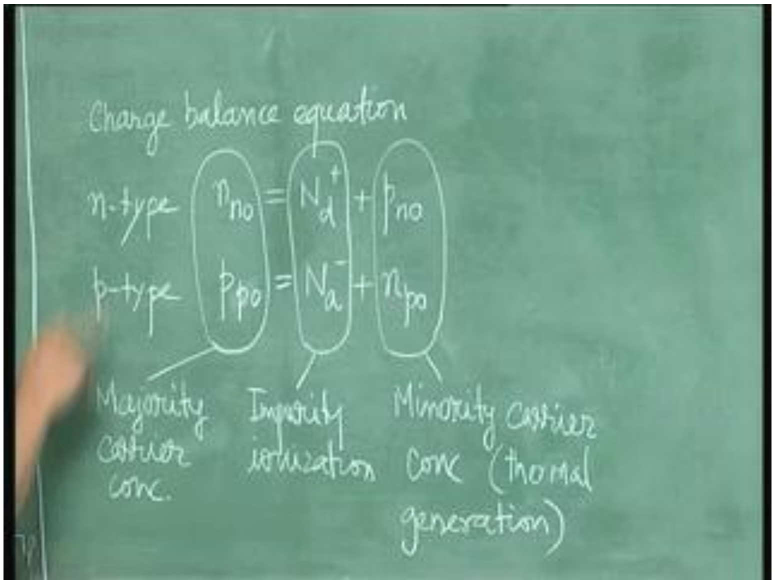
Solid State Devices
Dr. S. Karmalkar
Department of Electronics and Communication Engineering
Indian Institute of Technology, Madras
Lecture - 10
Equilibrium and Carrier Concentration (Contd…)

This is the 10th lecture of this course and the 8th lecture on equilibrium carrier concentration. In the last class we considered the bond model of the extrinsic semiconductor and we showed that we can write a charge balance equation relating the carriers in the semiconductor, concentrations of these carriers.

(Refer Slide Time: 01:16)



Charge balance equation

n-type $n_{no} = N_d^+ + p_{no}$

p-type $p_{po} = N_a^- + n_{po}$

Majority carrier conc.

Impurity ionization

Minority carrier conc (thermal generation)

So, equations for n and p-type are shown here. This equation shows that majority concentration can be written as the ionized impurity concentration plus the thermally generated concentration or the minority carrier concentration. The behavior of these concentrations as a function of temperature is as shown in this graph. We have explained this particular graph which says; as the temperature increases from 0 the carrier concentration starts rising.

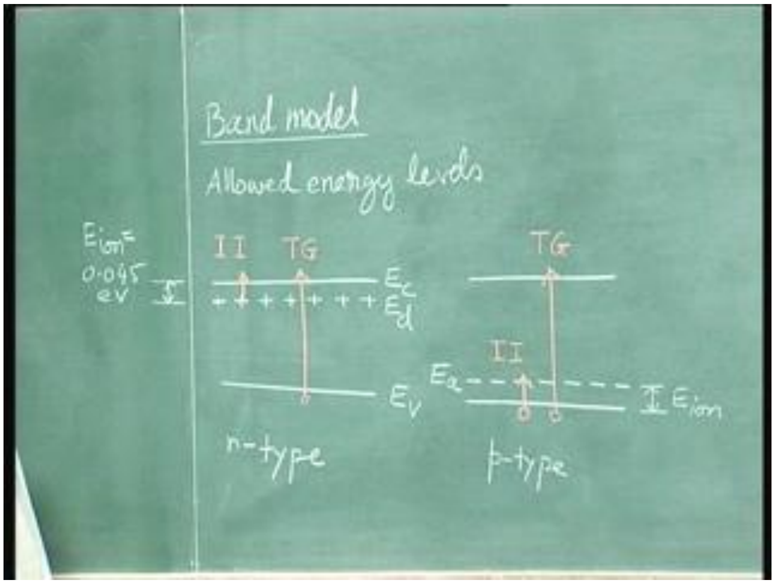
We are plotting the majority carrier concentration here in this line, this concentration starts rising because of increase in the extent of impurity ionization and ultimately by this temperature all the impurities are ionized and the majority carrier concentration just saturates. Then at very high temperatures the majority carrier concentration starts increasing because even though the impurities which are ionized have saturated, i.e. the impurities have completely ionized, the minority carrier concentration has started increasing with temperature which was very low in this temperature range. And so you have the three ranges; the partial ionization range, then the extrinsic range here and the intrinsic range. The approximate values of the concentrations in these three ranges are written here.

For an n-type semiconductor, the majority carrier concentration is approximately equal to the ionized impurity concentration in partial ionization range which is this range. Then in the extrinsic range, the majority carrier concentration is approximately equal to the impurity concentration itself because impurities are fully ionized. In the intrinsic range, the concentrations of electrons and holes both tend towards n_i. So this is what we saw in the last class. The bond model however has weaknesses.

Another important thing to know from the point of view of the operation of the semiconductor devices is, finding out the minority carrier concentration at any temperature that we are not able to calculate using the bond model. Also the majority carrier concentration at any general temperature is difficult to find out.

For example, if you want to know majority carrier concentration here, you must know the extent of impurities ionization which will not be provided by the bond model. Similarly, if you want to know when the intrinsic range starts, even this cannot be told by the bond model and also it cannot give you the concentration here when both the minority carrier concentration and the majority carrier concentration are becoming comparable at higher temperatures. So for these reasons one needs to go to the energy band model for an accurate calculation of the concentration, either of majority carriers or of minority carriers at any temperature and doping. So this is the topic in this particular lecture, that is, the Band model of the extrinsic semiconductor.

(Refer Slide Time: 04:37)



We proceed in the same way as we did for the intrinsic semiconductor. We basically need to see three aspects; the allowed energy levels, then the density of states function and the fraction of occupied states function. And then we need to put these three aspects together to get the concentration. Let us start with allowed energy levels.

Supposing we consider an n-type semiconductor to begin with, we make the dilution approximation, this dilution approximation was discussed earlier. That is, we assume that the impurities do not affect the crystal structure and secondly the concentration of impurities is so small that one impurity atom does not interact with another impurity atom. So, the consequences of these two approximations can be shown on the energy band as follows; Since the crystal structure is unaffected by impurity concentration, the difference between conduction band edge and valance band edge which is the energy gap does not change with doping. Even after you have added any impurities the energy gap of the semiconductor is the same as that of an intrinsic semiconductor provided that doping is moderate so long as the dilution approximation is valid, this is E_g.

Now, the impurity atoms do not interact with each other. This means the energy level corresponding to the electrons occupying the impurity sides for all the different atoms will be the same because the impurity atoms are not interacting with each other. So you do not have an energy band corresponding to the impurity atoms, you only have a single level.

Now, where do we show the single level on this energy band diagram?
Supposing you take phosphorous impurity, this is n-type we are drawing. The phosphorous impurity has ionization energy of 0.045 electron volts. This means that an electron which is occupying a donor impurity gets 0.05 electron volts of energy it will get into the conduction band. This means you can locate the energy level corresponding to a phosphorous impurity somewhere here above which is located at a distance of 0.045 electron volts below the conduction band edge. This is nothing but the ionization energy of the impurity, E_{ion}.

So, because of a dilution approximation it is a single level corresponding to all the impurity atoms and this energy level is located at a distance equal to ionization energy below the conduction band edge. This means that introduction of impurities result in extra allowed energy levels within the energy gap of a semiconductor. So that is the consequence of introduction of impurities in a semiconductor on the energy band diagram. This particular level is represented e suffix d and it is called the donor level, this d stands for donors.

On this diagram if you want to show the processes that contribute to carriers in the semiconductor, they can be shown as follows; the impurity ionization can be shown as an arrow like this where an electron jumps from the impurity site or the impurity level to the conduction band edge so this is impurity ionization so I will call it II.

Incidentally the positive sign here indicates that this is a donor level because a donor is positively charged when it releases as an electron, so a positive sign means it is a donor level. As we will see if for an acceptor impurity you have a negative sign to show the acceptor like nature of the impurity. The other process of thermal generation involves an electron jumping from the valance band to conduction band so this is the process of thermal generation; these two together contribute to majority carriers.

So, in an n-type semiconductor electrons will be contributed by the impurity and as well as by thermal generation, the sum of these two processes. On the other hand the holes will only be due to thermal generation. One can similarly draw the allowed energy level diagram for an accepted type impurity, so these are E_c and E_v.

Supposing the acceptor impurity is boron the boron has ionization energy of 0.045 electron volts, this means that when a boron impurity accepts an electron from the valence band and in the process creating the hole, the electron from the valence band has to gain energy of 0.045 electron volts to get into the impurity side. This means you can show the acceptor impurity at a distance of 0.045 electron volts from the valence band edge. So this is E_a the acceptor level.

The negative sign here indicates that this is an acceptor level i.e. this particular level where if an electron jumps into this level it becomes negatively charged and that is why it is acceptor time. So this distance again equal to the ionization energy of the acceptor impurity which is 0.045 electron volts. Now note that, if we are talking of silicon the energy gap is 1.1 electron volts so this is really very small compared to this distance. What we have shown is not drawn on to scale, this must be remembered.

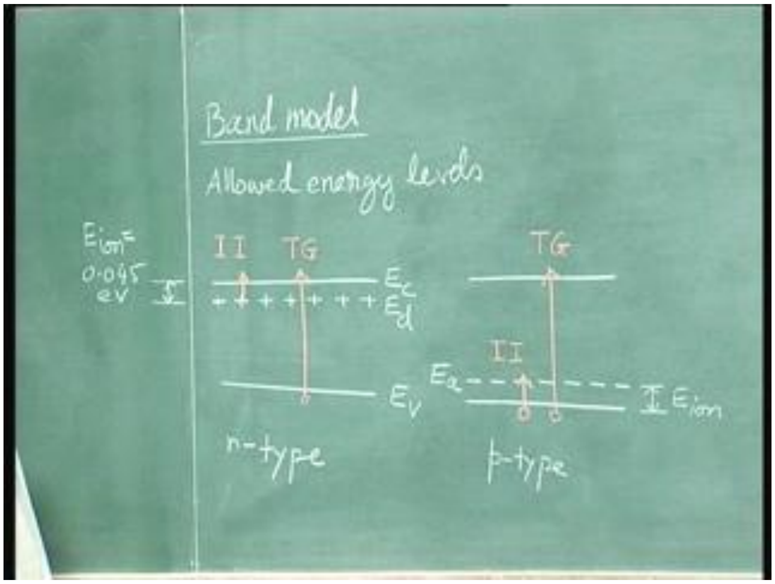
Now in this particular diagram if you want to show the processes contributing to the carriers the impurity ionization process can be shown using this arrow and electrons jumping from valence band to the impurity level leaving behind a hole. This is impurity ionization and you also have a process of a thermal generation, an electron jumping from valence band to conduction band this also leaves behind a hole. So holes are in majority in p-type semiconductor, they are contributed by impurity ionization as well as thermal generation whereas electrons are minority and are only contributed by thermal generation.

I would like to emphasize here that you have a single level corresponding to all the impurity atoms because of the dilution approximation which says that acceptor impurities do not interact with each other. So this is the allowed level picture for an extrinsic semiconductor n or p-type. In this particular case we have shown this picture for phosphorus and boron. If you have other impurities depending on the ionization energy of those impurities, the particular impurity level will be located in a different location.

It is not necessary that for example donor level should be close to the conduction band edge. You may have some impurity for which donor level is here because the ionization energy is very large. It could be closer to the valence band edge, it could be somewhere in the middle and the same thing applies to an acceptor level. Similarly you can have an impurity which gives both donor and acceptor levels this is also possible. Or it gives raise to two donor levels and two acceptor levels.

Towards the end of the discussion on the energy band model we will see examples of other impurities which have different levels within the energy gap. The next topic that we need to discuss is the density of states function. In an extrinsic semiconductor how is the density of states function look like?

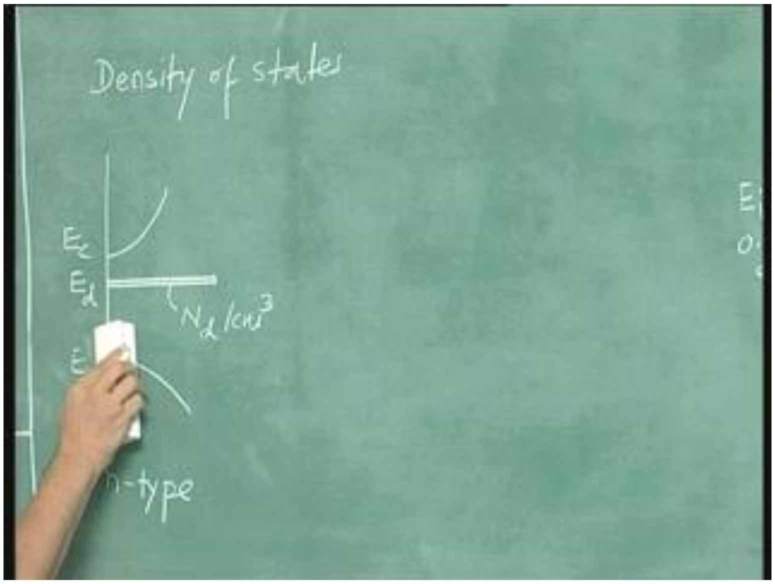
(Refer Slide Time: 14.23)



You start with a picture for an intrinsic semiconductor, if this is E_c and this is E_v then you know that the density of states function is close to parabolic near the edge of E_c and similarly near the edge of E_v, so this is the density of states function in an intrinsic semiconductor. Now, by adding impurities what we have done is that we have created an allowed level somewhere here as shown for n-type semiconductor so all the states are corresponding to those impurities.

Supposing the impurity concentration is N suffix d in this case then there will be N suffix d states one because of each impurity and location of these states will be at the donor level, so we can show this picture as follows. It is something like a delta function, all the N_d states are at the energy level E_d. So this is N_d per centimeter cube so this is an n-type semiconductor. So introduction of impurities has created N_d per centimeter cube states at the energy level E_d in an n-type semiconductor.

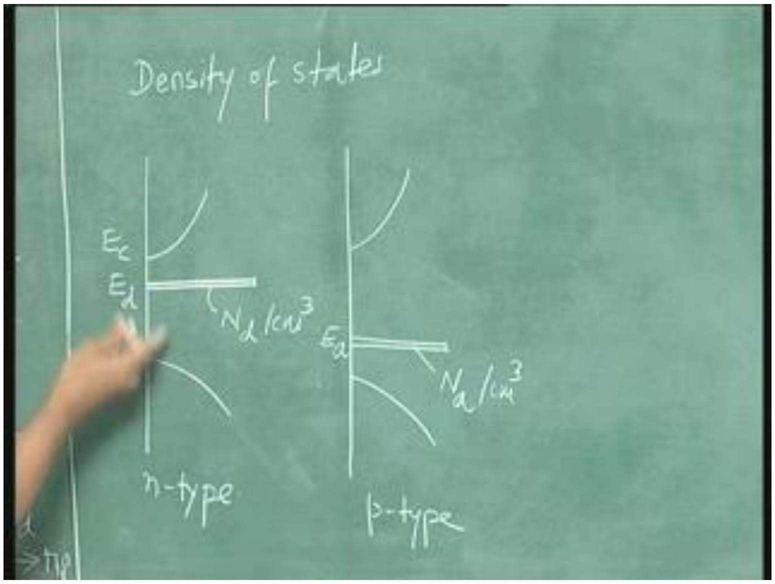
(Refer Slide Time: 17:01)



If you introduce acceptor type impurities, for example, boron then you will have an additional number of states at energy level E_a, so this is N_a. Now notice that if I show both the types of impurities here in a single semiconductor it means this is the semiconductor in which both the n-type dopant as well as p-type dopant, this is not necessary. You may have the semiconductor in which n-type dopant is there in which case this E_a picture will not be there. On the other hand, if it is p-type semiconductor, since I am writing here n-type may be I will show this on a separate diagram, the p-type impurities. This is p-type this is E_a and this is N_a per centimeter cube. This is the density of states picture, in p-type in which only acceptor impurities are present.

Now, if you have a semiconductor in which both impurities are present as I had shown earlier, you can have E_d as well as E_a and that semiconductor could be either n-type or p-type, it depends on the relative magnitude of N_d and n. We will not discuss this point further because we have discussed compensated semiconductor to some extent earlier.

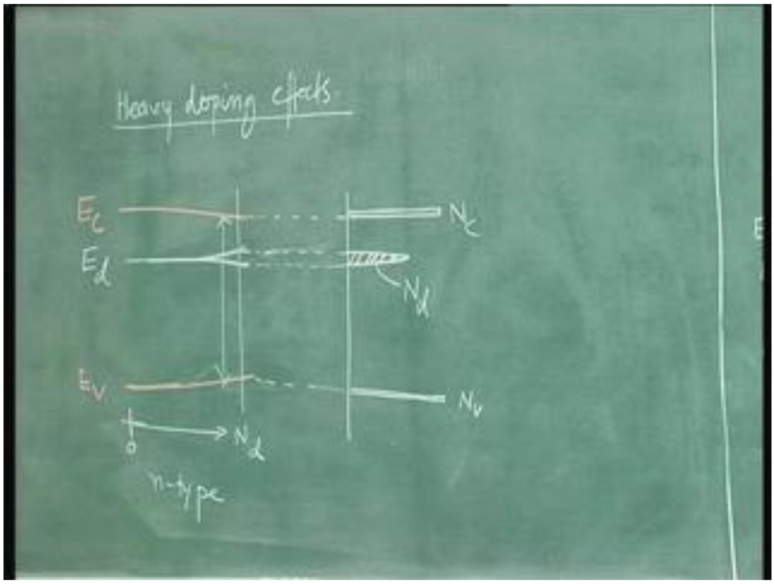
(Refer Slide Time: 17:42)



Now this is the actual picture of the density of states, we can also draw an effective picture. For example, for n-type semiconductor an effective picture would be, at E_c you have N_c states per centimeter cube and you have N_d states per centimeter cube at E_d and you have N_v states per centimeter cube at E_v. This is an effective picture, a simplified diagram. So we have discussed the effective density of states picture for intrinsic semiconductor. The only addition here is the impurity states corresponding to this N_d, donor impurities. I leave it as an exercise for you to draw the effective picture for p-type semiconductor. This is an actual picture and that is an effective picture. So that is the density of states picture for an extrinsic semiconductor.

These pictures have been drawn under the dilution approximation as we have said, it is a single impurity level and the energy gap of the semiconductor is equal to that for intrinsic semiconductor. If the dilution approximation is violated which happens when your doping is very heavy then these conditions will not hold, so what will be the picture under that case, let us see.

(Refer Slide Time: 20.27)



This is heavy doping effects, in fact here these diagrams were for dilution approximation to be valid or moderate doping. Similarly, this is also valid only for moderate doping or dilution approximation. Now we are talking of heavy doping effects, when dilution approximation fails what happens? Let us plot, supposing we take an n-type semiconductor, as a function of N_d what is the allowed levels picture? So allowed levels picture would be something like this.

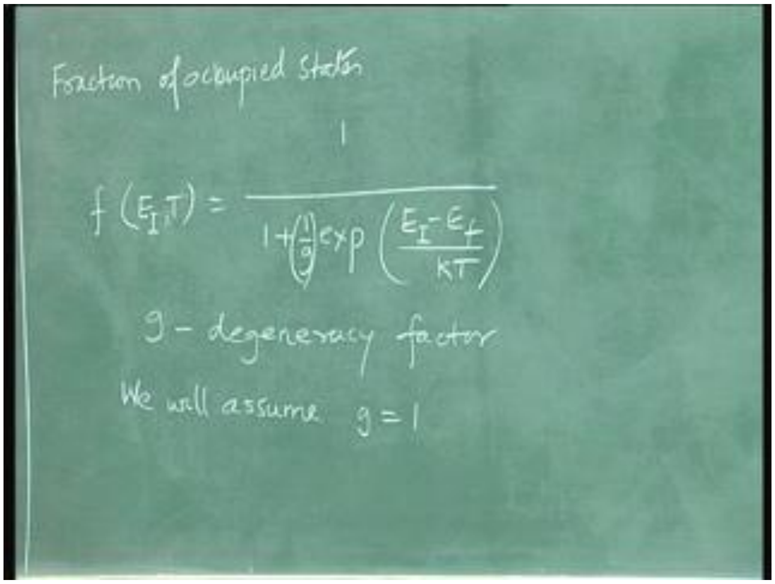
The valence band edge will start moving closer to the conduction band edge, so this is E_v. And similarly conduction band edge also starts moving close to the valence band edge meaning that energy gap of the semiconductor decreases for heavy doping.

Now what about the impurities? Now the impurity starts interacting when the doping is heavy because the distance between the impurities starts reducing. As a result a single level corresponding to the impurity will spread into a band which can be shown here. Supposing it was a donor impurity for low doping, this is doping zero and for low doping it is a single level and for high doping it spreads into a band. So this is E_d, the impurity band.

This is the picture, these are the heavy doping effects, this is the band gap and what you find is, it is reducing as you increase the doping. So this is called band gap narrowing and this is called impurity band, spreading of the impurity level in to impurity band, these are the two effects. Supposing somewhere for this doping we want to show the density of states picture then it would be something like this. So N_c is coming here the delta function and N_v is also delta function but for N_d you now have a band. So N_d will look like this, it is an impurity band so N_d states are now distributed over energy, so it will not be a delta function. This is the density of states picture for heavy doping effects.

We will not discuss the topic of heavy doping effects any further because in this course we will consider semiconductors in which doping is moderate. Having discussed the allowed levels and the density of states picture we have to now look at the fraction of occupied states function and then we have to put together these three aspects; density of states, allowed levels and the fraction of occupied states function to get the carrier concentration.

(Refer Slide Time: 25.02)



Fraction of occupied States

$$f(E_I, T) = \frac{1}{1 + \left(\frac{1}{g}\right)\exp\left(\frac{E_I - E_f}{KT}\right)}$$

g - degeneracy factor

We will assume g = 1

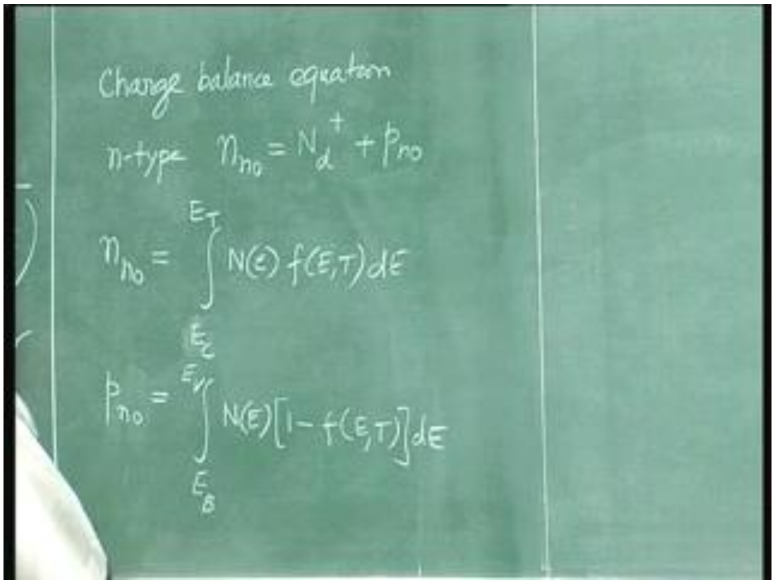
Fraction of occupied states:

We had said that in the intrinsic semiconductor the fraction of occupied states under equilibrium conditions at any temperature is given by the Fermi-Dirac function; let me write that function here. Fermi-Dirac function is a function of energy and temperature, it is given by; f(EjT) is equal to 1 by 1 plus exp(E_i minus E_f by KT).

Now, the same function will describe the fraction of valence band or conduction band states occupied in an extrinsic semiconductor. However, when you want to find out the fraction of occupied states at any impurity level which is present within the energy gap the function is slightly different. So it turns out, I will write suffix $E_{I,}$ for an impurity level: f(E_iT) is equal to 1 by 1 plus (1g)exp(E_i minus E_f by KT). If you want to know the fraction of occupied states then this function is given by the same Fermi-Dirac function with a constant introduced here with the exponential factor 1 by g where g is called the degeneracy factor. This particular g is different for different impurities.

For donor and acceptor impurities they can be different. In this course however we will ignore the degeneracy factor and continue to use the Fermi-Dirac function to describe the fraction of states occupied even at the impurity level. So, impurity level means we are talking of this level E_d or E_a; the levels introduced by impurity within the energy gap. So, for us we assume g is equal to 1. Now how do you put together the information that we have so far to calculate the concentration of electrons and holes which is the next aspect that we want to discuss.

(Refer Slide Time: 28.23)



Charge balance equation

n-type $n_{n0} = N_d^+ + p_{n0}$

$$n_{n0} = \int_{E_c}^{E_T} N(E)\, f(E,T)\, dE$$

$$p_{n0} = \int_{E_B}^{E_V} N(E)\,[1 - f(E,T)]\, dE$$

Now we take up the charge balance equation to do this calculation. For an n-type semiconductor your charge balance equation is: n_{n0} is equal to N_d to the power plus plus p_{n0}. We will assume only one type of impurity that is donor impurity and this is the charge balance equation. Now, in this equation the various terms can be expressed as follows n_{n0} is the electron concentration in the conduction band. So from the energy band model we can write this concentration as integral n_{n0} is equal to E_T integral E_c to N(e) f(E_iT)d so E_c to E_t N(e) that is density of states into fraction of states function into d integrated over the conduction band. That gives you the electron concentration. And similarly if you want to know the hole concentration it is given by integral E_b that is bottom of the valence band to top of the valence band that is P_n0 is equal to E_v integral E_b N(E) [1 minus f(E, T)]dE.

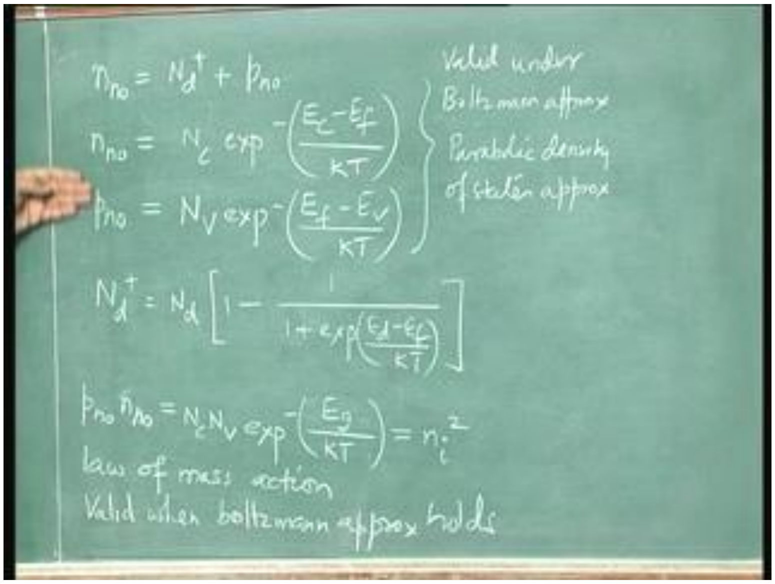
So you know this term and this term, and this is the way to estimate. Now what about this term N_d to the power plus? I will shift this charge balance to the top. Now I write it as n_{n0} is equal to N_d to the power plus plus p_{n0}. Now I can write N_d to the power plus is given as N_d into the fraction, here N_d to the power plus is the number of states which have lost electrons. In other words, number of unoccupied states so this is N_d to the power plus is equal to N_d[1 minus f(E_d, T)] where this represents the fraction of states occupied at E_d corresponding to $N_{d,}$ this is the function. So we have for all the three terms of this equation these are the equations.

How do you proceed further from here?
We need to write down a close form equation that results from this and this to start with. Now if you follow the same approximation we did for the intrinsic semiconductor namely, the term N_e is given by the parabolic density of states approximation then the

term f(E, T) the Fermi-Dirac function is approximated by the Boltzmann approximation and Et is made to tend to infinity, replace E_t by infinity. These are the three assumptions we had made to simplify this particular function for intrinsic semiconductor. We can make the same assumptions in this case.

Of course we will have to check whether the assumptions will be valid. But if you make these three assumptions namely parabolic density of states, Boltzmann approximation and E_t turning to infinity then this formula for n_{n0} reduces to with the negative sign.

(Refer Slide Time: 32.35)



$$n_{no} = N_d^+ + p_{no}$$

$$\left.\begin{aligned} n_{no} &= N_c \exp -\left(\frac{E_c - E_f}{KT}\right) \\ p_{no} &= N_v \exp -\left(\frac{E_f - E_v}{KT}\right) \end{aligned}\right\}$$

Valid under
Boltzmann approx
Parabolic density
of states approx

$$N_d^+ = N_d\left[1 - \frac{1}{1 + \exp\left(\frac{E_d - E_f}{KT}\right)}\right]$$

$$p_{no} n_{no} = N_c N_v \exp -\left(\frac{E_g}{KT}\right) = n_i^2$$

law of mass action
Valid when boltzmann approx holds

Hence n_{n0} is equal to N_c exp minus (E_c minus E_f by KT) is the result of this particular integration after making approximation. One can similarly make approximations for this $p_{n0,}$ same approximation and then you will get relation p_{n0} is equal to N_v exp minus (E_f minus E_v by KT) with a negative sign. Note that I am just writing the same equations that I have derived earlier for the intrinsic semiconductor because this Mathematics is the same. Then one may get a doubt, after doping the carrier concentration has changed the electron concentration.

For example, the electron concentration is much more than the hole concentration in n-type semiconductor, then how is that the equation is same? The equation is same but the Fermi-level is not the same as in the intrinsic semiconductor so this E_f is different. The E_f has changed, equations have not changed. We will shortly see the location of E_f in an extrinsic semiconductor. Now, having discussed this we can also write this particular term N_d to the power plus in terms of the various parameters. So, replace this by its Fermi-Dirac fraction, the equation that is given here, this particular function.

We will assume g is equal to 1 as we have said. If you do that then I can write N_d to the power plus is equal to N_d[1 minus 1 by 1 plus exp(E_d minus E_f by KT)]. Now, using this charge balance inequality and these equations we can solve for the concentration. Of course it is not a simple affair but at least we have moved one step closer to the solution.

I will write the charge balance equation again over this so that all the equations are available in one place. So this is valid under Boltzmann approximation and parabolic density of states approximation. I am not listing the approximation of making the top of the conduction band tending to infinity or e b tending to minus infinity. That approximation will be more or less hole so I am not listing it separately, it is not a significant approximation.

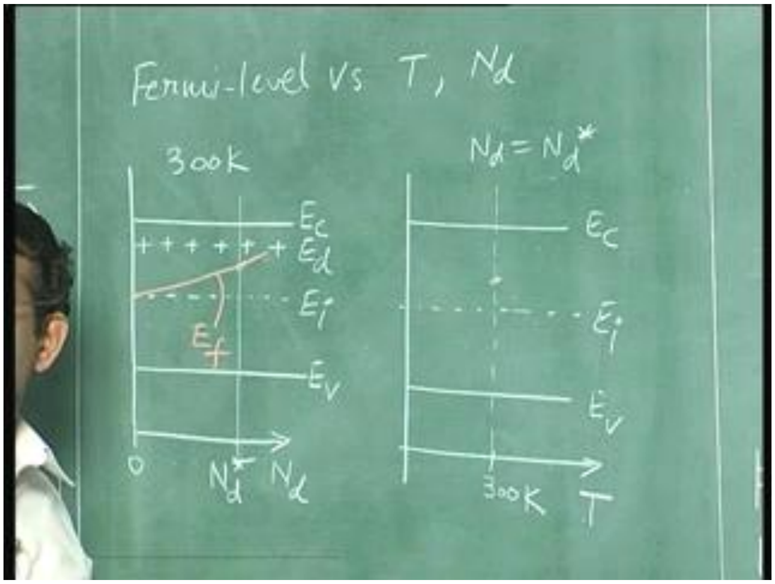
How do we proceed from here to find out the carrier concentration? One thing that strikes us is that if you take these two equations then you find that p_{n0} into n_{n0} is equal to, multiply these two and you will get N_c and N_v and what will happen is, this E_f will get cancelled and you will get exponential E_c minus E_v upon KT with the negative sign. But E_c minus E_v is nothing but the energy gap. So I can write this as minus E_g by KT. And you know that this is nothing but n_i square 2 of the intrinsic concentration. This is a very interesting relation we get under equilibrium conditions: P_n to the power 0 n_n to the power 0 is equal to N_cN_v exp to the power minus (E_g by KT) is equal to n_i square 2. The product of the hole concentrations and electron concentrations in any semiconductor is equal to the square of the intrinsic concentration, so p_n is equal to n_i square is a very important relation.

What we have done for n-type we can also do for p-type and you will get the same relation because these formulae are not going to be any different for holes and electrons in p-type semiconductor. This particular relation is called the law of mass action that the p_n product is only a function of temperature in any semiconductor. So, if you change the doping and as a result you try to change one concentration, you increase one concentration then other concentration decreases that is what it shows in a way in which the product of concentrations remains the same as an intrinsic semiconductor, the square of that. It is important to note that this formula has been derived under Boltzmann approximation so this approximation has been listed here, the Boltzmann approximation is very important. So this is valid when Boltzmann approximation holds.

As we will see the Boltzmann approximation may fail if the doping is heavy because the Fermi-level will move close to either conduction band or valance band. Therefore in moderate doping level conditions this is valid and this is the meaning. In other words, when dilution approximation is valid this formula will hold. There is an interesting analogy to this formula. In dilute solutions in chemistry you might have come across the concept of solubility product.

The concept of solubility product says that the product of the positive ions and the negative ions of any particular species is a constant in dilute solutions. It is something similar here. This can be regarded as positive ions and this can be regard as negative ions and their product is a constant. Let us now see the Fermi-level location in a semiconductor because these formulas are valid only when Boltzmann approximation holds and Boltzmann approximation whether it holds or not if you want to check you must know where is the Fermi-level. Let us now discuss the location of Fermi-level at a qualitative plane in the semiconductor. How does it change when you change the temperature or when you change the impurity concentration? And after discussing that, we will come back to this problem of this derivation of the concentrations in the doped semiconductor.

(Refer Slide Time: 40.17)



Fermi-level versus temperature and doping: Let us assume n-type semiconductor, you can do a similar exercise for p-type semiconductor. Let us first discuss Fermi-level versus doping. So this is the E_c and this is E_v and let us say we are varying the doping from 0 onwards so this access is doping and this is doping equal to 0. How will the Fermi-level change? Let us also show the donor level. When the doping is 0 you know that the Fermi-level will coincide with the intrinsic level. The Fermi-level in intrinsic semiconductor and this is E_i. We start from here.

As you increase the doping now we have to check whether the Fermi-level will move up or will it move down. That can be easily seen from this formula. When you increase the donor doping your majority carrier concentration will go on increasing. So the left hand side of this equation is increasing which means the right hand side the difference E_c minus E_f should decrease. E_c minus E_f is positive quantity and this positive quantity should decrease for left hand side to increase. This means the E_f is moving closer to E_c as you increase the doping. This is the way your E_f is going to change so this is E_f as a function of doping. So we are talking at room temperature.

Obviously you have to fix the temperature when you are discussing the Fermi-level versus doping. What we see is the Fermi level can move close to the conduction band and in fact it can even get into the conduction band if you go on increasing the doping this Fermi-level will be going up. When this happens there is a chance of Boltzmann approximation failing if the doping is very high, if the distance between E_c and E_f

becomes lesser than 3 into KT because as we know the Fermi-Dirac function can be approximated by the Boltzmann approximation which is given here i.e. 1 is neglected, if this is much more than this then this term can neglected and you get the Boltzmann approximation. This is valid for e minus E_f greater than or is equal to 3(KT).

If your E_f becomes close to E_c, if this difference becomes lesser than 3(KT) then for calculating the electron concentration for the conduction band we cannot use the Boltzmann approximation. So at room temperature for heavy doping the Boltzmann approximation may fail depending on the doping level and location of the Fermi-level.

Now let us look at Fermi-level as a function of temperature and see how it changes. Now, in this case we have to fix the doping. Let us say we fix the value of doping corresponding to something like this, this is the doping so this is the location of the Fermi-level. Here we are plotting as a function of temperature and let us say this corresponds to room temperature, this is room temperature so this is E_i, I take this same distance here and place it here, that is the location of the Fermi-level at 300 k corresponding to this particular doping level and we are fixing the doping level.

Supposing this is something N_d to the power star, so I am assuming here N_d is equal to N_d to the power star some value that is fixed. Now the question is, as you increase the temperature what happens to Fermi-level, in which way does it move? Does it move up from here or does it move down now? This can be very easily understood that when you go to very high temperatures the semiconductor becomes close to intrinsic because of thermal generation increasing the hole and electron concentration tend to become equal therefore the Fermi-level will tend towards the intrinsic level for very high temperatures. So this is the way the Fermi-level will move. Obviously this means that you can extent backwards, extrapolate and the Fermi-level is going to move up as you reduce the temperature.

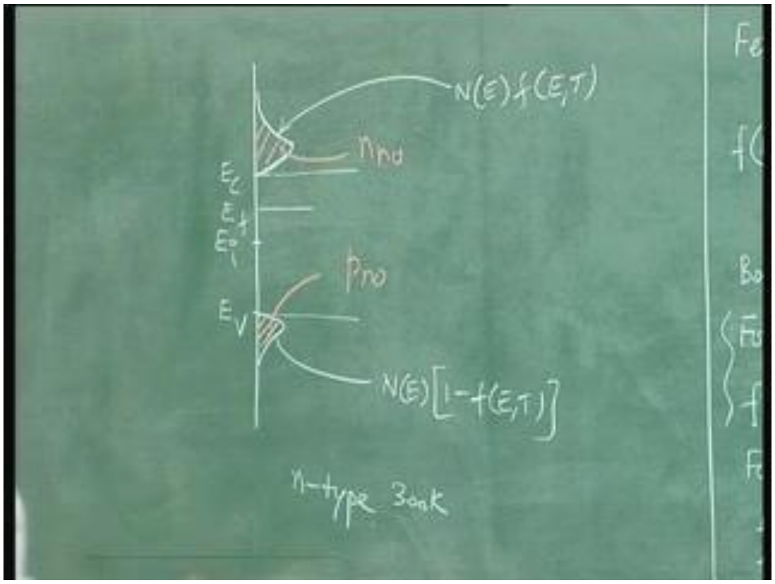
Now the question is at T is equal to 0 where is the Fermi-level going to be? So let us locate the donor level also here and now we can locate the Fermi-level by simple logical arguments as follows. At t is equal to 0 there are no free electrons or holes which means the entire donor levels should be occupied at t is equal to 0.

If the donor levels should be occupied then the Fermi-level cannot get below E_d because by definition of the Fermi-level at t is equal to 0 all states above Fermi-level are unoccupied and all states below Fermi-level are occupied. So, if the Fermi-level is located below E_d then all the states would be unoccupied which is wrong because all the states are occupied so Fermi-level is not below E_d.

Similarly, Fermi-level cannot be above E_c because if it is above E_c then some conduction band states would be occupied at t is equal to 0 by definition of Fermi-level. That means there are free electrons at t is equal to 0 so that is also not possible. So E_f has to lie between E_c and E_d. It turns out that if you do a rigorous analysis it lies exactly in the middle of this particular difference E_c and E_d so this is the way your Fermi-level moves as a function of temperature. So this is the variation of Fermi-level with temperature and doping.

What is important to note from here is at around room temperature or in extrinsic range for heavy doping the Boltzmann approximation may fail. And in that case these two equations and this particular law of mass action will not be valid. So moderate in doping levels will work. Now having discussed the Fermi-level we can come back to this and discuss how you estimate the carrier concentration.

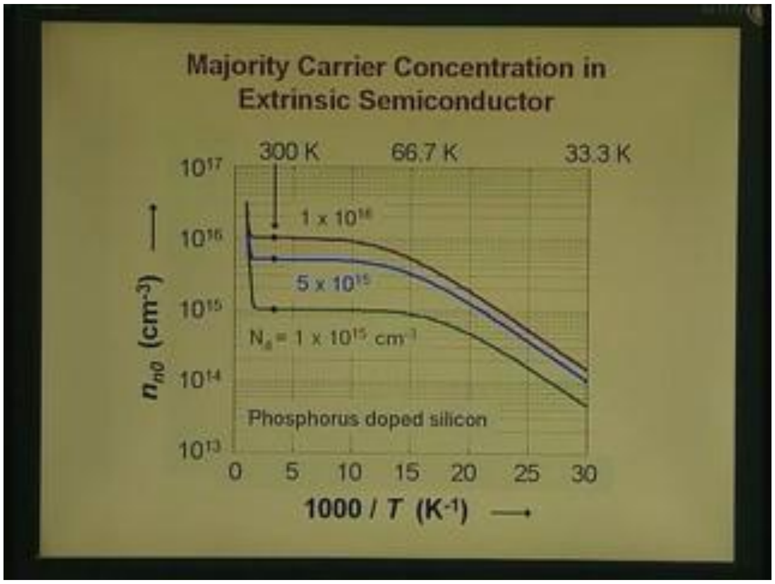
(Refer Slide Time: 47.40)



We could draw the distribution of carriers as follows: For an n-type semiconductor at 300k the picture would be like this: This is E_c, this is E_v and if you take the function N(E)f(E, T) then that function would be; the n(E) goes like this but f(E, T) when you multiply you get the separation of the function and this is the function N(E)f(E, T). This is E_i in an n-type semiconductor the Fermi level is above somewhere here, so this is E_f, this is your E_c, and this is E_v.

Now since Fermi-level is above E_i this function N(E)f(E, T) will be something like this and for valence band it will be something like this. This is N(E)[1 minus f(E, T)] the unoccupied states. And if the Fermi-level is above E_i then here these two areas represent the electron and hole concentrations. This area will be more than this area because the electrons are majority carriers. So this is your n_n to the power 0 and this is your p_n to the power 0. We have the two areas which are being estimated using these two formulae under the Boltzmann approximation. Let us proceed further and write down simplified expressions for the concentrations.

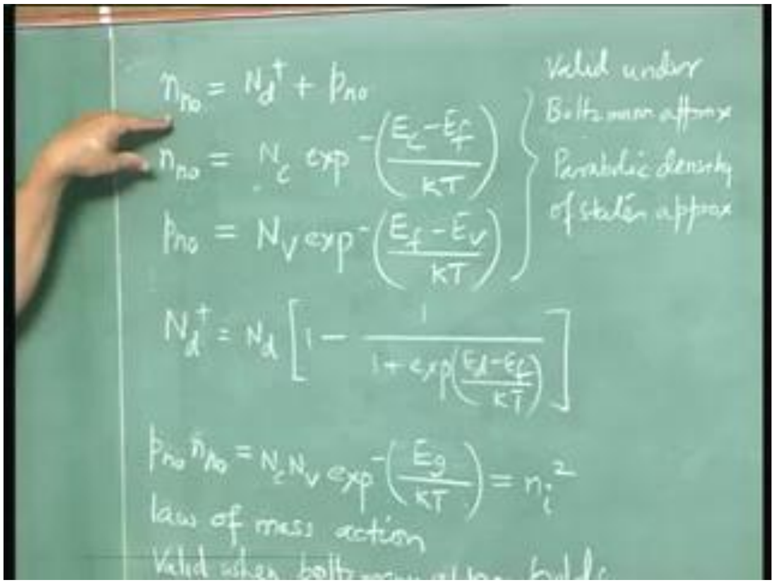
(Refer Slide Time: 49.45)



Majority Carrier Concentration in Extrinsic Semiconductor

Let us look at the slide which shows the concentration behavior let us start from first expression for a concentration the extrinsic range. You can see here in this formula.

(Refer Slide Time: 49.59)



$$n_{n0} = N_d^+ + p_{n0}$$

$$n_{n0} = N_c \exp^{-}\left(\frac{E_c - E_f}{kT}\right)$$

$$p_{n0} = N_v \exp^{-}\left(\frac{E_f - E_v}{kT}\right)$$

Valid under Boltzmann approx
Parabolic density of states approx

$$N_d^+ = N_d\left[1 - \frac{1}{1 + \exp\left(\frac{E_d - E_f}{kT}\right)}\right]$$

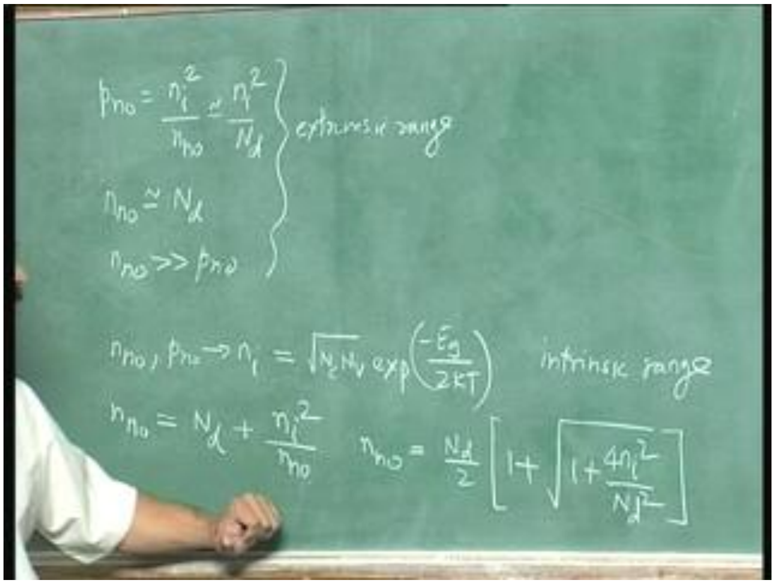
$$p_{n0} n_{n0} = N_c N_v \exp^{-}\left(\frac{E_g}{kT}\right) = n_i^2$$

law of mass action

Valid when

In the extrinsic range for phosphorus impurity all the donors will be ionized so N_d to the power star would be approximately equal to N_d and the minority carrier concentration, thermal generation will be very small. Notice from here that you can find out now the minority carrier concentration using the law of mass action if you know the majority carrier concentration.

(Refer Slide Time: 50.35)



$$\left.\begin{array}{l} p_{n0} = \dfrac{n_i^2}{n_{n0}} \simeq \dfrac{n_i^2}{N_d} \\ n_{n0} \simeq N_d \\ n_{n0} >> p_{n0} \end{array}\right\} \text{extrinsic range}$$

$$n_{n0}, p_{n0} \rightarrow n_i = \sqrt{N_c N_v}\, \exp\left(\frac{-E_g}{2KT}\right) \quad \text{intrinsic range}$$

$$n_{n0} = N_d + \frac{n_i^2}{n_{n0}} \qquad n_{n0} = \frac{N_d}{2}\left[1 + \sqrt{1 + \frac{4n_i^2}{N_d^2}}\right]$$

Then here it is P_{n0} is equal to n_i square 2 by n_{n0}. This is the way to find out the minority carrier concentration. The majority carrier concentration is given by this particular equation and N_d plus is approximately is equal to N_d and naturally that will be much greater than the minority carrier concentration. So you have n_0 approximately equal to N_d this is an extrinsic range so this is approximately equal to n_i square 2 by N_d. So clearly n_i square 2 by N_d will be much lesser than N_d, n_n0 will be much greater than p_{n0}. We really do not have to use this equation and the other equations to find out the concentration. It is a very simple situation for extrinsic range when complete impurity ionization is present.

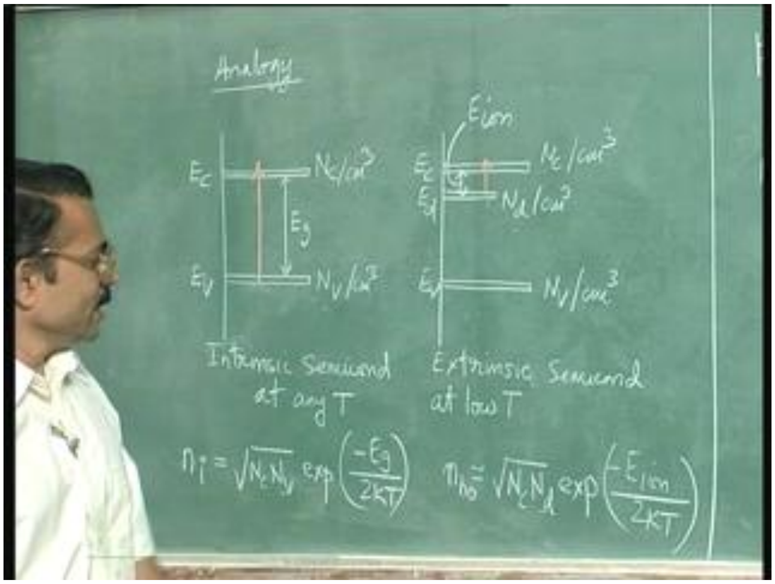
Now let us move to this particular region, the intrinsic range is of very high temperature. At very high temperature the concentration is given by n_i is equal to square root of N_cN_v exp(minus E_g by 2KT). That is the intrinsic range which gives you this particular linear segment here. Now what happens if you are somewhere here where the majority carrier concentration and minority carrier concentrations are comparable?

In other words, in this formula your N_d to the power plus is N_d because complete ionization is there. But the p_{n0} also is becoming comparable to n0 this is because your n_i is increasing. So in such a case you can write a formula, the charge balance equation can be written as: N_{n0} is equal to N_d plus n_i square 2 by n_{n0}. You can use a complete quadratic equation because you do not want to neglect the thermal generation, you do not want to neglect this p_n to the power 0 which were neglected in extrinsic range and this will be a quadratic equation in n_n to the power 0 and the solution for this is: N_n to the power 0 is equal to N_d by 2 [1 plus square root of 1 plus $4n_i$ square 2 by n_d square 2]. So you can do it as an exercise and show this. If your temperature is high and your n_i is high you can use a quadratic formula to get an accurate estimate of the concentration.

Now let us come to the low temperature range here. Here we find it is again a linear segment.

Actually the derivation of the formula for this temperature range should be done from all these three equations where you solve these three equations, you substitute these three equations into this charge balance equations and then you will have only an unknown E_f Fermi-level as all other things are known. You can solve for the Fermi-level then you can get the Fermi-level and substitute in these equations to get back the n_n to the power 0 and p_n to the power 0. That is the way to solve for the concentrations rigorously. But we can simplify this particular calculation using an analogy.

(Refer Slide Time: 54.41)



Now, the situation in an intrinsic semiconductor at any temperature t is analogous to the situation between an extrinsic semiconductor at low temperatures in some sense.

What is the analogy or similarity?
It is as follows: In an intrinsic semiconductor at any temperature the electron jumps from the valence band to conduction band and that is how the electron hole pairs are created or electrons are created. So electrons are jumping from N_v available states to N_c available states. The distance between these two states is E_g, the formula in such a situation for the electron concentration is given by; n_i is equal to square root of N_cN_v exp(minus E_g by 2KT) where E_g is the difference between the energy levels corresponding to N_c and N_v.

Now come to the extrinsic semiconductor at low temperature in this case the electrons we have considering an n-type semiconductor the electrons in the conduction band come from the donor level because of ionization. This is the process shown here by this arrow which give rise to free electrons, there is also thermal generation process but that is very negligible as compared to this. Here you have electrons jumping from N_d states at E_d to N_c states available at E_c this difference is ionization energy. Now we can write down in a formula for n_{n0} that is this electrons as follows. You replace the N_v by N_d and you replace the energy gap by the ionization energy and then you get the relation n_n to the power 0 is equal to square root of N_cN_d exp(minus E_{ion} by 2KT), it is very clear that the N_v here is replaced by N_d and Eg is replaced by E_{ion}. So this is the formula for electrons in an n-type semiconductor and this is this behavior here that is shown.

So log of n_n to the power 0 is linear, a state line function as a function of reciprocal of temperature just as in the intrinsic range you also have a linear segment. So with this we have shown how energy band diagram can be used to find out the carrier concentrations at various temperatures such as low temperature high temperature and the extrinsic range. So for low temperature it is this relation and extrinsic range it is this relation, minority carrier concentration is obtained from n_i square 2 by majority carrier concentration.

Majority carrier concentration is the impurity concentration and high temperature is this quadratic equation which can be simplified to Ni if the temperature is really very high. At any temperature if you want to find out the concentration in a rigorous manner you can simultaneously solve these equations, at any temperature that is not covered under these formulae that I discussed right now. From here we will proceed further in the next class.