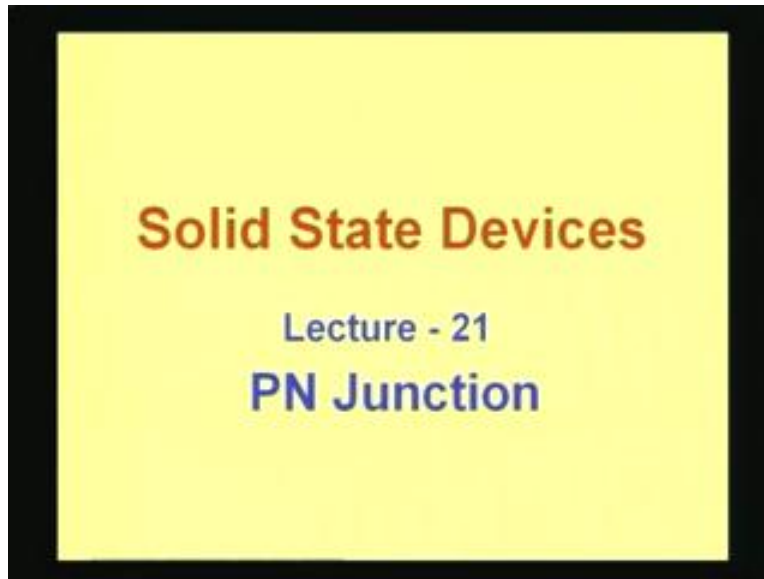**Solid State Devices**
Dr. S. Karmalkar
Department of Electronics and Communication Engineering
Indian Institute of Technology, Madras
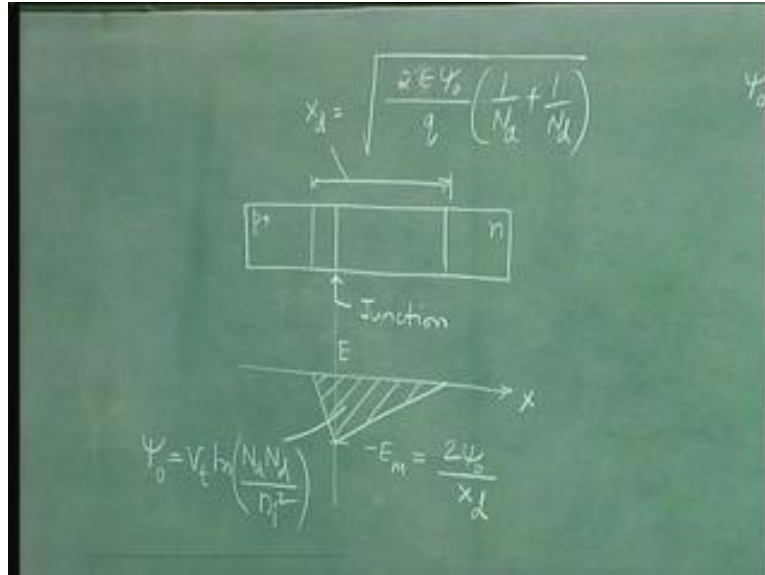**Lecture - 21**
PN Junction (Contd…)

This is the 3rd lecture on the PN junctions and the 21st lecture of this course.

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In the previous lecture we had completed the equilibrium analysis and we had obtained the concentrations of electrons and holes and then the electric field distribution in a space charge layer that is located near the junction. We also showed that the space charge layer can be assumed to be depleted of free carriers. Since J_n and J_p the electron and holes current densities are 0 under equilibrium we did not draw the distribution of J_n and J_p separately. The result of our analysis can be illustrated using this diagram.

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This is the depletion layer, this is the junction and this is the electric field picture under the depletion approximation. In this lecture we will continue with the equilibrium analysis. To start with what we will do is we will try to estimate the various parameters namely the depletion width, the peak electric field and the built-in potential for a typical situation. So let us look at the slide which gives the details of the solved example that we are going to consider.

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You have an abrupt and uniformly doped p-n junction with doping levels of 10 to the power 16 cm cube on the p-side and 10 to the power 15 cm cube on the n-side, We want

to estimate the depletion width, built-in potential, peak electric field and finally we will also estimate the average diffusion current densities of electrons and holes in the depletion layer. These diffusion current densities will be very useful starting points for the non-equilibrium analysis or the analysis under bias.

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The built-in potential psi_0 is equal to $v_t ln(N_a N_d$ by n_i square) so we can calculate this as follows at room temperature. So t is equal to 300 K that is the assumption. So we can write this as $v_t ln(10$ to the power 16 plus 15 minus 20 by 1.5 square) by n_i square. Here n_i is 1.5 into 10 to the power 10 for silicon. In the denominator you get 1.5 square (10 to the power 20). We will shift this power up here so that calculations become easy. It is useful to keep all the powers of 10 together and all the numbers separately as this avoids mistakes in calculation. So this is equal to $v_t ln$ 15 minus 20 minus 5 plus 16 is equal to 11 so 10 to the power 11 by 1.5 square. So you will get the result as 24.5 into v_t therefore you get about twenty five times thermal voltage built-in potential. Substituting v_t as 0.26 volts you get the result as 0.638 mV or 0.638 V which is about 0.65 volts which is the built-in potential of a p-n junction with 10 to the power 16 and 10 to the power 15 cm cube doping levels. If the doping levels change and they are more obviously your built-in potential will be more but note that the doping levels come in the logarithm.

Therefore the change is not so significant and you will find at the built-in potential hovers around 0.65 to 0.75 volts for various junctions in silicon. The depletion width 2 into epsilon the epsilon for silicon is this epsilon is epsilon silicon ϵ_0 the electric constant of silicon is 12 and this ϵ_0 is 8.85 into 10 to the power minus 14 Farad cm. When you multiply these two the result is approximately equal to 10 to the power minus 12 Farad cm. This is the number one can remember as epsilon for silicon. Since we want to collect all the powers of 10 together we will write this power here and also, do not forget to put the units separately. Now psi_0 is equal to 0.638 volts then comes the q(1.6 into 10 to the power minus 19) and in the denominator this will go up and it will become plus 10 to the power 19. Then 1 by N_d and this should be 1 by N_a plus 1 by N_d. So 1 by N_a will be 10 to the power minus 16 and 1 by N_d will be 10 to the power minus 15.

Notice that we can take out 10 to the power minus 15 out of the bracket so the result will be 1 here and 0.1 here so it is multiplied by 1.1. In fact it is a common practice when you have one side doping more than ten times the other side you can neglect the heavily dope side level. For example here N_a is 10 into N_d so one can neglect this. The error because of that will be only 5%.

Generally it is a practice that if N_a is more than 10 into N_d then you neglect N_a. The heavily doped side doping level is neglected. That is the depletion width X_d depends on the doping on the lightly dope side predominantly. This is the important conclusion that we get from here. So let us put the units for 1 by N_a and they will be cm cube. Let us put the unit for q that is Coulomb. In fact we must put the unit as when you put the number so here Farad into volt is Coulomb, this Coulomb gets cancelled and leads to cm square. If you take the square root you will get cm, so the units are correct because X_d is the depletion width. The result of this is that we collect these terms; 10 to the power minus 12 plus 19 minus 15 and it results in 10 to the power minus 8 and when you take the square root you will get 10 to the power minus 4 cm. Then we take this number and make

a calculation and you will find, this is 0.97 micrometer because 10 to the power minus 4 cm is 1 micrometer. This is the order of the depletion width. Therefore one can remember the depletion width to be of the order of a micron for a p-n junction doping levels of 10 to the power 16 and 10 to the power 15 on both the sides. If your doping levels are higher clearly your depletion width will be smaller. It is useful to remember values for one particular doping level because then you can find out the values for any other doping level very quickly.

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Finally the E_m the peak electric field is 2 into psi_0 by X_d is equal to 2 into 0.638 volts by X_d is 0.97 into 10 to the power minus 4 cm. If you shift this up you will get 10 to the power 4 which means tens of kilo volt and that is why your unit will be kilovolt cm for the peak electric field and the value turns out to be 13.2. Remember, the peak electric field to be the order of 10 to the power 4 volt cm or tens of kilovolts cm is the kind of electric field so it is a very high electric field. I emphasize that this high electric field is not resulting in any current because the current drift is balanced by the current because of diffusion. So that is why there is no current for either electrons or holes under equilibrium conditions.
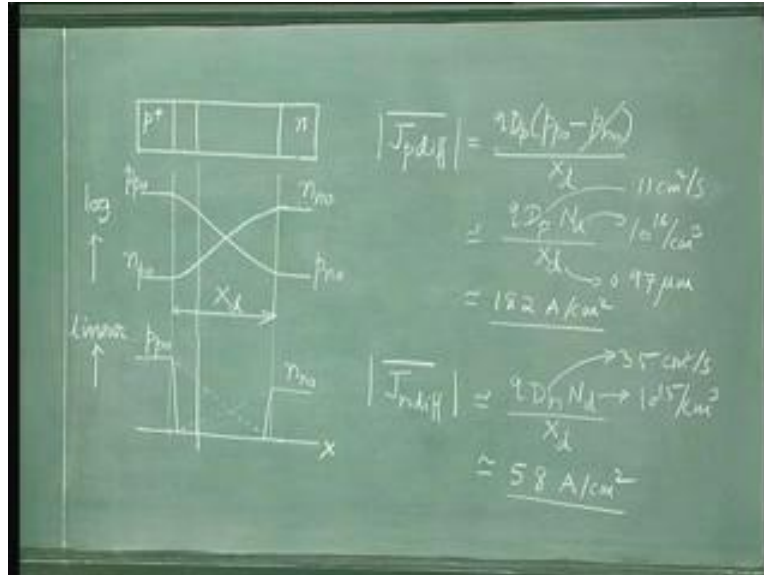
Finally we have the average diffusion and diffusion currents for holes and electrons. Let us look at this diagram which gives you the variation in concentrations on a log scale within the depletion layer. From here one can write the average diffusion current as, for example for holes J_p diffusion we will put a bar here to show that these are average values $qD_p(P_{p0}$ minus $P_{n0})$ by X_d so P_{p0} is the hole concentration on this side P_{n0} is the hole concentration on the other side and the gradient therefore can be written as P_{p0} minus P_{n0} by X_d. Now here the P_{n0} is much less than P_{p0} so you can neglect this. Further P_{p0} is approximately equal to the doping level N_a.

We are assuming complete ionization and neglecting thermal generation. So q D_nN_d by X_d is the average drift current. You can substitute the various values as N_a is n to the power 16 cm cube, this is 0.97 micrometer and this Dp for 10 to the power 16 cm cube we have to use the mobility versus doping graph or the equation and we will find this is approximately 35 cm square sec, this is for electrons and for holes it is about 11 cm square sec so you substitute everything and this will be about 182 ampere cm square which is a very high current density for diffusion currents. Again this is balanced by the drift current. We have shown that the field is very high so you are getting a high drift current and that is counter balancing this diffusion current.

Similarly, one can do for electrons N_{n0} minus N_{p0} is the difference in concentrations over the distance X_d and that will again reduce to q D_nN_d by X_d following the same approach as for holes. So this average current if you substitute the various value is 10 to the power 15 cm cube and D_n is 35 in cm square for sec for doping level of 10 to the power 15 from the mobility versus doping equation.

Substituting the various values the result is about 58 ampere cm square. This is the average diffusion current of electrons. We must emphasize that this average value should be used with caution. That is because if you see the concentrations on a linear scale

which correctly shows the variation then you find that the electron concentration is dropping rapidly and the hole concentration also dropping rapidly at the depletion edge.

The estimations we have done for J_p diffusion and J_n diffusion are based on the slope of these lines and these lines. So these estimates will be way off in fact the diffusion current is much higher. You can see the gradient is more here in these regions whereas in this region you will find the diffusion current values will be much less. That is what we mean by saying that average current which we have calculated should be used with caution. They are only very crude estimates to get a feel for the fact that the equilibrium situation in this case implies very high drift and diffusion currents exactly in balance within the depletion layer. Now let us summarize some of the quantitative values from this example and carry out to the rest of the course.

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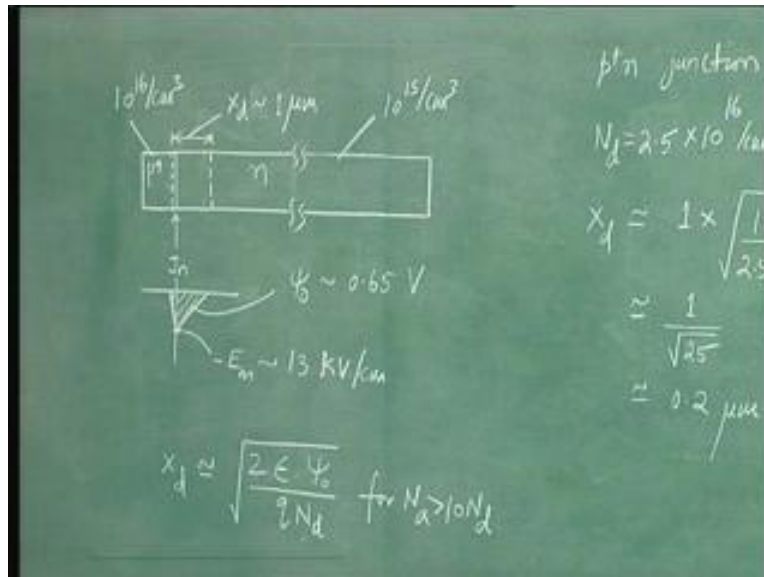
For your p to the power plus n junctions the p-side is of the order of microns whereas the n-side is of the order of several tens of microns. In fact we saw this was 1 micron and this side was 100 micron in our real device structure. As we have said we will be assuming that these two regions are sufficiently longer than their diffusion lengths.

Now, the depletion width here is of the order of 1 micron for 10 to the power 16 cm cube and 10 to the power 15 cm cube doping levels. So, if we remember the values for this particular doping level then we can make calculations for other doping levels very easily. This is the electric field picture and the area under this is a build-in potential that is of the order of 0.65 volts for the doping levels under consideration and the peak electric field is tens of kilo volt cm. This is what one must remember.

This is a diagram which is not exactly to scale but it is somewhat to scale in the sense that here you can see the depletion width and the width of the p-region are almost of the same

order whereas this n-region width is very long compared to the depletion width. This is the practical situation in p plus n junction that you encounter.

Finally the depletion width formula for p to the power plus n junction is where N_a is more than 10 into N_d again this is the kind of situation we will encounter in practice. It is very simple as it is 2 into epsilon to the power star psi_0 by q into N_d. This formula is very easy to remember because the q always goes with the doping because this is the ionized impurity charge and this is the space charge in the depletion layer. And then you know that the depletion width will be more if the built-in potential is more. This is very easily understood based on our knowledge of Physics. We also know that the depletion width will be less if the doping is more. So the doping is in the denominator and the built-in potential is in the numerator. The q always goes with the charge.

Now, to get the correct dimensions you can see that the dielectric constant should appear along with the square. This factor of 2 comes because the electric field is triangular in shape and the area under the electric field consists of half of depletion width into the peak electric field so that half is coming here on this side as a 2 factor and then you have a square root. Now let us see that if you know these values how you can make a quick estimate for other conditions. One example is shown here.

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Supposing you have an example of p to the power plus n junction with n-side doping as 2.5 into 10 to the power 16 then how do we quickly estimate the depletion width without doing detailed calculations?

Now we remember that for 10 to the power 15 cm cube it is 1 micron and we know that the depletion width is inversely proportional to the square root of doping and the depletion will be less if the doping is more. For this case the depletion width has to be less than 1 micron case. Therefore in the square root you put 10 to the power 15 in the

numerator and this value in the denominator and you end up with 1 by square root of 25 which is about 0.2 micrometer. So, one can make this kind of a calculation by inspection. That is the advantage of remembering this approximate formula for a p plus n junction.

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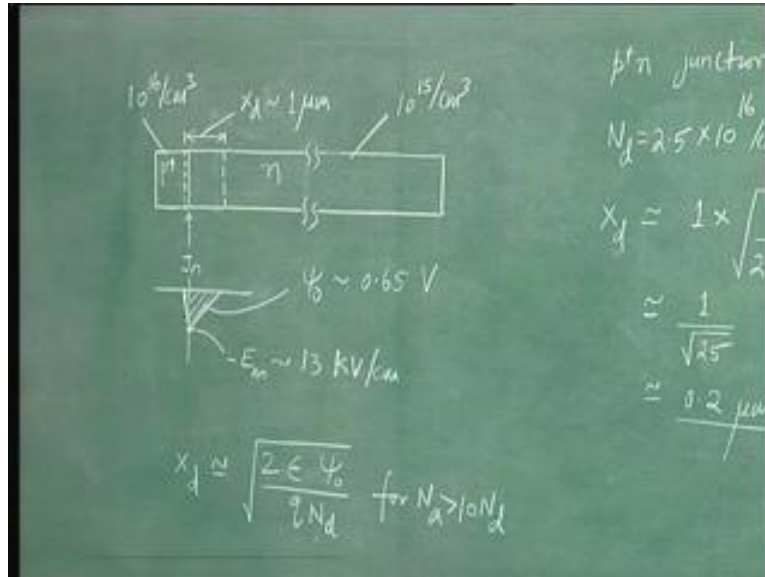
You can always translate this formula for n to the power plus p junction. For example, if N_d is more than 10 into N_a then this N_d here will be replaced by N_a. Always the depletion width depends on the lightly doped region. So that is the end of the solved example. So far in our analysis we have not used the energy band diagram. From the five basic equations we have derived the information about n, p, J_n J_p and e. We have estimated the built-in voltage, the depletion width, the peak electric field and so on. Now we would like to look at the energy band diagram under equilibrium for a p-n junction. Then we will see what advantage the energy band diagram offers and what new information does it give.
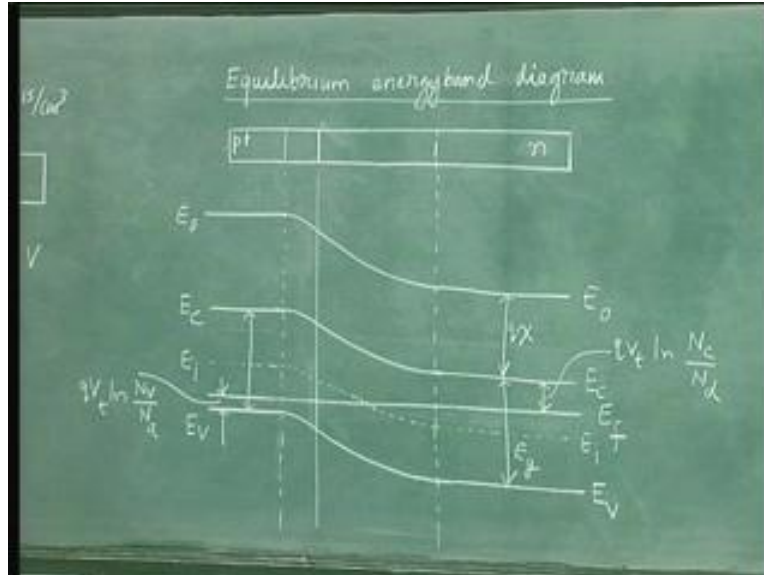
To draw the energy band diagram what is the starting point. The starting point is a constant E_f as a function of x. You would recall that in an earlier lecture when we were discussing the procedure for device analysis we considered the equilibrium situation and then we showed that if the transport equations are written in terms of the Quasi Fermi-level gradients then J_n is equal to 0 and J_p is equal to 0 in equilibrium translated to the gradient of the Quasi Fermi-levels is equal to 0 which means that the Quasi Fermi-levels are constant with distance and since the electrons and the hole Quasi Fermi-levels are same in equilibrium it amounts to saying that the Fermi-level is constant with distance. So this is the starting point for drawing the energy band diagram. Once this is done the next step is to draw the energy band diagrams in the neutral p and n regions. So using the doping levels we can sketch the E_c and E_v. So on the n-side for example the E_c will be close to E_f but since this is a neutral region and is uniformly doped the E_c will be flat.

You can look at the E_v below this at a distance of energy gap. We can repeat a similar thing on the p-side since the p-side doping level is more than n-side doping level. E_c minus E_f will be more than E_f minus E_v so E_v will be closer to E_f than E_c to E_f on the n-side. This is your E_v and then we can place the E_c at a distance of 1 energy gap so this is E_c.

Now we need to join this end of E_c to this end of E_c and we need to join this end of E_v to this end of E_v. While joining this we must remember that the E_c and E_v must be continuous even though they vary with distance. This is because any gradient of E_c represents the electric field. This point was discussed in lecture on Carrier Transport that the gradient of the band edges E_c and E_v represents the electric field. Now, you cannot have infinite electric field anywhere so E_c and E_v variations cannot have any discontinuities. So you must draw continuous variations so that is the E_c and E_v variations in the depletion layer. The next is we can draw E_I that is the intrinsic level in between E_c and E_v.

The intrinsic point is somewhere here where E_I crosses E_f. It lies on the lighter doped side. Now we can also draw the vacuum level E_0 at a distance electron affinity from E_c. Let us start from this end so this is E_0 and in the same distance you should have E_0 and now we join them by a continuous line that is E_0 where this distance is electron affinity and this distance E_g energy gap. This electron affinity is in volts so normally you multiply by q to indicate that it is energy and this is energy gap. Now this E_c minus E_f depends on a doping level. So this E_c minus E_f is nothing but q into $v_t \ln N_c$ by N_d. On the other hand this E_f minus E_v is nothing but $v_t \ln N_v$ by N_a a multiplying factor of q because it is energy. So this energy band diagram of the p-n junction under equilibrium.

Now this has been sketched based on two principles; one is constancy of E_f and second is the continuity of the band edges i.e. E_c E_v and E_0. In this particular case since we are considering a homo junction i.e. this side and the other side p and n sides both are made of the same material E_0 E_c and E_v all three were discontinuous and in fact E_0 was not necessary while drawing the energy band diagram of E_c E_v and E_f so E_0 was drawn at the end.

However, in the case of hetero junctions the picture is somewhat different because when the materials are different on the p and n side your energy gaps are not the same on p and n sides and therefore if you look at this diagram if this energy gap is not the same as this energy gap then obviously somewhere there will be a discontinuity either in E_c or in E_v or in both. Now that being the case it is important to note that in such situations it is the E_0 which should be maintained continuous because E_0 represents the energy corresponding to an electron which is just outside the crystal.
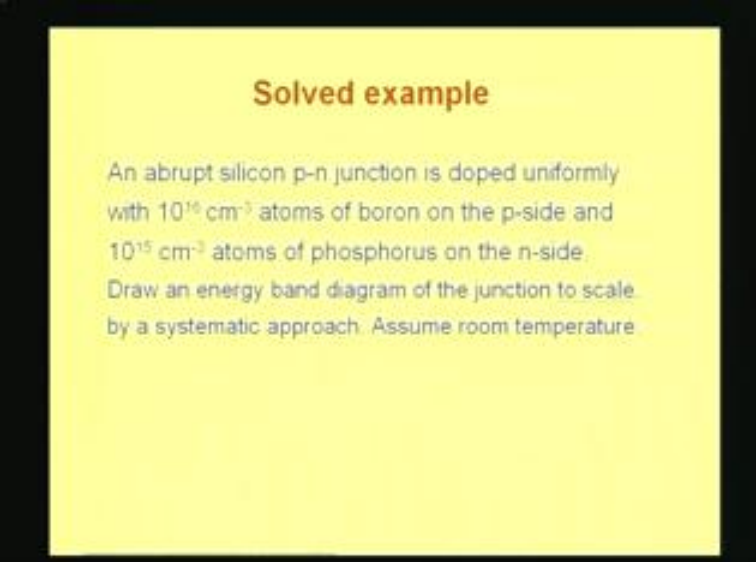
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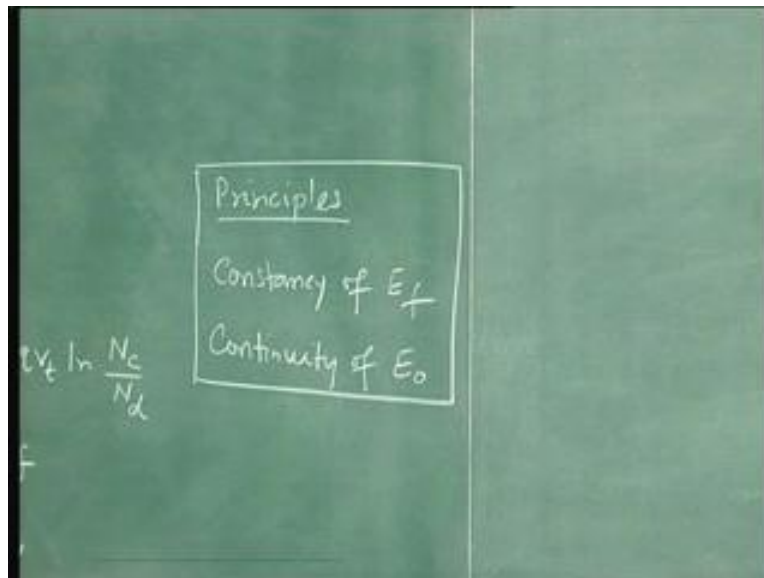
So electron here is the p-n junction. Now if you move the electron along this junction touching this junction from outside it is obvious that this electron should not encounter any infinite electric field here as that will be non physical. Since the gradient of E_0

represents the electric field at this location here therefore the E_0 should be a continuous line because any discontinuity in E_0 would mean an infinite electric field for this electron wherever the E_0 is discontinuous and that is not possible. That is why one states the two conditions for drawing energy band diagram in general, under equilibrium conditions and it means E_f should be constant and E_0 should be continuous.

The fact that E_c and E_v are continuous in a homo junction follows directly from the continuity of E_0 because E_c should be drawn here where the electron affinity is below the E_0 so E_c is parallel to E_0. And since E_0 is continuous and electron affinity is the same throughout then E_c which is parallel to E_0 should also be continuous and E_v parallel to E_c should also be continuous because energy gaps are also the same. We can summarize this by writing the two principles: Constancy of E_f and continuity of E_0. They are the principles for drawing energy band diagram. Now we will work out an example to give you a feel for the values of some of the parameters on the energy band diagram and also how an energy band diagram can be drawn for a given condition.

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An abrupt silicon p-n junction is doped uniformly with 10 to the power 16 cm cube atoms of boron on the p-side and 10 to the power 15 cm cube atoms of phosphorous on the n-side. Draw an energy band diagram of the junction to scale by a systematic approach. Assume room temperature. Note that the phrase to scale has been underlined so various parameters should be drawn to scale to get a correct picture.
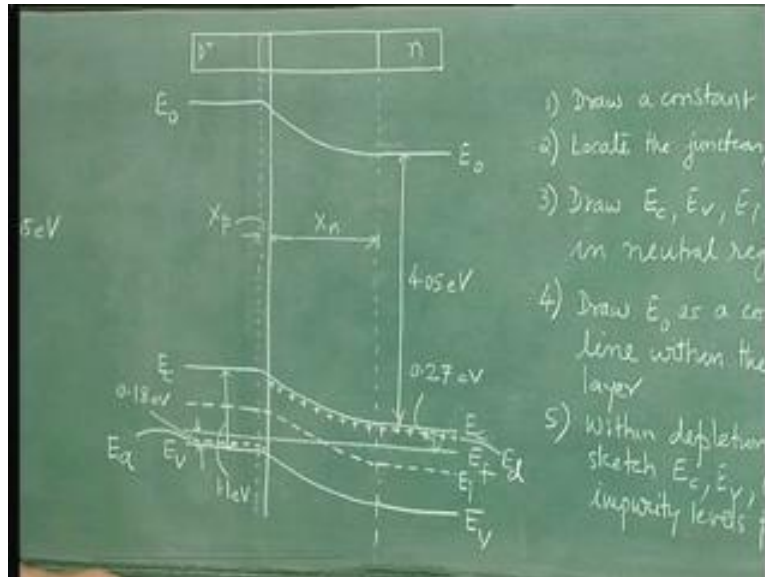
The first step in the drawing of energy band diagram is a constant E_f, so draw a constant E_f. The second step is to locate the junction. Here the junction has been located on this constant E_f line. And then after locating the junction we should locate the depletion edges. In this case it is the depletion edge on the p-side which is located at a distance of X_p. Before drawing the energy band diagram the calculations of the depletion width, built-in potential etc should be completed. So X_p is on this side and X_n on the other side so this is X_n.

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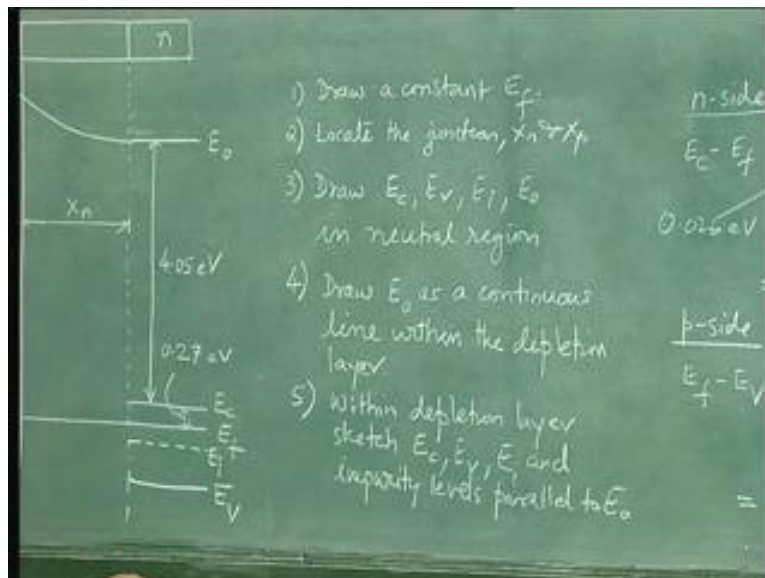
Locate the junction X_n and X_p. So X_n and X_p represent the depletion edges. After doing that draw the diagram in the neutral regions p plus and n regions. So, the third step is to draw E_c E_v E_I and E_0 in neutral region. So to do that we must know where E_c is on the n-side, this can be done as follows.

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You can use the formula E_c minus E_f on the n-side is equal to KT l_n N_c by N_d. We are assuming complete ionization and neglecting thermal generation so the majority carrier concentration is equal to the doping. Now you substitute the values, for silicon this is 10 to the power 19 into 2.8 cm cube and this is 10 to the power 15 cm cube and KT at room temperature is 0.026 electron volts. The result is 0.27 electron volts. So we locate E_c here and the 0.27 electron volts is the difference. If this is 0.27 we must locate the E_v at one energy gap below so energy gap is 1.1 that is almost four times this. Since we want to draw it to scale we must choose four times this and somewhere here is your E_v and E_I is about halfway and it is here, this is E_I.

Now we should complete a similar exercise on the p-side. So, for p-side you have E_f minus E_v is equal to KT l_n N_v by N_a. When you substitute the value of N_v which is 10 to the power 19 cm cube in silicon and N_a is 10 to the power 16 cm cube and so the result is at room temperature this is 0.18 electron volts. Since we want to draw it to scale notice that 0.18 is 2 by 3 of 0.27 so it is 2 by 3 of this distance you must locate the E_v here so this is E_v. And now we should locate the E_I and E_c, E_I and this is half of energy gap about that distance and then this is E_c. So this difference is 0.18 electron volts and this difference is 1.1 electron volts energy gap. We should also locate E_0 so E_0 minus E_c is electron affinity. The electron affinity chi into q is 4.05 electron volts for silicon so this distance is 1.1 and 4.05 would be little less than times this distance. So we must locate four times this distance here and somewhere here is your E_0 and this difference is 4.05 electron volts. We do the same thing here, this is 1, this is 2 and this is about 2½ so 1 then

2 and 2½ is somewhere here i.e. E_0. So on this side also the difference is 4.05 electron volts.

The next step is to complete the band diagram in the depletion layer. So the fourth step is to draw E_0 as a continuous line with the depletion layer. So you can draw a line like this, now what is the shape of this line?
This line indicates the potential within the device. We have already sketched the potential as the function of x since the electric field is linear the potential is parabolic so you have one parabola here and another parabola here. They are the two parabolas, so this follows exactly the shape of the potential expect that the potential which you plot for a positive charge where as this energy band diagram is for negative charge, that is the difference so having drawn E_0 you can draw E_c E_v and E_I as parallel to E_0. Within the depletion layer sketch E_c E_v E and impurity levels parallel to E_0.

Let us do that exercise, so parallel to E_0 we sketch E_c then sketch E_v and E_I and also the impurity levels. Where will the impurity levels be? On the n-side the donor level is close to E_c but at a distance of 0.05 electron volts. If you compare 0.05 electron volts which is the ionization energy of phosphorus with 0.27 is little more than 5 into 0.05 which means that 1 by 5 of this distance between E_c and E_f is the impurity level. So approximately we are locating here so this is the donor level. In the entire region the donor level is very close to the conduction band edge so this is E_d. similarly one can draw E_a on this side so E_a is also at 0.05 electron volts from the valance band edge, E_a is the acceptor level. And notice the acceptor level has to be drawn parallel to E_v so donor level is also going to bend like the conduction band edge and other levels like E_I and E_d. So with this the energy band diagram is complete. This is how energy band diagram will look like when drawn to scale for a silicon p-n junction.
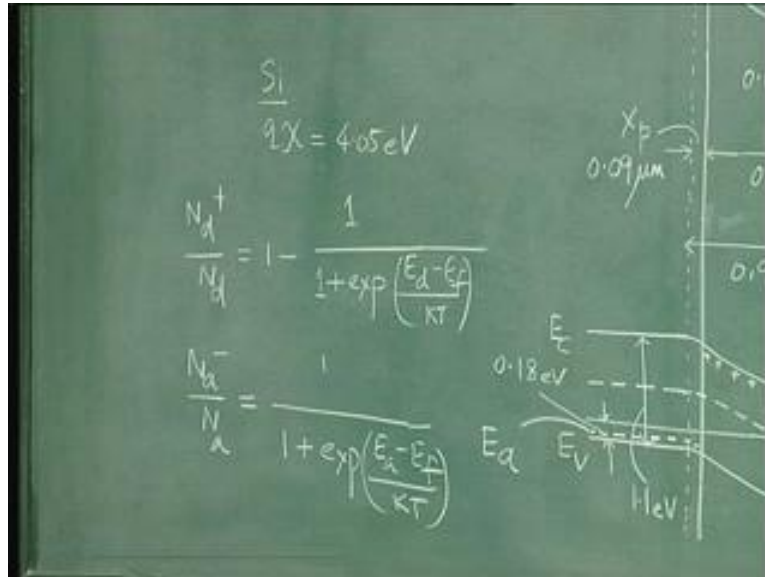
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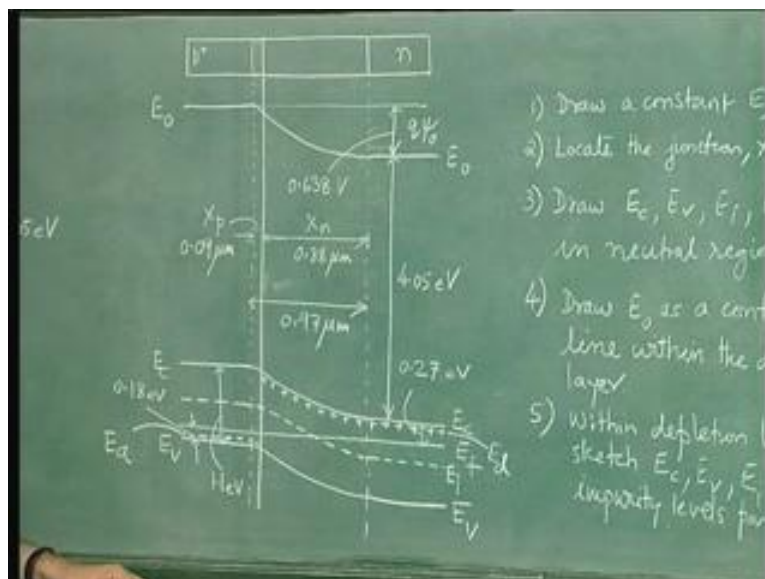
Now let us look at the some of the advantages of this diagram. What information does it give that we have not obtained from our analysis based on the five basic equations where we drew n, p, J_n J_p and e as a function of x.

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First thing here is the built-in potential is the difference between the E_c on the two sides or it could be E_v on the two sides or E_I on the two sides or it could also be E_0 so this difference is a built-in potential which is 0.638 volts. We have already calculated this value, what about this distance X_n and X_p? This X_n plus X_p total distance is 0.97 microns as we have already calculated this.

Now, if doping is ten times on the p-side as compared to n-side you will end up getting a value of X_n as 0.88 micrometer and you get X_p to be 0.09 micrometer. Coming back to our discussion on what advantage does the energy band diagram offer, notice that the donor level is varying with distance here. It means that the extent of ionization is changing with distance within the depletion layer because the difference between E_d and E_f is what shows the extent of ionization. You know that N_d plus by N_d is equal to 1 minus 1 by 1 plus exp of E_d minus E_f by KT. Here you have to multiply by degenerating factor but we are ignoring this degenerating factor in our course as we decided in the beginning. This is the equation that we have written down in one of our earlier lecture for ionized impurity concentration.

Similarly one can write the expression for N_a to the power minus by N_a and this is nothing but 1 by 1 plus exp of E_a minus E_f minus E_f by KT. So the difference E_d minus E_f decides the extent of ionization and the difference E_a minus E_f decides the extent of ionization for boron. Since E_d minus E_f is changing with distance it means that the extent of ionization is changing. In our problem the extent of ionization is almost complete. This is not really giving any significant information because already the extent of ionization is complete. And if it changes a little bit it does not make any difference.

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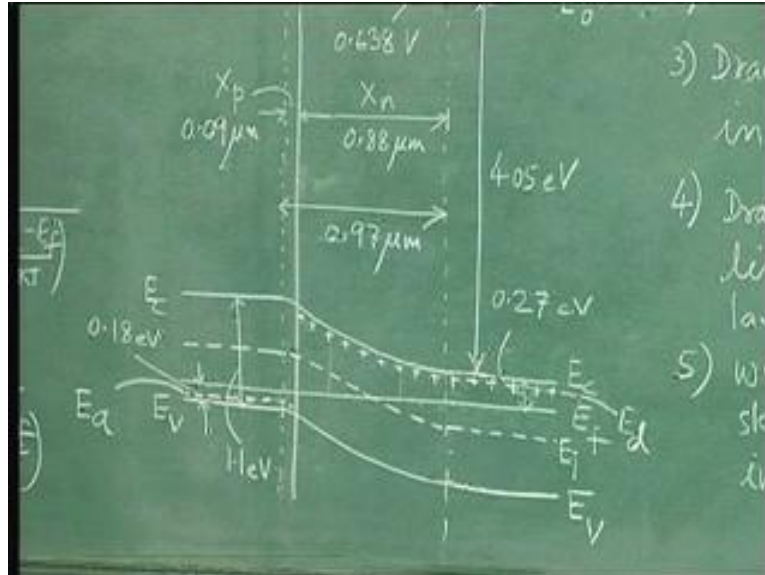
But you will note that if the doping level is heavy and if this Fermi-level moves close to the donor level then the extent of ionization changed shown by this difference E_f and E_d will be significant and your space charge picture will be therefore affected. In other words, in the space charge region your space charge concentration will not be exactly q into N_d but it will be q into N_d. Let me show this with an example, Apart from the shallow donor level which is present there let us say there was a deep level also called as the donor level, and the deep donor level is E_I the intrinsic level itself then what would happen? Look at the diagram.

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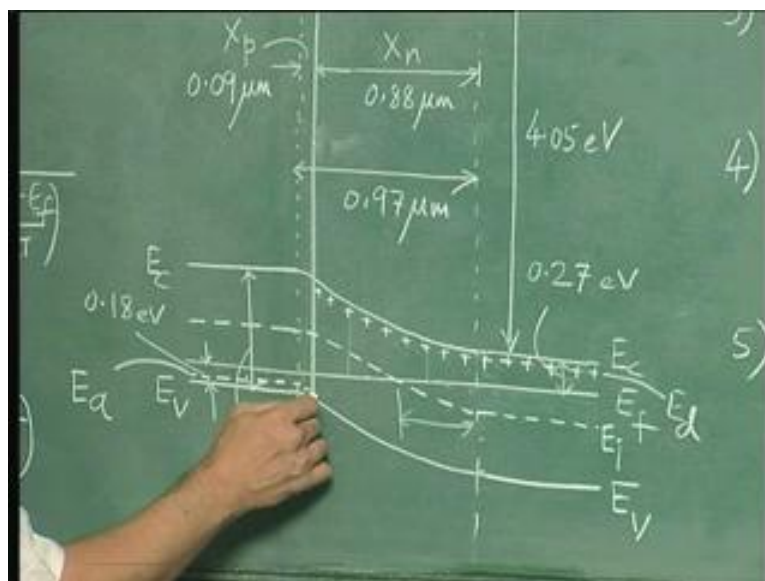
So if you have a donor level along E_I then you can see that in this portion of a depletion layer the deep donor level is below E_f. On the other hand, in the remaining portion of the depletion layer on the n-side, this donor level would be above E_f. Since the levels below E_f are all occupied a donor level below E_f means an occupied donor level which also means unionized donor impurity.

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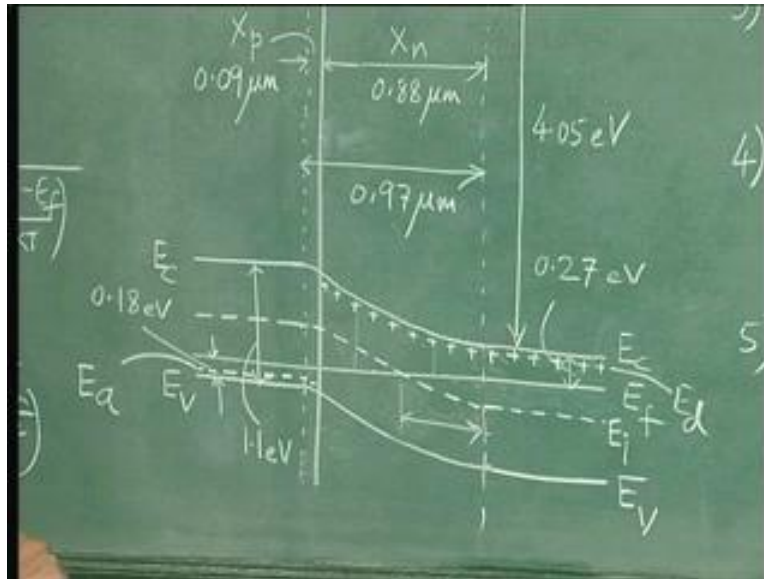
So in this region you will have partial ionization. In fact ionization will be very poor. Gradually the ionization will improve as you move and will become very good near the junction. So from partial ionization to complete ionization the state of ionization of impurity is changing within the depletion layer. Therefore your space charge picture also will be affected. Let us draw this space charge picture for an impurity that is crossing the Fermi-level.

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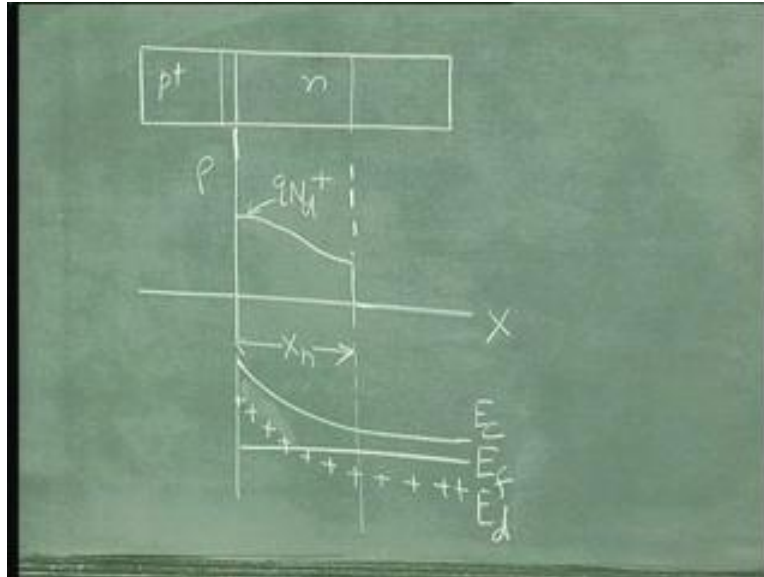
We are assuming a donor impurity which crosses the Fermi-level and the picture being as shown in the band diagram here. So a portion of the band diagram which shows E_f crossing E_d is shown here. This is the depletion region, this X_n is the portion of the depletion layer on the n-side. If you draw the space charge picture it would be something like this; q into N_d to the power plus where N_d to the power plus the ionized impurity concentration. So information as such is revealed by the energy band diagram which is something cannot be obtained without the help of the energy band diagram. Let us summarize how the information given by the five basic equations is reflected in the energy band diagram.

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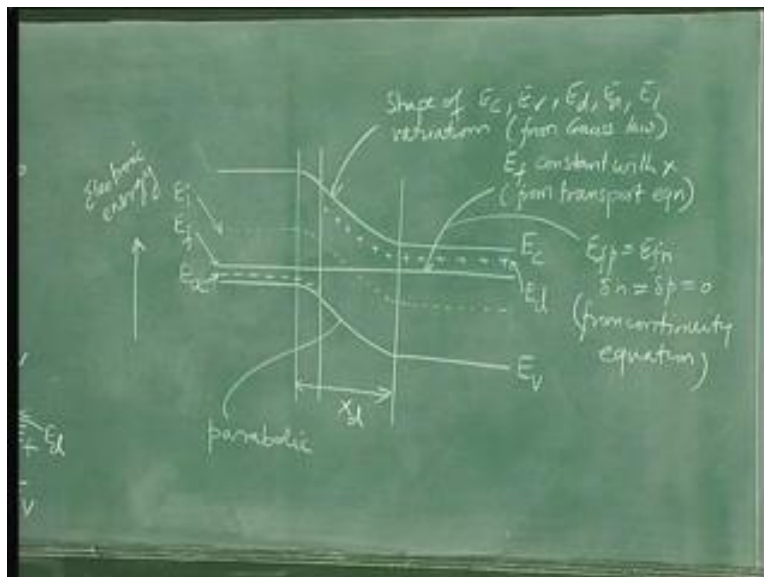
Let us look at this picture. The fact that J_n is equal to 0 and J_p is equal to 0 that is the information from the transport equations is reflected in the fact that the Fermi-level is constant. The Fermi-level is constant with x so E_f is constant with x. This is the information from the transport equation. Next is the fact that there are no excess carriers that is the same E_f applicable for both electrons and holes so E_{fp} is equal to E_{fn} or delta$_n$ is equal to delta$_p$ equal to 0. This is the information from the continuity equation so the effect of the continuity equation is again shown in the same Fermi-level which is drawn here from the fact that the same Fermi-level is being used for electrons as well as for holes.

What does the Gauss's law say or how information is reflected in this energy band diagram from the Gauss's law? This is reflected in the shape of the E_c E_v and other levels in the energy gap shape of E_c E_v E_d E_a and E_I variations. This is obtained from Gauss's law. For example in this region the E_c is flat which means there is no electric field here and also there are no space charges where the shapes will be parabolic. So this parabolic shape is again obtained from Gauss's law. These are electronic energies and within this region the shape is parabolic. This is again obtained from the Gauss's law. This is how the information from the five basic equations is clearly reflected in the energy band diagram. So there is a one to one correspondence between various aspects of energy band diagram and the five basic equations.

With this we come to the end of these lectures wherein we considered the equilibrium analysis quantitatively. We solved an example to show the typical values of depletion width, built-in voltage and the peak electric field. Then we also saw how we can draw energy band diagram under equilibrium conditions.