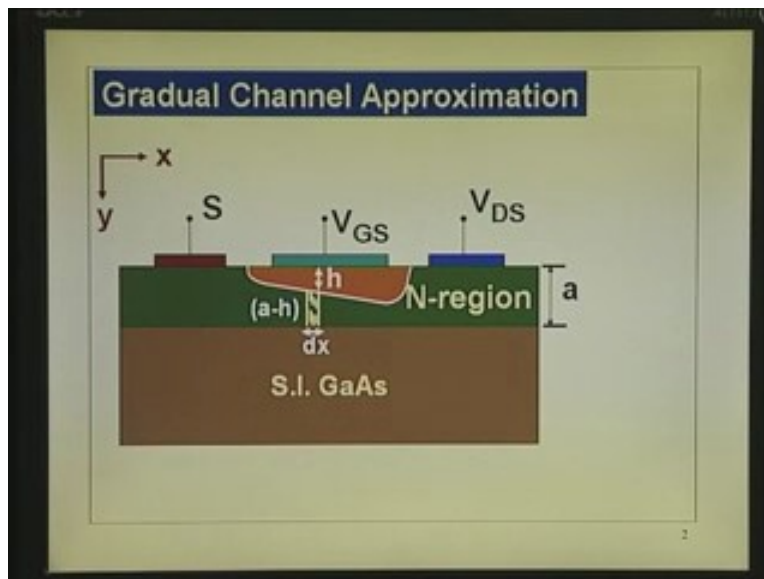


High Speed Devices and Circuits
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Lecture – 23
MESFET
Velocity Saturation Effect
On Drain Current Saturation

We have been discussing the drain saturation current using the Shockley's model.

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There assumption is that the drain current saturates when pinch off occurs at the drain edge. That is actually the voltage across this, the total potential drop there is V_{p0} . When V_{p0} is the voltage across of that, when you have $V_{DS(sat)}$ here.

$V_{DS(sat)}$ plus V_{bi} minus V_{GS} is equal to V_{p0} , that is the condition.

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Velocity Saturation Effect

Shockley Model assumes:

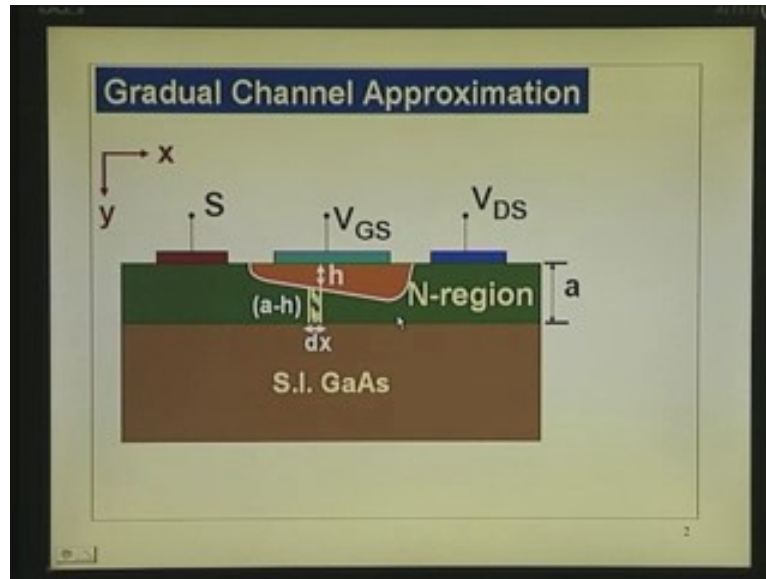
1. Low Field Mobility
2. I_D saturates due to channel pinch off at the drain end

$$V_{bi} - V_{Gs} + V_{DS(sat)} = V_{po} \quad \text{--- (1)}$$

Now, what we have discussed later on is that the just couple of things. So, assume it actually Low Field Mobility that is OK, if the fields are high and I_D saturates due to the channel pinch off at the drain end and just putting reverse whatever I said now and that is the equation that we used for saturation voltage.

V_{bi} minus V_{Gs} plus V_{DS} is the potential drop across the depletion layer that is managed V_{po} to current saturates.

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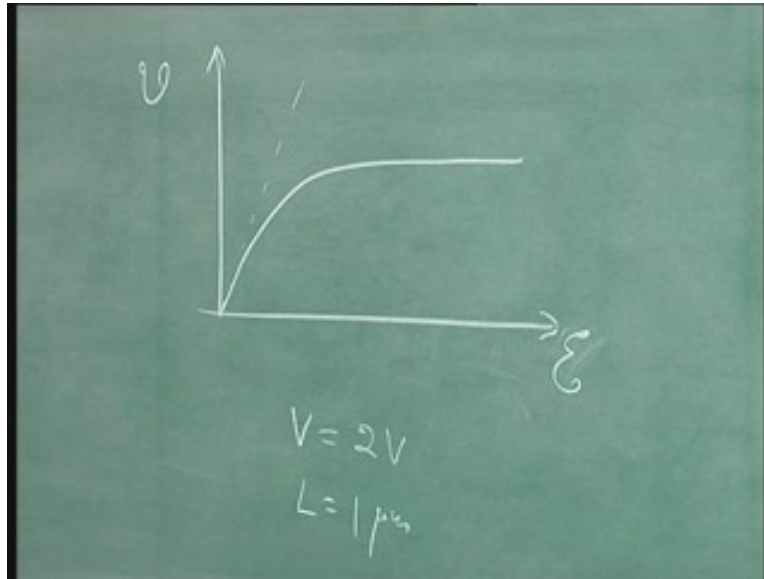


Now, we discussed yesterday that such a situation is questionable because; as you move from the source end to the drain end, the current density keeps on increasing because of the current density increases, velocity increases. In fact, electric field increases, because after all what we have used is the equation: j is equal to qn into v .

Now area comes falls, so v increases, the electric field increases and the region is at this end, you will have actually velocity saturation. What you are telling is, the real saturation in current terms of we discussed in the last time real saturation is due to velocity saturation at this end. Whether pinch off takes at that point or not? That issue we will see.

It could be show that, when the velocity saturation takes place, it is almost closing off at the drain end that is the pinch off. So let us see that, what we did is the electric field effect can play a very dominant role in saturating current.

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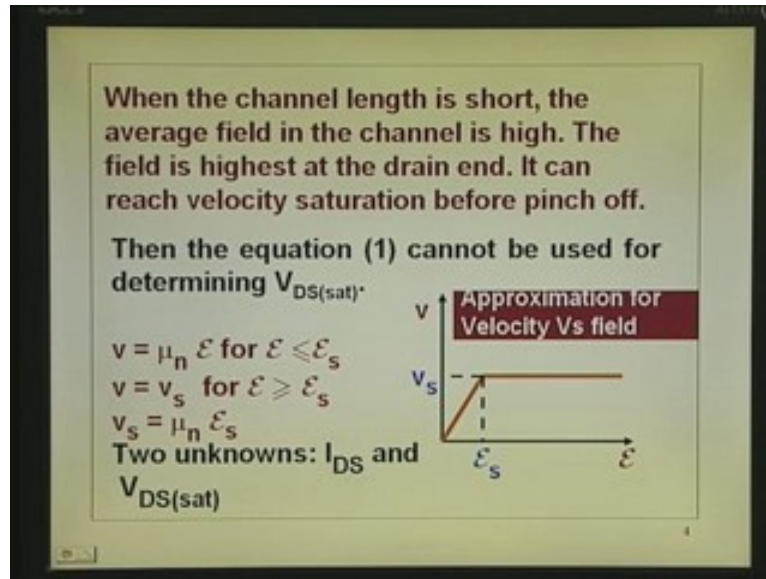


Strictly speaking, the velocity field characteristic is actually something like this; the electric field versus velocity. In silicon, if you take you know that it is not going up like that, but it goes up like that. After 20, 30 kv per centimeter of volts then it saturates. That can be met very easily.

If the channel length is 1 micron, if the voltage drop is 2 volts; V is equal to 2 volts and if L is equal to 1 micron then V by L is equal to 20 kv per centimeter that can be met very easily when you go to channel length which is of the order of 1 micron and if you go down below that, it is sure that you have very high fields. But what you are trying to find out is how to incorporate this curve saturation result? It becomes very complicated if you take the entire distribution. If you take Gallium Arsenide, it will go like that; it will saturate may be even at a small value and if it comes like this, it may be matching close to that.

It will really complicate in those situations to get an idea to bring out the effect of velocity saturation that is first order think to do analytically you can bring all this effects by numerical methods later on. What you do is, instead of taking all these things; take one of the equation characteristics as shown in the graph there. We will go back to this graph here.

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Instead of plotting it all those variations, **dupe field approximation** that is what done here. Award all those speaking and everything; take it linear up to certain electric field. Velocity is proportional to the electric field or E less than E_s and till E equal to E_s , E is a saturation field and E is equal to saturation velocity for electric field greater than saturation electric field.

So, at this point, the velocity is μ_n into E_s , saturation velocity is obtained right from there. So, to make this assumption, now you find out what is the Saturation current? What is the Saturation voltage? Would it correspond to the channel pinch off? And when channel pinch off occurs, definitely there must be velocity saturation.

Other question you are asking is, when the velocity saturation takes place, will there any channels pinch off at all? So, those are things which we are going to examine. But now, in the case of Shockley analysis, you had that advantage of assuming what is the saturation voltage is.

That is $V_{DS(sat)}$ plus V_{bi} minus V_{GS} is V_{p0}

V_{p0} is thickness and doping and a squared by epsilon zero is V_{p0} that is a known quantity

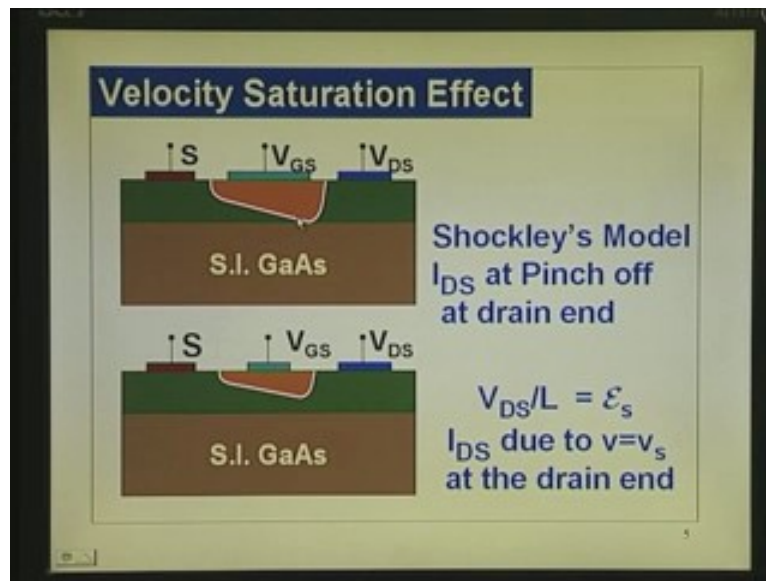
For a given V_{GS} and V_{bi} immediately from that equation, you know that $V_{DS(sat)}$. Once you know $V_{DS(sat)}$, you substitute in equation from the current where the V_{GS} is there, that is all you got that things.

There are two unknowns I_{DS} and $V_{DS(sat)}$ both are there. If you have one of them, you can find out the other one or there must be two equations which we can solve to that I_D and $V_{DS(sat)}$, both.

Here, we do not know what point to $V_{DS(sat)}$ is taking place but we know that it would saturate from the velocity saturates at some point, that concept is $V_{DS(sat)}$.

Two unknowns I_{DS} and $V_{DS(sat)}$. What we do now is, divide the channel into two regions.

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Let go back to that. Here, that is exam in the two cases. This case, I have drawn that depletion layer; Shockley's model, I_{DS} is a pinch off. Drain current saturates when the pinch off takes place. What we want to examine is, whether we get the same condition for the situation where the velocity saturation takes place. In fact, there is no doubt that current saturates because of velocity saturation. Otherwise, if it is closing completely as sum of your raising some time bad, current is found out to be zero.

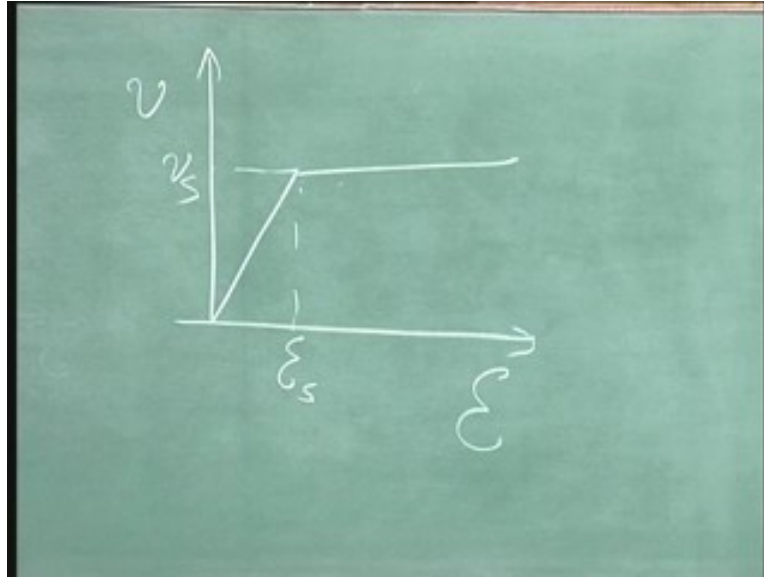
We said qualitative leads a dynamic equilibrium and that dynamic equilibrium have come due to velocity saturation here. It could happen so that this width is closed to A. Now other condition is I_{DS} due to v equal to v_s at the drain end. Please understand once again. In both the cases, the current saturates due to velocity saturation. I pointed out on yesterday even without a junction when you go to high fields, current can saturate in the resistor. We examine that in last lecture. Here, what we tell is, I have plotted two things: one, a channel which is long and other one, the channel which is shorter.

So, the field in this region should be higher compared to the field in this region. It could so happen that velocity saturation can take place here even without pinching off.

What you are telling is, V_{DS} drops across the channel divided by the length is a field that could be closed to the saturation field. The difference between these two is no difference; both of them current saturate due to velocity saturation. But in this case, the current has saturated before pinch off because before pinch off itself, there is enough voltage drop and there is enough field; whereas, in this case it has allowed the pinch off. That is the difference.

Now, we will take that equation which is there.

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What we are trying to do now here is this. We will take the equation which we used from this end to this end same as what we have used with v is equal to μ into E because we are now taking the case like this; that is what we are doing. That means, till the electric field is equal to saturation field, I can write v is equal to μ into E and from the point at which E is equal to saturation field beyond that point, velocity saturation is there. Beyond that point, the channel cannot string. This is the condition which is equivalent to the pinch off that is what you are telling is from here to here as you move, field is not here as you gone for larger and larger. At a particular point at which velocity saturates, E is equal to E_s and from that point onwards we cannot write E is equal to μ into T and from this point you write, current density is equal to qn into velocity saturation. In this point, you write current density is equal to q_1 into v_1 into v and that is the equation at you derived

(Refer Slide Time: 12:30)

In the regions where $\mathcal{E} \leq \mathcal{E}_s$,

$$I_{DS} = G_0 \left[V_{DS(Sat)} - \frac{2}{3\sqrt{V_{po}}} \left\{ \frac{(V_{bi} - V_{GS} + V_{DS(sat)})^{3/2}}{(V_{bi} - V_{GS})^{3/2}} \right\} \right]$$

$$I_{DS} = G_0 V_{po} \left[\frac{V_{DS(Sat)}}{V_{po}} - \frac{2}{3} \left\{ \frac{(V_{bi} - V_{GS} + V_{DS(sat)})^{3/2}}{V_{po}} - \left(\frac{V_{bi} - V_{GS}}{V_{po}} \right)^{3/2} \right\} \right]$$

Normalizing I_{DS} , $V_{DS(Sat)}$ and $(V_{bi} - V_{GS})$

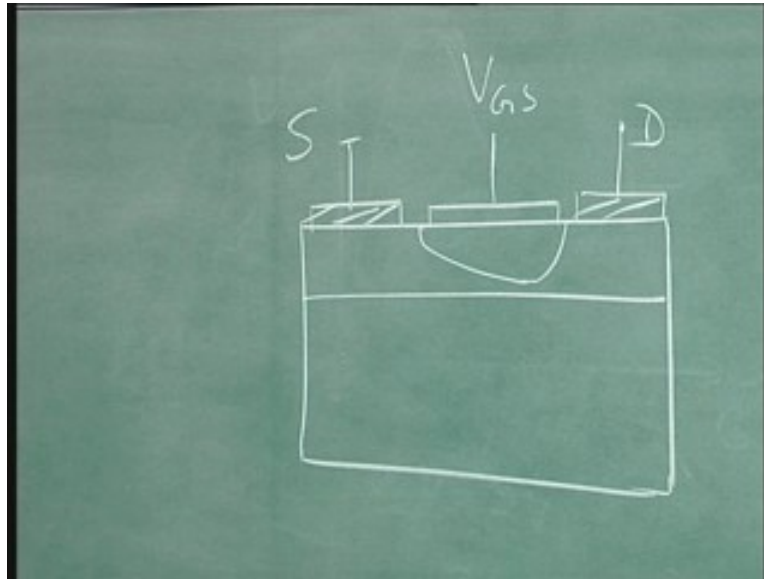
So, in the channel region not at the wavelet end, other portions; we can still write this equation which we have derived in the previous two lectures that is G_0 into $V_{DS(sat)}$, we are talking of the saturation current. In fact, what we are doing is that will go back to that. What you are doing is, you are taking a situation where this velocity saturation is present here and in this portion, there is no velocity saturation.

Up to this portion, I write the equation. Under that condition, current has saturated because velocity saturation has taken place at the drain end. Now, it is the very familiar equation for you.

It is written by writing j is equal to qn into v into area.

I am rewriting that equation. All that we are trying to do is writing the two equations because after all we need to find the I_{DS} and $V_{DS(sat)}$, we need two equations.

(Refer Slide Time: 13:48)



So, we write one equation in that region, let me put that instead show it to you on the board. That is a source, that is the drain and that is the gate. This is V_{GS} , so what we are trying to say is, you have a depletion layer like that, it may or may not be pinched off; whatever saying is, for this condition that we have drawn here velocity has saturated here and beyond this point, v is equal to μ_n into v that is what you are trying to point out.

So in this equation, I write the same equation which we are used for Shockley's model where v is equal to μ_n into E . I get one equation for I_D and when the velocity is saturated here, the current has saturated. In that equation; I say whatever V_{DS} is there is $V_{DS(sat)}$ and whatever I_D is there is I_{DS} .

But I do not know anyway both of them.

To know both of them, I write one more equation at this point. At this point, I do not use this equation but we use is, this portion. So, at this point current density is equal to qn into velocity saturation. We will see how it works out.

And current is equal to qn into velocity saturation into this area and once you find out this area that is, this height is equal to A minus H and H is related to $V_{DS(sat)}$. To get one equation from this portion writing v is equal to μ_n into E for the current equation and

other equation between I_D and V_D writing to the current especially for current here using that portion. That is all what we are doing; we are trying to get two equations.

(Refer Slide Time: 16:15)

In the regions where $\mathcal{E} \leq \mathcal{E}_s$,

$$I_{DS} = G_0 \left[V_{DS(sat)} - \frac{2}{3\sqrt{V_{po}}} \left\{ \left(V_{bi} - V_{GS} + V_{DS(sat)} \right)^{3/2} - \left(V_{bi} - V_{GS} \right)^{3/2} \right\} \right]$$

$$I_{DS} = G_0 V_{po} \left[\frac{V_{DS(sat)}}{V_{po}} - \frac{2}{3} \left\{ \frac{\left(V_{bi} - V_{GS} + V_{DS(sat)} \right)^{3/2}}{V_{po}} - \frac{\left(V_{bi} - V_{GS} \right)^{3/2}}{V_{po}} \right\} \right]$$

Normalizing I_{DS} , $V_{DS(sat)}$ and $(V_{bi} - V_{GS})$

The first equation is like this; now instead of keeping all writing this long equation again and again, what we do is normalize I_{DS} , saturates the current. Please note I do not know what is I_{DS} is; I just set I_{DS} is the value you get when V_{DS} is equal to $V_{DS(sat)}$ that is all what I have done. So, I_{DS} is equal to G_0 I pull out this, multiply by V_{po} , so I take inside also divide by V_{po} , multiply and divide, so you get $V_{DS(sat)}$ by V_{po} .

2 by 3 square root of V_{po} to the power of 3 by 2 because this is pulled inside. See, multiplied here by V_{po} , that we divide everywhere by V_{po} , so you get 2 by 3 V_{po} to the power of 3 by 2 that is taken inside.

So, what is happening now is, the terms inside the bracket each one of them is divided by V_{po} . So, you normalize this with respect to V_{po} because V_{po} is the quantity which is decided by the particular device doping and thickness of the layer. So, you have got that terminal, I call it as u_{ds} . Instead of writing $V_{DS(sat)}$ by V_{po} , I call it as u_{ds} normalized value of drain source voltage and I_D , I can normalize. What is this quantity? All these are normalized with respect to these ratios.

These are dimensions of G_0 into V_{p0} it terminates the current, voltage to the conductance. So, I_{DS} divided by G ; this quantity that is normalized value of the current, normalized value of drain source voltage or normalized value of voltage at the source end; you call it as u_{GS} , the whole thing and this, you call it as u_{GS} plus u_{ds} .

(Refer Slide Time: 18:32)

The slide contains the following content:

$$i_{ds} = \frac{I_{DS}}{G_0 V_{p0}} = u_{ds} - \frac{2}{3}(u_{ds} + u_{GS})^{3/2} + \frac{2}{3}u_{GS}^{3/2} \quad \text{--- (A)}$$

Current Saturation

It is due to velocity saturation at the drain end

$$I_{DS} = qN_D v_s W[a - h]$$

$$= qN_D \mu_n \mathcal{E}_s W a \left[1 - \frac{h}{a} \right]$$

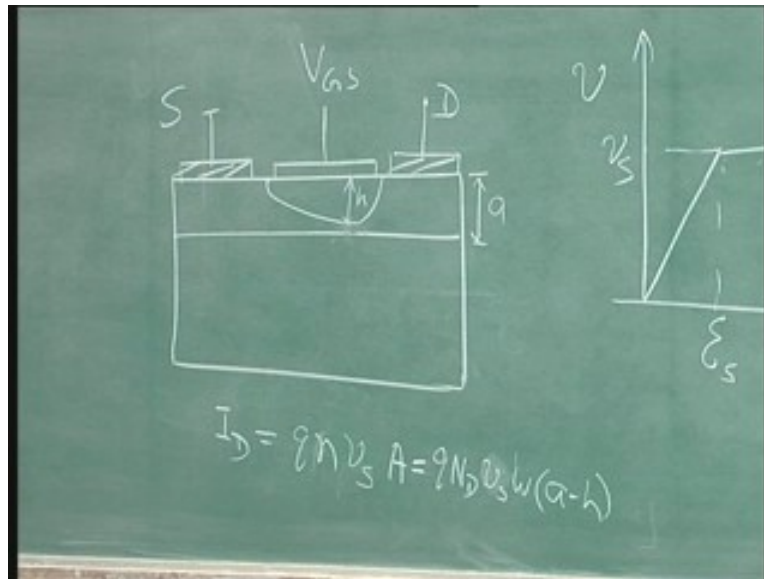
$$= \frac{qN_D \mu_n W a}{L} \mathcal{E}_s \left[1 - \sqrt{\frac{V_{bi} - V_{GS} + V_{DS(sat)}}{V_{p0}}} \right]$$

I rewritten that equation, go back to the equation and see whether we written correctly. I_{DS} by $G_0 V_{p0}$ that is normalized value of I_{DS} is equal to u_{ds} which is nothing but $V_{DS(sat)}$ divided by V_{p0} , the first term (Refer Slide Time: 18:58). Minus 2 by 3 into u_{ds} plus u_{GS} to the power 3 by 2, just go back once and see. Minus 2 by 3 into u_{GS} is this quantity and this is u_{ds} , V_{DS} by V_{p0} . That is why you get that. Plus V_{bi} minus V_{GS} divided by V_{p0} to the power of 3 by 2 which is that. All that you have done is the whole thing comes in one line now. The whole thing it comes to be so terrifying with all these half a page occupying here.

Now, single line but the identity is maintained normalized with respect to that, you can write simpler equations in the simple form by normalizing.

This is the one equation that we have written for a current in this portion

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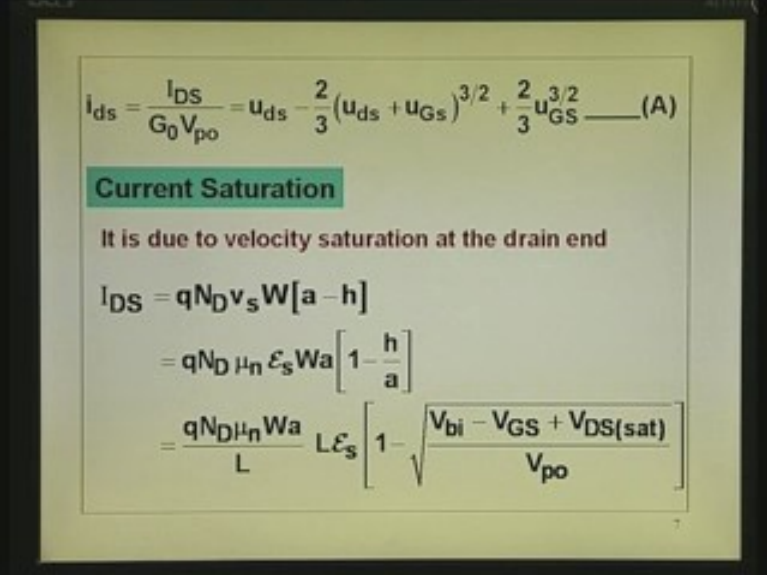
I have both I_{DS} and u_{ds} , both are unknowns I write other equation here. How do you write the current? Because here the velocity is not given by this, that is given by velocity saturation. So, what we do now is, write the current at the drain end; it is due to the velocity saturation at the drain end.

What is the current actually? J or I_D is equal to qn carrier concentration into velocity saturation into area. This I am writing it as, what is n ? Electron concentration which actually equal to doping concentration that is N_D velocity saturation of course which is μ_n into E_s , we can write it afterwards. Area, this height into W and that height is actually equal to, this is h at the drain end and this is a . So, this is actually h minus a into W . So, (21:33) that is equal to qN_D into W into V_s into W into a minus h that is the area. That is what you have written there.

This equation put down here where h is actually the depletion layer width at the drain end. Shockley's analysis said h is equal to very close to a and it cannot be a . You can see the current becomes equal to 0. Now, we can further simplify. I can write all that we do here is qN_D is retain velocity, I can write it as μ_n into E_s because we are looking into saturation velocity which is μ_n into E_s , if you are looking to this particular graph.

V_s is μ_n into E_s and we are substituting in that particular velocities saturation μ_n into V_s .

(Refer Slide Time: 23:00)



The slide contains the following content:

$$i_{ds} = \frac{I_{DS}}{G_0 V_{po}} = u_{ds} - \frac{2}{3}(u_{ds} + u_{GS})^{3/2} + \frac{2}{3}u_{GS}^{3/2} \quad \text{--- (A)}$$

Current Saturation

It is due to velocity saturation at the drain end

$$I_{DS} = qN_D v_s W[a - h]$$

$$= qN_D \mu_n \mathcal{E}_s W a \left[1 - \frac{h}{a} \right]$$

$$= \frac{qN_D \mu_n W a}{L} \mathcal{E}_s \left[1 - \sqrt{\frac{V_{bi} - V_{GS} + V_{DS(sat)}}{V_{po}}} \right]$$

Let us go back to this slide now and see we have got velocity is that V_s is not there actually may be I think I should cut it off right away here.

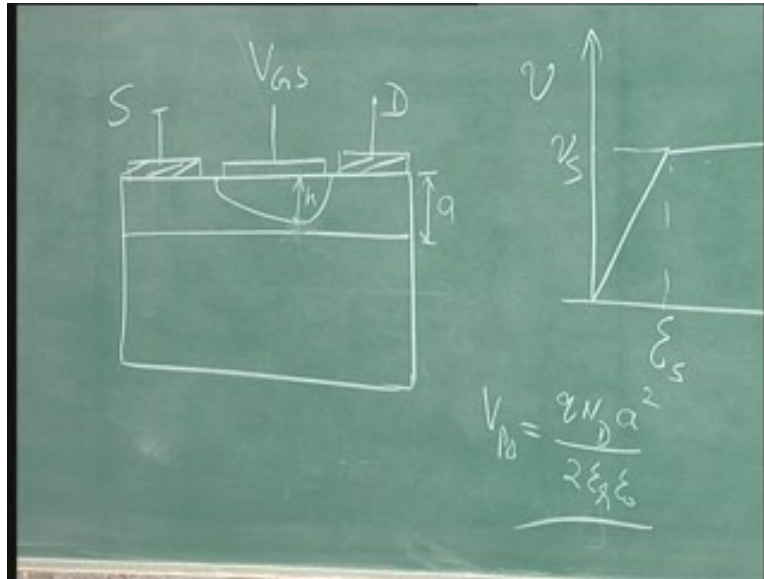
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$$= qN_D \mu_n \epsilon_s v_s W a \left[1 - \frac{h}{a} \right]$$

I will take a unit for mean but I think we will take it off right away here. Because we substitute already for that, so it is alright now. We have got this, $qN_D \mu_n \epsilon_s W$ into a into 1 minus h by a . There is a problem of cut and paste; you know cut and paste and add that μ_n into V_s anyway now you removed that. qN_D into μ_n into W into a . I retain all those terms. Now, the rest is just manipulation. You have written by f 2 expressions for current, one is this and other one is this. Here you wrote velocity saturation go back to that (Refer Slide Time: 24:28). So, what you have done is you have done this substituted V_s is equal to μ_n into E_s and then what I do is, go ahead further I just remove that E_s there pull all this together $qN_D \mu_n$ into W into a ; just I remove that and put it here, multiplied by L divide by L and this is obvious to you, why I am doing, what is this particular term?

This is the channel conductance G_0 , this is actually sigma, this is area of the channel divide by length; sigma area by L that is the conductance and this is $L E_s$ into 1 minus...

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Now, we write for an expression for h by a . What is a ? Just to recall once again, I will put it down here, V_{p0} is just standard formula. So, a is proportional to root of V_{p0} and h here is proportional to whatever voltage is present across that, square root of that. So, ratio of h and a , h divide by a is square root of whatever voltage is present across that divided by V_{p0} . What is the voltage present across that? V_{bi} minus V_{GS} plus whatever voltage is present across that and we are saying that is the voltage activity current is saturating. So, that is why at the drain end, you got h is equal to V_{bi} minus V_{GS} plus $V_{DS(sat)}$ divided by V_{p0} . That is simple.

(Refer Slide Time: 26:35)

$$I_{DS} = G_0 V_{p0} \left(\frac{L E_s}{V_{p0}} \right) \left[1 - \sqrt{u_{ds} + u_{GS}} \right]$$

$$i_{ds} = \alpha \left[1 - \sqrt{u_{ds} + u_{GS}} \right] \text{---(B)}$$

At saturation i_{ds} given by (A) & (B) are the same. Hence we can write,

Now, you are writing a normalized equation. What I did is, this is G_0 multiplied by V_{p0} divide by V_{p0} . $G_0 V_{p0}$ into $L E_s$ by V_{p0} and terms in within the bracket, 1 minus, I will write that so that it is not clear to hear.

(Refer Slide Time: 27:08)

$$I_{DS} = G_0 L E_s \left[1 - \sqrt{\frac{V_{bi} - V_{GS} + V_{DS(sat)}}{V_{p0}}} \right]$$

$$= G_0 V_{p0} \frac{L E_s}{V_{p0}} \left[1 - \sqrt{u_{GS} + u_{ds}} \right]$$

See, I_{DS} is G_0 into $L E_s$ into 1 minus square root of V_{bi} minus V_{GS} plus $V_{DS(sat)}$, not over step into that, divided by V_{p0} . What we do is the quantity is G_0 into V_{p0} into $L E_s$ divided

by V_{p0} , multiplied by that quantity and what is inside is equal to 1 minus V_{bi} minus V_{GS} divide by V_{p0} is V_{GS} and $V_{DS(sat)}$ divide by V_{p0} is u_{ds} That is what you have written there.

(Refer Slide Time: 28:21)

$$I_{DS} = G_0 V_{p0} \left(\frac{L E_s}{V_{p0}} \right) \left[1 - \sqrt{u_{ds} + u_{GS}} \right]$$

$$i_{ds} = \alpha \left[1 - \sqrt{u_{ds} + u_{GS}} \right] \quad \text{--- (B)}$$

At saturation i_{ds} given by (A) & (B) are the same. Hence we can write,

$$\alpha \left[1 - \sqrt{u_{ds} + u_{GS}} \right] = u_{ds} - \frac{2}{3} (u_{ds} + u_{GS})^{3/2} + \frac{2}{3} u_{GS}^{3/2}$$

$$\alpha = \frac{u_{ds} - \frac{2}{3} (u_{ds} + u_{GS})^{3/2} + \frac{2}{3} u_{GS}^{3/2}}{1 - \sqrt{u_{ds} + u_{GS}}}$$

Now, I think we should have reason to be happy now, why? Not because of the lecture is over because you have got two equations A and B. A is this one which relates i_{DS} with u_{ds} and B is other equation which relates i_{ds} with u_{ds} . Two unknowns, we can evaluate them.

This is the statement which is understand; we are computed the current here by using the linear relationship. You have computed the current here using velocity saturation. A is written for this portion, current. B is equation written for this portion. Now the current computed using that and this are the same thing. So, equate both of them, we can find out $V_{DS(sat)}$. So, that is right hand side of equation B and this is right hand side of equation A, that portion. So, currents are same thing in a same device.

Now, what is alpha here? It is again writing one term for this particular quantity, alpha is $L E_s$ by V_{p0} . This quantity is refer to alpha just for writing purpose; there is no, its not like the current gain nothing like that. It is actually channel length to the saturation voltage, how much it is compared to the pinch off voltage? So that place, this term alpha which is the ratio of this channel length into the critical electric field or saturation field divided by

V_{p0} place a very important role in deciding whether whatever we have writing down earlier, Shockley's model is correct or not? Come straight from here.

Alpha therefore is equal to this quantity, entire big term. Now, if u_{ds} plus u_{GS} is equal to 1, what happens to alpha? Infinite.

(Refer Slide Time: 31:26)

The image shows a chalkboard with the following handwritten equations:

$$u_{ds} + u_{GS} = 1 \quad \text{given } \alpha \rightarrow \infty$$

$$\text{i.e., } \alpha \gg 1$$

$$\frac{V_{DS(sat)}}{V_{p0}} + \frac{V_{bi} - V_{GS}}{V_{p0}} = 1$$

$$\frac{L E_s}{V_{p0}} \gg 1$$

$$V_{DS(sat)} + V_{bi} - V_{GS} = V_{p0}$$

u_{ds} plus u_{GS} equal to 1, it gives alpha, actually alpha is equal to infinity, I am putting alpha tends to infinity or if you want to be soft on that, alpha is very much greater than 1 that is the meaning of that or you can put it other way, if alpha is very much greater than 1, u_{ds} plus u_{GS} is equal to 1.

This indicates $L E_s$ divided by V_{p0} , he is very much greater than 1 that is the meaning of that. What is this condition? This condition that you have written here turns out to be true and that is satisfied or in closer and closer to be true, this is larger and larger.

What is this quantity? Write down, put it in that and remove the normalized thing you recognize it. u_{ds} is $V_{DS(sat)}$ divided by V_{p0} that is u_{ds} . The second term is, what is that? V_{bi} minus V_{GS} ; there is a potential drop at the channel and this potential drop across the depletion layer at the source end divided by, that is what you have written. What is this condition? This condition is, take this V_{p0} to the other side, V_{bi} minus V_{GS} plus $V_{DS(sat)}$ is

equal to V_{p0} . This is what you recognize. So, this condition turns out to be $V_{bi} V_{DS(sat)}$, I am write it an angle $V_{DS(sat)}$ plus V_{bi} minus V_{GS} equals V_{p0} .

That is the condition. What we are telling is, when α is very much large compare to 1 that is $L E_s$ by V_{p0} is very much greater than 1; velocity saturations takes place, that condition may use. But when the velocity saturation takes place, this condition is satisfied. That means long channel devices; the channel length is long and whatever Shockley's has been telling or writing the condition for saturation out's good, pinch off takes place or at least which closed to pinch off. Its real pinch off takes that that will be infinity but now this larger than 1 May be 3 or 4 like that, so what we are telling again is, if the channel length is long, by the time you read that field there.

Let us say E_s is a field that is average field, L into E field. So, that is long that means it is very large by that time the high field is there. That voltage becomes large and it reaches pinch off. Either way it is true, supposing pinch off voltage is small even if a channel length is small; if this is very small, you can satisfy that condition.

So, what we are trying to just go back to that graph and see it once just maybe I do not do that. So, what we are trying to say is that drop here is sufficient enough so it is closed this, if L is long we know that, if the channel length is small, that will what happen but if the channel length is small, if the small voltage **your h will** (36:26) achieve it may not close at all that is the other condition. Obviously, when the $L E_s$ is smaller compare to V_{p0} , you will not have this condition satisfied. We will see what happens there.

If that, we will have a situation where the general equation can be derived and people have played games in the sense game is not real game; played at the equations and estipulate interpolated etc., caught from analytical expressions with look same as that $\mu_n E_s W$ by L into V_{GS} minus u_{ds} whole squared with the factor, we will see that.

(Refer Slide Time: 37:08)

$$(i) \alpha = \frac{L\mathcal{E}_s}{V_{po}} \gg 1 \Rightarrow u_{ds} + u_{GS} = 1$$

$$\frac{V_{bi} - V_{GS} + V_{DS(sat)}}{V_{po}} = 1 \text{ Shockley Model}$$

$$(ii) V_{bi} - V_{GS} + V_{DS(sat)} < V_{po}$$

$$\therefore V_{DS(sat)} \ll V_{po}$$

$$\frac{V_{DS(sat)}}{V_{po}} \ll 1$$

$$\frac{L\mathcal{E}_s}{V_{po}} \ll 1 \quad \alpha \ll 1$$

ID saturates before V_{po}

This is Shockley's model we are discussed just now that condition one; alpha very much greater than 1. The other condition see, if this becomes equal to 1 that is Shockley's condition. If this term when the voltage reaches the saturation and it is less than the V_{po} , the channel is not closing down. So, if this is less than V_{po} , $V_{DS(sat)}$ alone will be even less than V_{po} .

The $V_{DS(sat)}$ plus that quantity is less than V_{po} . But we are telling is $V_{DS(sat)}$ plus V_{bi} is less than V_{po} that is channel is not closing down; that is situation this plus that is less than V_{po} , means this portion is definitely less than V_{po} .

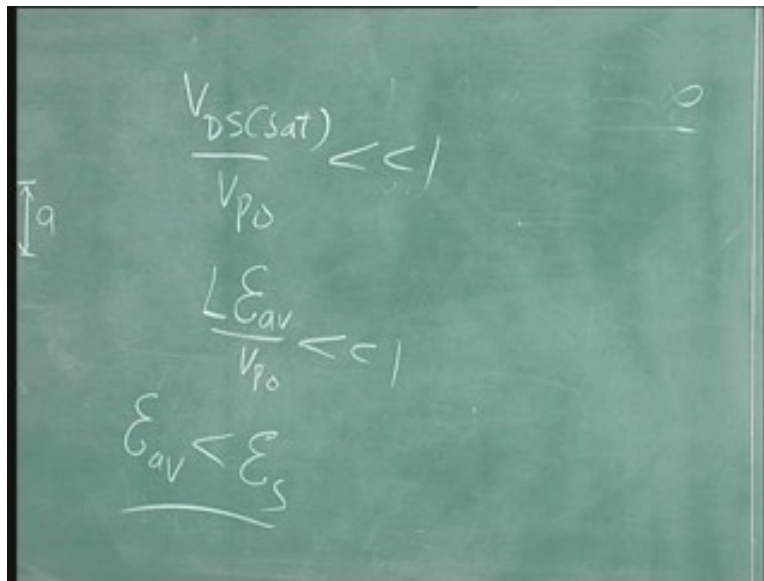
Three times are there out of the two knock out that is much much smaller, that is this quantity is less than 1. Now what we are writing is $V_{DS(sat)}$ approximately we can put it as L into E_s . This is of course assuming electric field is everywhere. But actually it should move from this end to that an electric field is E_s closer to this point.

$V_{DS(sat)}$ actually is less than L into E_s because E is less than E_s here. $V_{DS(sat)}$ is voltage across that and it is L into an average electric field there. The average electric field is less than E_s because E_s is highest here. L into E_s by V_{po} is less than 1, is it clear now? See we arrived at, I will go through quickly once if you gone through fast on that.

These three terms equal to be V_{p0} is Shockley's condition. That is valid when α is greater than 1, $L E_s$ by V_{p0} is greater than 1. We are trying to see, what is the situation when L into E_s is less than 1? In fact, we are going over to other way because we understand this term clearly. Out of these quantities, I remove this quantity. So, all the three times together is less than V_{p0} . Therefore this term alone should definitely much less than V_{p0} .

Now, I am removing this taking into other side $V_{DS(sat)}$ may be by V_{p0} is very much less than 1. What is that condition we are trying to see?

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Handwritten equations on a chalkboard:

$$\frac{V_{DS(sat)}}{V_{p0}} \ll 1$$

$$\frac{L E_{av}}{V_{p0}} \ll 1$$

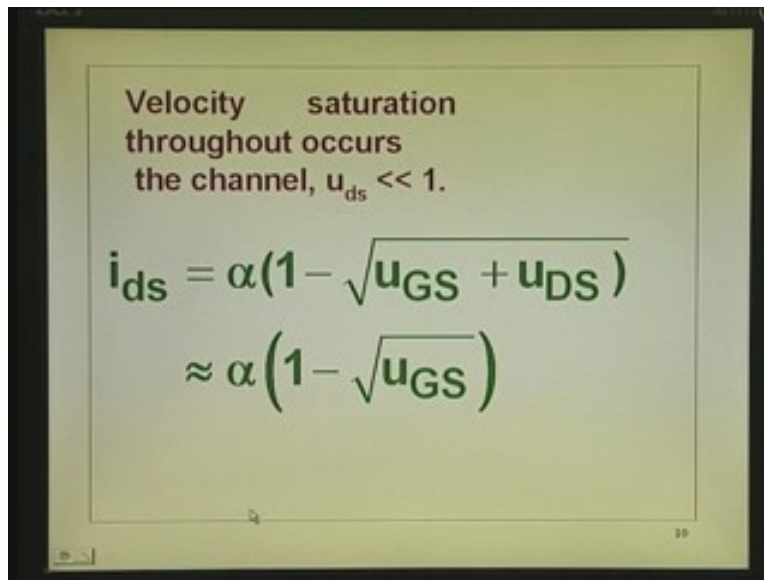
$$E_{av} < E_s$$

If $V_{DS(sat)}$ is very much less than V_{p0} , I will put there on the board. So, we are taking a situation where $V_{DS(sat)}$ by V_{p0} is very much less than 1. What is $V_{DS(sat)}$? It is a voltage drop across this portion and I can say it is channel length into the average electric field and what is the average electric field here? That is less than saturation field because peak electric field is maximum here, it is less than that so average electric field is less than E_s . See, this is less than that which means actually L into electric field average by V_{p0} is less than 1, average electric field is less than that. So, this is less than 1, average electric field is less than E_s , so L into E_s is definitely if I replace this by this, that will be given from that. An average what you can say is $V_{DS(sat)}$ by V_{p0} is less than 1.

So, L into E_s even if it is a maximum thing, L into E_s is less than 1. At the best case, you can have this quantity less than 1. In other words, α less than 1 or $L E_s$ less than 1 correspond to that situation. We are indirectly going backwards. See, I could written straight away this and according to that but I have done is, I have taken that equation where the pinch off does not take place and I write this condition pinch off does not take place velocity saturation, that condition corresponds to that.

We can take the $V_{DS(sat)}$ is equal to that maximum field itself $L E_s$ that is less than 1. Go back to that velocity saturation throughout, now what is the situation? We say, this is much less than 1 and now let go back to that.

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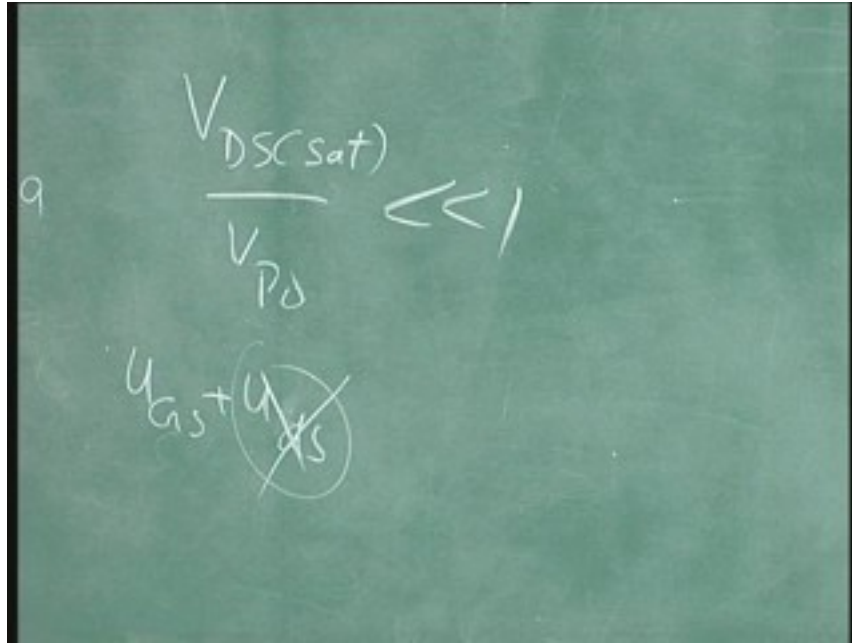
Velocity saturation throughout occurs the channel, $u_{ds} \ll 1$.

$$i_{ds} = \alpha(1 - \sqrt{u_{GS} + u_{DS}})$$

$$\approx \alpha(1 - \sqrt{u_{GS}})$$

An i_{ds} is actually equal to, what is i_{DS} ?. This I hope agree because this equation that you have derived by writing the current here.

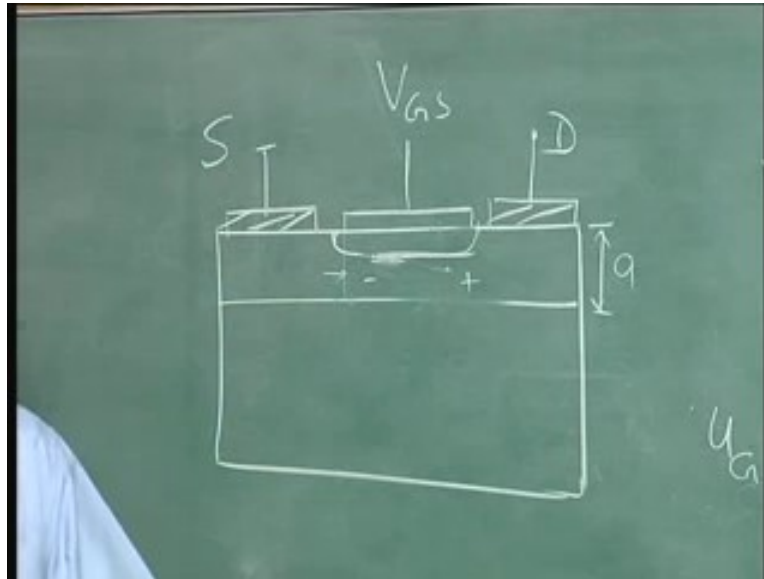
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$$\frac{V_{DS(sat)}}{V_{PD}} \ll 1$$
$$u_{GS} + u_{DS}$$

Now, what we are telling is, u_{ds} is very much less than 1 because we are replacing $V_{DS(sat)}$ by almost equal to L into E_s . So, this is less than 1. Now, when you say u_{GS} plus u_{ds} , this term is that quantity and this quantity is very much less than 1. I am knocking it out because there is very much small compare to 1.

So, what we are telling now is, I can write an equation for i_{ds} with involved through the thing but not very complicated, all that you realize which is small quantity. So, I am neglecting this if I write like this, what is the meaning of this? The meaning of that is, we have got velocity saturation throughout the channel.

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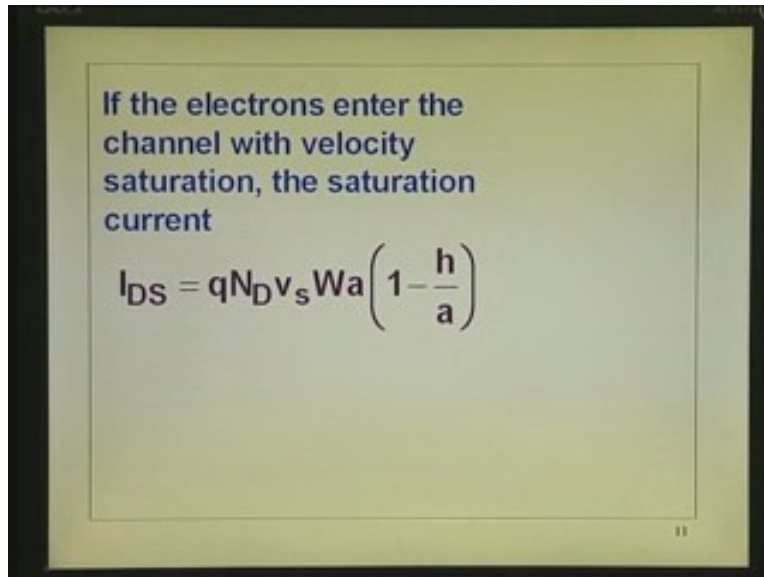


Let us put that down here now, things get little look bit blink because you are looking into an extreme situation. The electrons when they enter, itself has reached the velocity saturation. They already explained velocity saturation and they are not just getting into the portion; they are getting from an inner portion that is the situation once reaches this point if there is velocity saturation, what will happen to the depletion layer afterwards?

This is velocity saturation and you go beyond that point velocity cannot increase, the saturation velocity. J is equal to qn into V_s , current is equal to qn into V_s into area. V_s does not change, they cannot change and this will remain the same thing. If it remains the same thing, what is the meaning of that? What is the drop here? Very small that is the meaning. If there is no drop there may not be any current flow, it is very small. That means you get very high fields with very small drop that is possible manner. You can get very high fields here because field is in this portion it must be greater than the saturation of field; we can get very high field, there is small voltage drops if length is very small sort channel devices. So, we are driving ourselves into device which we have got channel length is so small such that even with small voltage drops across that, you will have velocity saturation then that will be the thing. Now you can **thing** (45:52) u_{GS} that quantity is V_{bi} minus V_{GS} compare to that, that drop is small.

So, this is a situation you have got velocity saturation right through the channel that is possible if launching the electrons right in to the channel with velocity saturation or they just came immediately the velocity saturation.

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If the electrons enter the channel with velocity saturation, the saturation current

$$I_{DS} = qN_D v_s W a \left(1 - \frac{h}{a}\right)$$

Few things now; I think I am just now we can zip through because the concepts are over. $L E_s$ that is $\alpha L E_s$ by V_{p0} .

If it is very large that is long channel devices or pinch off voltage small such devices velocity saturation takes place and pinch off takes place. They coincide. If velocity saturation and pinch off coincide, that is why, Shockley's theory survived so long because we are talking of channel length is long or which are pinch off voltages are small but the motive go to shorter channel devices; you will have velocity saturation but pinch off voltage will not take place. Here, pinch off have not taken place at all, where there is saturation right through, there is absolutely no pinch off. So, pinch off need not take place of current saturation. There is this situation and we can write this entire thing, I am just rewrite that, we can derive it in different approaches. I write the whole thing again that I can skip all these because you get the same equation whatever approach you write.

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$$i_{ds} = \alpha (1 - \sqrt{u_G})$$

$$V_{DS(sat)} = L \mathcal{E}_s \ll V_{po}$$

$$\alpha = \frac{L \mathcal{E}_s}{V_{po}} \ll 1$$

$V_{DS(sat)}$ is actually $L E_s$ which is less than V_{p0} . In fact, there is no need of going through and I just rewrote it in different approach but we have got the whole thing condition satisfies before that.

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General Expression for I_{DS}

In General,

$$i_{ds} = u_{ds} - \frac{2}{3}(u_{ds} + u_{GS})^{3/2} + \frac{2}{3}(u_{GS})^{3/2}$$

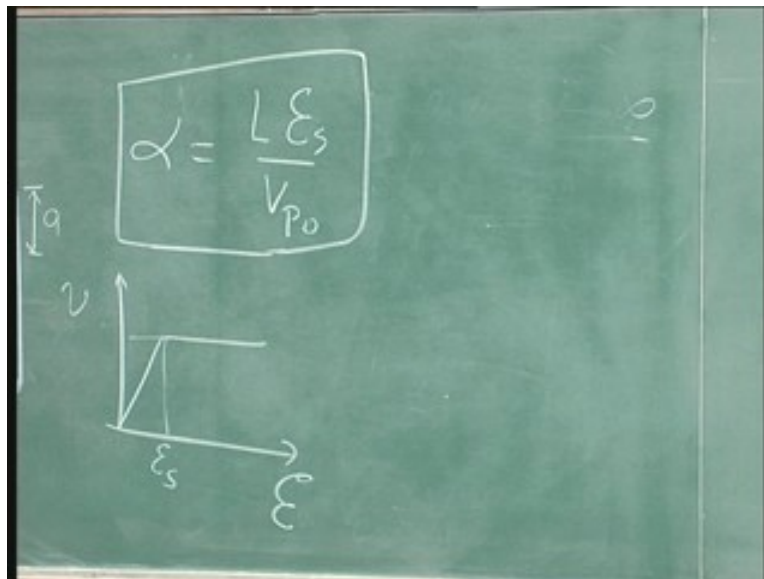
$$= \alpha \left[1 - (u_{ds} + u_{GS})^{1/2} \right] \text{----- (2)}$$

$$\alpha = \frac{u_{ds} - \frac{2}{3}(u_{ds} + u_{GS})^{3/2} + \frac{2}{3}(u_{GS})^{3/2}}{1 - \sqrt{u_{ds} + u_{GS}}}$$

Now, let us see the general case. The most general case is obtained you have to solve this equations. You have got velocity is equal to μ_n into E , this portion at the drain end, this

is the portion. I have put those two things together. You have to equate these two and find out u_{ds} from these equations. Numerical solution you have to do. If I have to find out u_{ds} , once I know u_{ds} , I can actually find out I_{DS} by substituting either this or this. In other words, we have two equations and equate two to get one equation. Alpha is this, you notice of this. Next, what we can do now is, we are interesting in finding out, let me quickly go through that; we are interested in finding out the value of u_{ds} by using equating these two. Wherever some clever people who did some solution but you do is to equate these **two to get** (49:20) alpha. These we have used already, now alpha depends upon L into E_s by V_{p0} , that is alpha.

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What we are trying to see is, as a function of alpha what will be the u_{ds} for a given V_{GS} ? Alpha will be decided when you decide the device, E_s is fixed. At the property of mid device material, that is fixed.

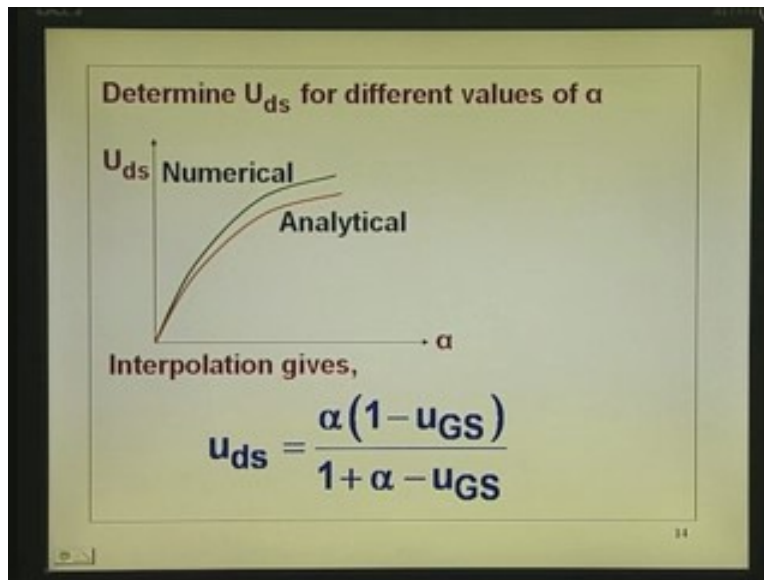
You have taken that by lean arising thing 20 kb per centimeter or 10 kb per centimeter depending upon whether you talk of silicon or Gallium Arsenide. So, that is fixed. This is fixed from fifth device. What about this quantity? That is also fixed. For a given device, if I am changing alpha for a given doping and thickness, I think change L and see.

So, one other way that people are done is, do that alpha versus u_{ds} . Using this equation, you substitute to do is for a given gate to source voltage u_{GS} , after all we cannot get very all the things.

You find out alpha versus u_{ds} , how to do that? Change u_{ds} and find out what is alpha is? It is straight or you find alpha. To decide alpha and find u_{ds} if more complicated; you have both numerator and denominator.

For given u_{GS} , put for different values of u_{ds} verse alpha, you get that.

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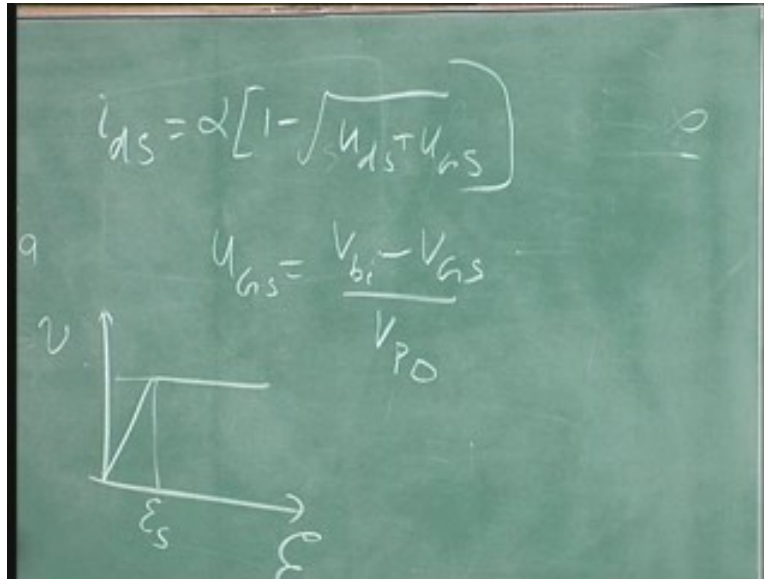
You can do numerically or you can do this substitution and find out and then that is virtually hooking up in the sense fitting the parameter. So, what people did in fact to be actual person who did Michele soore from redlich of techniques? We have published for some time back with result because to simplify to get an analytic position. Numerically; if I do not have to go for numerical, analytically we can do by substituting u_{ds} different values to find what is alpha?

For a given drain to source voltage, you found that you can fit into this expression. You can fit in this expression because after all, each curve is started for one u_{GS} . You can get number of curves like that I have plotted one curve for one u_{GS} and put that an analytical

expression corresponding that first fit and once get an expression for u_{ds} in terms of u_{GS} , you go back and substitute the equation 2.

What is the equation 2? Let go back this equation 2. i_{ds} is equal to, see what we have got now is, for a given alpha we know what is u_{ds} is from that expression and in terms of u_{GS} so you get i_{ds} verses u_{GS} . By substituting for u_{ds} in that equation,

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The image shows a chalkboard with handwritten mathematical expressions and a graph. At the top, the equation $i_{ds} = \alpha [1 - \sqrt{u_{ds} + u_{GS}}]$ is written. Below it, the equation $u_{GS} = \frac{V_{bi} - V_{GS}}{V_{p0}}$ is written. To the left of these equations is a graph with a vertical axis labeled v and a horizontal axis labeled ξ_s . The graph shows a piecewise linear function that starts at the origin, increases linearly to a certain point, and then continues as a horizontal line.

I think propagate down here we will get i_{ds} equals alpha into 1 minus root of u_{ds} plus u_{GS} . I have got an expression for u_{ds} from here which is alpha into 1 minus u_{GS} divide by 1 plus alpha minus u_{ds} . So, I removed this quantity in terms of u_{GS} , that means I get i_{ds} verses u_{GS} . That means, once I get no, what the gate voltage u_{GS} ? What is u_{GS} ? V_{bi} minus V_{GS} divide by V_{p0} and i_{ds} is actually drain current, i_{ds} divide by $G_0 V_{p0}$. So, what I am trying to point out is, you know u_{ds} in terms of u_{GS} by establish interpolation etc., you know i_{ds} in terms of u_{GS} . You can get the transfer characteristic that is what you have got.

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Substitute for u_{ds} in (2)
Interpolation Formula

$$i_{ds} = \frac{\alpha}{1+4\alpha} (1 - u_{GS})^2$$

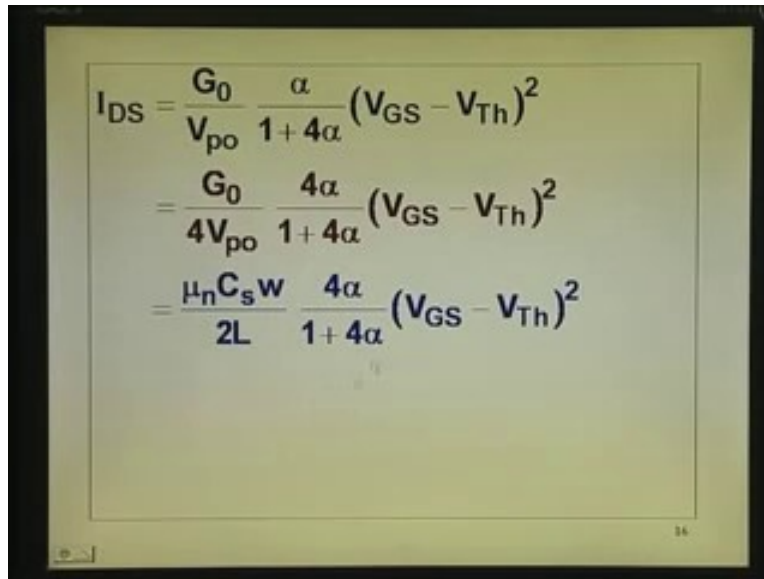
$$I_{DS} = G_0 V_{po} \frac{\alpha}{1+4\alpha} \left(1 - \frac{V_{bi} - V_{GS}}{V_{po}} \right)^2$$

$$= \frac{G_0}{V_{po}} \frac{\alpha}{1+4\alpha} (V_{po} - V_{bi} + V_{GS})^2$$

Substitute there again an approximation is done, you get this. Here after, it is very simple and you do not have to worry about the path is that of establish interpolation to get a fit into that complicated formula simplified, you get this expression. Now, we can do that. I multiplied by this, normalized value; I_{DS} divide with G_0 into V_{p0} . I remove the normalized quantities and write it in terms of that. Go back to the original equation and then I get G_0 by V_{p0} into. All that I have done is pull this V_{p0} out.

So, I get V_{p0} gets cancel of that 1 by V_{p0} squared; I get this, just simple. I have not made any modification there and same equation rewriting this, you get that. I multiplied this by 4 divide by 4, I get again. What is this quantity? Minus $V_{threshold}$ voltage, this whole thing is V_{GS} minus $V_{threshold}$ whole squared and this term is 4 alpha by 1 plus 4 alpha and G_0 by 4.

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$$\begin{aligned} I_{DS} &= \frac{G_0}{V_{po}} \frac{\alpha}{1+4\alpha} (V_{GS} - V_{Th})^2 \\ &= \frac{G_0}{4V_{po}} \frac{4\alpha}{1+4\alpha} (V_{GS} - V_{Th})^2 \\ &= \frac{\mu_n C_s w}{2L} \frac{4\alpha}{1+4\alpha} (V_{GS} - V_{Th})^2 \end{aligned}$$

What is that? What is this first term? I multiplied again by that, this quantity is familiar term for you and go through is again in next time because what you are showing is ultimately you get the same expression that you get using the Shockley's equation except that you got this particular term to start right from here next time going into this.

Then we will see compare with some experimental results. In fact, it is a pair that these things are described right in discussion when we bring the measurements of results.

So, I will stop on that in this discussion.