Solid State Devices
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Lecture - 18
Procedure for Device Analysis (Contd..)

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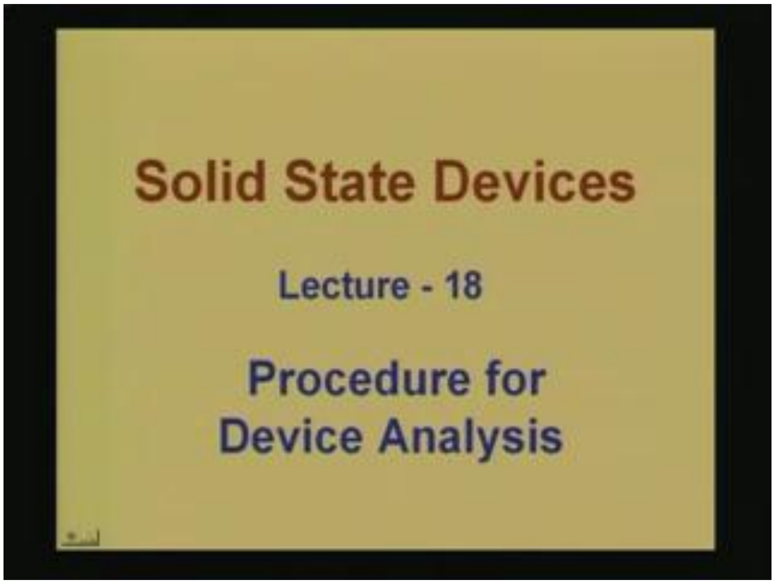

Solid State Devices

Lecture - 18

Procedure for Device Analysis

In the last lecture we began a discussion on the Procedure for Device Analysis. In this lecture we will continue with this discussion. In the last lecture we introduced the continuity equation and then summarized the five basic equations of device analysis.

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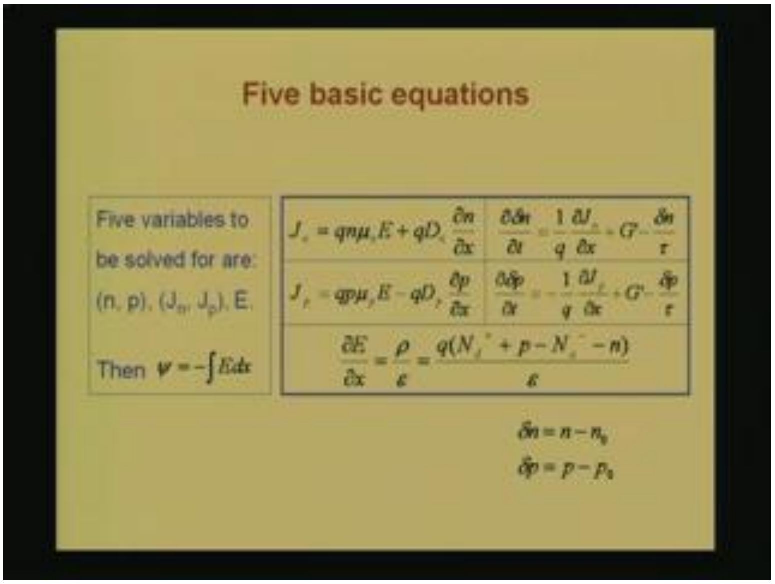

Five basic equations

Five variables to be solved for are: (n, p), (J_n, J_p), E. Then $\psi = -\int E dx$	$J_n = qn\mu_n E + qD_n \frac{\partial n}{\partial x}$	$\frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G - \frac{\delta n}{\tau}$
	$J_p = qp\mu_p E - qD_p \frac{\partial p}{\partial x}$	$\frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G - \frac{\delta p}{\tau}$
	$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon} = \frac{q(N_d^+ + p - N_a^- - n)}{\varepsilon}$	

$\delta n = n - n_0$
$\delta p = p - p_0$

We saw some methods of approximating these complicated looking equations. Some of the approximations we saw were the equilibrium approximation in which the current J_n and the current J_p can be assumed to be 0.

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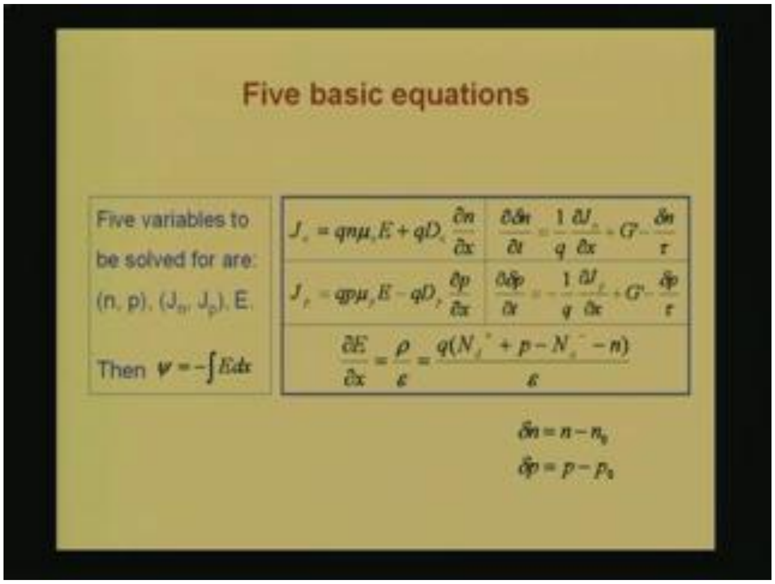

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$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon} = \frac{q(N_d^+ + p - N_a^- - n)}{\varepsilon}$

$\delta n = n - n_0$
$\delta p = p - p_0$

Then we saw a deviation of this approximation called the Quasi Equilibrium when the equality signs can be replaced by approximately equal signs and then these currents J_n and J_p can be set is equal to 0 approximately. Then we saw the steady state approximation or assumption according to which this particular time varying term in the continuity equation for electrons and a similar term for the holes can be set is equal to 0. In a deviation of this approximation called the Quasi Steady State approximation; we can still set these two terms as 0 but the equality sign is replaced by an approximately equal sign here. That means the right hand side of these two continuity equations are approximately equal to 0 in Quasi Static approximation. Then we saw the diffusion approximation in which the currents for any particular type of carrier under certain situation can be assumed to be solely due to diffusion. So for electrons it would be this term and for holes it would be this term. Under the diffusion approximation and steady state conditions and when the semiconductor is uniformly doped we have seen that the continuity equation simplifies to an equation whose solution is exponential or hyperbolic in nature. In this particular lecture we will continue with the other approximations which are possible. Let us look at the approximation related to the space charge.

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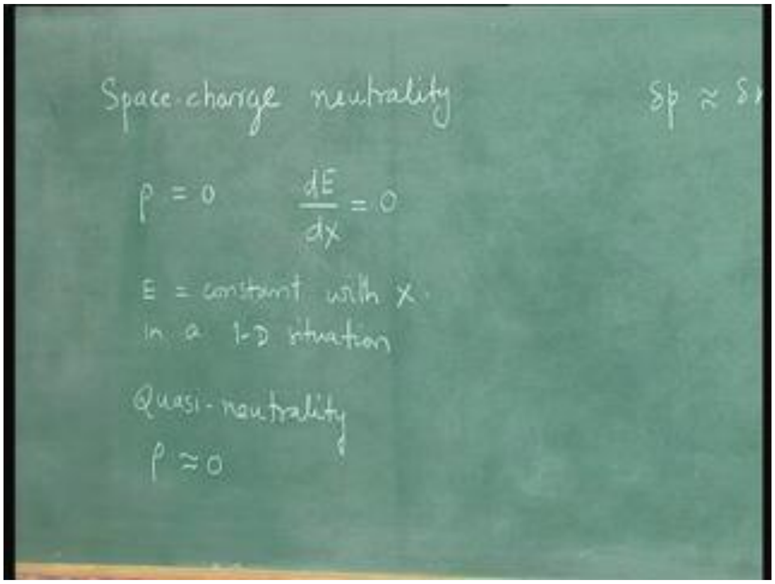

Space-charge neutrality

$\delta p \approx \delta$

$\rho = 0 \qquad \frac{dE}{dx} = 0$

E = constant with x

in a 1-D situation

Quasi-neutrality

$\rho \approx 0$

Space charge neutrality: What does it mean? So space charge neutrality means that the term rho can be set is equal to 0 in the Gauss's law where rho is the space charge. So if it is a one dimensional situation then it means dE by dX is equal to 0 which in turn means that the electric field constant with x in a one dimensional situation. So in any region of the semiconductor if the space charge is 0 it means that the electric field is constant with x if it is a one dimensional situation. Now this charge neutrality condition which holds the many situations is the Quasi neutrality condition. As per which even if rho is not exactly is equal to 0 it can be assumed to be approximately is equal to 0. Now, what is the implication of this Quasi neutrality or charge neutrality conditions? How does it help us to simplify the analysis? According to these Quasi neutrality condition for example the Excess Carrier Concentration of holes and electrons are approximately equal.

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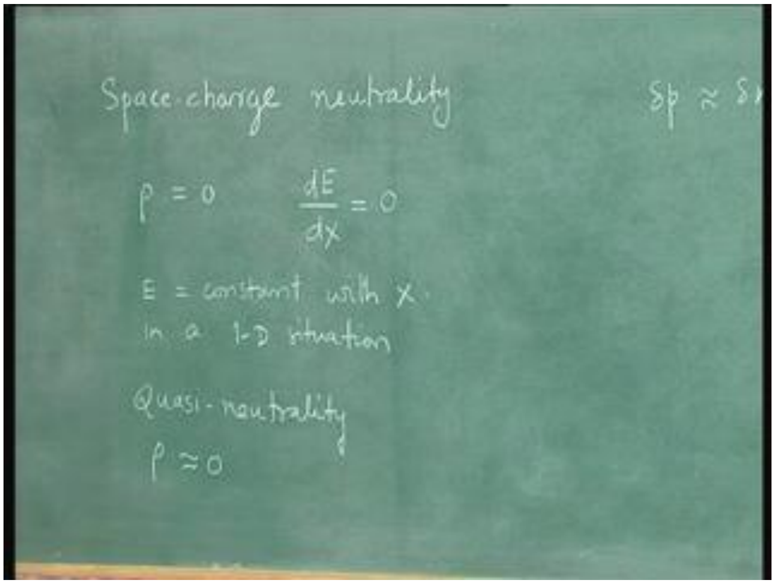

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$\delta p \approx \delta$

$\rho = 0$ $\frac{dE}{dx} = 0$

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in a 1-D situation

Quasi-neutrality
$\rho \approx 0$

If the semiconductor under equilibrium conditions is obeying space charge neutrality and now if you are creating Excess Carriers in the semiconductor then even if your Excess Carriers delta p and delta n are not exactly equal but they are approximately equal which means if Quasi neutrality holds then delta p is approximately equal to delta n and in such a situation the simplification is that you only need to solve either for delta p or for delta n and not for both.

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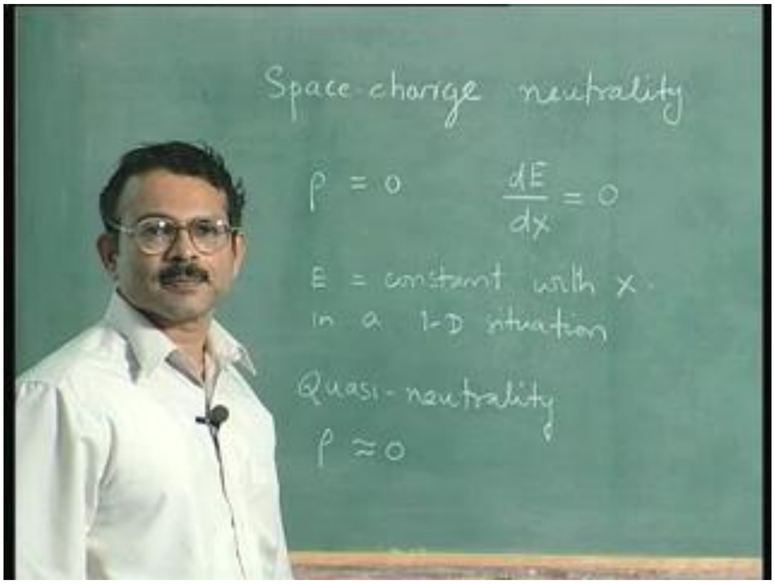

Space-charge neutrality

$\rho = 0$ $\frac{dE}{dx} = 0$

E = constant with x.
in a 1-D situation

Quasi-neutrality
$\rho \approx 0$

Therefore noting the fact that the Gauss's law is also simplified under the neutrality or Quasi neutrality condition that is we do not have to solve the Gauss's law equation at all.

We have removed Gauss's law equation that is one equation out of the five. And further since delta p is equal to delta n approximately we can either solve for holes or electrons which means we need not solve for hole continuity equation and hole transport equation and electron continuity equation and electron transport equation. Instead we could solve any one pair of equation. So the Quasi neutrality condition reduces the complexity of the problem since we only need to solve two equations out of the five, any pair of equations either for holes or for electrons. By pair I mean the transport equation and the continuity equation.

Now let us understand this delta p approximately is equal to delta n clearly. In keeping with the explanation that we have given for Quasi Equilibrium and Quasi Steady State or Quasi static approximations delta p approximately is equal to delta n means that the individual terms delta p and delta n the values of these are much greater than the difference between these.

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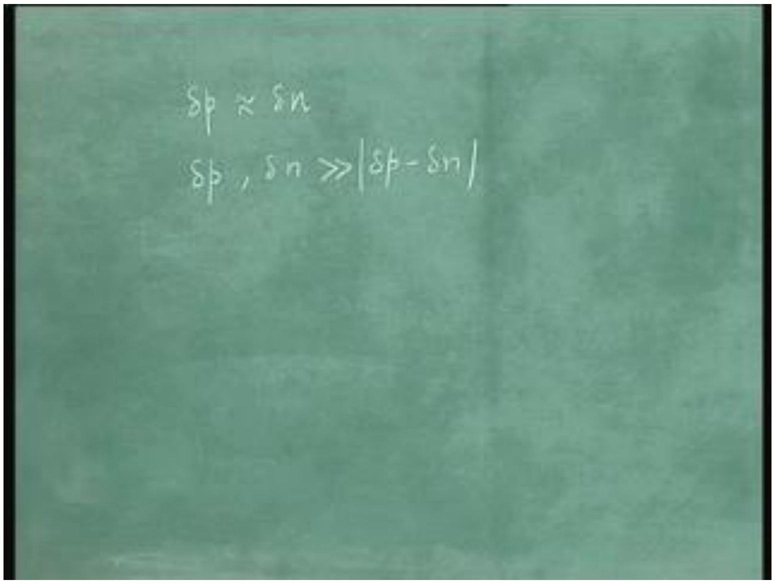

$\delta p \approx \delta n$

$\delta p\, ,\ \delta n \gg |\delta p - \delta n|$

So delta p and delta n are individually very large and the difference between them is very small as compared to this. This is very important to remember because you may find the delta p minus delta n may be a significant quantity in absolute terms. But still you can say Quasi neutrality holes if this difference is much less than the individual magnitude of delta p and delta n. We will be illustrating some of these ideas with the help of example a little later.

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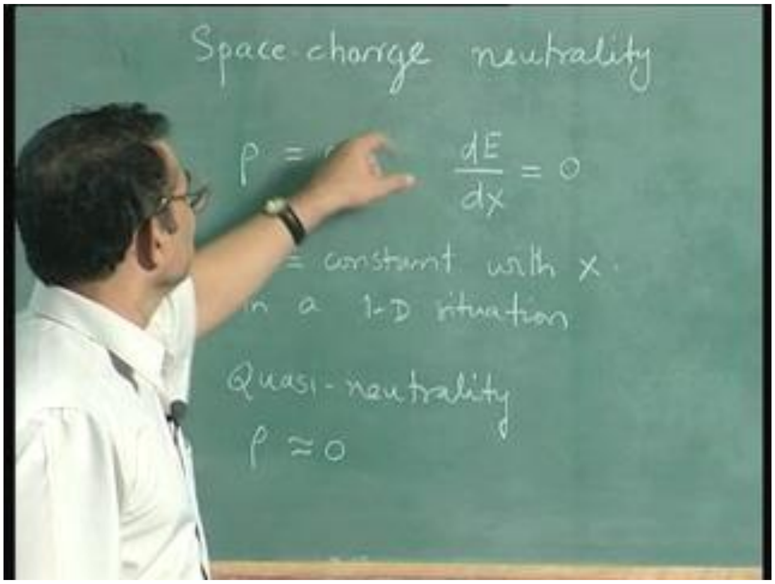

Space charge neutrality

$\rho =$ $\quad \frac{dE}{dx} = 0$

$=$ constant with x.

n a 1-D situation

Quasi-neutrality

$\rho \approx 0$

Now let us look at another approximation related to the space charge and this is the depletion approximation.

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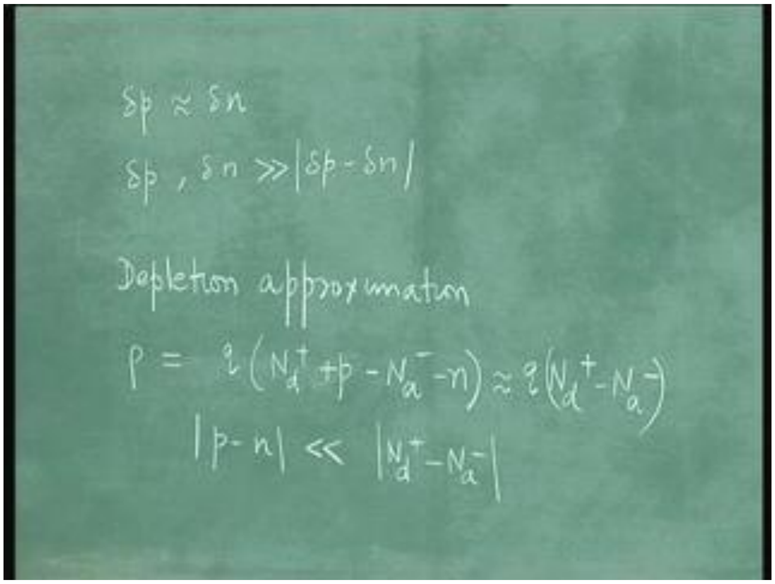

$\delta p \approx \delta n$

$\delta p \,,\, \delta n \gg |\delta p - \delta n|$

Depletion approximation

$\rho = q(N_d^+ + p - N_a^- - n) \approx q(N_d^+ - N_a^-)$

$|p - n| \ll |N_d^+ - N_a^-|$

The depletion approximation means that it can happen that in a given region of the semiconductor the free carrier concentration is very small. So, in terms of the equation this could mean when we write rho is equal to q (N_d to the power plus plus p minus N_a to the power minus minus n) means you can have a situation where this is approximately is equal to q(N_d plus minus Na to the power minus) so this is the meaning that the free carrier concentrations p and n are negligible. Again let us understand this meaning of p and n being negligible. Actually you can see from this equation that it is not strictly the p and n in individually that need to be very small because what matters is p minus n. If this p minus n is much less than this quantity N_d plus minus N_a minus you can say the depletion approximation holds. So this |p minus n| in magnitude is much less than |N_d to the power plus minus Na to the power minus|. So this is the meaning of depletion approximation. Now the depletion approximation is often used to describe space charge layers which occur near junctions such as PN junctions, the MOS junction and so on. We will see this when we take up these devices later on. As a corollary to the depletion approximation we will introduce the concept of what is called the Debye length.

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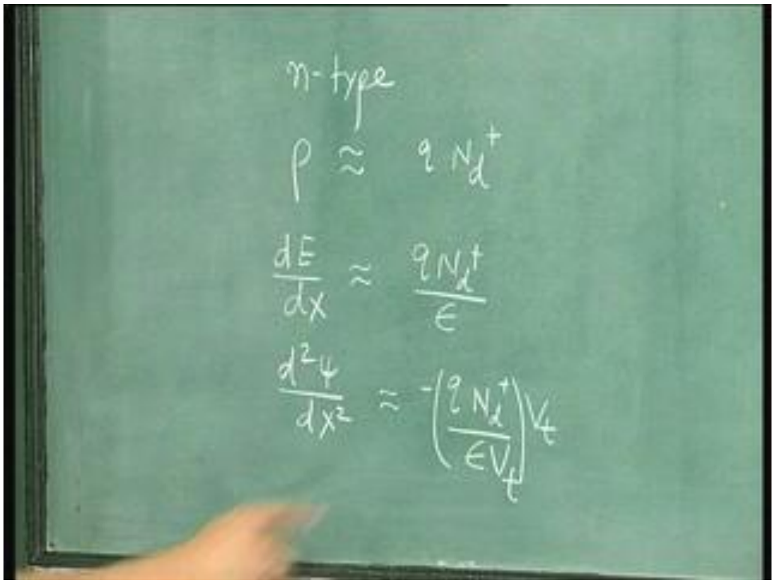

n-type

$$\rho \approx q N_d^+$$

$$\frac{dE}{dx} \approx \frac{qN_d^+}{\epsilon}$$

$$\frac{d^2\psi}{dx^2} \approx -\left(\frac{q N_d^+}{\epsilon V_t}\right) V_t$$

The Gauss's law can be written in the case of depletion approximation as follows: Let us take n type semiconductor. Further let us assume that there is only one type of doping that is donor doping there is no acceptor doping. If that is the case then under depletion approximation the space charge can be written as q into N_d to the power plus. So this is an expression for the space charge under the depletion approximation. We have removed the p and n terms because the region is depleted of free carriers and there is no N_a minus.

So Gauss's law becomes dE by dX assuming a one dimensional situation is equal to q into N_d to the power plus by epsilon. We can write the dE by dX in terms of the potential. In other words, we can write the Gauss's law as the Poisons equation. So when we do that you put E is equal to minus dpsi by dx you will get d square psi by dx square is minus qN_d to the power plus by epsilon. We can transform this equation by multiplying and dividing by the thermal voltage. Now let us look at this term in brackets. The dimension of this term is 1 by L_D square because dimension of V_t is potential and left hand side you have potential by length square so this term q N_d to the power plus by epsilon V_t is 1 by L_D square.

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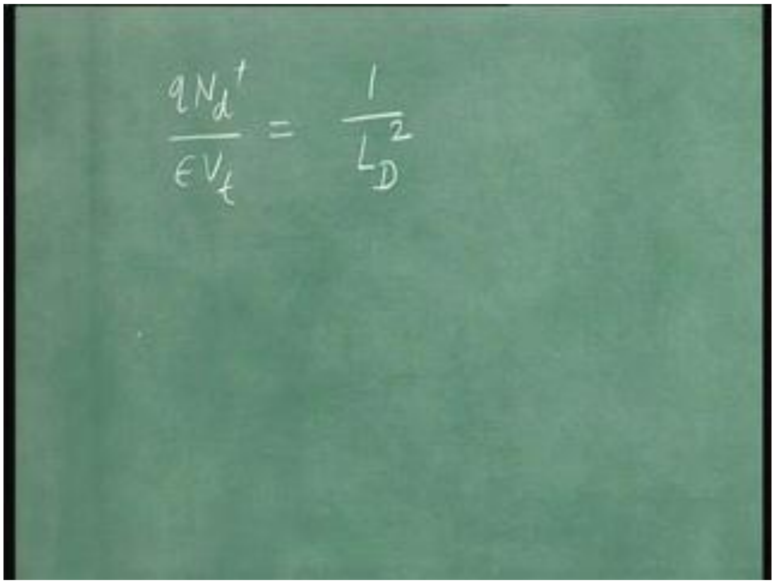

$$\frac{qN_d^+}{\epsilon V_t} = \frac{1}{L_D^2}$$

So we write it here as q N_d to the power plus by epsilon V_t is equal to 1 by L_D square and this characteristic length is the Debye length L_D square.

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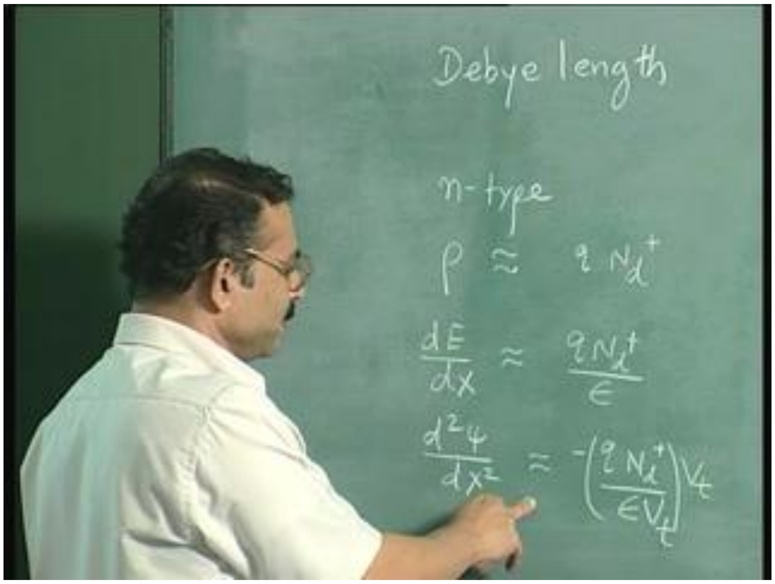

Debye length

n-type

$$\rho \approx q N_d^+$$

$$\frac{dE}{dx} \approx \frac{qN_d^+}{\epsilon}$$

$$\frac{d^2\psi}{dx^2} \approx -\left(\frac{qN_d^+}{\epsilon V_t}\right)V_t$$

So this equation can be rewritten in terms of the Debye length as d square psi by dx square is equal to minus v_t by L_D square.

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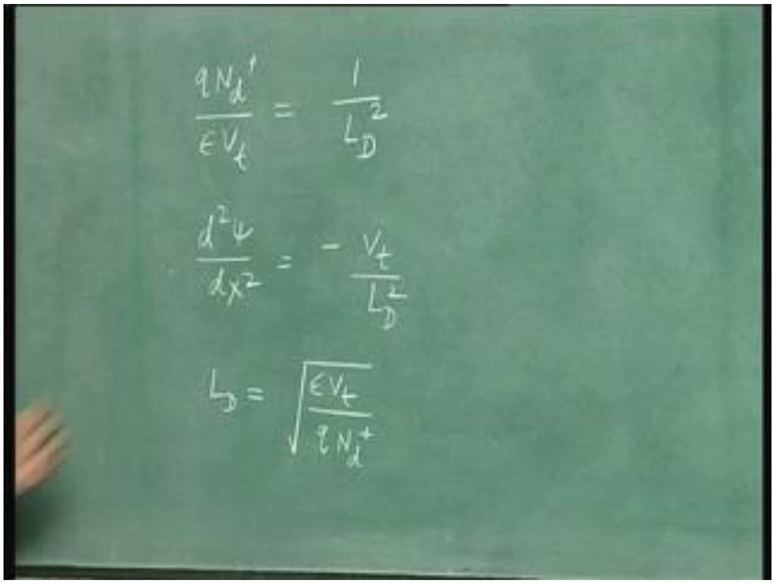

$$\frac{qN_d^+}{\epsilon V_t} = \frac{1}{L_D^2}$$

$$\frac{d^2\psi}{dx^2} = -\frac{V_t}{L_D^2}$$

$$L_D = \sqrt{\frac{\epsilon V_t}{qN_d^+}}$$

So the Debye length in this particular semiconductor is given by epsilon V_t square root of this term square root of epsilon V_t by qN_d to the power plus this is the Debye length where we are assuming a uniformly doped n-type semiconductor. So you see that it depends on the doping level.

What does the Debye length indicate?

What is the physical significance of the Debye length? It is very clear from here that the Debye length tells you how the potential varies with distance in a depletion region. Wherever the space charge region is there how the potential varies with distance. So this is the characteristic length that describes this variation. So like the diffusion length which we introduced in the previous class describes how the Excess Carrier Concentration decays with distance when the carriers are diffusing and recombining.

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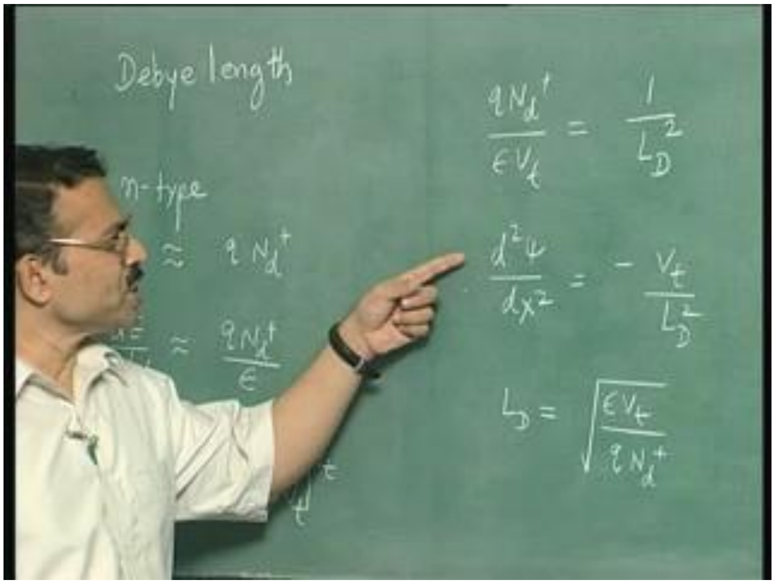

Debye length

n-type

$\approx qN_d^+$

$\approx \frac{qN_d^+}{\epsilon}$

$$\frac{qN_d^+}{\epsilon V_t} = \frac{1}{L_D^2}$$

$$\frac{d^2\psi}{dx^2} = -\frac{V_t}{L_D^2}$$

$$L_D = \sqrt{\frac{\epsilon V_t}{qN_d^+}}$$

The Debye length describes how the potential varies with distance in the depletion layer. Now, we will discuss two other approximations related to the Gauss’s law or Poisons equations which are employed. One is called the Charge sheet approximation and the other is called the Gradual channel approximation.

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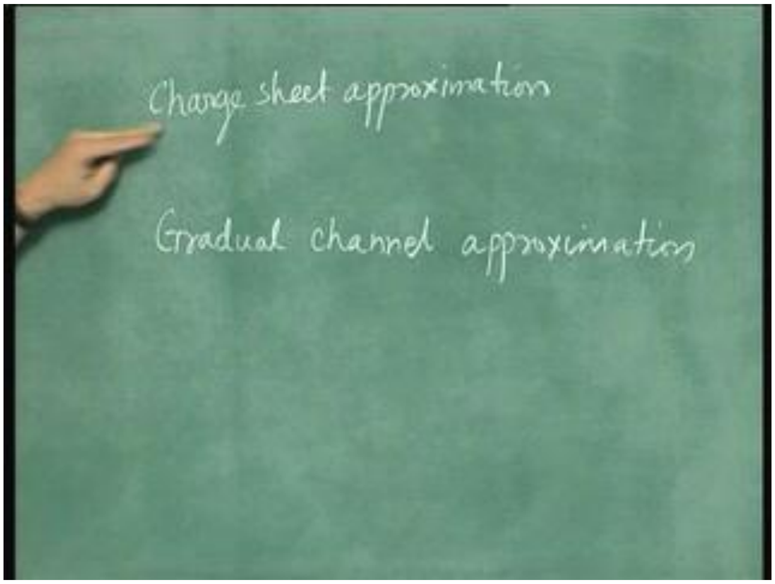

Charge sheet approximation

Gradual channel approximation

The charge sheet approximation is basically used to describe the charges due to free carriers near the interface or surface.

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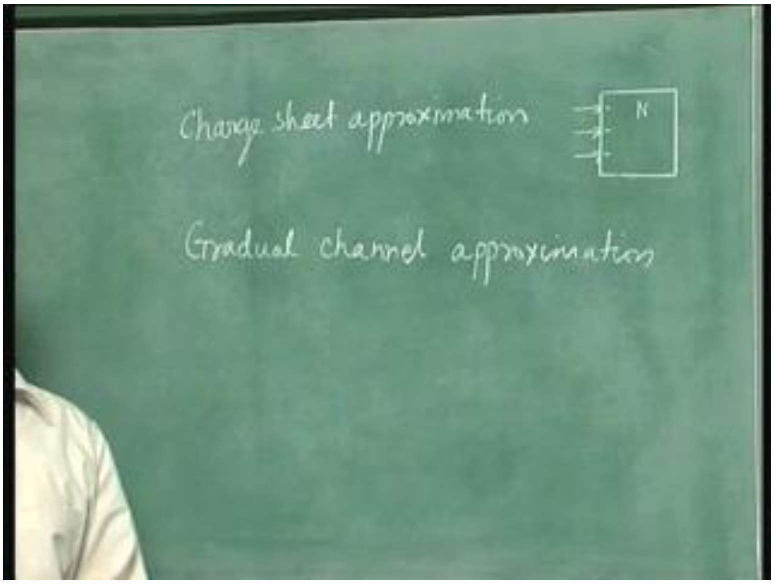

When free carriers are accumulated near the surface for any reason, supposing you have a semiconductor in which somehow there is an electric field on the surface and the free carriers are accumulating, this is an N type semiconductor, now electrons will be attracted here to the surface. Since they are mobile carriers they will try to stick very close to the surface so in such a case we can assume that the charge due to mobile carriers is entirely near the surface or it is the charge sheet, this is what is meant by charge sheet approximation.

Although in practice the charge will be high here and then gradually the charge will reduce. But the thickness over which this change occurs is so small that you can assume the charge region to be a charge sheet. We will discuss more about this when we take up the MOS capacitor. This other approximation called the gradual channel approximation is used to convert a two dimensional situation into a one dimensional situation approximately.

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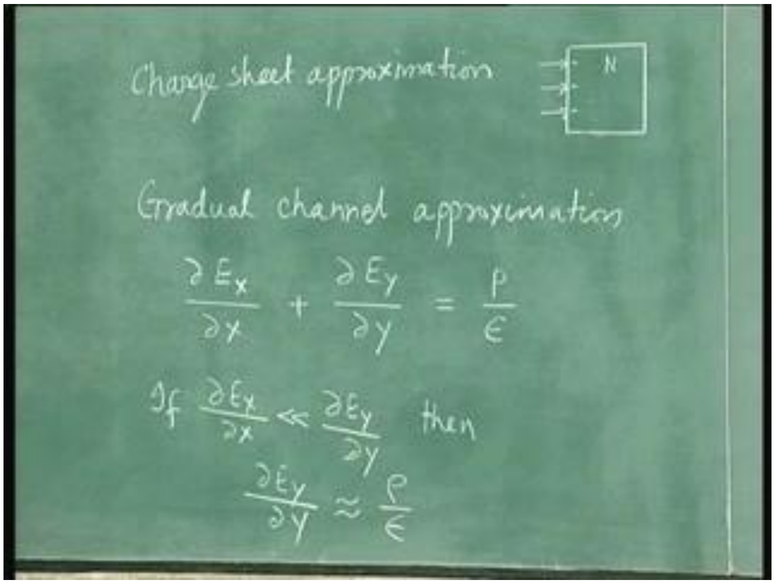

Charge sheet approximation

Gradual channel approximation

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \frac{\rho}{\epsilon}$$

If $\frac{\partial E_x}{\partial x} \ll \frac{\partial E_y}{\partial y}$ then

$$\frac{\partial E_y}{\partial y} \approx \frac{\rho}{\epsilon}$$

So, why exactly this approximation is called gradual channel will become clear when we take up the MOSFETS where this approximation is used. But here we will mention what this approximation essentially involves. If the situation is two dimensional the Gauss's law can be written as dou E_x by dou x plus dou E_y by dou y is equal to rho by epsilon, only a two dimensional situation where you have E_x as well as E_y. Now it can happen that one of these terms can be very small compared to the other term, So for example dou E_x by dou x may be much less than dou E_y by dou$_y$ in which case we can neglect this term and then your Gauss's law can be written as dou E_y by dou y is approximately is equal to rho by epsilon.

Although the situation is two dimensional because a variation of the x component of the electric field with x is slower than the variation of the y component of the electric field with y we can neglect this term. Please understand that we are not talking of the absolute values of the x component and y component but rather the variation of the x component in the x direction is being compared with the variation of the y component in the y direction. So it is conversion of this two dimensional situation into one dimensional situation simplifies the analysis.

We will discuss more about this approximation when we take up MOSFETS where this approximation is used. Now, the various approximations that we have employed where one must understand one thing about them and that is throughout the device volume a single set of approximation may not be valid. So how do you analyze a device using these approximations in practice? This is a very important thing that we must understand. So let us look at this slide which summarizes the procedure for Device Analysis.

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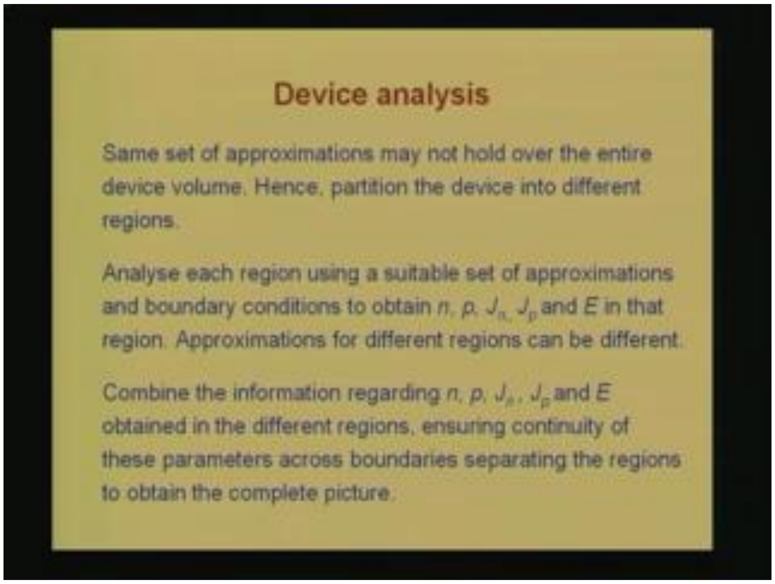

Device analysis

Same set of approximations may not hold over the entire device volume. Hence, partition the device into different regions.

Analyse each region using a suitable set of approximations and boundary conditions to obtain n, p, J_n, J_p and E in that region. Approximations for different regions can be different.

Combine the information regarding n, p, J_n, J_p and E obtained in the different regions, ensuring continuity of these parameters across boundaries separating the regions to obtain the complete picture.

As is written here, the same set of approximations may not hold over the entire device volume. Hence partition the device into different regions. This is the first important step in analysis. So you know as the word analysis implies analysis means separation of the hole into parts, understanding the parts in isolation and then combining the understanding so obtain to understand the hole. So please understand that analysis always involves some separation of the hole into parts or separation of the device into various regions.

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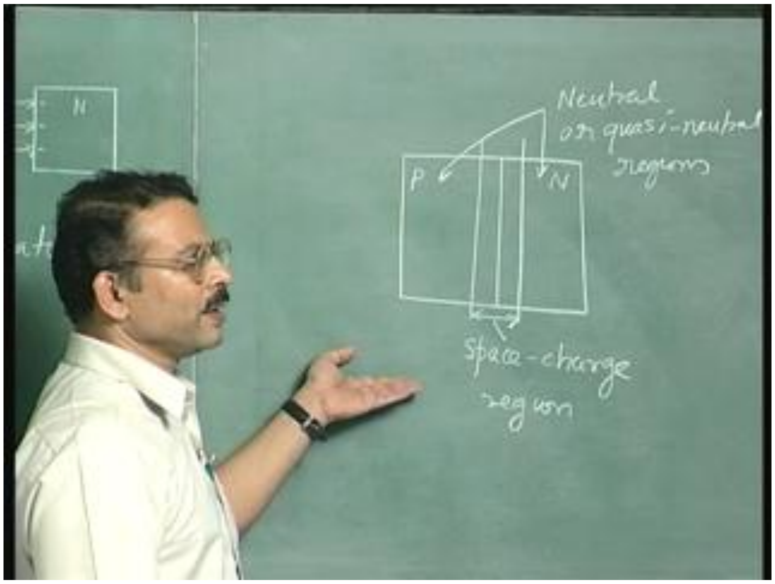

Supposing this is our device then this device will be separated into various region. For example, if we take a PN junction you have P and N regions here and this is our device, so for the purpose of analysis this will be separated further into regions like this for example we will isolate this region called the space charge region. Here I am not discussing the PN junction analysis in detail. Please understand I am only trying to show what is meant by partitioning a device. And how you decide how to partition a device is the separate topic we must address when we consider a particular device.

Now we are only illustrating what is meant by partitioning of the device into various regions. So, PN junction, for example is partitioned like this where near the junction you have a region that is the space charge region and these two regions are the neutral regions so neutral or Quasi neutral if it is non equilibrium. This is one way of partitioning this device PN junction for analysis. Now the boundary between the Quasi neutral or neutral regions and the space charge region is not abrupt in practice but we have to identify some boundary. This is one of the approximations that one has to do so this is the first step in device analysis.

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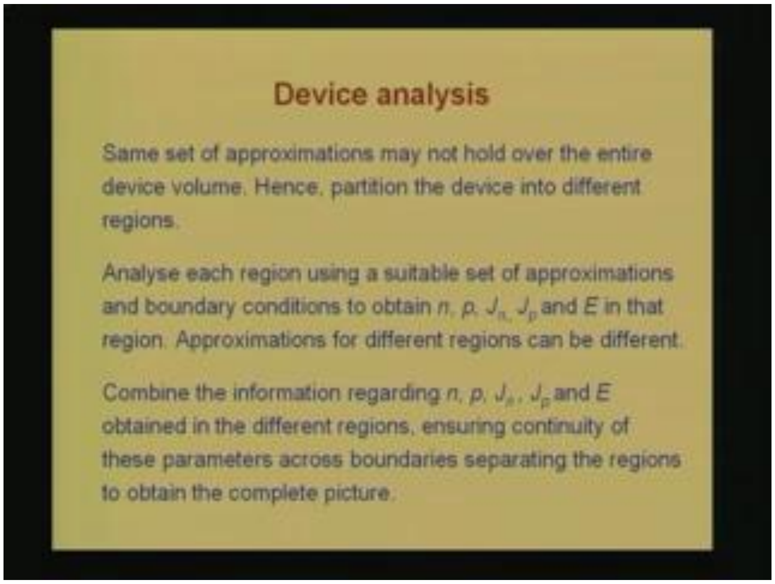

Device analysis

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Analyse each region using a suitable set of approximations and boundary conditions to obtain n, p, J_n, J_p and E in that region. Approximations for different regions can be different.

Combine the information regarding n, p, J_n, J_p and E obtained in the different regions, ensuring continuity of these parameters across boundaries separating the regions to obtain the complete picture.

Now let us look at the next step as shown in the slide. Analyze each region using a suitable set of approximations and boundary conditions to obtain the electron concentration n, the hole concentration p, the current distribution J_n, current density for electrons, the current density for holes and the electric field. Obviously all these are obtained as the function of the distance in that region. Approximations for different regions can be different. This is a very important thing one must note.

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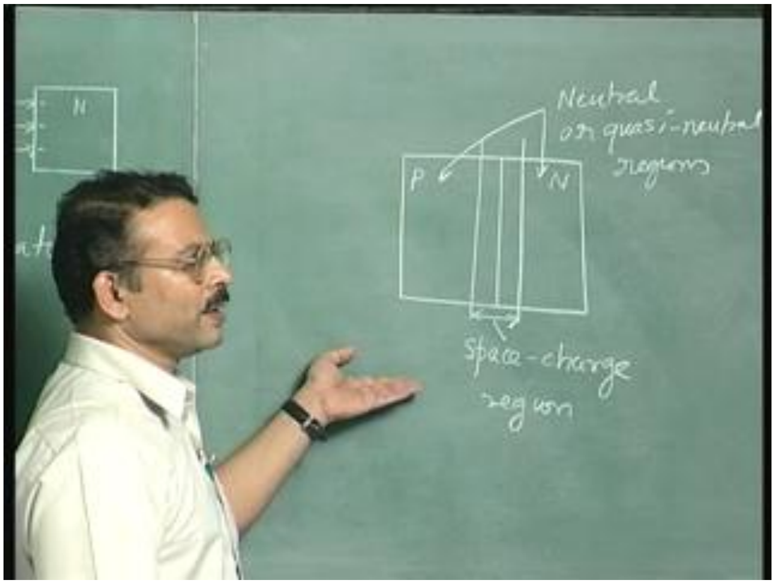

For example, in this case of the PN junction you may use one set of approximations in the space charge region and a different set of approximations for the p and n neutral regions as it means here. So using different sets of approximations you analyze each of these three regions to obtain n p J_n J_p and E in those regions which is the second step. Now let us look at the final step.

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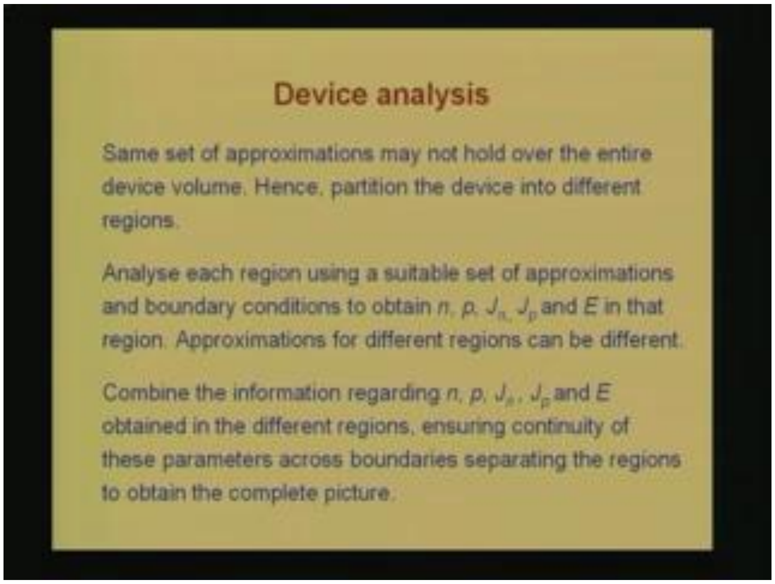

Device analysis

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Analyse each region using a suitable set of approximations and boundary conditions to obtain n, p, J_n, J_p and E in that region. Approximations for different regions can be different.

Combine the information regarding n, p, J_n, J_p and E obtained in the different regions, ensuring continuity of these parameters across boundaries separating the regions to obtain the complete picture.

The final step is combine the information regarding n, p, $J_{n,}$ J_p and E obtained in different regions ensuring continuity of these parameters across the boundaries separating the regions to obtain the complete picture. These are the three important steps in device analysis. The first step is partitioning the device into different regions, the next step is analysis of each region using suitable set of approximations and boundary conditions to obtain n, p, J_n, J_p and E in that region, different approximations may be employed in different regions and final step is combining the information regarding n, p, J_n, J_p and E obtained in the different regions so as to obtain the complete picture. And when you are combining the information you must ensure that the parameters n, p, J_n, J_p and E are continuous across the boundaries. Now we will take a simple example to illustrate some of the steps in the analysis of devices that we have discussed and how the approximations can be applied.

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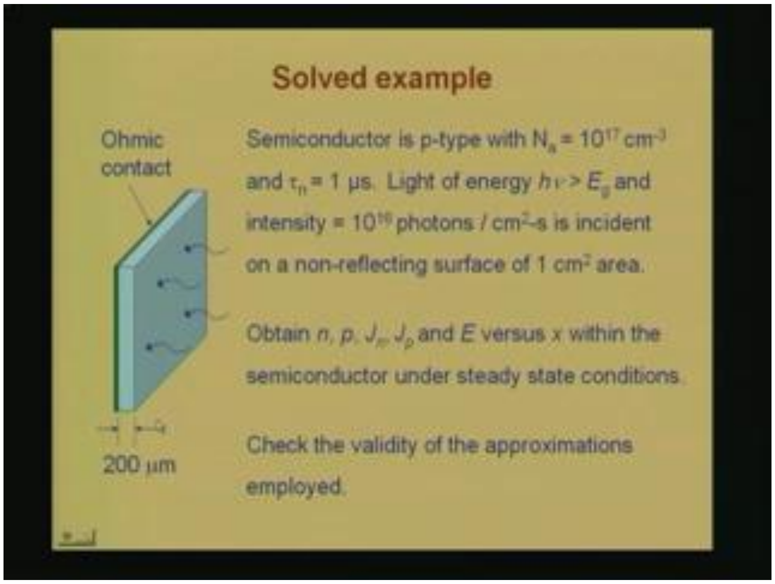

Solved example

Semiconductor is p-type with $N_a = 10^{17}$ cm^{-3} and $\tau_n = 1$ μs. Light of energy $h\nu > E_g$ and intensity = 10^{16} photons / cm^2-s is incident on a non-reflecting surface of 1 cm^2 area.

Obtain *n*, *p*, J_n, J_p and *E* versus *x* within the semiconductor under steady state conditions.

Check the validity of the approximations employed.

So this example is as follows: You have a semiconductor which is terminated on an Ohmic contact at one end and the other surface the opposite surface is illuminated with light. The thickness of the semiconductor is 200mu. Let us look at other details of the semiconductor and what is the expected to be done. The semiconductor is p-type with doping level of 10 to the power 17 per cm cube, the lifetime of minority carriers i.e. the electrons is one micro second. Now the light energy is greater than the energy gap of the semiconductor. Let us assume that the semiconductor is silicon in this case.

And intensity is 10 to the power 16 photons by cm square by second. This is incident on a non reflecting surface of 1 cm square area. So what it means is the area of this surface which is illuminated is 1 cm by 1 cm so this is 1 cm and that is also 1 cm and the surface is non-reflecting so the entire light is observed nothing is reflected. This is not a good assumption at all but we are making a simplifying assumption so that the problem becomes simple.

Now, the thing to be done is obtain n, p, J_n, J_p and E versus x within the semiconductor under Steady State conditions and check the validity of the approximations employed. Now here x direction is along the thickness of the semiconductor. Let us do this example.

The first step in the solution of any problem in Solid State Devices is a qualitative analysis. This means that we must sketch the distributions of various parameters as a function of distance based on simple logical reasoning without writing any equations using knowledge of the physical situation. So the situation is depicted on the diagram.

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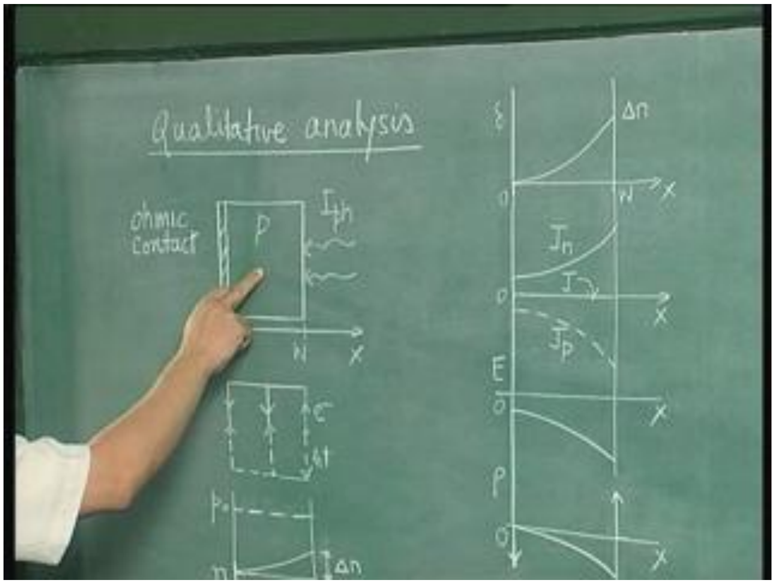

This is a sample and light is falling on it on one surface the intensity of the light is I_{ph}, this is the symbol we used to indicate the intensity of light that is number of photons by cm square by second. Then you have an Ohmic contact on the left hand side that is the contact between metal and the semiconductor. The situation in the sample is depicted in this flow diagram. On this phase here you have electrons and holes being generated this is surface generation i.e. the unit of generation is per cm square per second so per unit area of the surface where area of this is in that direction which we are discussing about.

Now all these carriers which are generated at the surface move inside because of diffusion because at the surface the concentration of carriers is high whereas within the bulk it is low so there is a concentration gradient. Now what is shown here is that all the carrier which is generated are moving inside. So these are electrons and these are holes. This is equivalent to saying that this surface has surface recombination velocity is equal to 0 because none of the carriers which are generated at this surface are recombining there. This may not be true in practice but we are making this assumption to simplifying the problem. Now carriers which are moving inside what happens is during their motion they also recombine and this is what is shown here.

Finally some carriers are still left by the time you come to this particular surface the contact where the Ohmic contact is present so all the carriers which are remaining until this point will recombine at this contact. Now let us sketch the concentration of holes and electrons based on this picture. We shall make the low level assumption which means that the excess minority carrier concentration is much less than the majority carrier concentration. Therefore when we sketch the majority carrier concentration that is hole concentration and minority carrier concentration on a log scale the majority carrier concentration will appear to be almost undisturbed. Please note that we must use the log scale because otherwise we cannot show both concentrations. So this is why it has been shown by a constant line because of the low level assumption.

Now minority carrier concentration of electrons however is disturbed at this end by a value is equal to delta n because there is a surface generation there and this is also shown in the flow diagram. So from this value delta n the excess electron concentration decreases progressively and when is come to this edge where there is an Ohmic contact the excess carrier concentration becomes 0. Please note that here there is an Ohmic contact. This is the consequence of statement of the Ohmic contact.

Let us try to see why Ohmic contact implies zero excess carrier concentration. Now Ohmic contact can be interpreted in terms of the surface recombination velocity as surface recombination velocity tending to infinity (∞) is the meaning of Ohmic contact. We will not discuss this point in detail as to why at an Ohmic contact surface recombination velocity is infinite. This is something to do with the structure of the interface between metal and semiconductor. And it is beyond the scope of this particular course. So you just accept it that Ohmic contact means infinite surface recombination velocity.

Now let us see how infinite surface recombination velocity is equivalent to zero excess carrier concentration. Recombination rate at the surface here is given by surface recombination velocity multiplied by the excess carrier concentration at that point that is here delta n is excess electron concentration so what we are sketching here is delta n so delta n is actually this difference. This is the equilibrium concentration and this is the concentration under illumination.

Therefore the difference between these two is delta n so delta n_0 the value of delta n at this point this is the equation for surface recombination. Now, surface recombination rate has to be finite no matter what is your s that is the surface recombination velocity. So we can write delta n_0 in terms of R_s and s as delta n_0 is equal to R_s minus s. Since R_s is finite it has to be finite but s is tending to ∞ because it is an Ohmic contact this means that the excess carrier concentration tends to 0 so delta n tends to 0 as s tends to ∞. So this is the reason why excess carrier concentration is 0 at an Ohmic contact. This is the distribution of minority carriers as a function of distance. I emphasize that delta n is equal to delta p therefore you also have excess holes.

But as we have discussed earlier, on the log scale under low injection level there will not be any disturbance seen. Hence delta p here almost appears to be constant equal to the equilibrium value on a log scale. In this case delta p here is on a log scale. Now let us only look into a representation of the Excess Carrier concentration. We are not interested in both majority carrier concentration and minority carrier concentration. For our analysis we need the Excess Carrier concentration.

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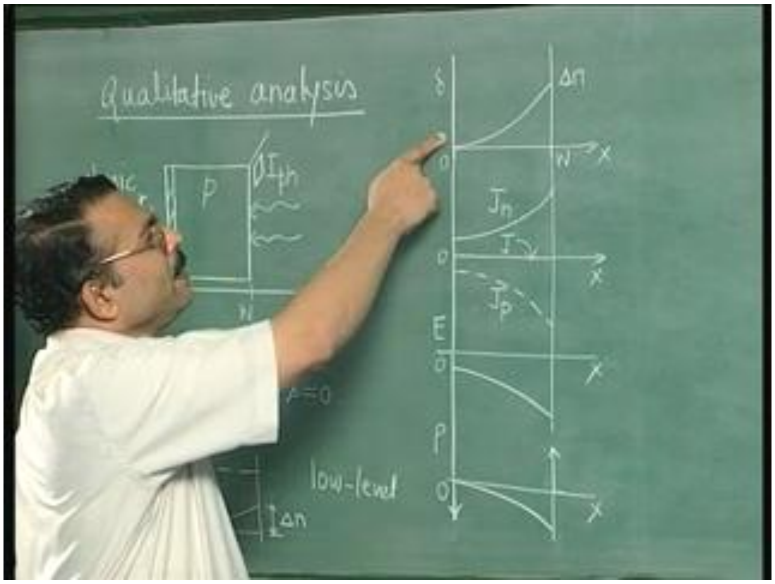

So this is shown here in a linear scale that delta is Excess Carrier concentration as a function of x. We are not putting a suffix p or n because delta p is approximately is equal to delta n which is the assumption of Quasi neutrality. We will prove at the end that these assumptions are valid namely low level, Quasi neutrality assumptions etc. Now continuing with this what we sketch here is the distribution on a linear scale delta n is the Excess Carrier concentration at the right hand illuminated surface and Excess Carrier concentration at this boundary is 0 and there is a gradual change from delta n to 0. We are expressing everything in terms of the excess minority carrier concentration.

Let us move to the current distributions. If you make the diffusion approximation for minority carriers J_n the electron current in a p-type semiconductor is approximately equal to a current because of diffusion and this is the assumption we are making so this is the diffusion approximation. Please note that this holds only for minority carriers. So, if you make this assumption then the electron current can be obtained from the slope of this distribution because diffusion current is the diffusion coefficient multiplied by the slope of the Excess Carrier concentration. So this is J_n note that J_n is not 0 here because the distribution of the Excess Carrier concentration is such that the slope here is not 0.

Earlier we have discussed that the slope of the Excess Carrier distribution is 0 only when the sample is infinitely long or very long compared to the diffusion length. Now what about J_p? How do you get J_p? Now you note the device is open circuited so J is J_n plus J_p is 0.

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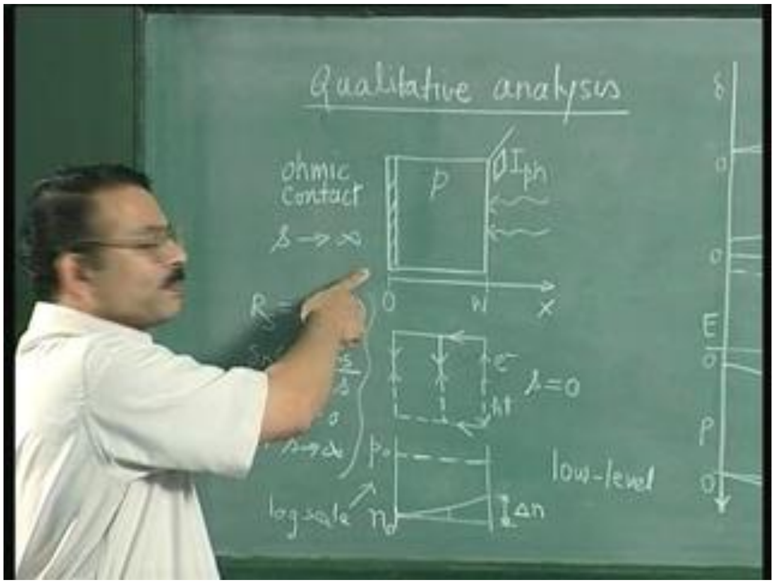

What is the meaning of open circuited?
If you come back here there is no wire connecting this end to the other end. If there was a wire connecting the illuminated surface to the Ohmic contact then we could say that the device is short circuited. Now there is no such connection so the device is open circuited.

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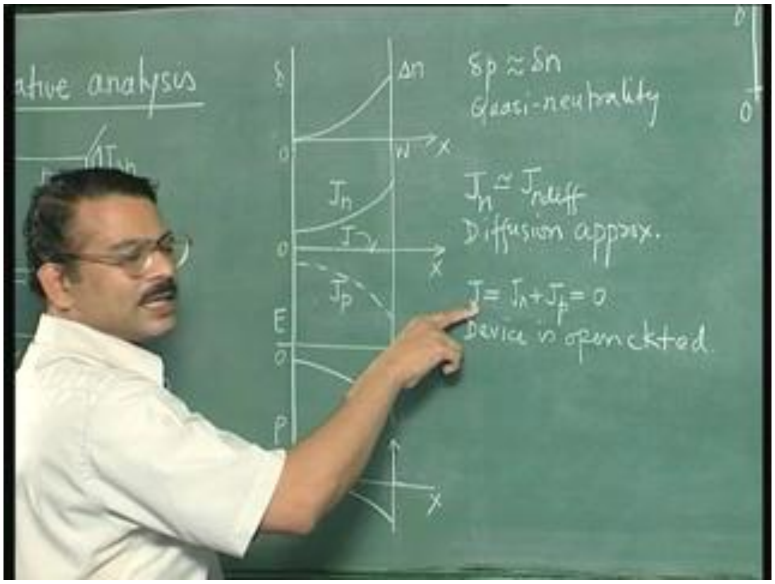

So when device is open circuited J_n plus J_p that is the current entering the terminal is 0 and since if the current entering the terminal is 0 since it is the Steady State condition we have shown earlier that the current is uniform throughout the device under steady state. You should see the consequence of steady state that we have discussed earlier so d_J by d_x

is 0. The total current in the device due to electrons and holes is constant. So whatever value is at the terminal is the same as the value inside the bulk and therefore it is equal to 0. Since this is true you get J_p is minus j_n. This is what is plotted here j is shown to be 0 throughout and J_p is minus j_n. So this is a curve that is similar to this but reflected on to this particular surface, this is the reflection of this. Now let us come to the electric field. Why is an electric field present in the device at all? The electric field can be traced to this particular condition J_n plus J_p is 0 how?

Now notice that if the currents are entirely because of diffusion for both electron as well as hole then the hole current would not be equal to the electron current because the hole diffusion coefficient is less than electron diffusion coefficient. So since D_p less than D_n, J_p diffusion in magnitude is not equal to J_n diffusion, But as shown here j has to be 0 so J_p has to be equal in magnitude to J_n although negative in sign.

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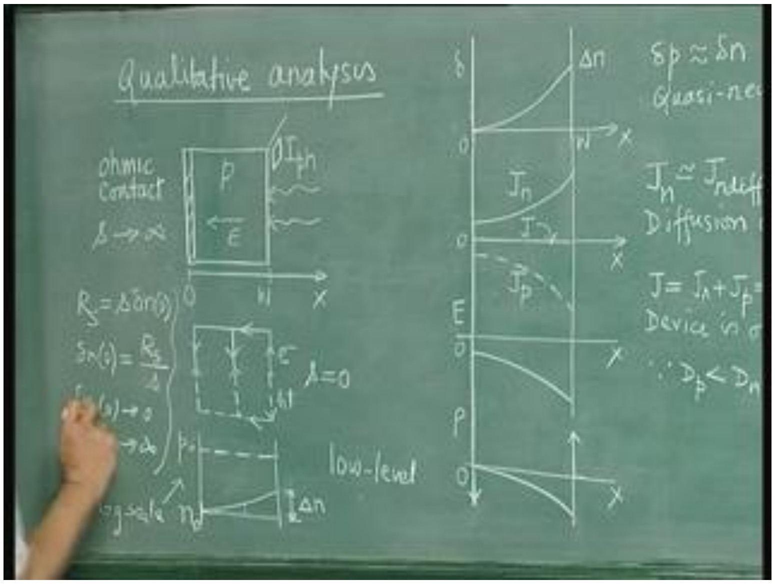

So how can you make these two currents equal? An electric field is set up in the device in this direction from the illuminated surface to Ohmic contact. Now what is the role of this electric field?

Notice that if the electric field is in this direction it will aid the movement of holes from right to left but it will oppose the movement of electrons from right to left. The same electric field will oppose the movement of electron from right to left but it will aid the movement of holes. We have shown here that electrons are also moving from right to left and holes are also moving from right to left. So this movement will be aided by this electric field but this movement will be opposed by this electric field.

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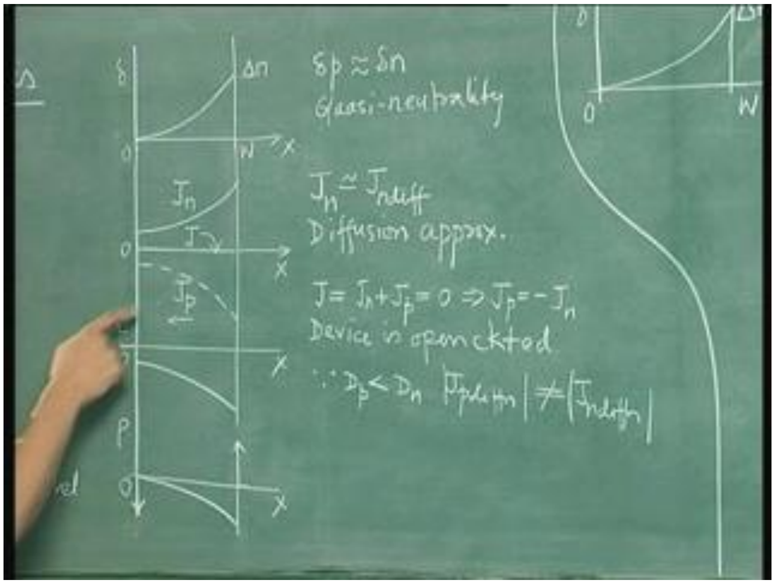

Thus in this manner the diffusion current of holes is aided by the electric field which is in this direction and the movement of electrons is slightly opposed so this is how the diffusion current of electrons is slightly reduced and the hole current of electrons is increased. In fact the effect of electric field on electrons is small because the electron concentration is small but the effect of electric field on a hole current is large. Therefore in effect what we can say is the electron current is almost equal to diffusion current but the hole current is because of both drift and diffusion and the drift of hole current is such that it adds to the diffusion so as to make this current equal in magnitude to J_n because that is the condition we should maintain because the device is open circuited. This is the role of the electric field. It is to bring the hole current in step with the electron current so as to maintain a total current j(0) value. So the electric current is in the electric field is in the negative x direction that is why it is shown on the negative axis.

Further whenever the electron hole currents are large to bring hole and electron currents in step you will need a larger electric fields and that is why the electric field is larger here and it decreases progressively. Note that here electric field is not 0 because the currents are not yet 0 so you need some electric field to keep these two currents in step. This explains the distribution of electric field. Now, we can readily derive the space charge distribution from the electric field distribution using Gauss's law. So Gauss's law says rho by epsilon is equal to dE by dX. So rho is equal to dE by dX into epsilon as shown here you take the slope of this you get this so rho is negative because you see that the slope is negative.

The fact that rho is negative is seen from here and then you can see that the space charge here is 0. This is because the Excess Carrier concentration is 0 at this point so when your excess hole and electron concentrations are 0 as shown here obviously the space charge should be 0. Now there are two impulses here, this impulse is because there is a step in the electric field like this so this is the positive charge sheet from where field lines emanate and they terminate throughout the bulk where the field decreases but some lines still remain when you come to this end and they terminate on a negative charge sheet that is shown here. So according to Gauss's law it is this step that is responsible for this particular impulse. So the area under this curve plus this will totally show the negative charge and this shows the positive charge and these two charges will be equal.

Based on qualitative analysis this is how we can draw the distributions of n and p J_n and J_p and E. Now the next step is to do a quantitative analysis so that we get the values of various critical parameters such as the boundary values of the electric field, the current and the Excess Carrier concentration and we also need an expression for these curves shown here. The Excess Carrier distribution obtained from the diffusion equation is the starting point of quantitative analysis.

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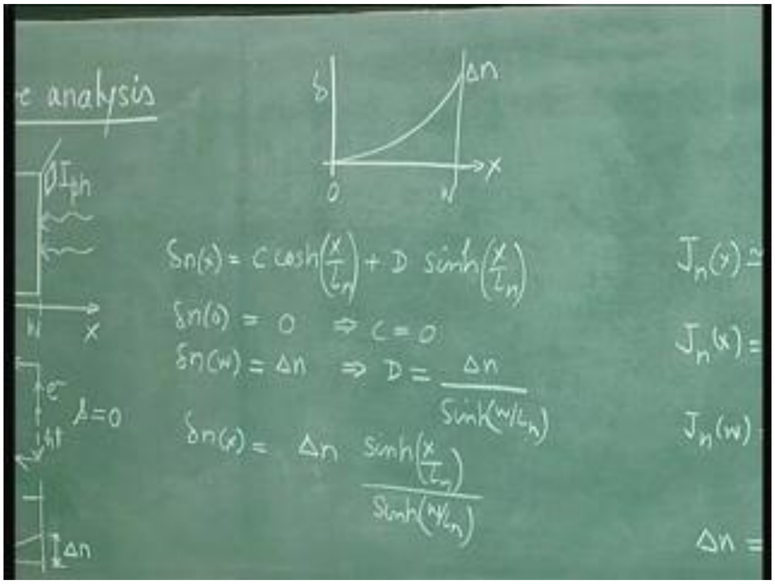

$$\delta n(x) = C\cosh\left(\frac{x}{L_n}\right) + D\sinh\left(\frac{x}{L_n}\right)$$

$$\delta n(0) = 0 \Rightarrow C = 0$$

$$\delta n(W) = \Delta n \Rightarrow D = \frac{\Delta n}{\sinh(W/L_n)}$$

$$\delta n(x) = \Delta n \frac{\sinh\left(\frac{x}{L_n}\right)}{\sinh(W/L_n)}$$

$J_n(x) =$

$J_n(x) =$

$J_n(W)$

$\Delta n =$

We use the hyperbolic form of the solution of the diffusion equation because the sample is finite. We have explained why for a finite sample a hyperbolic form of equation is more suitable. It will also become clear in this analysis how the hyperbolic form is more suitable. The value of delta n at 0 where x is equal to 0 as shown here. So, if you substitute this condition here you will get the result that c should be is equal to 0, this is because when you put x is equal to 0 this term will vanish and the left hand side is 0 but this term cosine hyperbolic of 0 is unity so c will remain here and you will get c is equal to 0. Using a similar approach you can write delta n and w is equal to Δn i.e. this delta n at w. And then since now you are left only with the sine hyperbolic term from this condition you will get the value of d to be delta n by sine hyperbolic w by L_n.

Hence, we can write the solution as delta n(x) the Excess Carrier distribution of electrons is Δn into sin h(X by L_n) by sin h(W by L). Now, we still do not know what is the value of delta n? To know this we should proceed to the current distributions.

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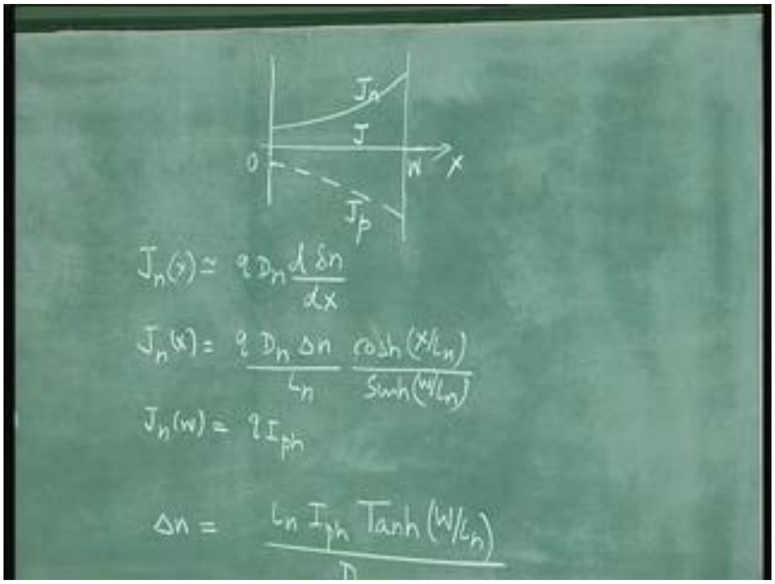

$$J_n(x) \simeq qD_n \frac{d\,\delta n}{dx}$$

$$J_n(x) = \frac{qD_n \Delta n}{L_n} \frac{\cosh(x/L_n)}{\sinh(W/L_n)}$$

$$J_n(w) = qI_{ph}$$

$$\Delta n = \frac{L_n I_{ph} \tanh(W/L_n)}{D}$$

Let us see how the current distribution will give us the value of delta n. Now, the $J_n(x)$ the electron current by diffusion approximation is given by qD_n the gradient of excess electron concentration. So substituting for delta n from here we can get this equation $J_n(x)$ is equal to qD_n Delta n by L_n into cos h(X by L_n) by sin h(W by L_n). So $J_n(x)$ is of the form of cos h(X by L_n) which is clear from here. You see the shape it is non zero at this point and then it goes on increasing so this is the shape of the cos h function. Now, the boundary value of J_n at x is equal to w that is J_n here, this is q into I_{ph} i.e. intensity of illumination which is given in the problem.

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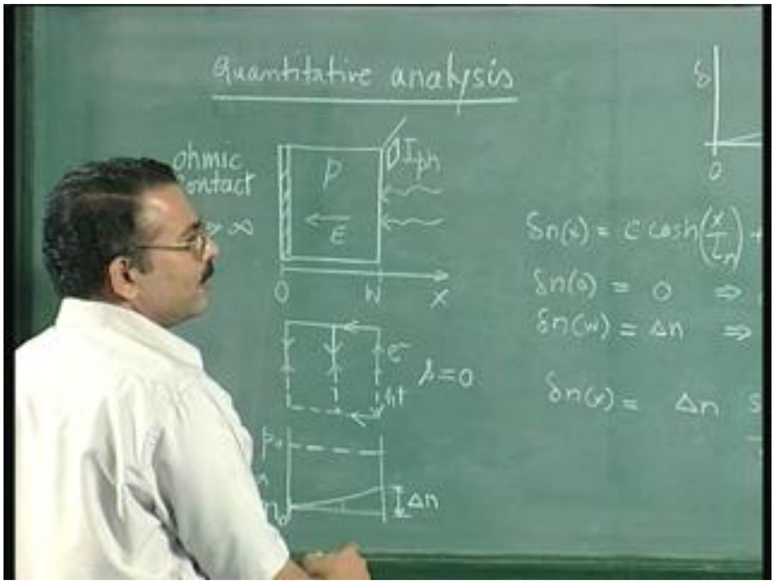

Let us look at this particular situation. The intensity of photons is I_{ph} and each photon is giving rise to an electron hole pair so this is how the current results or current density results. So q into the photon intensity is simply the magnitude of the current density of electrons at that end so that is how you get this.

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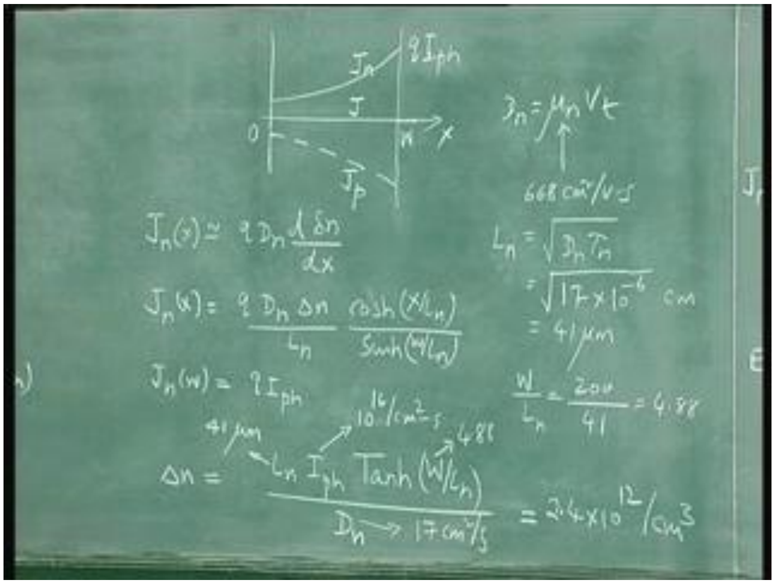

$$J_n(x) \simeq qD_n \frac{d\delta n}{dx}$$

$$J_n(x) = \frac{qD_n\Delta n}{L_n}\frac{\cosh(x/L_n)}{\sinh(W/L_n)}$$

$$J_n(W) = qI_{ph}$$

$$\Delta n = \frac{L_n I_{ph} \tanh(W/L_n)}{D_n} = 2.4\times10^{12}/cm^3$$

$$D_n = \mu_n V_t$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{17\times10^{-6}}\ cm = 41\,\mu m$$

$$\frac{W}{L_n} = \frac{200}{41} = 4.88$$

Now you substitute x is equal to w in this equation and then equate the result with this and then you solve for delta n and the result will be delta n is equal to L_n into I_{ph} intensity of photons by D_n into tan h(W by L_n). Now we can evaluate delta n by substituting the various values. D_n is 17 cm square by s and how do we get this? D_n is equal to mu_n into V_t for 10 to the power 17 by cm cube doping given in the problem and this mu_n turns out to be 668 cm square per volt second from the mobility versus doping formula. V_t is the thermal voltage so you multiply V_t and 668 you will get 17 cm square by second.

Now how do we get L_n? We write L_n is equal to square root of D_n tau_n which is equal to square root of D_n is 17 cm square by second and tau_n is 10 to the power minus 6 seconds 1 micro second so the result is square root of 17 into 10 to the power minus 6 cms and this turns out to be 41mu so this L_n is 41 micrometer. I_{ph} is 10 to the power 16 by cm square by s. Now W by L_n is equal to 200 mu by 41mu so this is about 4.88. So you substitute all these values and you will get the result delta n is equal to 2.4 into 10 to the power 12 by cm cube.

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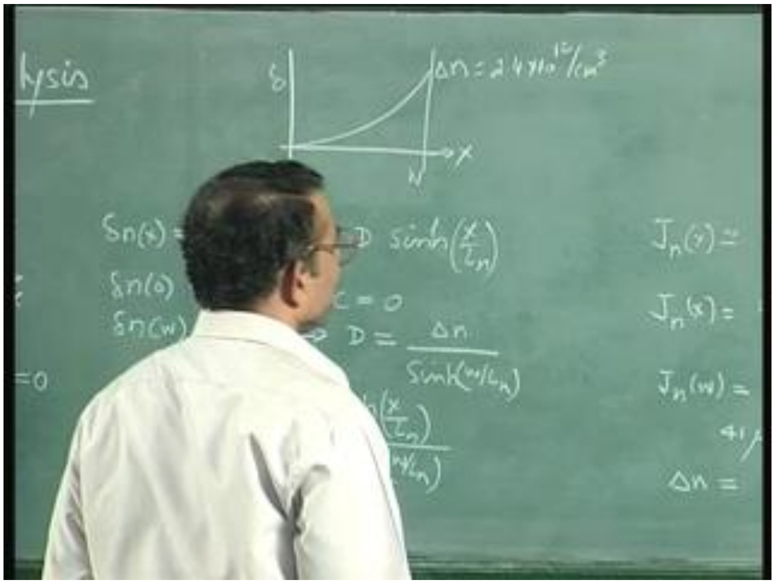

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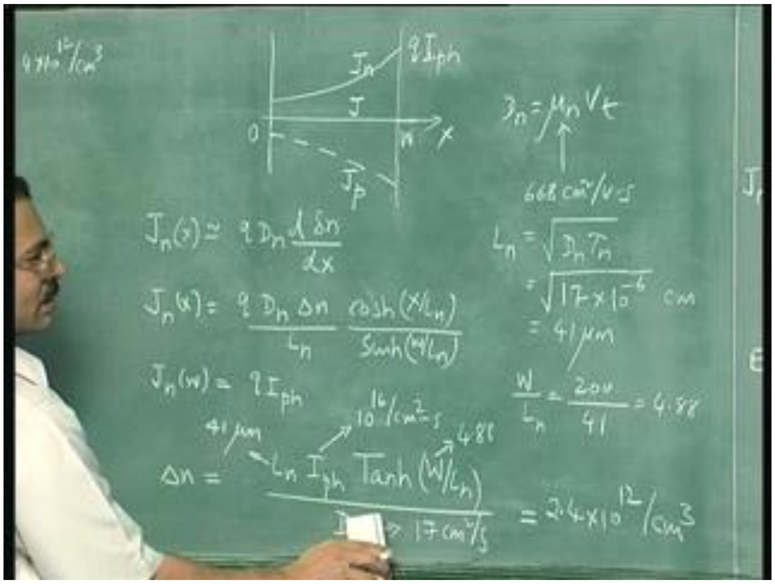

Now substituting this delta n in this equation we can also get the equation for $J_n(x)$ in quantitative terms. So it turns out that after substituting the various values you will get the result 24 cos h(X by L_n) micro amps per cm square which means this is 24 micro amp per cm square and this value is 1.6 milliamps per cm square. Now, let us go to the electric field distribution.

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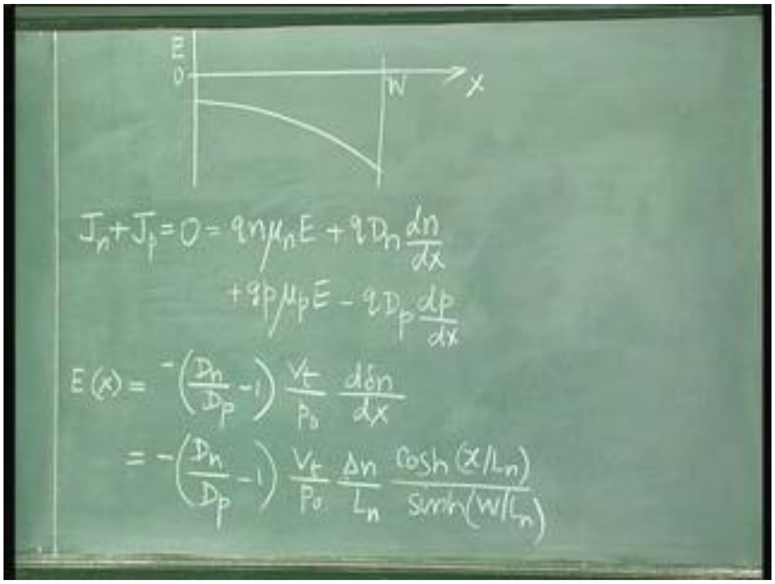

$$J_n + J_p = 0 = q\cancel{n}\mu_n E + qD_n \frac{dn}{dx} + q\cancel{p}\mu_p E - qD_p \frac{dp}{dx}$$

$$E(x) = -\left(\frac{D_n}{D_p} - 1\right)\frac{V_t}{p_0}\frac{d\delta n}{dx}$$

$$= -\left(\frac{D_n}{D_p} - 1\right)\frac{V_t}{p_0}\frac{\Delta n}{L_n}\frac{\cosh(x/L_n)}{\sinh(W/L_n)}$$

Electric field distribution is obtained from the condition J_n plus J_p is equal to 0. So we write the various terms for J_n and J_p so J_n drift J_n diffusion J_p drift and J_p diffusion. Now in this whole equation you note that dn by dx and dp by dx both can be approximated to be ddelta n by dx because of Quasi neutrality assumption. Further because of the low level assumption this p which is the majority carrier concentration is approximately equal to p_0 the equilibrium concentration of holes and the n is much less than p because n is minority carrier concentration. In fact we can neglect this term.

Now we can simplify this hole equation and noting that left hand side is 0 you will get the equation is E(x) is equal to minus(D_n by D_p minus 1) into V_t by P_0 into ddelta n by dx. See that dimensionally this equation is electric field, V_t is volts and p_0 and delta n have the same dimension so they cancel and dx is volts by cm and this is the electric field, Now you can substitute for delta n which we have obtained earlier and you will get this particular equation D_n by D_p minus 1 into V_t by p_0 into Delta n by L_n into cos h(X by L_n) by sin h(W by L_n). Substituting the various values it turns out that this is equal to minus 4 cos h (X by L) microvolt by cm. So this is minus 4 microvolt by cm and this is minus 270 microvolt by cm after evaluation. Now finally we need to check the approximations that we have made namely the low level, the Quasi neutrality and the diffusion approximation for minority carriers.

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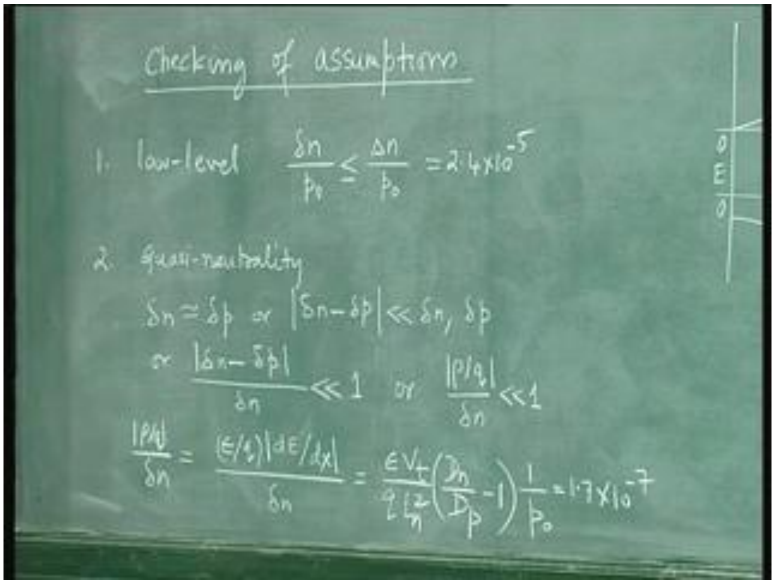

Checking of assumptions

1. low-level $\frac{\delta n}{p_0} \le \frac{\Delta n}{p_0} = 2.4 \times 10^{-5}$

2. Quasi-neutrality

$\delta n \simeq \delta p$ or $|\delta n - \delta p| \ll \delta n, \delta p$

or $\frac{|\delta n - \delta p|}{\delta n} \ll 1$ or $\frac{|\rho/q|}{\delta n} \ll 1$

$$\frac{|\rho/q|}{\delta n} = \frac{(\epsilon/q)|dE/dx|}{\delta n} = \frac{\epsilon V_t}{q L_n^2}\left(\frac{D_n}{D_p} - 1\right)\frac{1}{p_0} = 1.7 \times 10^{-7}$$

So it is checking of assumptions or approximations. If the low level approximation is valid then delta n by p_0 then the excess electron concentration or the excess minority carrier by equilibrium majority carrier concentration should be much less than 1. Now this is less than or equal to the maximum value of delta n by p_0 so the maximum value of delta n is this.

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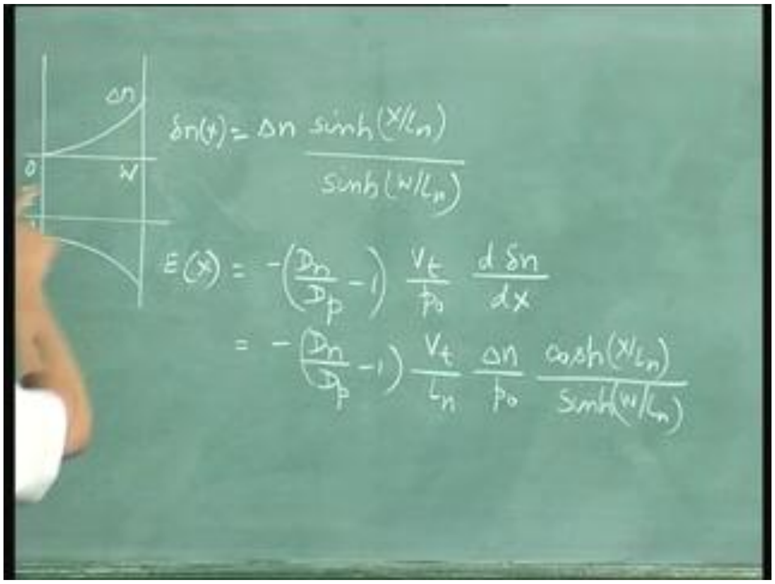

$$\delta n(x) = \Delta n \frac{\sinh(x/L_n)}{\sinh(W/L_n)}$$

$$E(x) = -\left(\frac{D_n}{D_p} - 1\right)\frac{V_t}{p_0}\frac{d\,\delta n}{dx}$$

$$= -\left(\frac{D_n}{D_p} - 1\right)\frac{V_t}{L_n}\frac{\Delta n}{p_0}\frac{\cosh(x/L_n)}{\sinh(W/L_n)}$$

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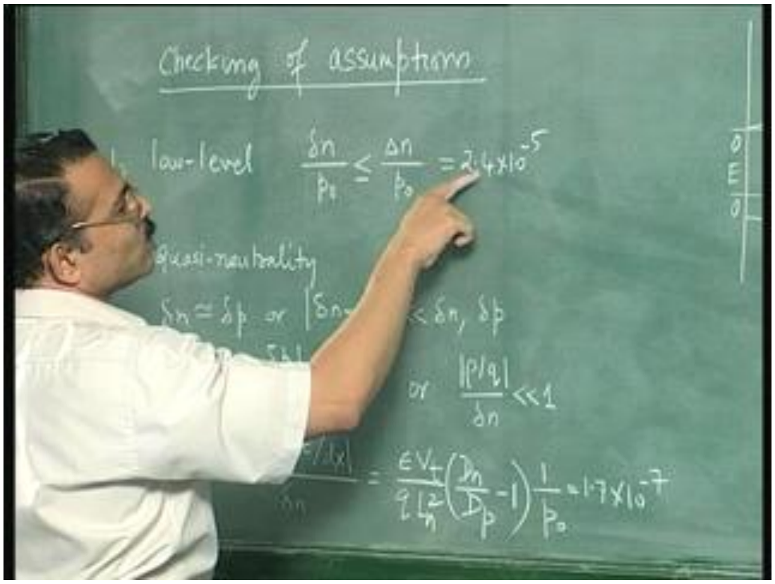

Now you substitute for delta n and p_0 which we have solved for and you will get the result 2.4 into 10 to the power minus 5 which is very clearly much less than 1 so the assumption of low level injection is very much valid. Next let us go to the Quasi neutrality assumption. Quasi neutrality means delta n is approximately is equal to delta p this is what we must prove which is equivalent to saying delta n minus delta p that is the difference of the two should be much less than delta n or delta p the individual components. This is the same as saying that delta n minus delta p the modulus of this divided by delta n should be much less than 1.

Notice that we are taking the modulus because delta n minus delta p can be either positive or negative but we are only interested in the magnitude. Now delta n minus delta p is nothing but the space charge divided by the q so showing this is equivalent to showing rho by q by delta n much less than 1. Now for rho by q by delta n their modulus of rho by q taken in the numerator can be written using the Gauss's law as epsilon by q de by dx by delta n. So rho by q is epsilon by q into de by dx. Now, we have solved for E as a function of x and we have solved for delta n as a function of x and these are the results we have obtained delta n is this and E(x) is this particular result.

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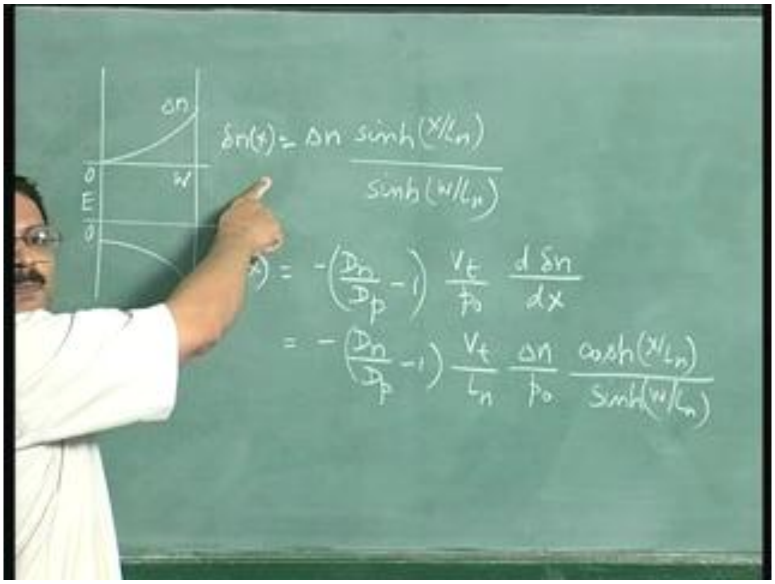

$$\delta n(x) = \Delta n \frac{\sinh(x/L_n)}{\sinh(W/L_n)}$$

$$x) = -\left(\frac{D_n}{D_p} - 1\right) \frac{V_t}{p_0} \frac{d\,\delta n}{dx}$$

$$= -\left(\frac{D_n}{D_p} - 1\right) \frac{V_t}{L_n} \frac{\Delta n}{p_0} \frac{\cosh(x/L_n)}{\sinh(W/L_n)}$$

So you can see from here that when you differentiate E with respect to x the cosine hyperbolic function will become sin hyperbolic function and therefore when you take the ratio of delta n to differential of electric field or de by dx the term that is dependent of x will cancel out and that is what is happening here.

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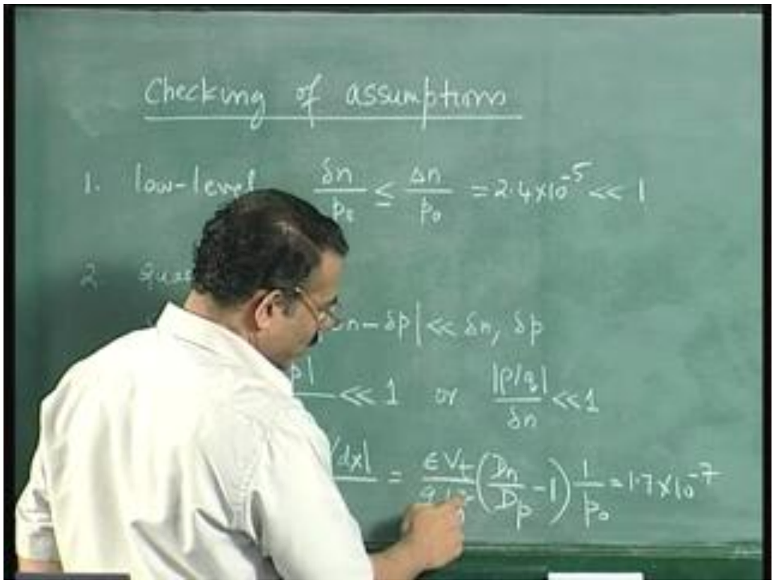

Checking of assumptions

1. low-level $\frac{\delta n}{p_0} \leq \frac{\Delta n}{p_0} = 2.4 \times 10^{-5} << 1$

2. Quas

$|\delta n - \delta p| << \delta n, \delta p$

$p| << 1$ or $\frac{|\rho/q|}{\delta n} << 1$

$/dx| = \frac{\epsilon V_t}{q L_n^2}\left(\frac{D_n}{D_p} - 1\right)\frac{1}{p_0} = 1.7 \times 10^{-7}$

You finally get an interesting and a constant result like this epsilon V_t by qL_n square into D_n by D_p minus 1 into 1 by p_0. Now if you evaluate this you will get it as 1.7 into 10 to the power minus 7 which is much less than 1 again. Now here one point I must emphasize is the value of D_n by D_p which we are used earlier also it was not clarified how we obtained so just as we obtained D_n for 10 to the power 17 by cm cube we obtained D_p also in the same manner. Then if you take D_n by D_p you will find that D_n by D_p is 2.85 which is the value used here.

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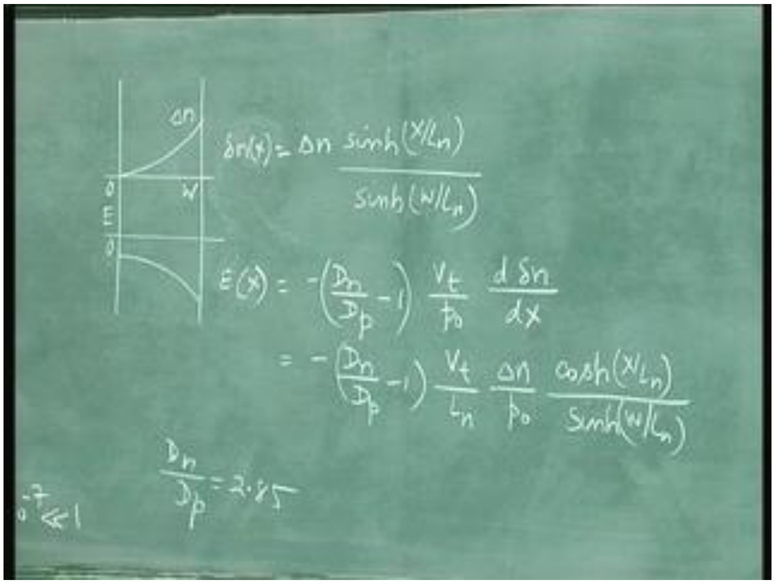

$$\delta n(x) = \Delta n \frac{\sinh(x/L_n)}{\sinh(W/L_n)}$$

$$E(x) = -\left(\frac{D_n}{D_p} - 1\right)\frac{V_t}{p_0}\frac{d\,\delta n}{dx}$$

$$= -\left(\frac{D_n}{D_p} - 1\right)\frac{V_t}{L_n}\frac{\Delta n}{p_0}\frac{\cosh(x/L_n)}{\sinh(W/L_n)}$$

$$\frac{D_n}{D_p} = 2.85$$

$$0^{-7} \ll 1$$

Finally let us discuss the diffusion approximation for minority carriers.

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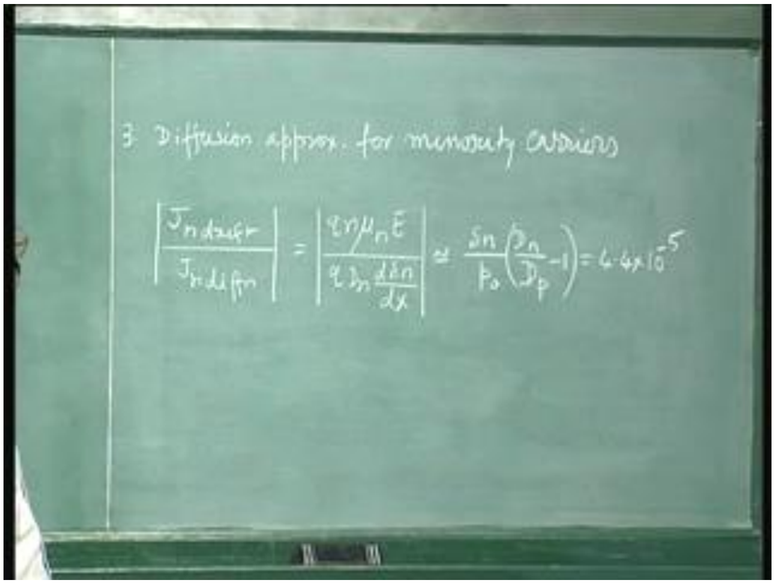

3. Diffusion approx. for minority carriers

$$\left|\frac{J_{n\,drift}}{J_{n\,diffn}}\right| = \left|\frac{q\cancel{n}\mu_n E}{q D_n \frac{d\delta n}{dx}}\right| \simeq \frac{\delta n}{p_0}\left(\frac{D_n}{D_p} - 1\right) = 4.4 \times 10^{-5}$$

So, to prove the diffusion approximation we must take the ratio of the drift current of electrons to the diffusion current of electrons and we must show that this ratio is much less than 1 therefore diffusion dominates over drift. This ratio can be written as qnmu$_n$E by q D_n ddelta n by dx this is drift and this is diffusion. Again we are taking modulus because the currents can be in opposite directions. And we do not want to use a negative sign for any reason because we are only interested in the magnitude. Now, we should find out E by ddelta n by dx, this q will cancel so it is E by ddelta n by dx.

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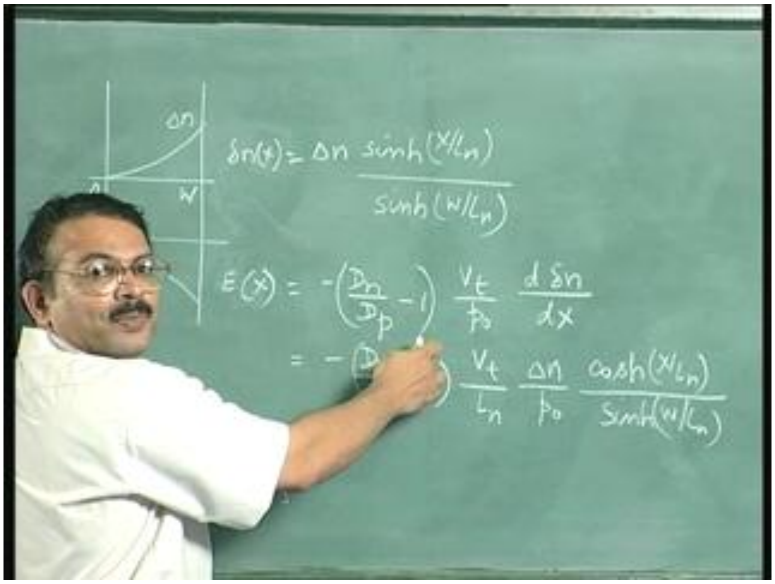

Now E(x) we have shown to be given by this particular formula. So E by ddelta n by dx is a very simple equation which is shown here.

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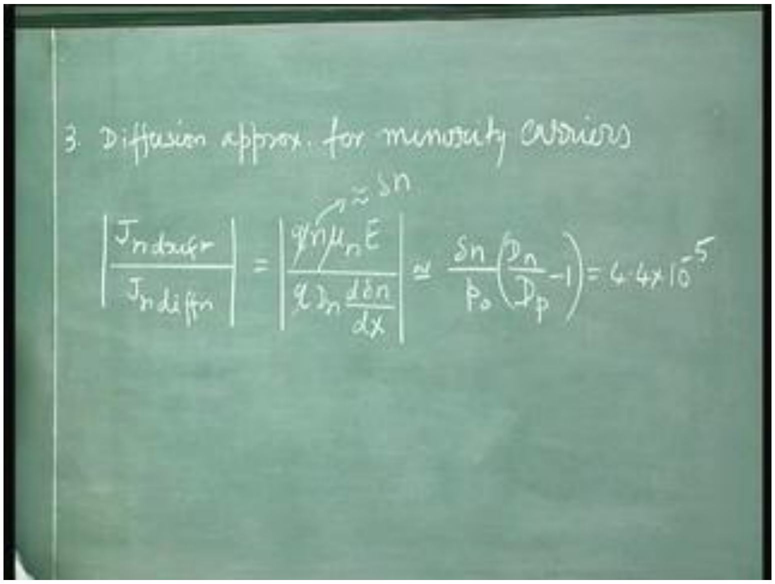

Now here n you must recognize is approximately is equal to delta n itself. This is because for minority carriers the Excess Carrier concentration will normally dominate over the equilibrium concentration so we can simply replace n by delta n approximately over most of the device and therefore your result will turn out to be of this form: delta n by p_0 into D_n by D_p minus 1.

Interestingly delta n by p_0 is the injection level so if the injection level is low definitely this ratio is going to be much less than 1. So you get this ratio 4.4 into 10 to the power minus 5 which is much less than 1 so diffusion approximation is justified. In fact this particular expression also shows how the diffusion approximation is linked to the low level approximation where if low level is valid then the diffusion approximation is also valid. Now we will summarize the discussions of the last two lectures.

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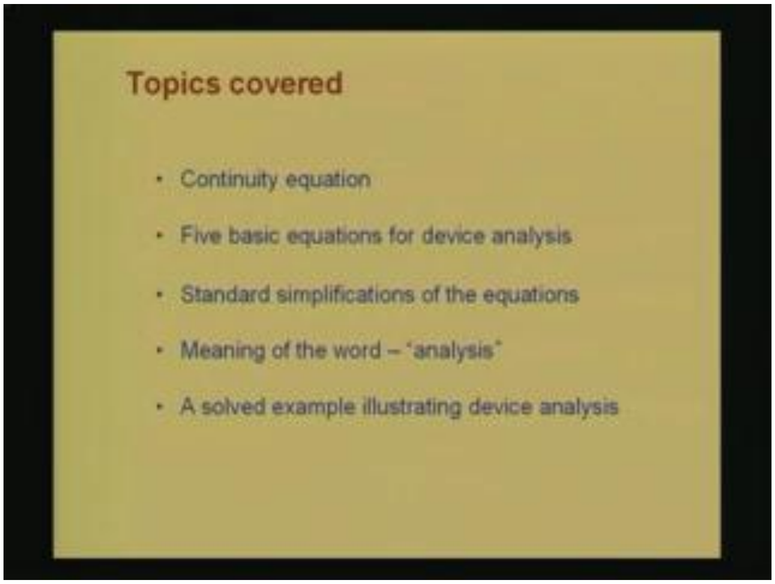

Topics covered

- Continuity equation
- Five basic equations for device analysis
- Standard simplifications of the equations
- Meaning of the word – 'analysis'
- A solved example illustrating device analysis

We have covered the following topics: First we considered the continuity equation for device analysis. Then we summarized the five basic equations for device analysis namely: the transport equations for electrons and holes, the continuity equations for electrons and holes and the Gauss's law. Then we discussed the standard simplifications of the equations.

We have explained the meaning of the word analysis i.e. the separation of the hole device into parts, understanding the various parts of the device in isolation and then combining the understanding so as to understand the whole of the device. So here we have explained that the analysis involves the solution of the five basic equations simultaneously to give the parameters namely; n, p, J_n, J_p and E as a function of distance then we illustrated the analysis using a solved example. Before we take up the next chapter which involves analysis of semiconductor devices such as PN junction, bipolar junction transistors, MOSFETS and so on it will be useful for us to summarize characteristic material parameters that we have come across in the last 18 lectures.

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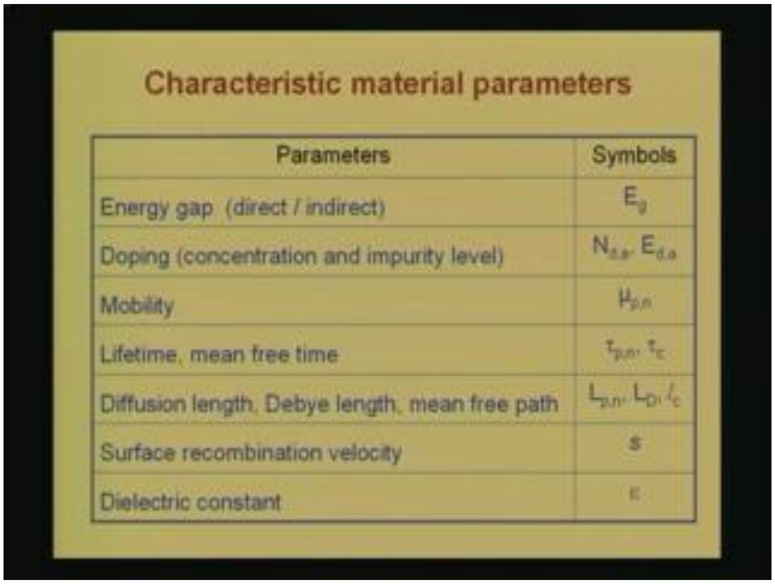

Characteristic material parameters

Parameters	Symbols
Energy gap (direct / indirect)	E_g
Doping (concentration and impurity level)	$N_{d,a}$, $E_{d,a}$
Mobility	$\mu_{p,n}$
Lifetime, mean free time	$\tau_{p,n}$, τ_c
Diffusion length, Debye length, mean free path	$L_{p,n}$, L_D, l_c
Surface recombination velocity	s
Dielectric constant	ε

Now these parameters are the energy gap, direct or indirect etc. The symbol for this energy gap is also shown there. In fact the symbol for all the important parameters had been shown. Then we have the doping. For doping there are two parameters; the concentration and the impurity level. Then the mobility of electrons and holes, then the lifetime and mean free time of carriers, then the diffusion length, Debye length and mean free path or mean free length the three characteristic lengths. And we saw surface recombination velocity and finally the dielectric constant. So whenever we derive equations for terminal characteristics in terms of the physical parameters these characteristic parameters will come into the picture.