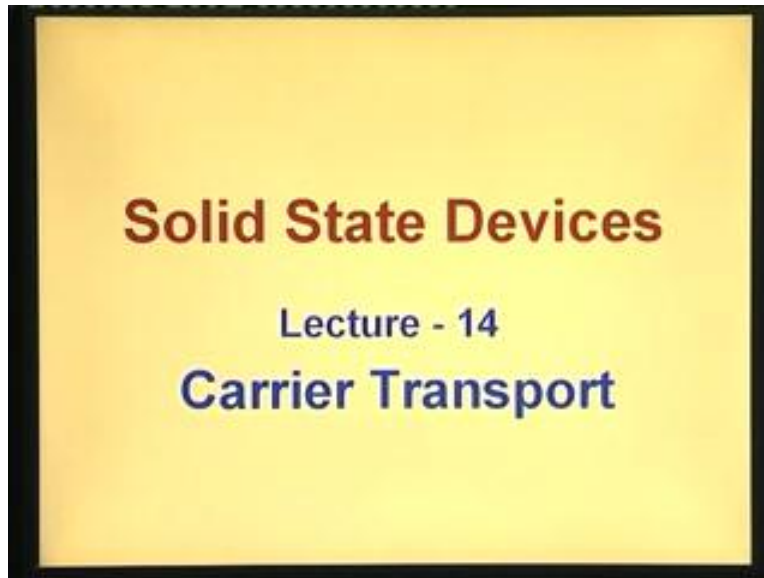


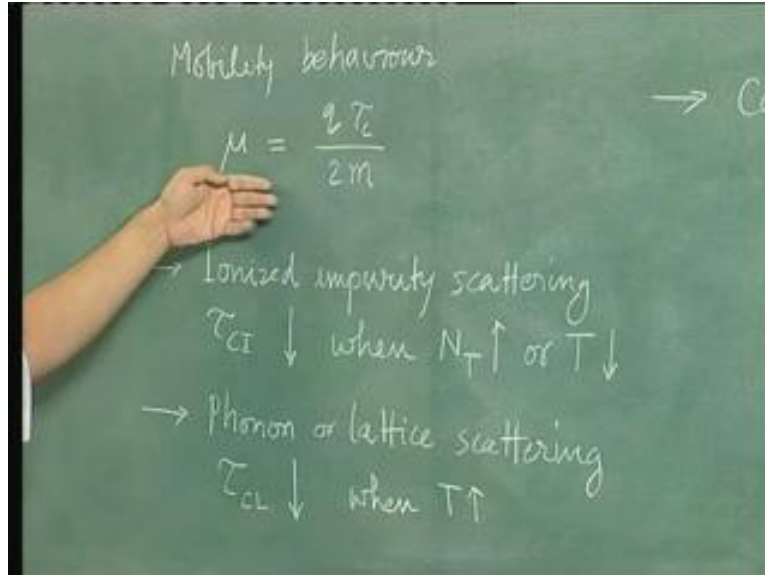
**Solid State Devices**  
**Dr. S. Karmalkar**  
**Department of Electronics and Communication Engineering**  
**Indian Institute of Technology, Madras**  
**Lecture - 14**  
**Carrier Transport (Contd...)**

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This is the 14th lecture of this course and the 3rd lecture on Carrier Transport. In this lecture we will be completing the discussion on this topic. In the last class we saw that for small electric fields the drift current is proportional to the electric field and mobility is the constant of proportionality coming in this particular relation between current and electric field. Similarly, we saw that for small concentration gradients the diffusion current is proportional to the concentration gradient and the constant of proportionality here which comes in is diffusion coefficient of course with the other terms also coming in. We also saw that the  $\mu$  and diffusion coefficient are related by the Einstein relation. And since the diffusion coefficient is related to mobility if you know the behavior of mobility as the function of doping, temperature and so on we can also know the behavior of diffusion coefficient. So the mobility as a function of doping on temperature is the behavior that one looks for characterizing the semiconductor.

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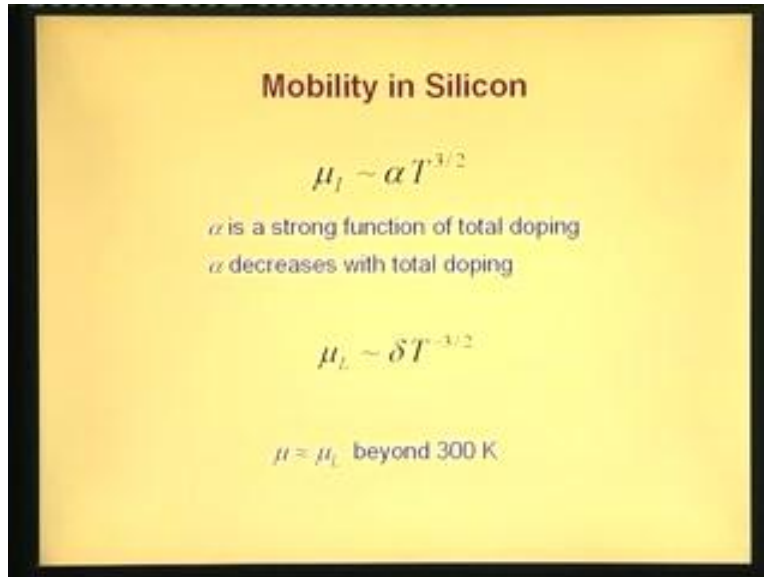
For this parameter  $\mu$  is equal to  $q \tau_c$  by  $2m$  we have seen the various scattering phenomenon and we showed that the mean free time between collisions for ionized impurity scattering decreases with increasing total doping or decrease in temperature. Similarly, the lattice scattering, the mean free time between collisions decreases when temperature increases. Then we came to carrier-carrier scattering and we said the carrier-carrier scattering does not affect the mobility of carriers if the carriers involved are of the same polarity. So, scattering of electrons by electrons and scattering of holes by holes will not affect the mobility of electrons or holes. However, the scattering of electrons by holes and vice versa can affect the mobility.

In a semiconductor if electrons are in a majority then the hole mobility will be affected by scattering with electrons. However the  $\mu$  of electrons will not be affected by scattering with holes because holes are in a small number. The carrier-carrier scattering affects only the minority carrier mobility in a semiconductor. If the carriers are almost in equal number and their concentration is very high which can happen if the semiconductor has been injected with large number of extra electron hole pairs in such a case also the carrier scattering can affect the  $\mu$  of both electrons and holes. Otherwise it affects only the  $\mu$  of minority carriers and that happens only at very high doping levels.

Taking ionized impurity scattering and lattice scattering as dominant scattering mechanisms we will ignore the carrier-carrier scattering in this course. We can write, taking these two into account, the overall  $\mu$  formula. How do we do that? We can write the equation 1 by  $\tau_c$  which represents the number of scattering events per unit time because  $\tau_c$  is the mean free time between collisions. So 1 by  $\tau_c$  that is the total number of scattering events is the sum of scattering events due to ionized impurity plus all the scattering events due to lattice scattering. Add up these two and you will get the total number of scattering event. This formula you can easily translate to  $\mu$  because you see from here that  $\tau_c$  is proportional to  $\mu$ . We can therefore write 1 by  $\mu$  is equal to 1

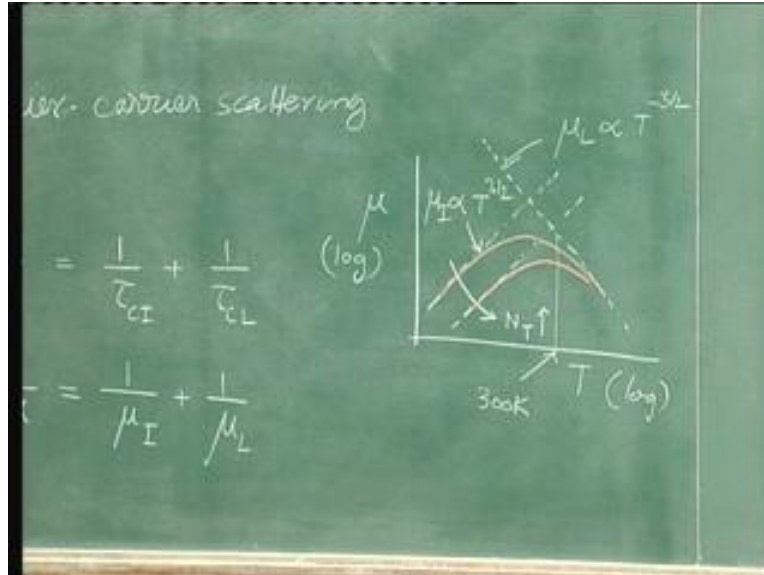
by  $\mu_i$  ionized impurity plus 1 by  $\mu_l$  due to lattice scattering, this is the formula. So if you know how  $\mu_i$  and  $\mu_l$  individually depend on doping and temperature then you know how the  $\mu$  depends on doping and temperature. Now it turns out that  $\mu_i$  and  $\mu_l$  individually depend on temperature as shown in this slide.

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So  $\mu_i$  tilde  $\alpha T$  to the power 3 by 2, this mobility increases with temperature because scattering reduces as a function of temperature. But scattering increases with doping so the coefficient  $\alpha$  is a strong function of total doping and  $\alpha$  decreases with total doping. For lattice scattering the formula is a constant  $\delta$  into  $T$  to the power minus 3 by 2; these powers 3 by 2 and minus 3 by 2 are derived from theory and in practice the powers are somewhat different. In the extrinsic range beyond 300 K the mobility is mostly governed by lattice scattering. So you can take these two segments of the mobility, and this formula shows that whichever mobility is lower will decide the overall mobility.

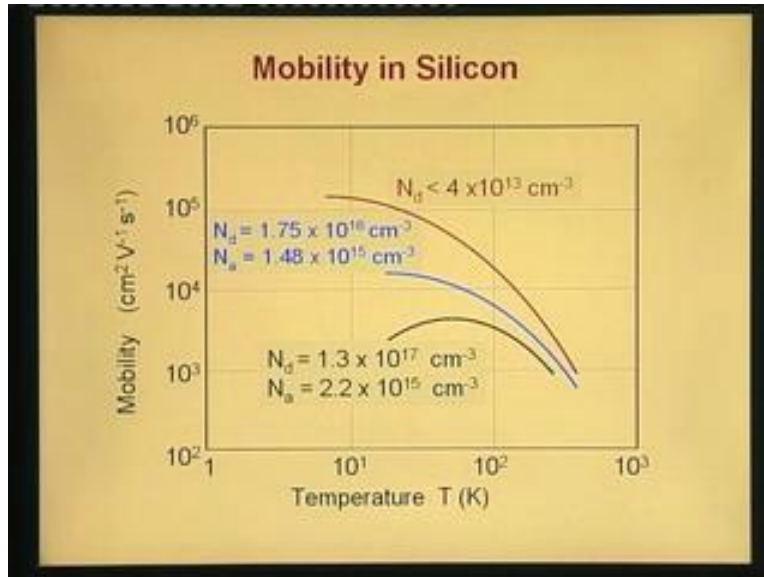
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Whenever  $\mu_I$  is much lower than  $\mu_L$  the mobility is approximately  $\mu_I$  and whenever  $\mu_I$  is much less than  $\mu_L$  the mobility is approximately  $\mu_L$ . So whatever we have written the behavior as a function of temperature and doping can be shown on a graph as follows. This is the line, here you have  $\mu$  on a log scale and temperature also on a log scale. We are plotting both on a log scale because mobility versus temperature is a power law behavior as we saw  $T$  power something. So when you plot log you will get a straight line. This is the straight line corresponding to the lattice scattering so this is  $\mu_L$  which is proportional to  $T$  to the power minus 3 by 2 and this is the line corresponding to  $\mu_I$  which is proportional to  $T$  to the power 3 by 2.

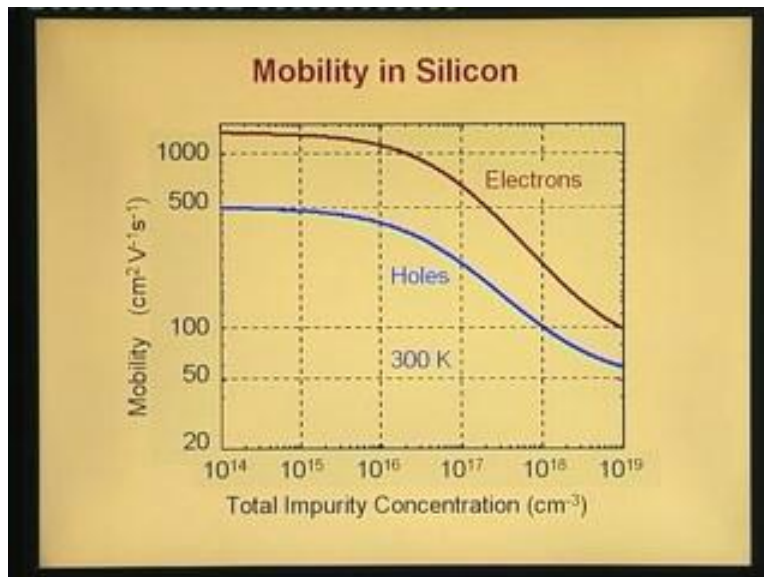
Now if you want to sketch the overall mobility for any given doping as a function of temperature it would be  $\mu_I$  for low temperatures and  $\mu_L$  for high temperatures so it will tend to go like that and this is the behavior of mobility. If you want the mobility for a higher doping then we said the ionized impurity scattering increases with higher doping so you will get another line here corresponding to a higher doping for ionized impurity scattering so your mobility for higher doping will tend to go something like this so this is increasing doping. And the temperature of 300 K would be somewhere here. That is to say that this is room temperature and beyond room temperature it is almost lattice scattering that is dominated. This is exactly the behavior as shown here.

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This is the practical behavior as a function of doping and temperature. If you are interested in knowing the mobility at 300 K then at 300 K the mobility as a function of doping is given by this particular graph its variation with doping mobility decreases doping and which can be captured in equations as shown here.

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Mobility in Silicon				
$\mu = \frac{\mu_{\max} - \mu_{\min}}{1 + (N_T / N_0)^\gamma} + \mu_{\min}$				
Parameter	$\mu_{\max}$ ( $\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$ )	$\mu_{\min}$ ( $\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$ )	$N_0$ ( $\text{cm}^{-3}$ )	$\gamma$
Electrons	1330	65.5	$8.5 \times 10^{16}$	0.76
Holes	495	47.7	$6.3 \times 10^{16}$	0.72

The  $\mu$  is equal to  $\mu_{\max}$  minus  $\mu_{\min}$  by 1 plus total doping divided by a constant and this ratio is raised to the power  $\gamma$  plus  $\mu_{\min}$ . For electrons and holes in silicon at 300 K the values are given in this table. So maximum mobility is 1330 for electrons and 495 for holes, the  $\mu_{\min}$  is 65.5 that is for very heavy doping levels, for electrons and 47.7 for holes. The unit for mobility is centimeter square per volt second. Similarly the value of  $n_0$  and  $\gamma$  are also given here. Use this formula to find out the mobility for any given doping conditions at 300 K. And then if you are interested in the mobility at any other temperature you could use the formulae that we showed earlier i.e.  $\mu$  variation with temperature and find out what is the mobility. We will consider a solved example to illustrate some of these ideas.

The mobility is the other important parameter that we have come across in this particular discussion. Earlier we had seen the parameters namely: energy gap of a semiconductor that was an important parameter so  $\mu$  after energy gap.  $\mu$  is the next important material parameter that affects the applications of the semiconductor.

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Mobility in Silicon and GaAs at 300 K			
Parameter	Material	Doping	
		$10^{14} \text{ cm}^{-3}$	$10^{19} \text{ cm}^{-3}$
$\mu_n$ ( $\text{cm}^2/\text{V-s}$ )	Si	1330	100
	GaAs	8500	1000
$\mu_p$ ( $\text{cm}^2/\text{V-s}$ )	Si	480	70
	GaAs	450	100

Let us look at a picture of mobility in silicon and gallium arsenide and see how the values of mobility affect the application of these two semiconductors. If you compare the mobility of electrons in gallium arsenide is much higher than mobility of electrons in silicon. You can see that at both doping levels 10 to the power 14 and 10 to the power 19. This has important consequences on the application.

For e.g. if you want to make devices which can carry high current or which can work very fast then gallium arsenide is better than silicon provided the devices are depended on only the movement of electrons. If you have bipolar transistors such as NPN transistors or you have MOSFETS or field effect transistors, we cannot say MOSFETS because it is difficult to make MOSFETS in gallium arsenide, so it is field effects transistors which depend on electrons that is n-type field effect transistors. Then these transistors in gallium arsenide can carry more current and will be faster than in silicon. That is the reason why people look for different semiconductors for making devices. So alternatives to silicon are being considered and gallium arsenide is one such alternative for high speed devices or high power devices.

However, you can see here that the hole mobility in gallium arsenide is comparable to that of silicon and in fact it is lower at 10 to the power 14 by cm cube doping level. This low hole mobility coupled with a very high electron mobility creates some problems. In the case of gallium arsenide if you are making circuits which require both n-type and p-type devices e.g. CMOS. If you want to make CMOS type of complementary devices in gallium arsenide in a circuit then there will be problem because of size of the p-type device will be much larger than that of the n-type device in order that the p-type device carries the same current as n-type device.

In complementary structures, that is in circuits, based on complementary devices there has to be matching between the performance of the p-type and n-type devices. To achieve



this matching the p-type device geometry in the case of gallium arsenide semiconductors will have to be much higher than the n-type geometry but this creates lot of problem in laying out. When you have a large number of n-type and p-type devices and you want to interconnect all of these then it creates problems. So, complementary type of circuits is not possible in gallium arsenide because of this problem. This is how the mobility behavior can affect the applications, the value of the mobility can affect the applications of the semiconductor.

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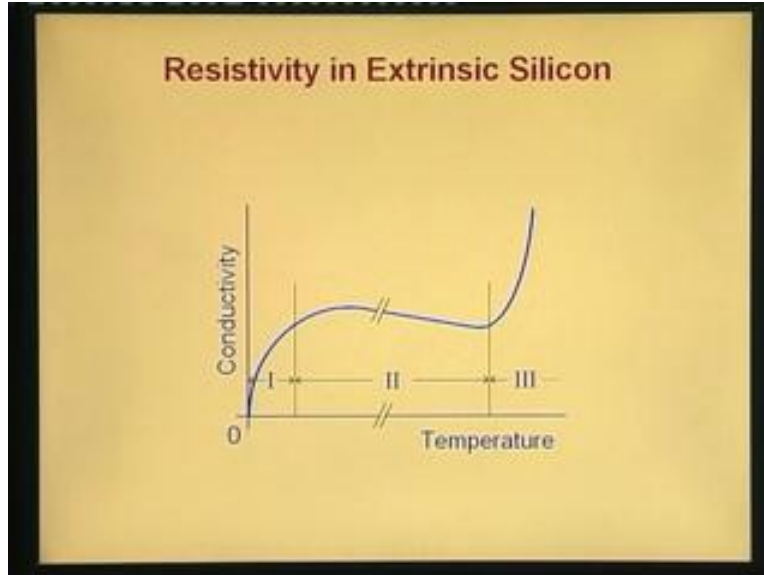
Effective mass in Silicon and GaAs			
Parameter	Symbol (Units)	Silicon	Gallium Arsenide
Conductivity effective mass	$m_n / m_0$	0.26	0.09
	$m_p / m_0$	0.38	---
Density of States effective mass	$m_n / m_0$	1.18	0.068
	$m_p / m_0$	0.81	0.56

Let me also just mention that the effective mass in silicon and gallium arsenide of electrons and holes you have to consider both types of effective mass: the conductivity effective mass and density of states effective mass. So what we have discussed so far in the context of carrier transport the conductivity effective mass is what is important whereas a density of states effective mass is important while estimating the carrier concentration namely effective density of states and so on. The differences in these two situations i.e. conductivity situation and density of states situation should be clear by now.

We will take up the topic of resistivity. Resistivity depends on both carrier concentration and mobility. If you combine mobility and carrier concentration dependence on what kind of dependence we get for resistivity as a function of doping and temperature? Let us start with extrinsic silicon first. This is the picture.

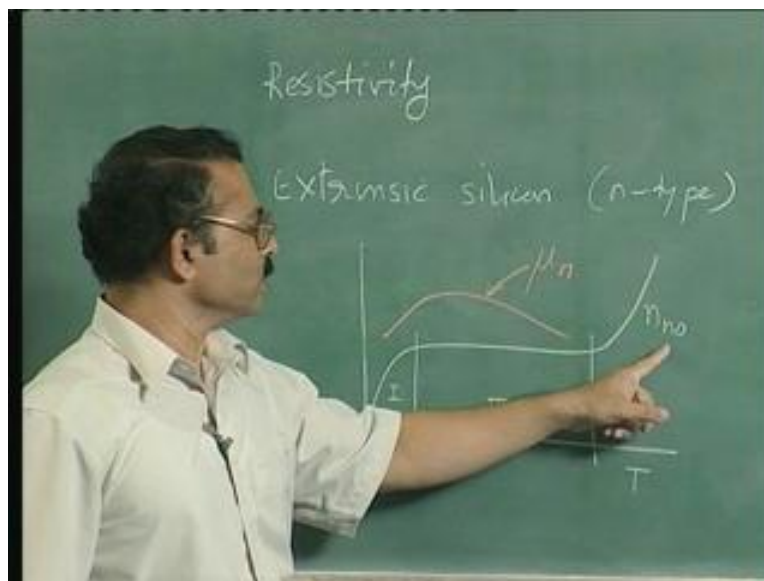


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Instead of resistivity we are plotting the conductivity which is nothing but the reciprocal of resistivity. So it is conductivity as a function of temperature where you find that the conductivity increases from 0 and the region 2 is the extrinsic range. Here in this range conductivity increases and then starts falling. And finally when it enters the intrinsic range it increases again. Let us explain this behavior by combining the behavior of mobility and carrier concentration.

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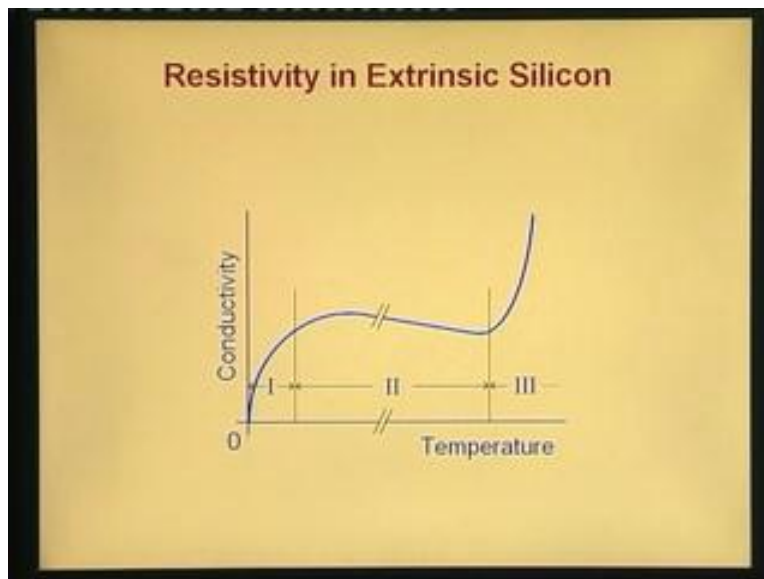
Extrinsic silicon: Supposing you take n-type the carrier concentration if you were to sketch as the function of temperature, so electron concentration  $n_{n0}$  then this behavior on

a linear scale is like this, the so called extrinsic range. This is the region 2. This is the partial ionization range and this is the intrinsic range.

Now if you sketch the mobility on the same graph, this is carrier concentration and if you sketch the mobility it will be something like this, it will increase and then fall and somewhere it is in maximum. For the same doping level we are plotting the mobility, this is mobility  $\mu_n$ . Then conductivity  $\sigma$  is equal to  $q n \mu_n$  plus  $q p \mu_p$ . If you are taking n-type semiconductor then this  $n$  is  $n_{n0}$  and we are considering equilibrium conditions or close to equilibrium conditions and this is  $p_{n0}$ . Now  $p_{n0}$  will be much less than  $n_{n0}$  because these are minority carriers so the  $\sigma$  in n-type semiconductor is approximately equal to  $q n_{n0} \mu_n$ .

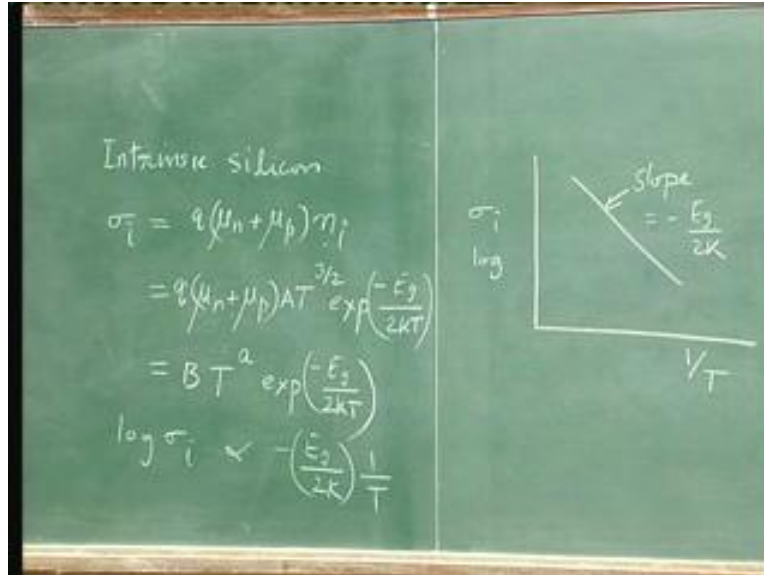
We need not bother about this hole concentration and the hole mobility. So that is why using  $\mu_n$  behavior and  $n_{n0}$  behavior all we need to do is multiply these two curves. It is evident that you will get a raise and then in this range a fall in conductivity because of fall in mobility which is happening because of lattice scattering and then because carrier concentration is increasing you again have a raise in mobility and this is something like this. So that is how one can explain this particular behavior.

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This fall here is because of mobility behavior, lattice scattering and this raise over a small region in the extrinsic range is because of ionized impurity scattering. In this range the Carrier Concentration is constant so whatever variation you see is mainly because of mobility. Now let us look at intrinsic silicon  $\sigma_i$ .

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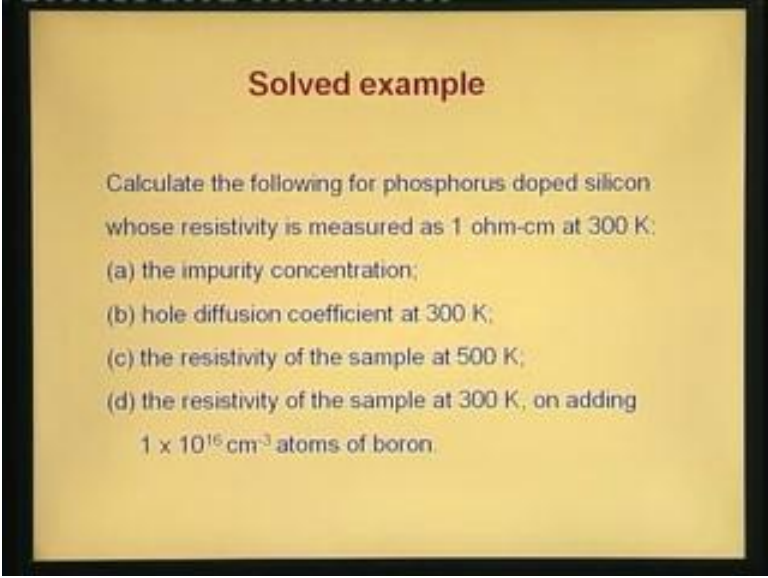
In this case sigma is equal to  $q(\mu_n + \mu_p)n_i$  because electrons and holes are in equal concentration. Now the behavior of  $\sigma_i$  will depend on  $\mu_n$ ,  $\mu_p$  and  $n_i$ . Now  $n_i$  is increasing continuously as a function of temperature right from 0 onwards to very high temperatures and this change in  $n_i$  with temperature is very rapid as compared to variation in mobility. If you combine these two:  $\mu_n$  and  $\mu_p$  have a power law depending on a temperature whereas  $n_i$  has an exponential dependence in addition to the power law. So we can write this as  $q(\mu_n + \mu_p) T$  to the power 3 by 2  $\exp(\text{minus } E_g \text{ by } 2KT)$  that is the behavior into some constant A.

Now, if you also include the power law behavior of mobility then this will be of the form, here q also you absorbed in a constant so this will turn out to be of the form  $BT^a \exp(\text{minus } E_g \text{ by } 2kt)$ . The resistivity of intrinsic silicon is not going to be very much depended on a mobility behavior. It is the carrier concentration behavior because this is the most dominant variation with temperature as compared to the power law variation  $T$  to the power a where a combines this 3 by 2 power as well as the power law because of the mobility. It turns out that 'a' is something like minus 1 if you combine all the power loss and together 'a' it is about minus 1. So if you sketch  $\sigma_i$  versus T the behavior will be the same as that of the carrier concentration behavior which means  $\sigma_i$  versus T is same as  $n_i$  versus T behavior.

In other words, if you sketch  $\sigma_i$  on a log scale versus  $1/T$  on the x-axis on the linear scale then you will get a behavior like this which is same as that of the intrinsic silicon. This will be a straight line because the mobility behavior only affects the T to the power a here, this term and that is why for intrinsic silicon if you sketch the measured resistivity as the reciprocal of temperature from the slope of that one can easily get the energy gap because this behavior is similar to that of the  $n_i$  behavior intrinsic carrier behavior multiplied by some constant.

If I sketch  $\log \sigma$ , it will go as proportional to  $-\frac{E_g}{2kT}$  which you get if you take the log. So slope of this is equal to  $-\frac{E_g}{2k}$ . From the slope of the resistivity versus reciprocal of temperature graph for intrinsic semiconductor one can get the energy gap. Now it is time to consider a solved example. You listed some of the ideas and to also show how formulae can be used to make calculations.

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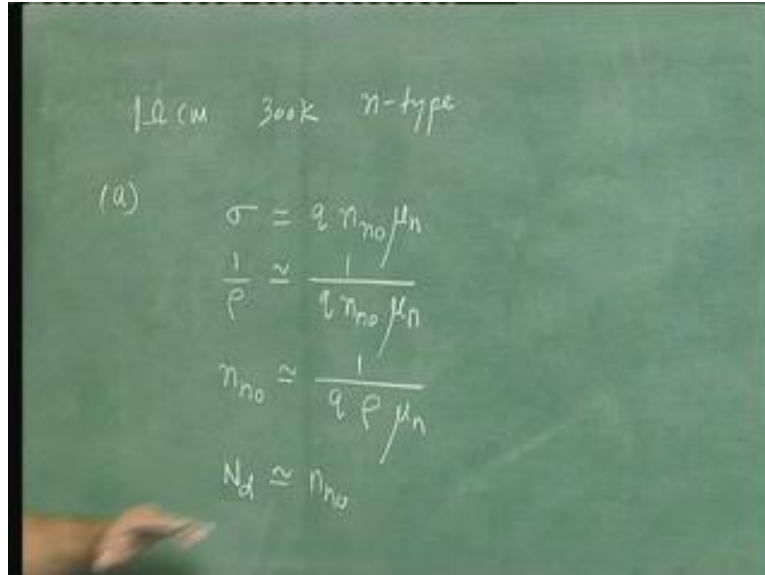
**Solved example**

Calculate the following for phosphorus doped silicon whose resistivity is measured as 1 ohm-cm at 300 K:

- (a) the impurity concentration;
- (b) hole diffusion coefficient at 300 K;
- (c) the resistivity of the sample at 500 K;
- (d) the resistivity of the sample at 300 K, on adding  $1 \times 10^{16} \text{ cm}^{-3}$  atoms of boron.

The example is, calculate the following for phosphorous doped silicon whose resistivity is measured as 1 ohm-cm at 300 K. What we are supposed to calculate are the following: the impurity concentration, the hole diffusion coefficient at 300 K, the resistivity of the sample at 500 K and the resistivity of sample at 300 K on adding  $1 \times 10^{16} \text{ cm}^{-3}$  atoms of boron. Basically you have an n-type semiconductor whose resistivity is 1 ohm-cm at 300 K.

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Handwritten equations on a green chalkboard:

$$1 \Omega \text{ cm} \quad 300 \text{ K} \quad n\text{-type}$$

(a)

$$\sigma = q n_{no} \mu_n$$
$$\frac{1}{\rho} \approx \frac{1}{q n_{no} \mu_n}$$
$$n_{no} \approx \frac{1}{q \rho \mu_n}$$
$$N_d \approx n_{no}$$

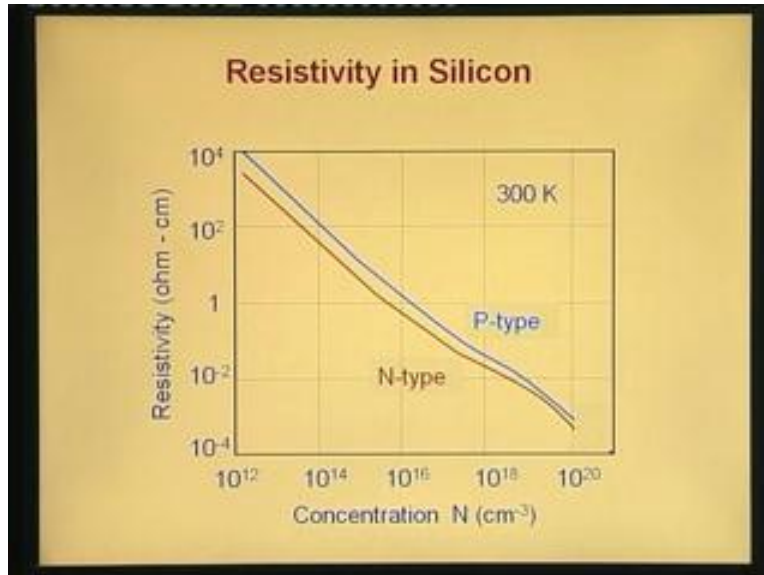
Let us start with the first part, what is the impurity concentration?

Data given is 1 ohm-cm at 300 K and it is an n-type phosphorous doped so what is the impurity concentration?

Now, as we have seen the conductivity of this semiconductor can be written as  $q n_{no} \mu_n$  we ignore the hole component of the conductivity because only majority carrier concentration decides the conductivity so the resistivity is 1 by  $\rho$  that can be written as 1 by  $q n_{no} \mu_n$ . We know  $\rho$  and we need to get  $n_{no}$  so  $n_{no}$  is equal to 1 by  $q$  into  $\rho$  into  $\mu_n$ .

Now the difficulty in applying this particular equation in a straightforward manner is that this  $\mu_n$  depends on the  $n_{no}$  which we want to determine so  $\mu_n$  depends on the doping. Since we do not know the doping in advance, a straightforward use of this relation is not possible and some involved calculation is required. The impurity concentration is  $n_{no}$  and we know that  $N_d$  will be  $n_{no}$  because we assume at 300 K complete ionization and the majority carrier concentration is almost equal to the doping level ignoring the thermal generation. One way to solve this problem is to look at a graph of doping versus resistivity so these kinds of graph are available where resistivity is given as a function of doping concentration  $N$ .

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This is the net doping for both p-type and n-type semiconductors. Here you can see the resistivity of 1 ohm-cm for n-type and read out the concentration. In this particular graph the finer divisions are not shown so it is difficult to find out. But in graphs available in books you will find these smaller divisions present and one can determine the concentration from there. Even then from the graph an accurate determination is very often difficult so even tables are available for resistivity as a function of doping which can be used to get the resistivity for a given doping or doping for a given resistivity. Now, alternately if you want to do a calculation using a computer then it can be done as follows.

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1  $\Omega$  cm    300 K    n-type

(a)

$$\sigma = q n_{no} \mu_n$$
$$\frac{1}{\rho} \approx \frac{1}{q n_{no} \mu_n}$$
$$n_{no} \approx \frac{1}{q \rho \mu_n}$$
$$N_d \approx n_{no}$$

What we do is we assume the mobility corresponding to the low doping or maximum value of mobility here and find out the concentration. Then we use this concentration and do iteration and find out the mobility corresponding to this concentration using the mobility versus doping formula. Then use the new value of mobility and determine the concentration again. When you repeat this calculation two times normally you get a very good result. If you do that then the calculation proceeds as follows. So  $\mu_n$  is equal to  $\mu_{n \text{ max}}$  is equal to 1330 cm square by v-s. If you do that then  $N_d$  is equal to 1 by  $q \rho$  into  $\mu_{n \text{ max}}$  is equal to 1.6 into 10 to the power minus 19 coulombs. You must also put the units simultaneously on the sides.  $\rho$  is 1-ohm cm and this is 1330 cm square by v-s this is the value of doping and these are the units. As you can see this gives raise to cm cube and coulomb per second is amperes. So ampere into ohm is equal to volt so this cancels and you have by cm cube as a unit and this will be equal to 4.7 into 10 to the power 15 by cm cube.

Now we use this value of doping and find out the mobility. You can write using this formula 1330 minus 65.5 by 1 plus (4.7 into 10 to the power 15 by 8.5 into 10 to the power 16) whole power 0.76 plus 65.5 cm square by v-s. This formula was shown on the slide earlier. So you use this formula for mobility as the function of doping at 300 K. Then the result will be a mobility of 1204 cm square by v-s, instead of a 1330 that we assumed earlier. Use this value of mobility instead of  $\mu_{n \text{ max}}$  and you will get the new doping. The new doping will be 4.7 into 10 to the power 15 into (1330 by 1204) and this is nothing but 5.2 into 10 to the power 15 cm cube which is the value of doping or impurity concentration.

Next you need to find out the diffusion coefficient of holes. To find out the diffusion coefficient of holes that is  $D_p$  you can use the Einstein relation  $\mu_p$  into  $V_t$  (thermal voltage) so you need to know  $\mu_p$ . Now  $\mu_p$  you can find out using a formula similar to this because you know the doping. Like you have formula for  $\mu_n$ , there is a formula for  $\mu_p$  so  $\mu_p$  is equal to 495 minus 47.7 by 1 plus (5.2 into 10 to the power 15 by 6.3 into 10 to the power 16) whole power 0.72 plus 47.7 cm square by v-s.

Please note that whenever you write the magnitude you must also write the units, dimensions, you should not forget that, otherwise would you go wrong in your calculations. This mobility turns out to be equal to 431 cm square by v-s. Therefore your  $D_p$  is 431(0.026), it is cm square by v-s and this is volts so volts will cancel and you will get cm square by sec as the unit and this will be 11.2 and that is the diffusion coefficient.



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**Solved example**

Calculate the following for phosphorus doped silicon whose resistivity is measured as 1 ohm-cm at 300 K:

- (a) the impurity concentration;
- (b) hole diffusion coefficient at 300 K;
- (c) the resistivity of the sample at 500 K;
- (d) the resistivity of the sample at 300 K, on adding  $1 \times 10^{16} \text{ cm}^{-3}$  atoms of boron.

Now part c is the resistivity of the sample at 500 K. To find out the resistivity at 500 K we need to check two things.

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1 ohm cm 300K n-type

Resistivity at 500K

$$n_i(500K) = n_i(300K) \left(\frac{500}{300}\right)^{3/2} \exp\left[\frac{-1.12}{0.052} \left(\frac{300}{500} - 1\right)\right]$$
$$= 1.78 \times 10^{14} \text{ cm}^{-3}$$
$$p_{no} = \frac{n_i^2}{n_{no}} \approx \frac{n_i^2}{N_d}$$

We need to check whether the minority carrier concentration important? The temperature is high so will the hole concentration be important and will it make a difference to the resistivity. For this purpose we need to find out the intrinsic concentration first so  $n_i$  at 500 K can be found very easily as we had done in this course earlier. We can write this in terms of  $n_i$  (300 K)  $(500 \text{ by } 300)$  to the power  $3 \text{ by } 2$   $\exp$  [minus 1.12 by 0.052(300 by

500 minus 1)] which is the energy gap divided by  $2(kT)$  where  $T$  is the room temperature that is 0.052 into room temperature which is 300 by the temperature of interest.

Make this calculation and you will get the result as  $(1.78 \times 10^{14})$  by cm cube. Now the minority carrier concentration would be  $p_{n0}$  is equal to  $n_i$  square by  $n_{n0}$  which is  $n_i$  square by the doping so which will be is equal to  $(1.78 \times 10^{14})$  whole square by  $5.2 \times 10^{15}$  doping and evidently since this is  $10^{14}$  to the power 28 and  $10^{15}$  your order of this particular result will be  $10^{13}$  so this is of the order  $10^{13}$  which is much less than  $5.2 \times 10^{15}$  by cm cube. You find that  $p_{n0}$  is much less than  $n_{n0}$ . So result is  $p_{n0}$  is much less than  $n_{n0}$ . We can still continue to use the relation  $\sigma \approx q \mu_n n$  to the power  $n_0$ . We are neglecting any contribution of  $p_{n0}$  to the resistivity. So this  $n_{n0}$  is nothing but the doping which we can replace by  $N_d$ .

However, the mobility at 500 K will not be the same as mobility at 300 K. We need to determine the mobility at 500 K and then we can easily determine the conductivity or resistivity. Mobility at 500 K by mobility at 300 K can be written as is equal to 500 by 300 to the power minus 3 by 2. This is because we are using the fact that, in the extrinsic range above room temperature the mobility approximately goes as  $T$  to the power minus 3 by 2 so  $\mu$  is equal to some constant into  $T$  to the power minus 3 by 2 so this is the relation we are using.

From here we find that mobility at 500 K is equal to mobility at 300 K for electrons. And the result at 300 K the mobility was 1204 cm square by v-s so this is equal to into 3 by 5 to the power 3 by 2 is equal to 555.6 cm square by v-s so that is the mobility. You can use this mobility here and then find out the resistivity So  $\rho \approx 1$  by  $q \mu N_d$ . And in fact we could directly substitute this value of mobility or alternately what we find is since the resistivity is depended on the mobility behavior we can write whatever we have written in terms of mobility directly as an equation in terms of resistivity.

For example, we can write  $\rho_{500\text{ K}}$  by  $\rho_{300\text{ K}}$  so  $\rho$  is inversely proportional to mobility so this is equal to the mobility at 300 K by mobility at 500 K of electrons in this case. This is equal to 5 by 3 to the power 3 by 2 and the result therefore is, resistivity at 500 K is equal to 300 K resistivity which is 1 ohm-cm multiplied by this and the result would be 2.15-ohm cm. So this is the way one can find out the resistivity at a higher temperature. You can find out the mobility at a higher temperature and find out the resistivity. But since we already know the resistivity at a lower temperature it is easier to write it in this simplified form and then get the resistivity. At a higher temperature actually the resistivity has increased because of the fall in mobility.

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**Solved example**

Calculate the following for phosphorus doped silicon whose resistivity is measured as 1 ohm-cm at 300 K:

- (a) the impurity concentration;
- (b) hole diffusion coefficient at 300 K;
- (c) the resistivity of the sample at 500 K;
- (d) the resistivity of the sample at 300 K, on adding  $1 \times 10^{16} \text{ cm}^{-3}$  atoms of boron.

Now we have the final part of the example that we have to show and this is the resistivity of the sample at 300 K on adding 1 into 10 to the power 16 cm cube atoms of boron. This means we have a compensated semiconductor with donor type impurity of doping 5.2 into 10 to the power 15 cm cube.

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$$\begin{aligned} N_d &= 5.2 \times 10^{15} / \text{cm}^3 \\ N_a &= 1 \times 10^{16} / \text{cm}^3 \\ \text{p-type} \\ p_{p0} &\approx N_a - N_d = 4.8 \times 10^{15} / \text{cm}^3 \\ p &\approx \frac{1}{2 p_{p0} / \mu_p} \end{aligned}$$

And an acceptor doping of 1 into 10 to the power 16 cm cube will help you find the resistivity. Clearly, this is a p-type semiconductor because  $N_a$  is more than  $N_d$  so the carrier concentration of holes i.e. the majority carrier concentration in this case would be  $p_{p0}$  and it can be written as  $N_a$  minus  $N_d$ . All impurities are ionized and now we can

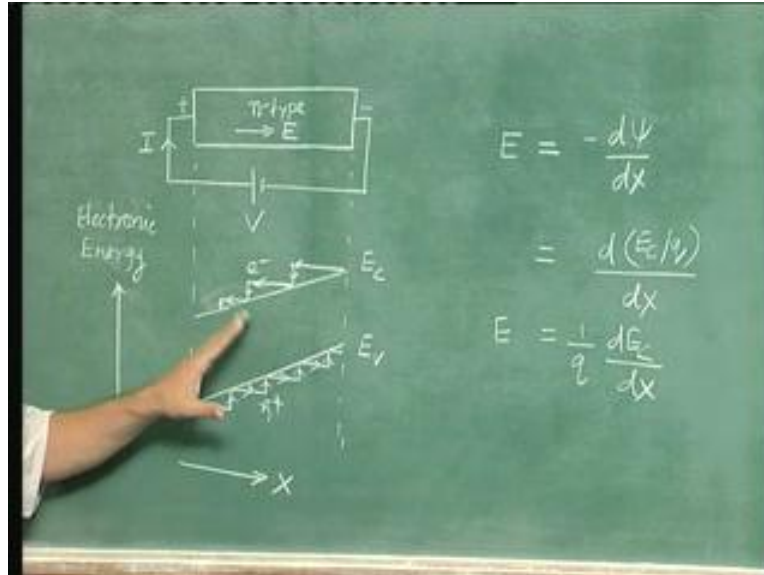
neglect the electron concentration because minority carrier concentration will be very small and this is equal to  $4.8 \times 10^{-15}$  by cm cube. This is the p-type semiconductor with this particular concentration of holes.

Now the resistivity can be written as  $1/q \times p_{p0} \times \mu_p$  where  $p_{p0}$  is given here. Now  $\mu_p$  we have to determine. We cannot use the hole mobility that we determined earlier in this case because now we have to use total doping of this plus this. The mobility of holes which we have determined earlier in context of the diffusion coefficient calculation was only because of this particular doping so we need to recalculate the hole mobility. So we use the formula  $\mu_p$  is equal to  $495 \times 10^{16} / (6.3 \times 10^{16} + 47.7 \times 10^{16})^{0.72}$  cm square by v-s is in the denominator and power is 0.72. In the numerator we will have to put a sum of this  $N_d$  plus  $N_a$  is equal to  $1.52 \times 10^{16}$  and plus  $47.7 \times 10^{16}$  cm square by v-s this is the hole mobility and the result is 377 cm square by v-s.

Now we can substitute this  $p_{p0}$  that is this  $4.8 \times 10^{-15}$  here and 377 cm square by v-s here and the resistivity would be  $1/q \times p_{p0} \times \mu_p$ . This is the resistivity and of course your unit will be ohm-cm because this is in coulombs cm cube and this is cm square by v-s. We have shown that if you use these units you will get resistivity in ohm-cm and the result is 3.46 ohm-cm. The resistivity has actually increased because the sample has become p-type and hole mobility is smaller than electron mobility and because of combination of these factors you are getting this resistivity. This is how one can make calculations of resistivity at different temperatures and different doping conditions. This example should clarify many of the ideas that we have so far discussed.

The next topic we will discuss before closing this particular Carrier Transport discussion will be on how do you see the mechanism of drift from the energy band point of view? Even diffusion can be seen from the energy band point of view but we will see how the drift is seen from the energy band point of view and one can work out for diffusion in the same manner.

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We need to draw the energy band diagram of a semiconductor in which a drift current is flowing. Suppose this is a semiconductor let us assume an n-type for simplicity, we can use a p-type it does not matter really. In the semiconductor the electric field is in this direction. Now what will be the energy band diagram under this condition? We will assume a uniform semiconductor which means that the electric field is also going to be uniform. And that means that the potential will vary linearly as you move from the left end to the right end so this is positive and this is negative. Now how do we draw the energy band diagram under these conditions?

Recall that the vertical axis in energy band diagram shows electronic energy. We have also mentioned earlier in the discussion of energy band diagram that  $E_c$  represents the potential energy of the free electron and  $E_v$  represents the potential energy of a hole. So if you want to draw the energy band diagram without an electric field it would look something like this where this is  $E_c$  and this is  $E_v$ . This is the potential energy of the free electron and this is the potential energy of the hole. And this is the first time that the diagram is drawn with distance as the x-axis. This is energy and this is distance along the sample, so this is the sample. No electric field is applied under equilibrium conditions.

The Fermi-level here would be somewhere here, this is the Fermi-level, these dotted lines show the intrinsic level. So  $E_c$  is the potential energy of the free electron and this is electronic energy which means that whichever side is negative, whichever position here in the sample is negative there the electronic potential energy should be shown higher.

Let us look at the sample now, this side is positive so here on this side your  $E_c$  will be lower so when you draw the energy band diagram it will be drawn as something like this. This is electronic energy, this is  $E_c$  potential energy of the free electron, this is lower and this is higher because this is more negative and as you move up you have higher and higher electronic energy. The valance band edge  $E_v$  will run parallel to the conduction

band edge because when you apply voltage the energy gap does not change in the semiconductor. The carrier concentration is also not changing so the structure of the silicon is not changing. The only thing that is happening is that you are superimposing an electric field so energy gap cannot change with distance and that is why this edge  $E_c$  is parallel to  $E_v$  and this difference remains equal to energy gap.

Now what is the slope of this particular diagram? Slope of this diagram is nothing but slope of the potential.  $E_c$  is energy and if you divide  $E_c$  by  $q$  you get the potential. Now your equation is electric field  $E$  is equal to minus  $d\phi$  by  $dx$  where  $\phi$  is the potential of a positive charge. So when you write electric field equal to gradient of potential this is potential of a positive charge. So if you want to write in terms of a negative potential then this negative sign will go away. It is important to note here that the  $E$  here is electric field whereas the  $E$  here is the energy. I hope there will not be any confusion because we have been using this nomenclature throughout the earlier lectures.

Therefore in terms of  $E_c$  if I want to write down replace  $\phi$  by potential  $E_c$  then I can write the same thing as this is equal to  $dE_c$  by  $q$  because I want to convert energy into potential so I have to divide by charge by  $dx$ . The negative sign has been removed because this  $E_c$  by  $q$  is electronic potential. In other words what we find is at the electric field is equal to  $1$  by  $q$  and  $dE_c$  by  $dx$ .

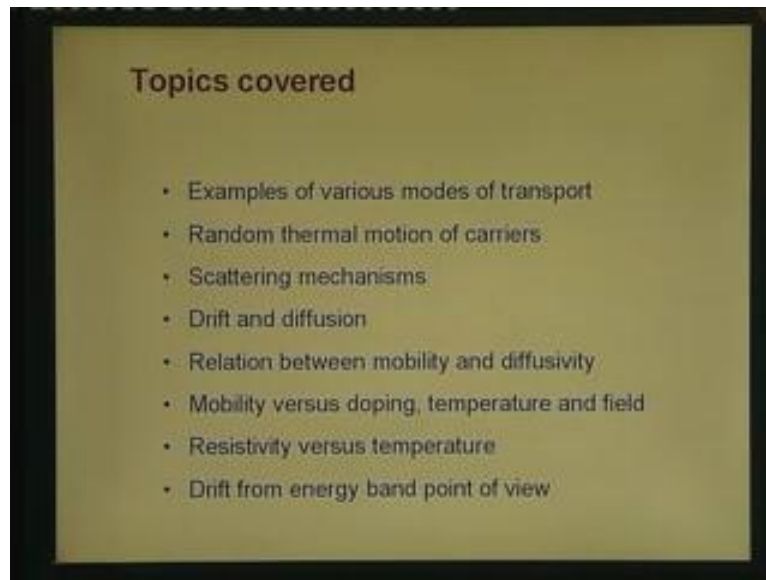
Clearly see that the slope of this line is positive  $1$  by  $q$   $dE_c$  by  $dx$  and therefore the electric field is in the positive direction from left to right. And the slope of this particular line gives you the electric field provided you take into account the charge  $q$ . So that is the important thing that you see for the energy band diagram when you apply an electric field. Now on this particular diagram how will you show the motion of an electron, electron will move from right to left because it will move against the electric field. This motion can be shown as follows.

Let us say electron starts from here and after collision it has started moving in this direction towards the left. So it moves like this and it encounters a collision somewhere here. At the end of the collision the energy gain in the direction of the electric field is randomized. In other words, we can show this as the kinetic energy has got dropped to  $0$  because this difference represents the kinetic energy of the electrons that it has gained from the electric field. Again it starts moving and encounters a collision and so on so this is the path.

Now, note here carefully that this is a horizontal line. This line is not going up nor is it moving down it is perfectly horizontal. This is because the total energy of the electron between two collisions remains constant. It is losing potential energy as shown by this line but it is gaining kinetic energy. Loss in potential energy is exactly equal to the gain in kinetic energy. Total energy being constant is shown as a horizontal line. So total electronic energy between collisions is constant but it is gaining kinetic energy seen by the fact that this line here for example is above this particular line and there is a gap and this gap represents the kinetic energy that is gain from the electric field.

One can similarly draw the movement of the hole on the energy band diagram which would be something like this the hole moves in this direction so this is electron and this is hole this is  $x$  in the sample. This is how one can look at the drift from energy band point of view. Note carefully that I have not shown the Fermi-level here because Fermi-level is strictly an equilibrium concept and under non-equilibrium conditions one can extend this idea but then one has to introduce what are called Quasi Fermi-levels so this aspect we will consider later. I am not showing the Fermi-level in this diagram. It is not necessary to show this level to understand the drift transport. Now let us summarize our discussion on Carrier Transport.

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We covered the following topics in order. First we saw the example of various modes of transport such as drift, diffusion, thermo electric current and current because of tunneling, random thermal motion of carriers, the semi classical phenomena of drift, diffusion and thermo electric current are based on this particular motion. Then scattering mechanisms which causes the random thermal motion and then we considered the drift and diffusion transport and how directed motion is superimposed over random motion by application of potential gradient or concentration gradient. Then we discussed relation between mobility and diffusivity, this is the Einstein relation. Then we discussed the mobility as a function of doping temperature and electric field. For small fields you have a linear relation between velocity and electric field but for large fields the velocity is constant so mobility varies with field. Then we discussed resistivity as a function of temperature and doping and finally we discussed the drift from energy band point of view. In the next class we will consider the topic of Excess Carriers.