Solid State Devices Dr. S. Karmalkar Department of Electronics and Communication Engineering Indian Institute of Technology, Madras Lecture - 34 Metal-Oxide-Semiconductor (MOS) Junction (Contd...)

In the last class we started our discussion on the MOS junction. We shall continue this discussion in this particular lecture. You would recall that we have set the goal of explaining the C-V characteristics shown on this slide.

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Now what we achieved in the last class is that for an ideal MOS capacitor we have explained the accumulation, depletion and inversion regimes of operation. We can show these various regimes of operation on a simple diagram.

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So let us draw a line indicating the various conditions in the semiconductor as far as the electron concentration at the interface is concerned. So this shows ns on a log scale.

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What is n_s ? If these are the hole and electron concentrations for any voltage this quantity is ns and this quantity is p_s and this bulk concentration of electrons we will denote as n_0 . Now suppose this is n_0 this is n_i and this is p_0 , so p_0 is this, the bulk concentration of holes. We can identify the regimes of operation in terms of these concentrations. When your n is less than n_0 that is this regime you have accumulation. When n starts becoming more than n_0 or correspondingly the hole concentrations starts becoming less than p_0 therefore here you have depletion. Now this depletion continues until your n_s is equal to p_0 . Now in this region you can separate and you can identify a weak inversion region when your ns is more than n_i but less than p_0 . So this is the weak inversion. Just as depletion implies removal of majority carriers (Refer Slide Time: 05:51)

Inversion is nothing but the pile up of minority carriers so that they dominate the bulk majority carriers. So whatever is the amount of majority carriers present in the bulk they are dominated by the minority carriers at the interface. So at the interface the minority carriers in the bulk are really the majority carriers because they are dominating. Therefore weak inversion and beyond ns is equal to p_0 here you have strong inversions. Now, in our first course what we said is that we will call this region as the inversion region. Whenever we use the word inversion it would imply strong inversion. The treatment of weak inversion is more complex so that is taken up in advanced courses on semiconductor devices.

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What is it that distinguishes strong inversion from weak inversion?

The pile up of the electron charge at the interface as shown in this diagram that is this area starts becoming significant in the strong inversion region. Now we can translate this diagram that you are able to see also to the voltage axis. For example, in terms of voltage if you show this is V the gate voltage, then this point when ns becomes equal to p_0 is said to be the threshold point and this voltage is called the threshold voltage.

On the other hand, this point where the surface concentration of the electrons is same as the bulk concentration or when the charge conditions in the device is 0 it is normally referred to as the Flat-band voltage. So this translates to this point in terms of the voltage, the conditions in the device translating to terminal voltage and this translates to threshold voltage point. Let us understand why this point is called the Flat-band point. To understand this we need to draw the energy band diagram for that condition. So this condition is also the condition V is equal to 0 for an ideal capacitor. So V_{FB} is equal to 0 for ideal capacitor. So what is this Flat-band and what is the meaning of this Flat-band?

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So, if I draw the energy band diagram in the semiconductor for V is equal to 0 which is a Flat-band for an ideal capacitor you will find that the conduction band edge and valance band edge both of these edges are flat they are horizontal. That is because there is no variation in the hole or electron concentration as a function of X for V is equal to 0. So recall that the hole concentration is flat and electron concentration is flat for V is equal to 0. So this flat hole and electron concentration to flatness of the conduction and valance band edges and therefore this condition is called the Flat-band condition.

Please note that we are considering a uniformly doped substrate. If the substrate is nonuniformly doped then even when voltage is 0 the doping will result in a non-uniform hole concentration and electron concentration and then the bands will not be flat. So for our case we understand why the voltage equal to 0 condition is referred to as the Flat-band condition. So accumulation implies voltage less than V_{FB} . From V_{FB} to V_T this is the depletion region and for gate voltage beyond V_T you have the inversion region. Now we need to find out what is this threshold voltage. We already know that the Flat-band voltage for an ideal capacitor is 0. Now, for this purpose we need to translate our hole and electron distributions in the semiconductor to charge an electric field condition. For the threshold condition the hole concentration is shown here as 4 and electron concentration also has been shown. It turns out that this electron concentration that is the area under the electron distribution is really small compared to the charge because of depletion of holes. So we can neglect this charge under threshold voltage conditions that is at threshold voltage conditions. If that being the case we can draw the charge on electric field conditions as follows:

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This our hole concentration on a linear scale. Now we draw the space charge. Since the charge is negative we draw it on the negative side. So that is the space charge where this value here is minus $q(N_a)$ approximately. We know from our PN junction theory that in depletion layer this space charge concentration is minus q times the ionized impurity concentration and we are assuming complete ionization.

Now we will assume the depletion approximation. That is, we will assume that this region has an abrupt ending for simplicity of analysis. Now based on this we can sketch the electric field. The electric field is on the positive axis because the electric field is positive from left to right, this charge is negative. Since this space charge is constant here be Gauss's law we will get a straight line for the electric field up to the interface. Now what happens in the oxide? When you move from one region to another region of different dielectric constant then the electric fields on the two sides of the boundary are related by the formula $epsilon_1 E_1$ is equal to $epsilon_2 E_2$ so these are the two electric fields; this is E_1 , this is E_2 and the dielectric constants are $epsilon_1$ and $epsilon_2$. (Refer Slide Time: 16:28)



Now, using this particular relation if you call this as the surface electric field in semiconductor, that is, we give the symbol epsilons here. In terms of this we can draw the electric field in the oxide. So, in the oxide layer the electric field would be a constant because there is no space charge and this electric field would be related to this electric field by this formula.

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So we can substitute the dielectric constants here and then we will find, this is E_{ox} therefore this is epsilon_{ox} and here it is epsilon silicon and this is E_s . So from here we have E_{ox} is equal to epsilon_s into epsilon silicon by epsilon_{ox}. Epsilon silicon is 12 and epsilon_{ox} is 4 equal to $3(E_s)$. Therefore you take three times this and that is your E_{ox} filled in the oxide.

Now what happens in the metal or gate?

Obviously you have the opposite charge so here you have the charge shown by a delta function on the positive side because you have a positive charge. So the electric field will abruptly drop to 0 in a small distance. The field is not of interest because it falls abruptly in the metal. Now this area under the electric field distribution is the applied voltage and that is the threshold voltage. So, to know the threshold voltage we need know this area. Now let us identify the components of the applied voltage across oxide and across semiconductor on the electric field versus x diagram.

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So here this area is a voltage drop across the oxide that will indicate as psi_{ox} . On the other hand, this area is a voltage across semiconductor and this we will denote as psi_s . So let us see in terms of our capacitor.

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What are these components? Now you are applying a voltage here and this voltage is falling across the oxide and across the semiconductor, this is the applied voltage V. So, in general you have V is equal to psi_{ox} plus psi_s . Now we need to identify psi_{ox} and psi_s , we need to get the values. Now it is interesting to note that you can get psi_s from the hole and electron concentrations versus distance diagram at threshold. So at threshold your concentrations of electrons and holes are as follows: So n_s is equal to p_0 that is a point here, this is hole concentration and this is electron concentration at threshold. You can use Boltzmann relation because this semiconductor is under equilibrium. So Boltzmann relation gives you the potential drop in this region if there is a variation as shown here.

Now this value here is n_0 and that is p_0 . So this is p_{s_i} the potential drop in the semiconductor which can be written as V_t into l_n of the concentration here that is ns by concentration in the bulk that is n_0 which is nothing but $V_t l_n p_0$ by n_0 which is the Boltzmann relation. Now we can simplify this equation because we know we can write n_0 in terms of p_0 so this is nothing but $V_t l_n$ of p_0 by n_i square by p_0 because $p_0 n_0$ is n_i square which will then be simplified to two times $V_t l_n$ of p_0 by n_i so that is your psis. So psis is equal to $2V_t l_n p_0$ by n_i . Now we know what is p_0 ? p_0 is approximately the doping of the substrate. Therefore p_0 is approximately equal to N_a . Therefore we have the relation, psis is approximately $2V_t l_n N_a$ by n_i .

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In other words, we know the value of psi_s at threshold in terms of the substrate doping. So this is psi_s at threshold. In other word, we have identified this area here.

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Please note that the psi_s will vary with voltage. We are talking about the conditions right at the threshold. Now once we know this we know this area also because we know this E_{ox} is three times $epsilon_s$ because of the dielectric constants of silicon being 12 and oxide being 4.

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Therefore now we can write an equation for threshold as follows:

So threshold voltage is equal to $psi_{ox} plus 2V_t l_n N_a$ by n_i where psi_{ox} is E_{ox} into t_0 is equal to $3E_s$ into t_0 because E_{ox} is $3E_s$. And for epsilons you have a relation. This area is $2V_t l_n N_a$ by n_i but this area we can write in terms epsilons and doping level itself using our analysis we did for PN junction. So for epsilons your relation is half of epsilons into the depletion width X_d , the depletion width X_d is this. So this is $2V_t l_n N_a$ by n_i . Now this X_d we can again write in terms of the charge so this X_d is nothing but the depletion charge which we denote as Q_d . We are putting a modulus because the depletion charge is negative.

What is the depletion charge?

Let us look at this diagram, this is the depletion charge Q_d . Of course at threshold this also happens to be the charge in the semiconductor q_s but that is a different issue. So now X_d is Q_d by doping into q. And this Q_d and epsilon_s are related by Gauss's law. That is, this epsilon_s and this Q_d , so what is the relation? The relation is Q_d is equal to epsilon_s into epsilon_s but epsilon_s is positive and the charge is negative. So Q_d is minus epsilon_s epsilon_s so modulus of Q_d is epsilon_s.

Here in this formula you can write this Q_d by $q N_a$ as epsilon_s epsilon_s by $q N_a$ so that now you can replace X_d in terms of epsilon_s and you have the relation, 1 by 2 epsilon_s square by $q N_a$ into epsilon_s. This is the left hand side so this is equal to $2V_t l_n N_a$ by n_i . Now let us indicate this particular voltage which is coming often with a symbol so that we do not have to write this whole term. Let us call this voltage as phi_t that is the surface potential at threshold. Note that the potential drop in the semiconductor psi_s found here is also referred to as surface potential because this psi_s is the potential of this point with respect to the bulk and this is this surface and that is why it is called surface potential. We are assuming that the bulk potential is 0 in the semiconductor, reference.

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From here we can derive the following formula for E_s ; epsilon_s is equal to square root of 2q N_a phi_t by epsilon_s. Now we can substitute this value of epsilon_s here and therefore get this psi_{ox}. And we will get the expression for threshold as V_T is equal to $3t_0$ into square root of 2q N_a phi_t by epsilon_s plus phi_t. Now this three is nothing but the ratio of the dielectric constants of semiconductor and oxide.

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So we can simplify by writing this 3 as $epsilon_s$ by $epsilon_{ox}$ and then we can cancel these terms and shift this $epsilon_s$ to the numerator and $epsilon_{ox}$ by t_0 is the so called oxide capacitance C_0 . So we can write this formula as square root of $2q N_a phi_t epsilon_s$ by C_0 plus phi_t where phi_t is given by this $2V_t l_n N_a$ by n_i . Now this is the equation, let us write it completely. So this is the equation for threshold voltage of an ideal MOS capacitor.

We have on the right hand side all the material parameters. So basically you see it depends on the doping level N_a the temperature t because that decides n_i and V_t and the oxide thickness which decides the C_0 . Of course you also have the dielectric constants coming in there. Now in this form you can identify the particular term the first term on the right hand side that is this term as follows: Since this is the dimension of holes the numerator is charge by unit area because charge by unit area divided by capacitance per unit area will give you voltage. Now here what is the charge we are talking about? This charge is nothing but this particular depletion charge here. So this is identified as Q_d at inversion with a modulus because the charge is negative.

Now this is again charge per unit area. We need to emphasize that the area we are talking about is perpendicular to the board. That is, the area is perpendicular to this. Now what does these Q_d by C_0 term indicate? It is like the voltage across the insulator of a parallel plate capacitor having charge Q. That is how this formula is very easy to remember. In this formula this is the potential drop in the oxide which is given by the parallel plate capacitor formula charge on the electrode of the oxide that is the semiconductor divided by the capacitance and this is the voltage drop in silicon. Therefore voltage drop in silicon plus the voltage drop in oxide at threshold you sum up and then you get the threshold voltage. Now it is useful to get some feel for what kind of values we get for the threshold voltage. So we will do a solved example.

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The solved example is:

Calculate the threshold voltage of an ideal MOS capacitor having the following parameters: p-substrate doping level of 1.45 into 10 to the power 16 by cm cube and oxide thickness of 0.2 micrometer. That is t_0 is equal to 0.2 mum and the temperature is 300 K. Basically these are the parameters corresponding to the experimental C-V curve we showed which we are going to explain. Let us start using this formula.

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If you substitute the various parameters what kind of values we get for the two terms on the right hand side. Let us calculate phi_t first. phi_t at 300 K is 2 into 0.026 volts into logarithm of 1.45 into 10 to the power 16 by 1.5 into 10 to the power 10. Now after calculating you find that this is approximately 10 to the power 6. So when you take log of 10 to the power 6 the 6 will come out and you will have l_n 10. So that 6 gets multiplied by 2 and you have 12 so twelve times 0.026 into l_n 10. It turns out that this quantity will be about 0.72 volts. So this phi_t value is somewhat like the value of the built in voltage of a PN junction about 0.7.

The next quantity to calculate is this C_0 which is capacitance of the oxide per unit area. So C_0 is epsilon_{ox} by t_0 which is 4 into 8.85 into 10 to the power minus 14 by 0.2 into 10 to the power minus 4 F by cm by cm. So you see the capacitance is F by cm square. Now this value will be of the order of Nano Farad's because this 10 to the power minus 4 goes up and then you have 10 to the power minus 10 but then you have 0.2 here so multiply numerator and denominator by 10 and this will become 2 and here you will have 10 to the power minus 9. So this value is 17.7 nF by cm square. Now it is useful to remember the typical value of C_0 like we remember the typical value of phi_t which is about 0.7.

Now please note that phi_t is logarithmly dependent on the doping level here. So even if the doping level changes the value of phi_t does not change very significantly. You will find it will remain between 0.6 and 0.8 volts so 0.7 is a good value to remember. Similarly for C₀ we find that for 0.2 micron thickness oxide the capacitance is 17.7 nF by cm square. It is useful to remember the value for 0.1 micrometer because 0.2 micrometer is not so easy to remember but relatively it is easier to remember the 0.1 micrometer thickness which is same as 1000 Angstroms in terms of Angstroms. Whenever you are handling small geometries you must be very comfortable with dimensions of the order of Angstroms and microns and you must be able to convert one from the other. So 0.1 micron oxide thickness is equal to 1000 Angstroms.



So for this 1000 Angstroms oxide thickness your C_0 would be double the value that you have obtained for 0.2 microns because the capacitance is inversely proportional to the oxide thickness. So, for this the C_0 is approximately 35.5 nF by cm square. So one remembers this value for t_0 is equal to 0.1 microns or 1000 Angstroms this is the capacitance. So, in practice whenever you come across any other oxide thickness you can always calculate the capacitance of that particular MOS capacitor by scaling the oxide thickness in appropriate manner starting with this value.

Next we need to calculate the numerator of this equation. Now we can write square root of 2q N_a epsilon_s into phi_t by C_0 is equal to square root of 2 into 1.6 into 10 to the power minus 19. We always collect all powers of 10 together and we also write the units here Coulomb. Then doping level is 1.45 into 10 to the power 16 so we put that here and this is for cm cube. Then comes epsilon_s. You recall from calculations done for PN junction that approximately we can assume the epsilon_s to be 10 to the power minus 12 F by cm for silicon. Basically this value 10 to the power minus 12 for epsilon is given by 12 into 8.85 into 10 to the power minus 14 because dielectric constant of silicon is 12. And then you have to multiply by phi_t that is 0.72 so 0.72 so we will shift this to the right a little bit and this into 10 to the power minus 19 plus 16 minus 12 and F by cm is volts. Now this divided by C_0 is 17.7 into 10 to the power minus 9 so this is F by cm square. So you have centimetre square in the numerator and Farad in the denominator.

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Now let us first check the units here. So this is Coulomb and Farad in to volt is also Coulomb. And now Coulomb square and cm power 4 in the denominator, so this centimetre power 4 in the denominator becomes centimetre square when it comes out of the square root and this becomes Coulomb. So this is the unit, and Coulomb by Farad is nothing but volts.

Dimensionally this equation is correct and we need to evaluate it numerically. So here you will find you have powers of 16 minus 12 is equal to 4 minus 19 plus 4 is equal to 10 to the power minus 15. Now you can make it 10 to the power minus 16 and multiply this quantity on this side by 10 so that what you get out from the square root is 10 to the power minus 8. Now bottom is 1.77 into 10 to the power minus 8. So 10 to the power minus 8 and 10 to the power minus 8 will cancel so you will get this of the order of volts. It turns out that this value is equal to 3.26 volts.

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Now you sum up 3.26 volts and 0.72 volts and you get V_T is equal to 3.26 plus 0.72 that is 3.98 volts that is about 4V. So you find that about 4 volts is the threshold voltage. That is the voltage you must apply to the MOS capacitor that is this voltage in order to achieve threshold voltage conditions in the semiconductor.



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Out of this approximate value of 4 volts please note that only about 0.7 is falling in the semiconductor that is this value here; phi_t is the psi_s potential drop in the semiconductor and this is potential drop in the oxide. So you find that most of the applied voltage falls across the oxide and a very small value falls across the semiconductor.

Now let us proceed further and see what happens beyond threshold because in fact from the point of view of operation of a MOSFET we will find that it is a regime beyond threshold that is of significant interest. So let us plot the concentrations in the p-type substrate for voltages beyond threshold. So it is V greater than V_T and supposing you vary this voltage what is going to happen?

First start from the threshold voltage point this is p_0 , this is n_0 so this is the hole concentration, this is the electron concentration at threshold. Beyond threshold what happens is this concentration rises even beyond this p_0 value and this will obviously fall. Now, this point is the so called intrinsic point because here the hole and electron concentrations are equal and therefore they will be equal to n_i . You will recall from the PN junction depletion region analysis that even there you had an intrinsic point.

Now, coming back to the concentrations, this is electrons and these are holes you find that n_s is now more than p_0 . Now what is interesting to note is that you will find that this depletion edge will not change very significantly beyond threshold. Here this is the edge of the depletion layer. This is an important point that we must understand that the depletion region almost ceases to expand beyond the threshold voltage point. Let us see why. Let us plot these same concentrations on a linear scale. This is the hole concentration and this is the electron concentration at threshold but beyond threshold this is somewhere here the electron

concentration is like this. This is when you plot on the linear scale. Please note that this is on a log scale whereas this is on a linear scale.

To understand why the depletion region does not expand significantly beyond threshold you must plot this concentration on a linear scale. And there you find that even a small charge here on the log scale in the electron concentration means a large change in the linear scale. This is almost a factor of 10; in fact we have not shown an increase of a factor of 10 here. If you really want to show a factor of 10 increase then this concentration should have been shown somewhere here or even beyond. So we are not able to show it to scale.

The point is that, now if you see this area under this curve this is going to rise rapidly beyond threshold. Therefore whenever you apply extra voltage you need extra charge in the semiconductor and beyond threshold what is happening is that the extra charge is coming from these electrons. The extra charge comes more easily from the electrons rather than by depleting the holes and exposing the negatively charged acceptor impurities. So, until the capacitor reaches the threshold when you increase the voltage the extra negative charge required in the semiconductor comes from the ionized impurities by expanding the depletion layer. But once you reach the threshold and go beyond this whenever you increase the voltage beyond threshold the extra negative charge required in the semiconductor comes from the semiconductor comes from the ionized impurities by expanding the depletion layer. But once you reach the threshold and go beyond this whenever you increase the voltage beyond threshold the extra negative charge required in the semiconductor comes from the mobile electrons which are very close to the surface.

Since this mobile concentration now can increase very rapidly there is no need to expand the depletion layer to provide for this negative charge. Now these aspects can be shown nicely on a graph. In fact one can do numerical calculation of this situation. The analysis of this to find out this particular charge under this area is not a straight forward analysis but it requires some intense numerical calculations because you have to solve a number of complicated equations. But we can sketch the graph qualitatively to show what is happening here. So, for that purpose we will translate this diagram on to a space charge versus distance diagram. So if you translate this diagram to space charge versus x this is how it looks.

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At threshold this is your space charge so rho versus x. And you are not able to show any electron charge here so this is that electron charge that is very small at threshold. But beyond threshold there is a very slight increase here but here the increase is very high. The dashed line shows beyond threshold. If you recall curve number 4 was threshold voltage so let us call this curve number 5. You should connect this diagram to the diagram we had drawn for carrier concentrations for voltage conditions below threshold then you will understand why this is 4 and this is 5. So here this white is 4 and this is 5. So 4 corresponds to V is equal to V_T and 5 corresponds to V greater than V_T . This is the charge we are talking about.

Now you need not have to increase this area under this rectangle to get the charge. The extra charge beyond threshold comes from increase in this negative charge because of electrons. What we will do is we will call this area under the electrons or the electron charge as the inversion charge and denote it by the symbol Q_i . Now the charge of electrons is negative so we will put a modulus here because this area is positive. Similarly, we will show the space charge because of electrons by a separate delta function here. This is like a delta function where it is showing electrons as a sheet of charge so this is Q_i .

Please note that space charge is on a negative axis and this area is the depletion charge. Now we sketch Q_i and Q_d as a function of voltage. As I have said earlier the calculation for Q_i and Q_d for V greater than V_T will have to be done numerically. We are only presenting the results here and based on it we can make a very simple approximation and we can get the charge conditions and calculate them. As a function of V supposing this is the threshold voltage point and this is the origin of Flat-band voltage then your depletion charge goes on increasing like this and almost saturates beyond this threshold voltage. So there is a small increase but the rate of increase is very small beyond this. So this is Q_d .

On the other hand, if you sketch this Q_i as a function of voltage you will get it is as follows. If Q_i is very small up to threshold and beyond threshold it starts increasing rapidly, so this is Q_i this is the threshold voltage point. This is the depletion region and beyond this you have the inversion region. Now we can complete the picture and show the accumulation charge on the other side. When your voltage is negative you have accumulation charge. Recall that the relationship between accumulation charges and voltage is a straight line and that charge is positive. So this is Q accumulation, suffix a stands for accumulation. The silicon charge stands for sum of Q_d and Q_i because this total thing is the silicon charge. So if you want to sketch the silicon charge that would look something like this and that we will show by dotted line, so this dotted line is Q_s .

Here of course the Q_s and Q_a are the same. Here the Q_s is obtained by summing up these two. So beyond threshold it is almost Q_i and below threshold it is almost Q_d so that is the charge condition. Now this explains why or how the depletion region stops expanding beyond threshold because with increase in voltage the extra charge in semiconductor comes from the mobile electrons and you are getting an inversion layer. Now, what we want to do is to derive expressions for Q_i and Q_d as a function of voltage. We already have an expression for Q_a as a function of voltage. So when you have expressions for Q_d and Q_i versus voltage we have the complete Q_v characteristics. So what we have drawn here is actually the Q_v characteristics that we wanted to derive for the MOS capacitor. So this is a charge in silicon versus the voltage. In the next class we will derive the equations for Q_d and Q_i as a function of voltage. Now to complete the explanation let us see what happens to the potential drop in silicon because we said the applied voltage drops across silicon and oxide.

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At threshold we have seen that psi_s is equal to phi_t that is about 0.7V. Now what happens beyond threshold? Since the depletion charge is almost saturating or the depletion layer is almost saturating what we find is that the surface potential or the potential drop in silicon psi_s also almost saturates at the value corresponding to the threshold voltage. So if you sketch psi_s as a function of voltage you will find the variation to be something like this. For V is equal to 0 psi_s is 0. At threshold it is phi_t and beyond threshold it almost saturates at phi_t 0. There is a small increase but that increase is very small. Therefore in inversion we can assume no matter what the voltage is so long as it is beyond threshold the surface potential is approximately equal to phi_{t0} .