is equal to L it is V_{DB}. Since this is not clear here we will rewrite it here to the right.

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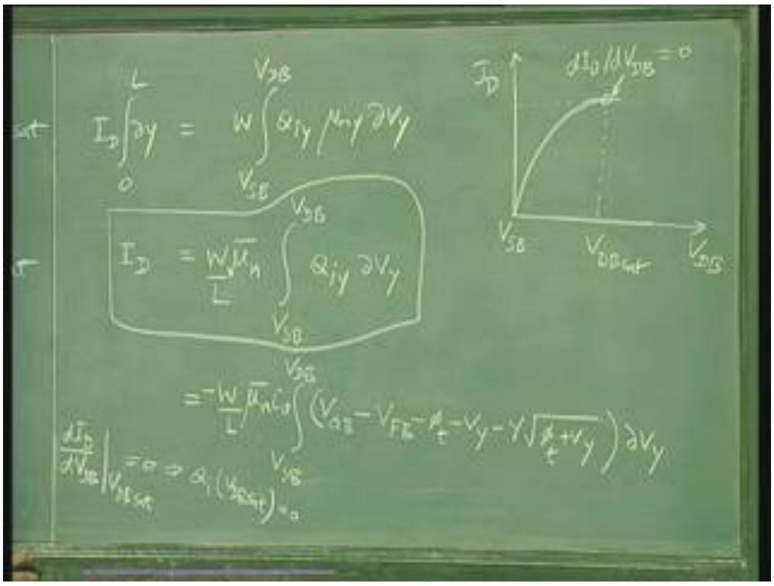

Here is V_{SB} to V_{DB}. Now mu_{ny} is not very simply known in terms of dV_Y or V_Y, mu_{ny} is not known simply in terms of V_Y. Now we can write an expression for mu_{ny} but for that we need to consider the various effects on the mobility of x directed electric field and y directed electric field. So a simple equation is obtained by using an average value of this mobility. Though mobility varies with y we use an average value and so we shift this out as follows: You write an average value of mu_n over the channel and then here you have V_{SB} to V_{DB} Q_{iY} dowV_Y. Now left hand side if you integrate since I_D is constant with y you simply get I_D into L. Now we can shift that L here in the denominator and we will have this. So this is your expression for the drain current: I_D is equal to average value of W by L mu_n V_{SB} to V_{DB} Q_{iY} dowV_Y.

Now in this we have to substitute Q_{iY} given by this formula and then we need to perform the integration. So we write the expression for Q_{iY} and this becomes, C_0 is a constant so we are moving it out into V_{GB} minus V_{FB} minus phi_t minus V_Y minus gamma into phi_t plus V_Y. So this whole thing with positive signs here is the threshold voltage V_{TY}. Now since V_{GB} minus V_{TY} all these signs have got converted to negative and then there is a negative sign which is coming out here.

Now what is the meaning of this negative sign?
The current being negative because all the other quantity is positive. The current being negative means simply that it is moving in the negative y direction. So please understand this here, the positive y direction is like this but drain is here and source is here so the current is

moving down and down means negative y. This is the reason why you are getting a negative sign so we need not be concerned about the negative sign and what we are interested is only the magnitude. Now here we should put the differential. We can very easily do the integration now. You will note that when you integrate this is a constant term multiplied by dV_Y so you will get a linear term, a term which is linear in the voltage when you integrate. This linear term will become a square law and square root term will become a 3 by 2 power law term.

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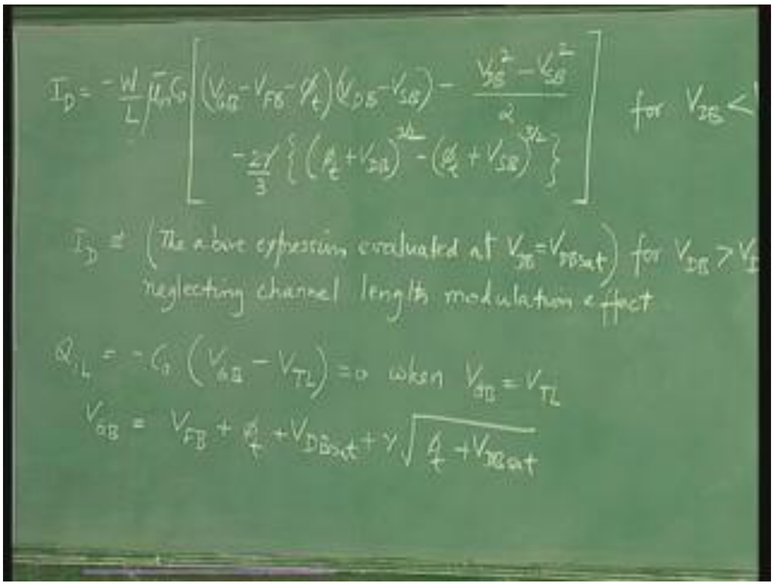

So we can write I_D is equal to minus W by L into average mobility into C_0 and now we open the bracket V_{GB} minus V_{FB} minus phi_t into V_{DB} minus V_{SB}. Then we have this square law term V_{DB} square minus V_{SB} square by 2 and the square root term gives rise to a 3 by 2 power law term so that we will shift it here so minus gamma to the power 2 by 3 (phi_t plus V_{DB}) whole power 3 by 2 minus (phi_t plus V_{SB}) whole power 3 by 2 divided by 3 by 2 and that divided by 3 by 2 has come here as 2 by 3 so we can put this in brackets like this.

So here this is the lengthy expression for current voltage characteristics: I_D is equal to minus W by L average mobility C_0[(V_{GB} minus V_{FB} minus phi_t) (V_{DB} minus V_{SB}) minus V_{DB} square minus V_{SB} square by 2 minus 2 gamma by 3{(phi_t plus V_{DB}) whole power 3 by 2 minus (phi_t plus V_{SB}) whole power 3 by 2 }]. This is the so-called 3 by 2 power law expression. It is a fairly lengthy expression and please note that this is valid only until the saturation is reached. Now please note saturation is reached when this inversion charge becomes 0. So this expression we have derived is valid only until inversion charge is 0. So saturation is reached when Q_i at L is equal to 0. Now this will happen for drain to bulk voltage, we will call that voltage as V_{DBsat}.

The lengthy formula is valid for V_{DB} less than V_{DBsat} so we will write this here. Now what happens for V_{DB} greater than V_{DBsat}. As we noted the current is almost constant at the value corresponding to the saturation. So, for V_{DB} greater than V_{DBsat} it is the same expression as this by substituting V_{DB} is equal to V_{DBsat}. So I_D is equal to the above expression evaluated at V_{DB} is equal to V_{DBsat}. So this is the result for V_{DB} greater than V_{DBsat}. Strictly speaking this is not correct because the current increases gradually because of channel length modulation so we will say it is approximately equal to neglecting channel length modulation effect.

Now how do you find out what is V_{DBsat}?
V_{DBsat} is found out by putting inversion charge at L is equal to 0 from that condition. Or it can also be found by differentiating this expression. If you differentiate this expression with respect to V_{DB} and set it equal to 0 and find out what is the value of V_{DB} in that equation that particular approach also will give you the same V_{DBsat} because of the following reasons: You know that your characteristics are something like this. The 3 by 2 power law expression we derived will go as something like this. This is V_{DBsat}, this is I_D versus V_{DB}, start from V_{DB} is equal to V_{SB} and at V_{DBsat} you will find that this slope becomes 0. So dI_D by d V_{DB} is equal to 0 at this point.

Now one can show this particular result mathematically also. that is, if I_D is given by this expression then dI_D by dV_{DB} differentiating I_D with respect to the upper limit of this integral and setting it equal to 0 whatever value of V_{DB} you will get that will be same as the V_{DB} you will get by setting this Q_{iY} which is a function of V_Y as 0 at V_{DB}. That is, dI_D by dV_{DB} at V_{DBsat} is equal to 0 implies Q_i as a function of V_{DBsat} will also be is equal to 0 if I_D is given by this relation. This approach of differentiating mathematical relation and finding out V_{DBsat} is more difficult than using this relation Q_i at V_{DBsat} is equal to 0. So we will use this particular approach to find out V_{DBsat}.

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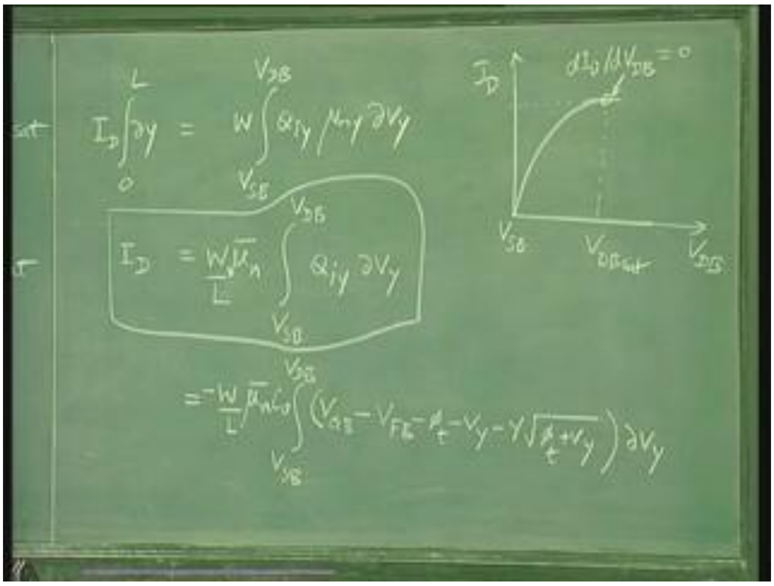

If you use that approach what is the relation we get? That is very simple because Q_i at L is equal to minus C_0 into V_{GB} minus V_T at L that is the formula. Now this will be is equal to 0 when clearly V_{GB} is equal to V_{TL}. Now we can very easily write an expression for V_{TL}. So this is same as saying V_{GB} is equal to V_{TL} is nothing but V_{FB} plus phi_t plus V_{DB} and we are considering the saturation point therefore we write V_{DB} as V_{DBsat} plus gamma into square root of phi_t plus V_{DBsat}. So this is the equation that you will have to solve for V_{DBsat}: V_{GB} is equal to V_{FB} plus phi_t plus V_{DBsat} plus gamma square root of phi_t plus V_{DBsat}. Obviously you will get a quadratic and you can solve it and it is again a complicated expression. So that expression for V_{DBsat} you will have to substitute here to get the saturation current by this approach. What we will do is we will consider the equation for a simple condition that is for source to bulk voltage is equal to 0.

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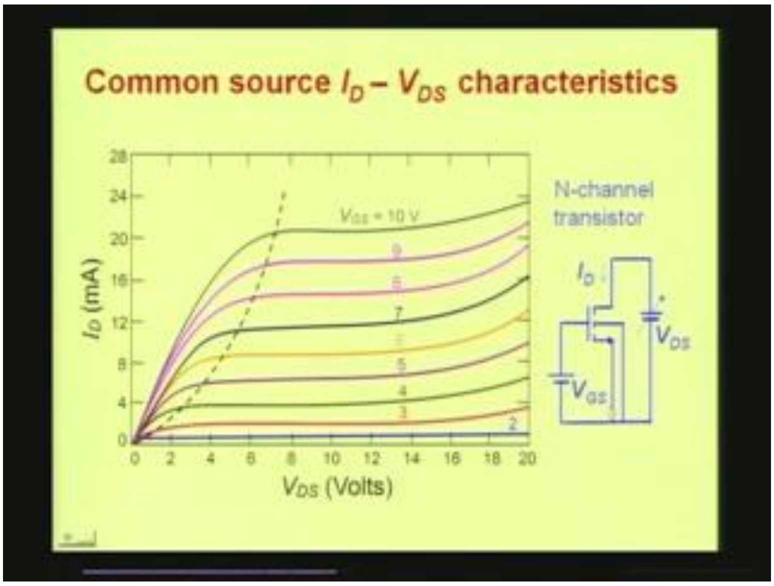

If you look at the slide here the characteristics have been shown for source to bulk voltage is equal to 0. In a number of applications we encounter this condition. Let us see the simplification result for this case.

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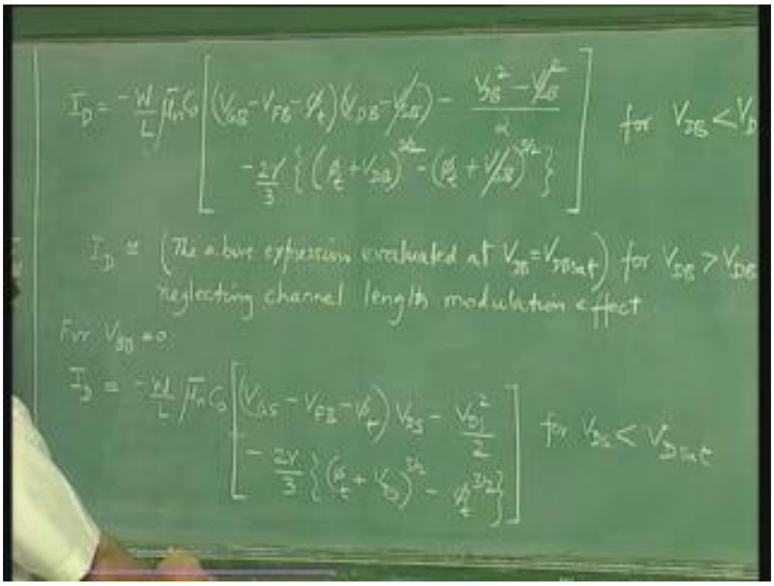

So when you put source to bulk voltage is equal to 0 this term here, this term here, this term will get cancelled and the bulk terms here can be replaced by source, the bulk letters can be replaced by a source because bulk to source voltage is 0 bulk and source are interchangeable. So your equation under those conditions will become for V_{SB} is equal to 0 I_D is equal to minus W by L mu_n C_0[V_{GS} so we are writing V_{GB} as V_{GS} minus V_{FB} minus phi_t into V_{DS}, this V_{DB} has become V_{DS} minus V_{DS} square by 2 this V_{DB} square has become V_{DS} square and similarly you replace V_{DB} by V_{DS} here and you get minus 2 gamma by 3{(phi_t plus V_{DS}) whole power 3 by 2 minus phi_t power 3 by 2}]. So the equation as a whole is, for V_{SB} is equal to 0 it is I_D is equal to minus W by L mu_n C_0[V_{GS} minus V_{FB} minus phi_t) V_{DS} minus V_{DS}

square by 2 minus 2 gamma by 3{(phi$_t$ plus V_{DS}) whole power 3 by 2 minus phi$_t$ power 3 by 2}]. And this is valid for, instead of V_{DB} it will be V_{DS} less than V_{DSsat}. So we do not want to put two s there so we will put V_{Dsat}. It is understood that when you write VD sat the source is the common terminal. Now, in the saturation all that you need to do is replace V_{DS} by V_{Dsat} and then you get the saturation current. But still you can see that we have the 3 by 2 power law terms. The equation is considerably simple but still you have the 3 by 2 power law terms.

Now in practice this kind of an equation is rarely used for a simple analysis. We use a square law expression. So how do you eliminate the three by two power law expression? This can be done as follows. So you use the following approximation. That is, you know that (1 plus x) to the power n is approximately is equal to 1 plus nx if x is much less than 1.

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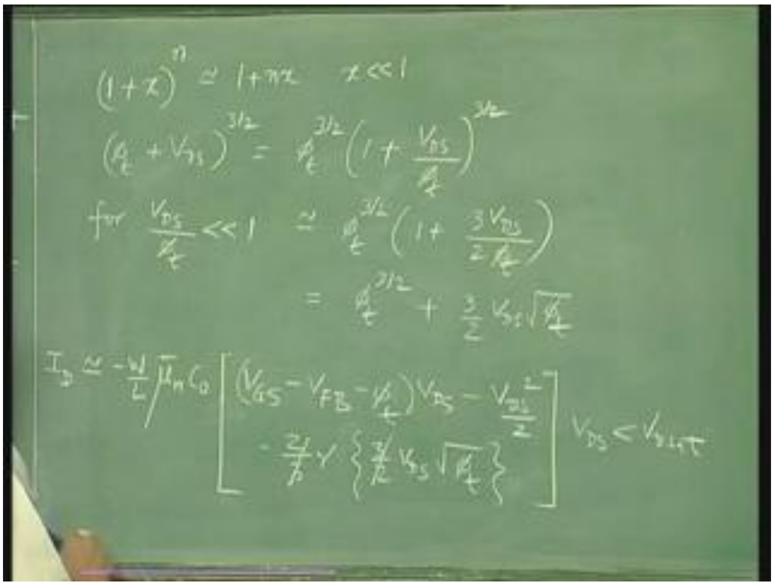

This is a mathematical approximation and if you apply this approximation (phi$_t$ plus V_{DS}) whole power 3 by 2 term than you can write this as follows: You remove phi$_t$ out which is the first step, so it is phi$_t$ to the power 3 by 2 (1 plus V_{DS} by phi$_t$) whole power 3 by 2. Now for V_{DS} by phi$_t$ much less than 1 the above can be approximated as phi$_t$ to the power 3 by 2 (1 plus 3V_{DS} by 2 phi$_t$) which is nothing but, this is equal to phi$_t$ to the power 3 by 2 plus 3 by 2V_{DS} square root of phi$_t$. So it is phi$_t$ to the power 3 by 2 phi$_t$ so you get square root of phi$_t$. Now, this is the result for (phi$_t$ plus V_{DS}) whole power 3 by 2. Now if you substitute this in this expression here then this phi$_t$ to the power 3 by 2 and the phi$_t$ to the power 3 by 2 coming from the approximation of this equation will get cancelled and this whole equation can be approximated as follows:

So we get I_D is approximately equal to minus W by L average mobility C_0[(V_{GS} minus V_{FB} minus phi$_t$) (V_{DS} minus V_{DS} square by 2 minus 2 by 3 gamma {3 by 2 V_{DS} square root of phi$_t$}]. This is valid for V_{DS} less than V_{Dsat}. So this cancels and now you can see that this is a term linear in V_{DS} so you can absorb gamma (square root of phi$_t$) here along with this and if you absorb this here you will get I_D is approximately equal to minus W by L average mobility C_0[(V_{GS} minus V_{FB} plus phi$_t$ plus gamma square root of phi$_t$) V_{DS} minus V_{DS} square by 2 for V_{DS} less than V_{Dsat}. This has been written here.

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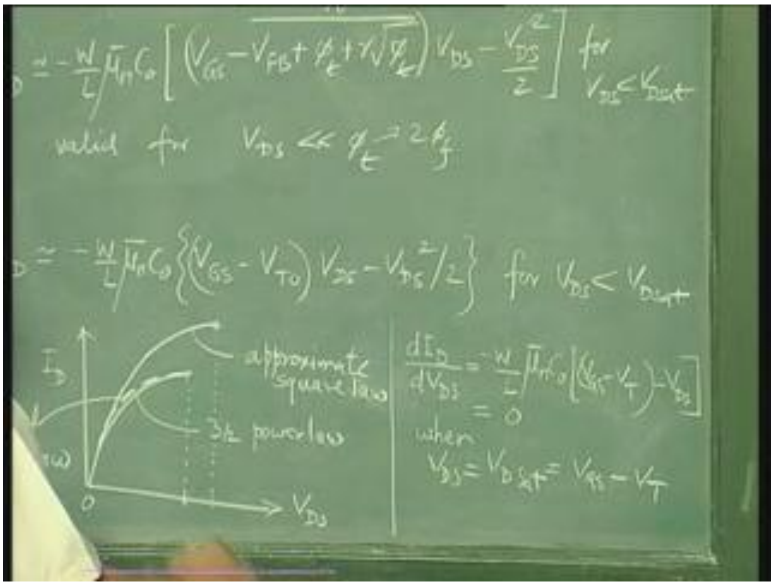

This is the square law expression and the 3 by 2 power has been eliminated. Now this is valid for V_{DS} much less than phi_t and this phi_t is nothing but $2phi_f$ this we have explained in the previous lectures. So only for very small value of V_{DS} the 3 by 2 power law reduces to this particular square law.

Now we will see that how you can still use this square law for higher values of V_{DS}. But before that we see the fact that this whole thing which we can write as V_{GS} minus within brackets these three terms (minus V_{FB} plus phi_t plus gamma square root of phi_t). When you put a bar here it means all these three terms are within brackets. You can identify this whole thing as the threshold voltage of the MOSFET without including the body effect. So this is V_T of the MOSFET for V_{SB} is equal to 0. So we can write it as V_{T0} because we are using V_{SB} is equal to 0 and so this expression becomes I_D is approximately equal to minus W by L mu n C_0{(V_{GS} minus V_{T0}) V_{DS} minus V_{DS} square by 2} for V_{DS} less than V_{Dsat}.

Now it is in a very simple form. Let us plot this approximation and the real 3 by 2 power law expression on a graph, how will it look? It will look as follows: Now we are plotting as a function of V_{DS} so we start from origin because V_{DS} is equal to 0 the current is 0. Let us say this is our 3 by 2 power law up to saturation point. As compared to that if you do the approximation the approximate current is almost same as the 3 by 2 power current in the beginning but there after it will be more. So this is the approximation and it will go up to higher values. Your saturation point will be shifted to the right in the approximate, so this is the approximate square law. So the square law expression predicts a higher current than the actual current.

Now what is normally done is that since we are anyway using an average mobility if you adjust this mobility a little bit more and reduce its value then this current will come and match with this current here. Of course the shape here and here will not match. So adjusted square law with mobility reduced so as to match the 3 by 2 power law you will get this kind of a shape shown by dotted line which is not very different from the 3 by 2 power law. So this dotted line is adjusted square law.

How do you do the adjustment?
We slightly adjust this mobility further. Obviously this is not physical this is an empirical adjustment. But since in this approach we can adjust the mobility and match the complicated 3 by 2 power by simple square law. This approach is often used and we use this expression to represent many real life experimental characteristics.

The only thing the mobility value that is used to match the experimental characteristics will be a little lower than what should be there physically. Now what we have still to do is to find out what is this V_{Dsat} for this square law. We have explained how the validity for very restricted values of V_{DS} can be actually dropped, this restriction can be dropped and square law can be used over a very wide range. Now, to complete the square law model we need to write an expression for the saturation voltage V_{Dsat}. That can be obtained very easily by differentiating this expression with respect to V_{DS} and setting it equal to 0.

In the 3 by 2 power law case we had evaluated the saturation point using the condition the inversion charge at drain is equal to 0 Q_{iL} is equal to 0 because that was easier to do. We can use that approach even here. But the mathematical approach here looks very straight forward for the square law case so we will do this approach and then we will see how this approach is similar to getting the saturation point Q_{iL} is equal to 0. So differentiating this you get dI_D by dV_{DS} is equal to minus W by L average mu n C_0 [(V_{GS} minus V_T) minus V_{DS}] this is the result of differentiating this. So you set it equal to 0 so this is equal to 0 when very clearly V_{Dsat} is equal to V_{GS} minus V_T. Therefore V_{DS} is equal to V_{Dsat} is equal to V_{GS} minus V_{T0}. That is the condition here. So we replace this V_{Dsat} by V_{GS} minus V
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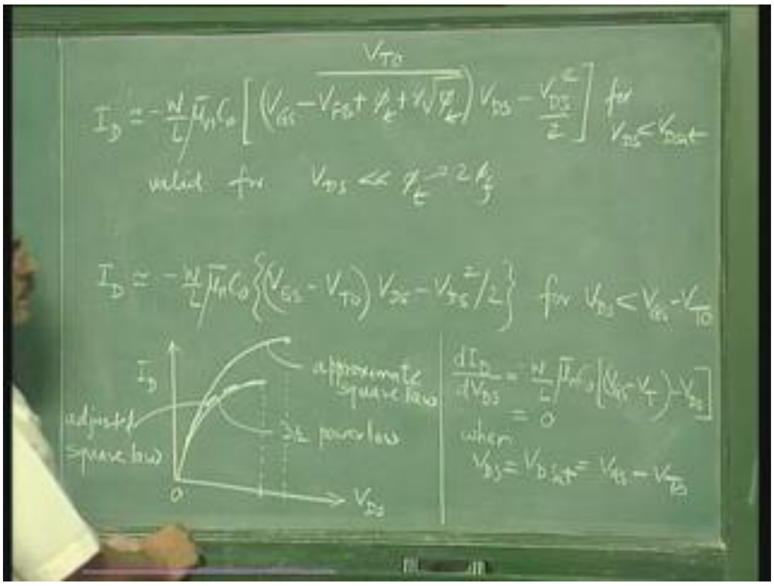

It is V_{T0} because we are considering bulk to source voltage is equal to 0. Now, having obtained this we can also write an expression for the saturation I_{Dsat} because all that we need to do is replace V_{DS} by V_{Dsat}. Now the complete square law model will therefore look like this:

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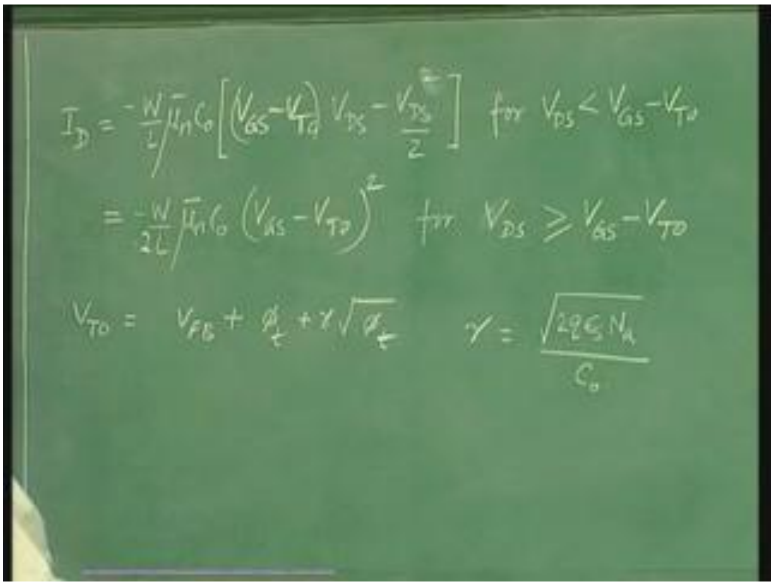

This is the expression for V_{DS} less than V_{GS} minus V_{T0} and you substitute V_{DS} is equal to V_{GS} minus V_{T0} to get the saturation current. So I_D is equal to minus W by L mu_n C_0 and then you will get two square law terms in V_{GS} minus V_{T0} this is half of this therefore you will get a 2 out and this expression will be simply this for V_{DS} greater than or equal to V_{GS} minus V_{T0} this is the saturation current. The expression is I_D is equal to minus W by L mu_n C_0 (V_{GS} minus V_{T0}) whole square. So that is the complete square law model where we are writing V_{T0} is equal to V_{FB} plus phi_t plus gamma square root of phi_t where gamma is nothing but square root of 2q $epsilon_s$ N_a by C_0 that is the complete model. Before we do some examples using this model let us explain the physical implication of the approximation we have made to replace the 3 by 2 power law terms and absorb these 3 by 2 power law terms into the square law model.

Recall that strictly speaking the square law model assumes that drain to source voltage is much less than phi_t and approximately 3 by 2 power law terms under that condition. Now if you trace your equations you will find that the 3 by 2 power law terms actually originate from the depletion charge expression in the threshold voltage. So, if you look at the depletion charge that is this charge here as shown by this region what the square law model is assuming is that even when your V_{DS} is large the 3 by 2 power law term which represents the depletion charge is not changing with y. That is, the depletion charge is constant from source to drain. This is shown by the dotted line here.

Though the inversion charge is changing the depletion charge is not changing this is an approximation. Now, evidently if you assume that the depletion charge to be constant at the value at which it is at the source then you will predict more inversion charge as you move towards the drain because you are predicting the same total charge based on the gate to bulk voltage and in the same total charge you have less depletion charge as compared to the actual condition. Therefore you will end up predicting a higher inversion charge and that is why the square law model gives you a higher current. So whatever we have seen mathematically so far we are now explaining physically. So physically the square law model approximation to the 3 by 2 power law is equivalent to assuming the depletion charge to be constant throughout the device from source to drain equal to the value at the source that is the meaning. Now one more point about the square law model. Though we have derived it assuming the condition bulk to source voltage equal to 0 people also use it to estimate the current approximately even when the bulk to source voltage is not equal to 0 that is even when the body effect is present. Therefore you will not find the suffix 0 here in the formulae when it is found in the books.

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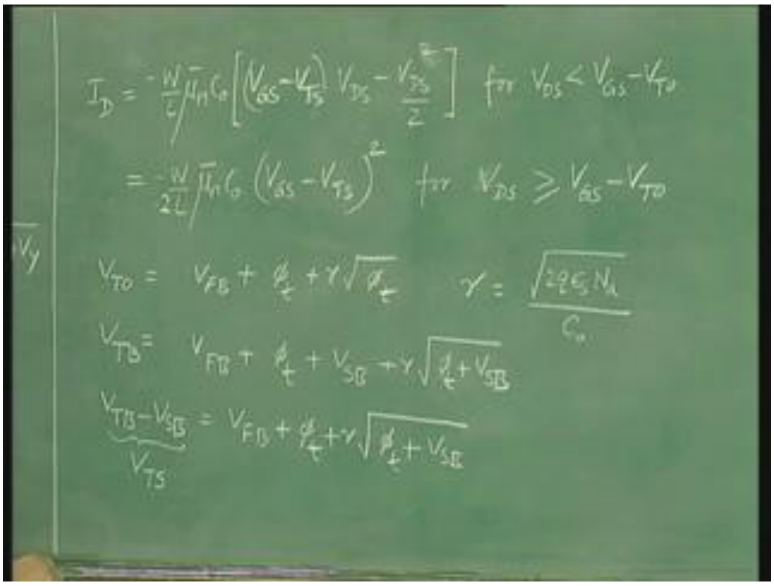

In other words, this is assumed to be valid or it is used even when the bulk to source voltage is not equal to 0. Now the question is, if V_{T0} is this then what V_T is when bulk to source voltage is not equal to 0. So let us write down the expression for V_T that should be used here in general. Let us recall the V_T expression we have written for the MOS capacitor assuming bulk as the common terminal and then convert that threshold voltage with respect to bulk to threshold voltage with respect to source. So V_T including body effect is given by V_{FB} plus phi$_t$ plus V_{SB} plus gamma square root of phi$_t$ plus V_{SB}. This is the expression for threshold voltage assuming bulk as the common terminal.

Now we want threshold voltage assuming source as the common terminal because all this is with source as the common terminal. Now that is readily obtained by shifting this V_{SB} to the left hand side and we get V_{TB} minus V_{SB} is equal to V_{FB} plus phi$_t$ plus gamma square root of phi$_t$ plus V_{SB}. Now V_{TB} minus V_{SB} is nothing but V_{TS} that is the threshold voltage with source as the common terminal. So this is the V_{TS} that is used in these formulae.

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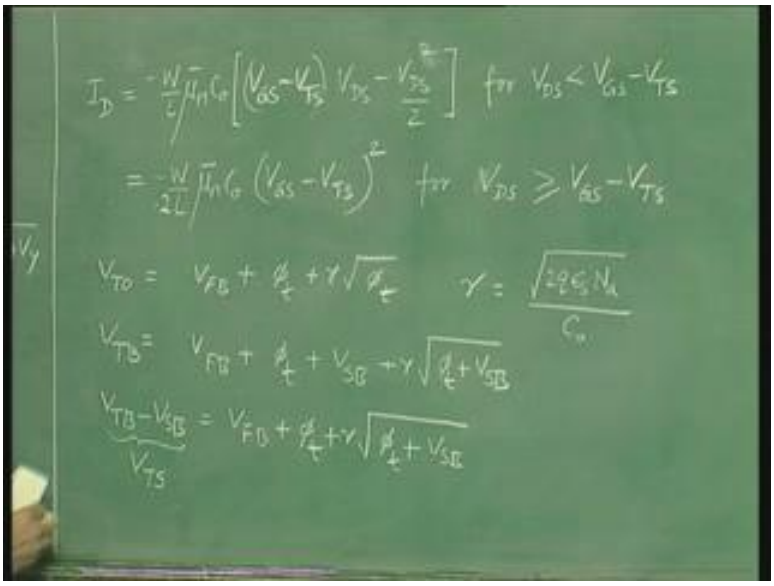

So here we replace 0 by S in general and this is the general expression. Now you see that, if V_{BS} is 0 then this term will be 0 and this will reduce to this expression.

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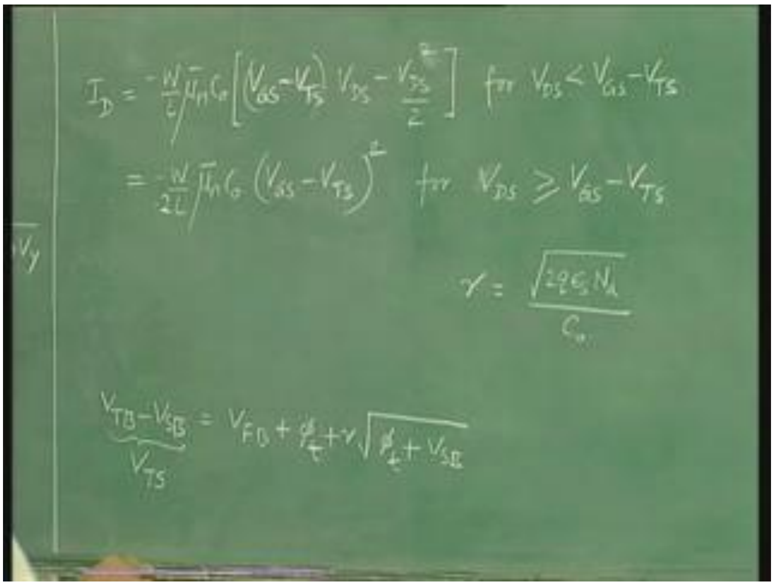

So we now do not need this V_{T0} expression so now in our power law we will have this expression.

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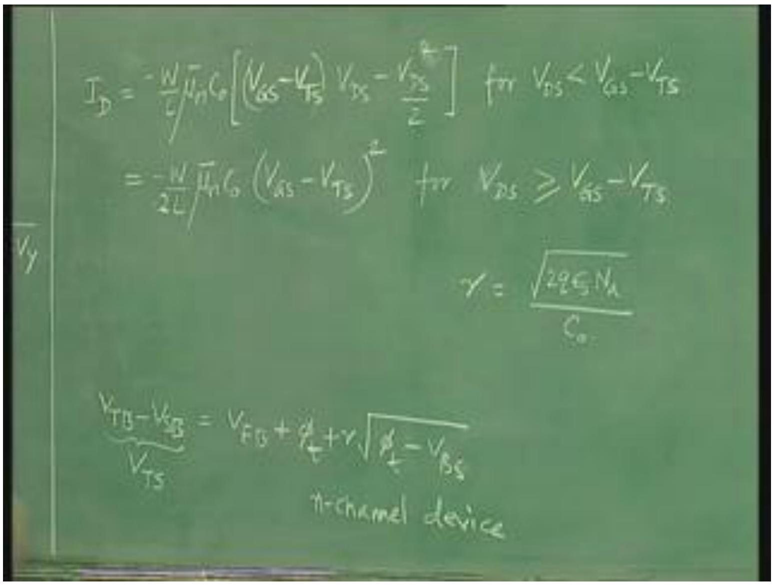

Since we want to write everything in terms of source as the common terminal you will find that in books this will be written as minus V_{BS} for an n-channel device. For a p-channel device of course the polarity will get adjusted or it will change. So here we will write n-channel device. Now with this we have almost completed the derivation of the current voltage characteristics. We still have to do the channel length modulation effect in saturation. Now these aspects will be discussed in the next class along with some other thing about the MOSFET operation.