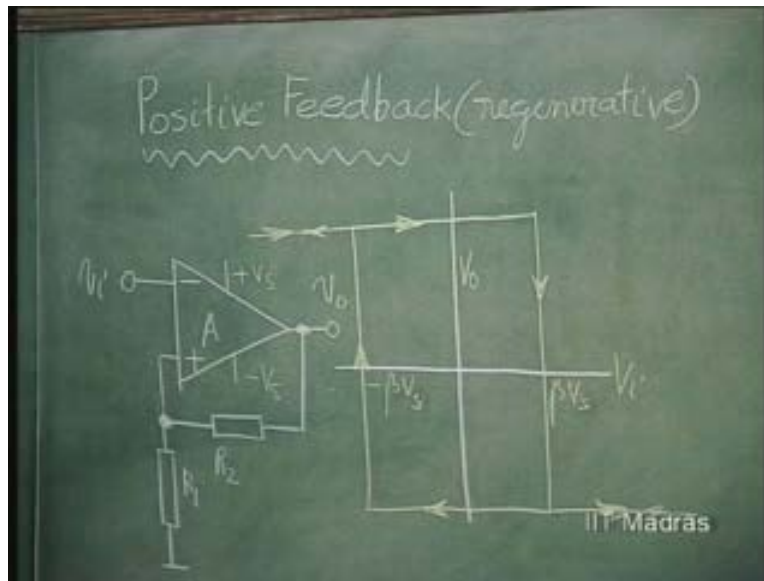


Electronics for Analog Signal Processing - II
Prof. K. Radhakrishna Rao
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Indian Institute of Technology – Madras

Lecture - 9
Positive Feedback (Regenerative)

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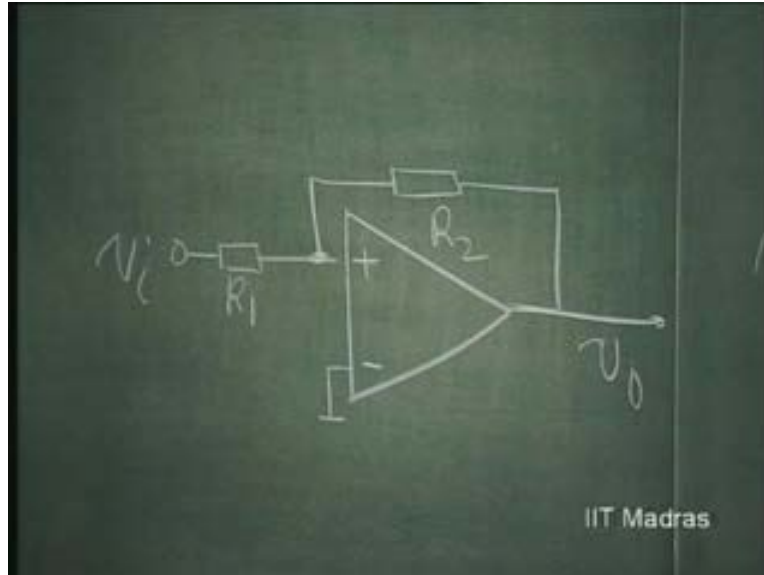


We had discussed about positive feedback which is regenerative. We said this is a very useful configuration with memory. That is hysteresis. The circuit understands how the input is varying; whether it is increasing or decreasing. If the input is increasing, it changes state at a particular point and if the input is decreasing, it changes state at another point. So, this is what is called as memory.

The same circuit with this end grounded and with input connected here... say this is R_1 , R_2 . R_2 is the resistance from the output coming to the non-inverting input and R_1 is the resistance which had gone to the ground. Instead of grounding it, we are connecting it to V_i and what has happened is, this inverting terminal is now grounded. So, what happens to this? This also retains the same feedback. It is still positive feedback; but what happens here is that V_o for large negative voltage here, large negative voltage here, has to

be negative because this is applied to non-inverting. If large negative voltage is applied, this has to be minus V_s .

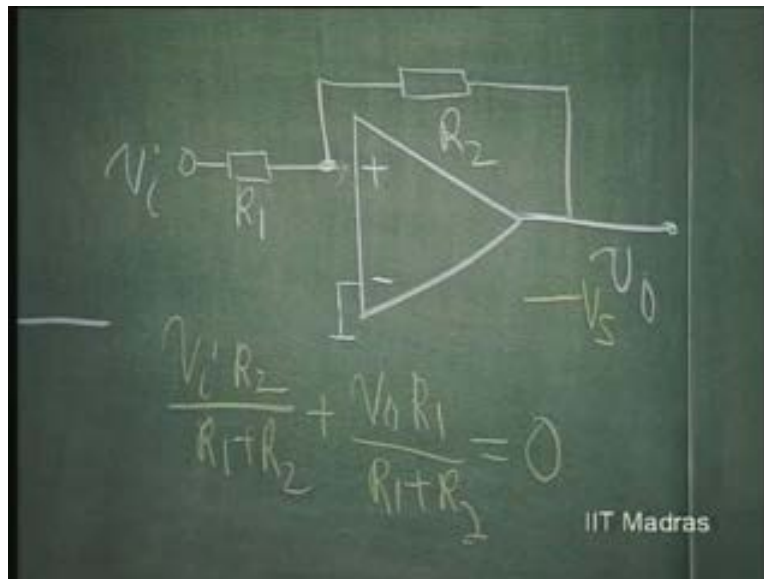
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So, it would start from minus V_s and this will start with minus V_s . The voltage here therefore, when it becomes equal to zero here... this is grounded; so, when this becomes close to zero, change of state should occur. What is that voltage which will make this voltage zero at this point? V_i . This voltage is V_i into R_2 divided by R_1 plus R_2 ; V_i into R_2 by R_1 plus R_2 ; plus V_{naught} into R_1 by R_1 plus R_2 .

If V_i and V_{naught} are the voltages here respectively, V_i into R_2 by R_1 plus R_2 plus V_{naught} into R_1 by R_1 plus R_2 is the voltage here. When this becomes equal to zero, change of state should occur.

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So, V naught at present is nothing but minus V_s . So, when V_i becomes equal to plus R_1 V_s by R_2 , change of state will occur.

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The diagram shows the same circuit as the previous slide, but with the node equation rearranged to solve for the input voltage V_i at the threshold:

$$\frac{V_i R_2}{R_1 + R_2} - \frac{V_s R_1}{R_1 + R_2} = 0$$

$$V_i = + \frac{R_1 V_s}{R_2}$$

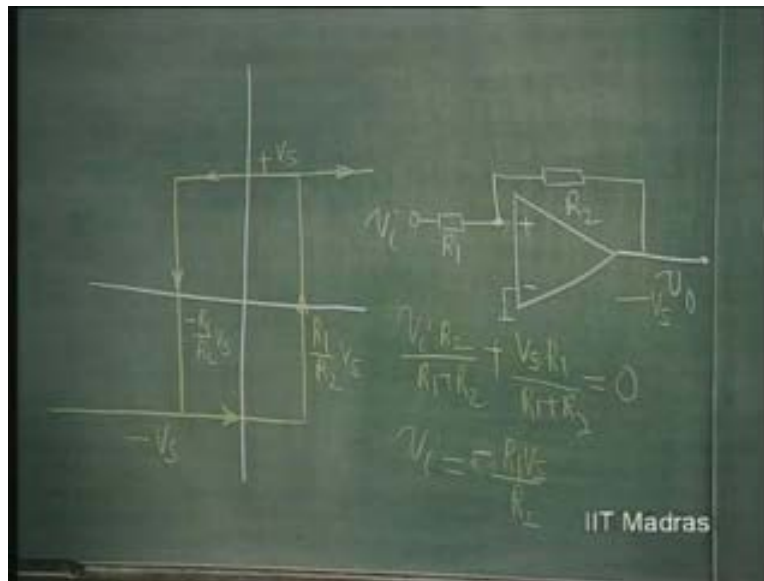
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So, this will go on like this and then change of state will occur from minus to plus at a voltage which is R_1 by R_2 times V_s . So, this is the change of state. It is going to now change state to plus V_s from minus V_s . Once this becomes plus V_s here, the next

change of state occurs only when V_i decreases; and that will happen when this becomes minus. Same equation is valid.

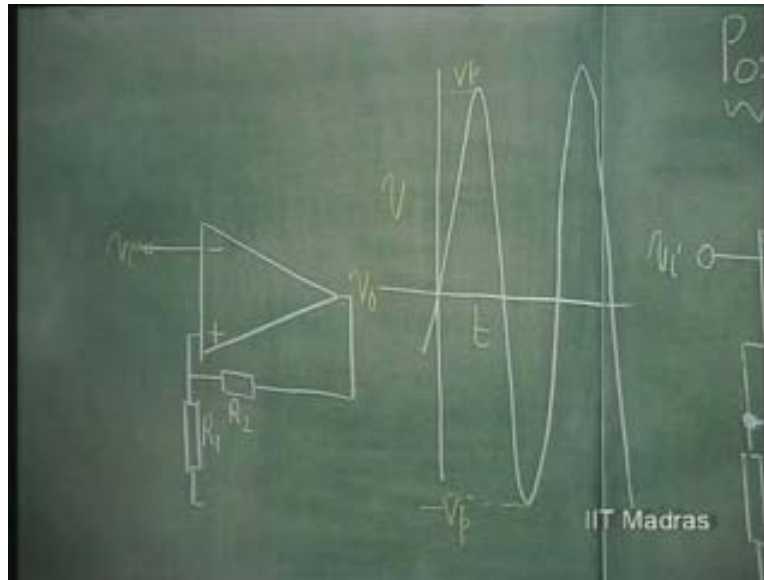
So, **so** again, you get the same hysteresis; but it is of another type. The output is at minus originally, goes to plus; whereas the output was minus originally, plus originally and goes to minus. So this is one type of Schmitt trigger or regenerative comparator; that is the other type of Schmitt trigger or regenerative comparator. These two types are commonly used in a variety of applications like signal generation and wave shaping, etcetera.

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We will work out an example in order to illustrate one of the applications of positive feedback using this circuit. So, consider the Schmitt trigger that we have earlier discussed. Let V_i be a sine wave, as shown here, being applied to the input of this Schmitt trigger.

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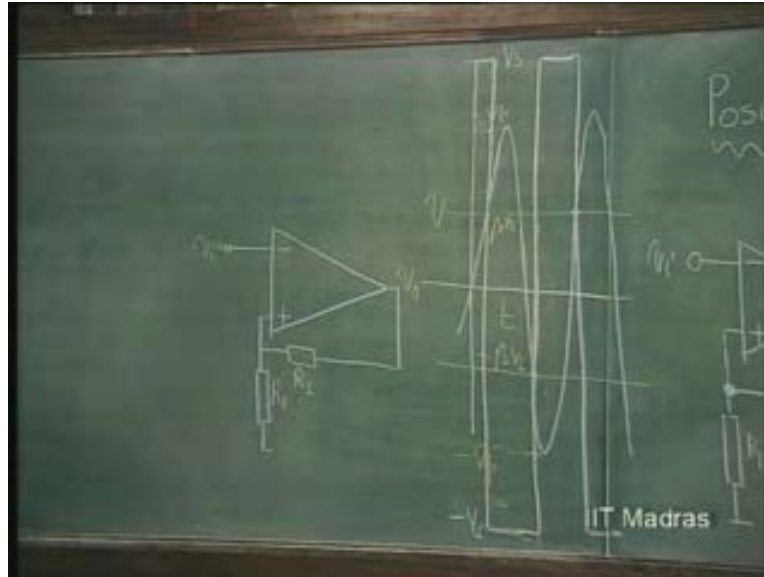
What is the output? We would like to see. So, this could be a sine wave or a triangular wave or an exponentially increasing and exponentially decreasing wave. I am typically discussing this for a sine wave.

So, as the input voltage here is increasing, this portion of the circuit, input voltage is increasing. So, this circuit says the change of state should occur when the input voltage is increasing at Beta V s. So, let us say this is Beta V s and this is minus Beta V s. Now, change of state should occur. Originally, when the input was negative, output was positive. So, it will start with positive voltage and at Beta V s, it will change state to negative. Let us say it is plus V s. Let us say V s is greater than V b. From there, plus V s, it will go to minus V s. So, this is the point at which the change of state will occur; that is Beta V s.

Thereafter, the input voltage is going on increasing. Nothing happens until now. It further starts decreasing. When it is decreasing, change of state will occur, not at this point; but at this point... because... at minus Beta V s. So, the next change of state is going to occur at this point to plus V s. Again, next change of state will occur at this point. So, you will see that for a sine wave input, you get a square wave output; but the change of state

occurs at levels fixed by the Schmitt trigger; uniquely fixed by the Schmitt trigger. Not at any other point.

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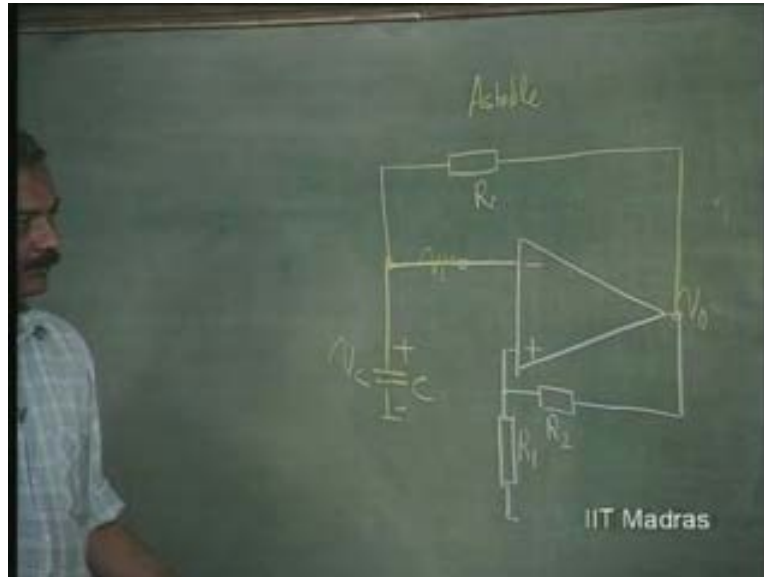


So, this is very useful in converting periodic wave form like sine wave or triangular wave to a very good square wave because the rate of increase of output voltage from this to this, now, totally depends not on input wave form but only on the maximum rate of rise possible or maximum rate of fall possible for the op amp or amplifier structure that is put here; comparator structure put here.

So, this is going to be very sharp transition here. So, in order to convert a stream of data which has got corrupted because of the line capacitors, etcetera, to exponential increase and exponential decrease, the so called square waves themselves may get distorted to exponential increase and exponential decrease because of line capacitors. Then, in order to convert them into sharp transition pulses, this Schmitt trigger is used. This is one application; one important application. Next application also, very simple.

Now what we do is... by itself it has two stable states or this is a bi-stable circuit. One is plus V_s ; another is minus V_s . Now, I am going to make it what is called as Astable circuit. Consider this circuit.

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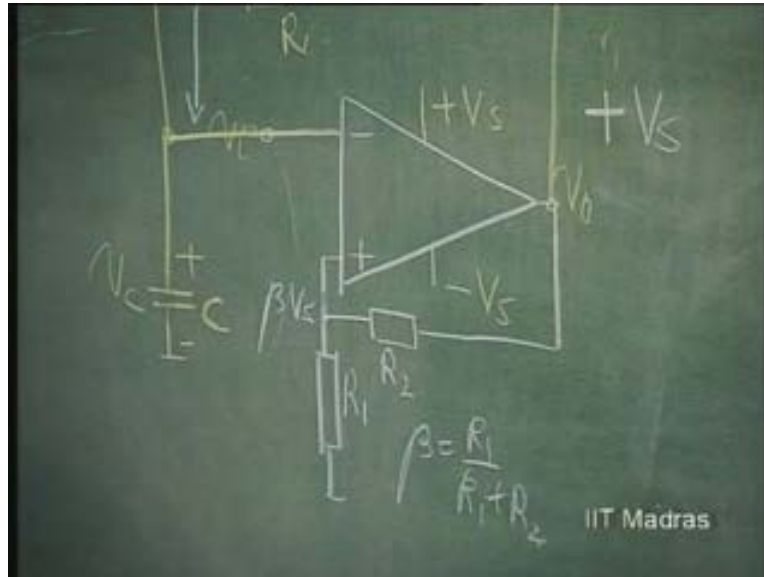
As far as this itself is concerned, this is a circuit with positive feedback here. So, it does not really know what is happening here, let us assume. But the capacitor is initially uncharged. We will consider capacitor is initially uncharged. I switch on the thing. Obviously, we say that this is a regenerative comparator and that output has to be either at plus V_s or at minus V_s . That is for sure.

So, output is at plus V_s , let us consider. If output is at plus V_s Let us put that down. What is happening here in this circuit? Let us independently consider. Capacitor is not charged. So, the capacitor will start getting charged through this resistance. So, that will be...that you know is an exponentially increasing wave form.

So, capacitor will start getting charged this way. What is this? This is going to be charged towards V_s . It will try to go towards plus V_s with a time constant equal to C into R . So, the tangent time constant is C into R . So, it is increasing. This capacitor voltage is

increasing with respect to time. So, this Schmitt trigger says, if a voltage is increasing at its input, I will change state, when the voltage reaches Beta times V_s , Beta being equal to R_1 by R_1 plus R_2 .

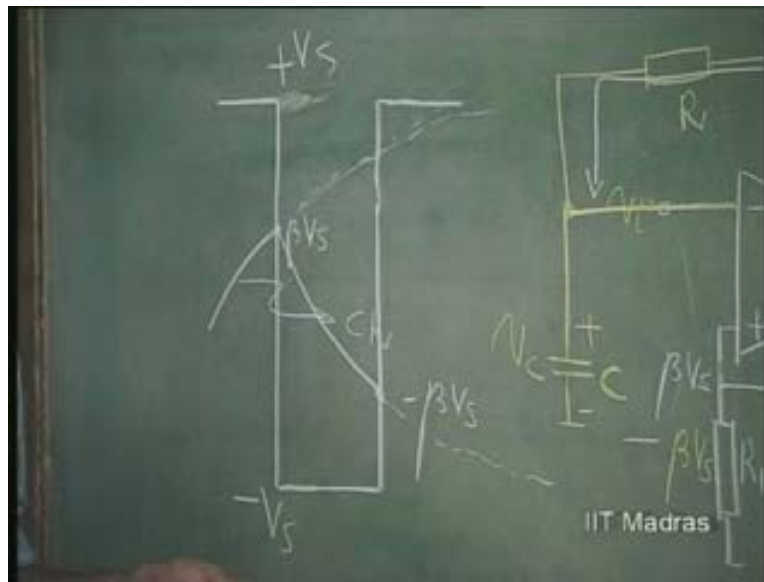
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So, since this is plus V_s , this is Beta V_s . Change of state will occur when this voltage reaches a magnitude of Beta V_s . How will the change of state occur? It will change state from plus V_s to minus V_s . Now, what will the capacitor see? This has changed over to minus V_s . This becomes minus Beta V_s .

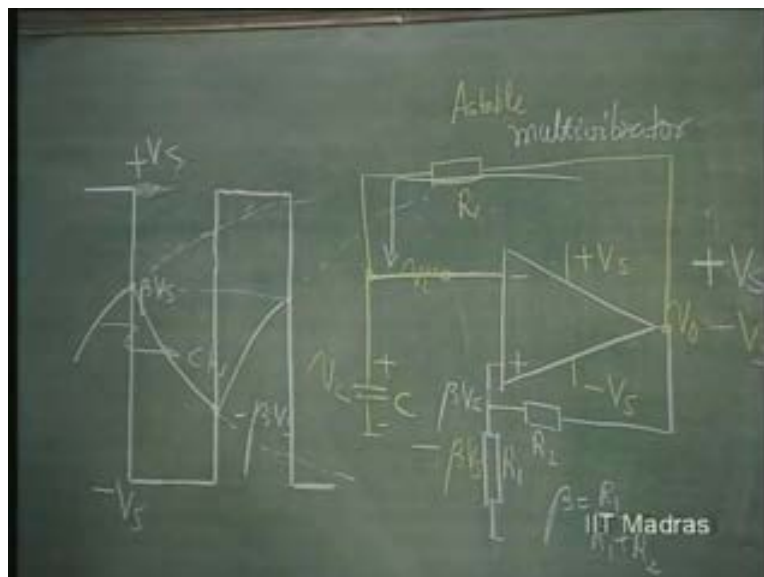
So, the capacitor will now charge towards minus V_s with the same time constant. So, the capacitor will keep charging towards minus V_s with the same time constant; but as soon as it reaches minus Beta V_s , because the voltage is decreasing now; this voltage is decreasing. So, this will say that I am going to change state the moment this voltage reaches minus Beta V_s . So again, it will go to plus.

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This story never ends because now the capacitor is going to charge towards plus V_s with the same time constant RC . As soon as it reaches βV_s , this will change state. This is what is called as an Astable multivibrator.

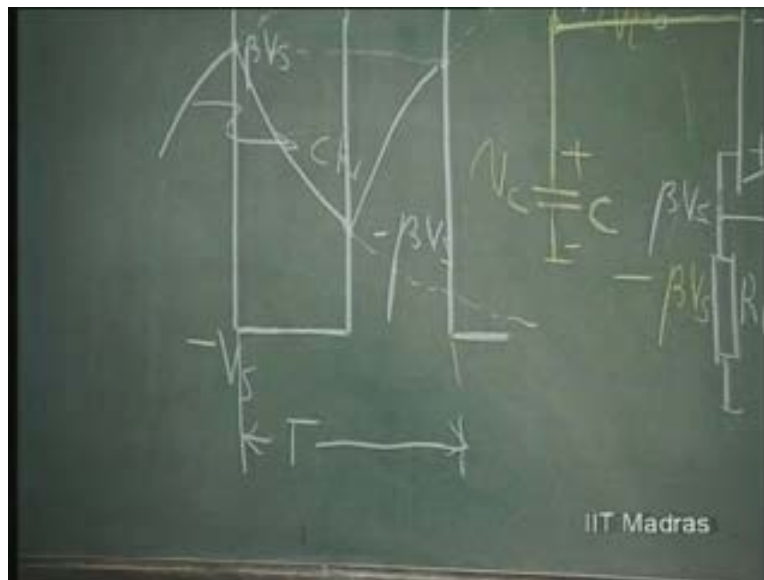
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It keeps changing a state from plus V_s to minus V_s to plus V_s , goes on on its own and therefore it produces a square wave at this point and an exponential increasing wave form and decreasing wave form at this point; this kind of wave.

In fact, you can therefore find out this time interval. This time interval and this time interval will be the same because the capacitor during this time has discharged with the same time constant, as it is getting charged. So, the discharging time constant is same as charging time constant; and the voltages changed will be from Beta V_s to minus Beta V_s . That means 2 Beta V_s is what is acquired by the capacitor. So, these two time intervals will be same. That means this is the period of the wave form T .

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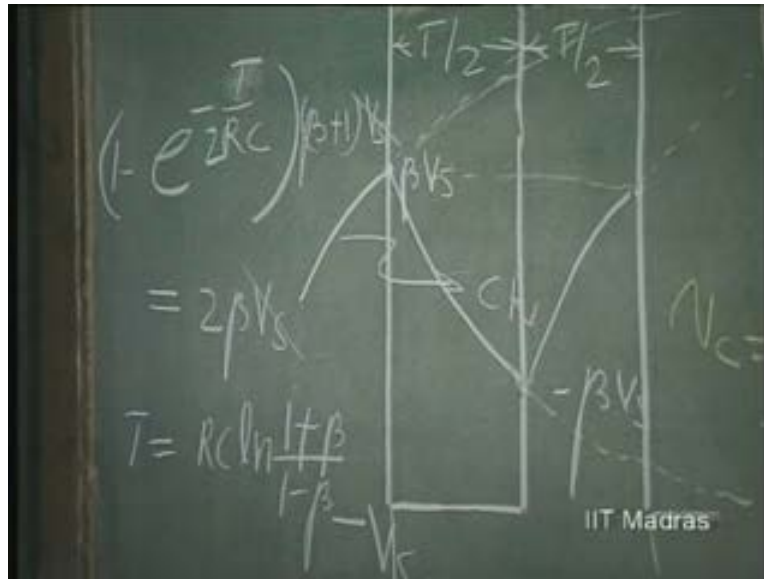


You can evaluate this by equating...let us say that this is exponentially changing; e to power minus t by $R C$. 1 minus e to power minus t over $R C$. This is the way voltage changes. The voltage applied here is...from minus Beta V_s , it is trying to go towards plus Beta V_s .

So, the total voltage applied is Beta plus 1 V_s from here to this point, assuming that this is zero. So, from here to the top point, it is Beta plus 1 V_s ; and it is going to change at 1

minus e to power t over RC . That kind of rate. But voltage acquired within T by 2 is twice βV_s . That means, during this time period T by 2, the voltage acquired is twice βV_s . From this equation, T comes out as... If we write down, V_s , V_s , gets cancelled. You have... you can equate this. T comes out as $RC \log 1 + \beta$ by $1 - \beta$.

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That is, this is T by 2. T by 2 comes out as $RC \log 1 + \beta$ by $1 - \beta$; or, T is equal to twice $RC \log 1 + \beta$... So, you can therefore evaluate the frequency of oscillation of this Astable multivibrator, if you know the resistance capacitance and the β value.

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This circuit is useful as a signal generator. It is commonly used in signal generators; and you can see that the basic principle is very simple. It is using some kind of integrator here. This is a rough integrator; capacitor getting charged through a resistor is equivalent to an integrator. And this is the Schmitt trigger. This integrator, Schmitt trigger, combination is a common circuit for what is called as function generation or wave form generation.

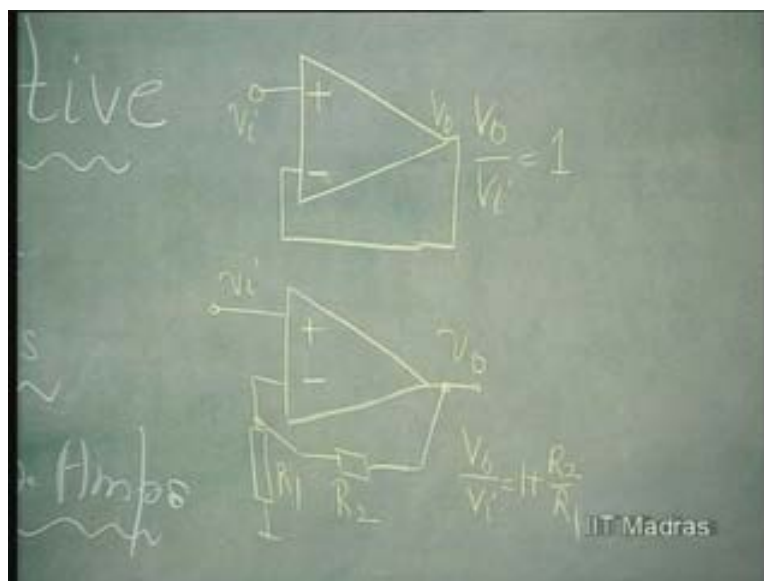
We had digressed a little bit. We were discussing negative feedback configuration. In order to differentiate it from positive feedback, we went over to positive feedback configuration.

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For example, this is negative feedback configuration. To show that structure looks similar with this minus, this plus; but that structure is entirely different. That is one with positive feedback, regenerative positive feedback and with hysteresis when this becomes plus and that becomes minus.

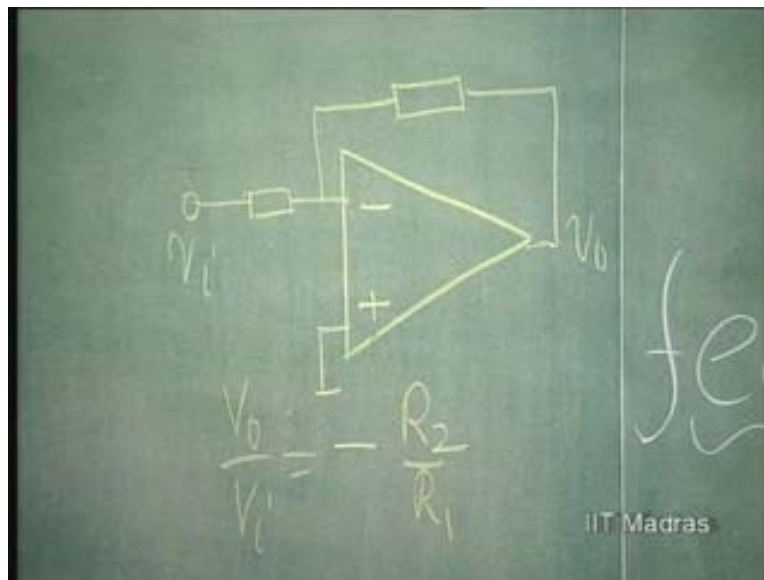
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Similarly, when this becomes plus and that becomes minus, that is another regenerative feedback configuration. So, this should be kept in mind that, the feedback changes, the characteristics becomes different. That is a non-linear circuit; this is a linear circuit with a gain of $1 + \frac{R_2}{R_1}$.

So, continuing with the basic negative feedback configurations, this is a buffer unity gain amplifier. This is a non-inverting amplifier of gain $1 + \frac{R_2}{R_1}$, which we had discussed; and this is another important configuration called inverting amplifier. So, just compare this with the two configurations for positive feedback that we had just now discussed. These are regenerative positive feedback with different characteristics. So, one was non-inverting Schmitt trigger and another was inverting Schmitt trigger.

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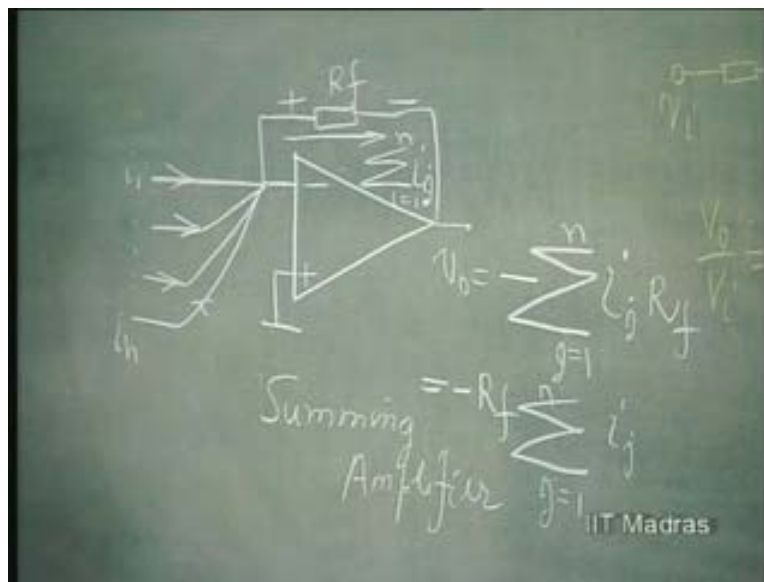
Now, coming back to the negative feedback, let us see how we can generalize this negative feedback configuration here.

This is an inverting amplifier with $\frac{R_2}{R_1}$. When you ground this, what we said was, always this is at virtual ground as long as output is connected to input. Please remember that. For this to be at a virtual ground, the condition is, output should be

somehow connected to the input. Somehow. It does not really, necessarily, get connected like this. It can be connected in any manner that there is negative feedback. Then, this can be at virtual ground. Such a situation the... this becomes a current summing **note** because at this point the potential is always zero and currents can get summed up.

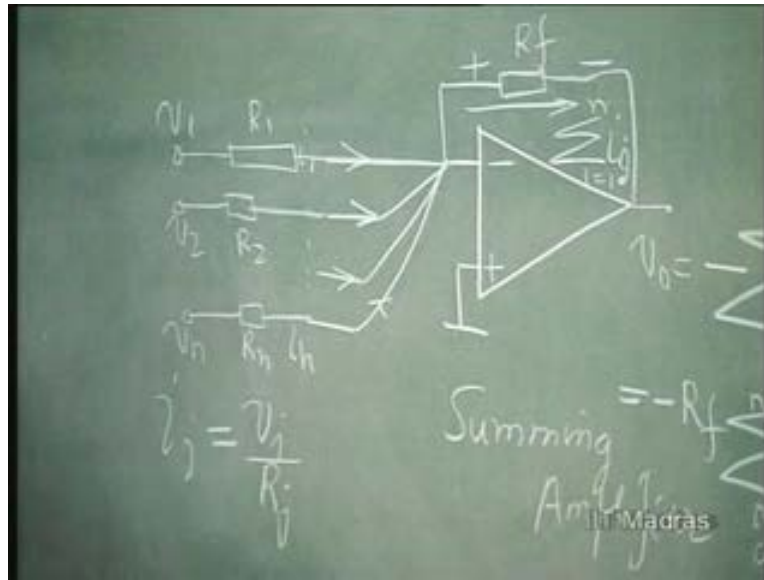
So, this property can be used in summing currents. So, this is actually I_1, I_n . So, the entire current is going to flow through this R_f , appear as this current is going to be $\sum I_i, i \text{ equal to } 1 \text{ to } n$; that therefore, the voltage here is going to be... that summed up current gets converted into a voltage which is minus because this is plus and that is minus. $\sum I_i, i \text{ is equal to } 1$. Or, we will call it i_j in order to differentiate. $j, j \text{ is equal to } 1 \text{ to } n$. So, this is... into R ; that is the scaling factor R_f . So, this is therefore going to be called as generalize, I mean, as a sum up summing amplifier. $\sum i_j \text{ into } R_f; j \text{ equal to } 1 \text{ to } n$.

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So, if I want, for example, resistors connected here, $R_1 V_1, R_2 V_2$, so on... $R_n V_n$, then all these currents, i_j s can be replaced by V_j s by R_j s because V_j is equal to... i_j is equal to V_j by R_j because this potential is zero.

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So, you can therefore get this output as minus R_f . Let us say, we will call this as Alpha 1 times R_1 , Alpha 2, Alpha 2 times R , this is Alpha n times R , scaling factor. Then, this will be sigma. This Alpha R will come. V_j divided by Alpha j ; j is equal to 1 to n .

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Handwritten notes on the slide:

$$\text{Summing Amplifier} = -R_f \sum_{j=1}^n i_j$$

$$= -\frac{R_f}{R} \sum_{j=1}^n \frac{V_j}{\alpha_j}$$

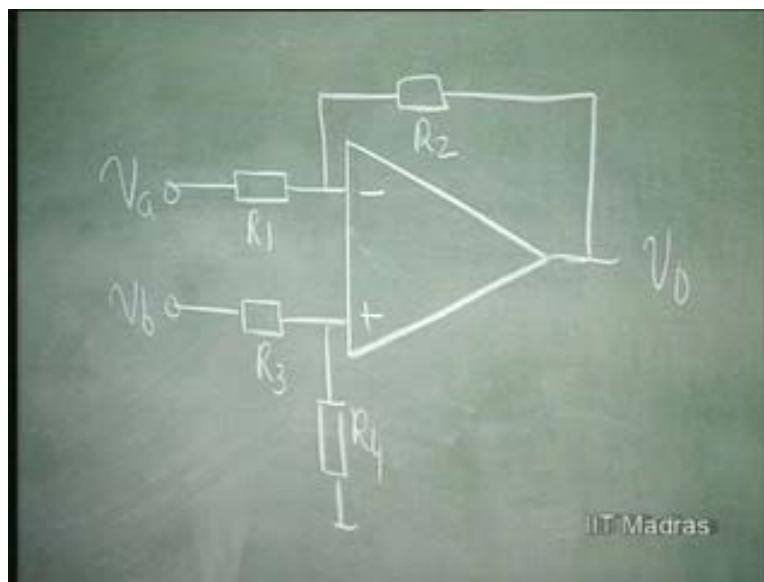
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So, if you have R_f by R equal to 1, then the co-efficients of V_j s can be conveniently chosen as $1/\alpha_j$. So, this becomes a very convenient summing amplifier.

Now, I am going to further generalize. I do not have any choice regarding the sign of the co-efficient here; but the value of the co-efficient can be greater than 1, less than 1, anything. So, it is a very simple design. Now, let us see whether we can have a means of controlling the sign as well as the magnitude so that it becomes a generalized summing amplifier.

Consider this circuit simply. At all times, we only give negative feedback. That you should remember. This is V_a , this is V_b . What is the output? V_o . If, let us say, this is R_1 , this is R_2 , this is R_3 and this is R_4 , that is a general configuration. Let us see how to evaluate this.

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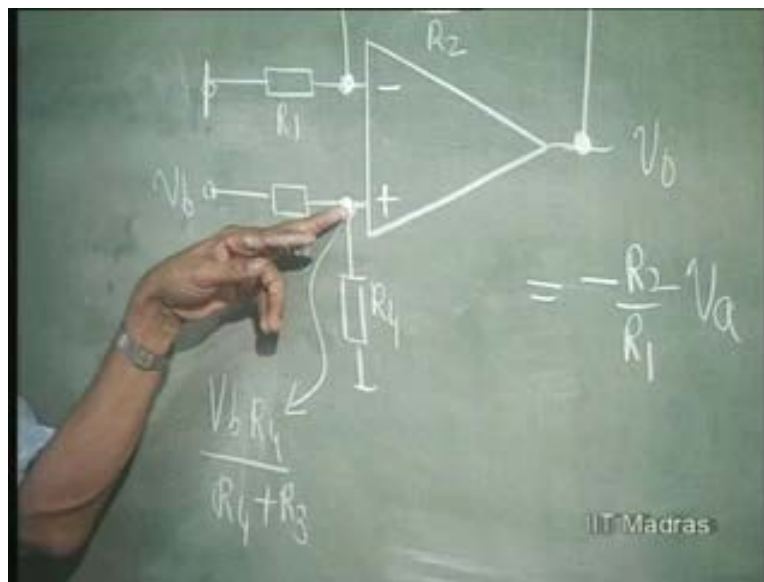
We can do this by super position theorem because there are two voltages. We can do this by super position theorem. First find out only for one voltage, other voltage being connected to ground; and then find out for the other voltage.

Now, considering that V_b is connected to the ground, let us find out the output for V_a . Now, this becomes nothing but the inverting amplifier shown here with R_2 and R_1 . If you are grounding this, this potential is zero. So, it is almost equivalent to this circuit. So,

this becomes an inverting amplifier. So, V_{naught} is nothing but minus R_2 over R_1 times V_a . With V_b connected to ground, it is equivalent to just an inverting amplifier there.

Next, let us lift this above ground. This is V_b . Connect this to ground. Now, what does it become? From here to here, it is an attenuation. So, the voltage here is V_b into R_4 by R_4 plus R_3 attenuator.

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And from here onwards, it is nothing but the non-inverting amplifier there. So, it is the same configuration. So, gain is 1 plus R_2 over R_1 . So, this is plus V_b into R_4 by R_4 plus R_3 into 1 plus R_2 over R_1 . So, this is the composite output of this amplifier. This is therefore equal to... This has a negative sign, you see; this has a positive co-efficient.

So, let us say, I would like to make this a difference amplifier. There are two voltages. I want to take difference of V_b and V_a . Therefore, I would like to take this as a common factor, let us say. R_2 over R_1 minus V_a plus this should have been plus V_b ; minus V_a plus V_b would have been the differential thing. So, I have taken out R_2 over R_1 . So, this becomes 1 plus R_1 over R_2 divided by 1 plus R_3 over R_4 .

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Handwritten derivation on a chalkboard showing the output voltage V_o in terms of input voltages V_a and V_b and resistors R_1, R_2, R_3, R_4 . The derivation is as follows:

$$V_o = -\frac{R_2}{R_1} V_a + \frac{V_b R_4}{R_4 + R_3} \left(1 + \frac{R_2}{R_1} \right)$$

$$= \frac{R_2}{R_1} \left(-V_a + V_b \frac{\left(1 + \frac{R_2}{R_1} \right)}{\left(1 + \frac{R_3}{R_4} \right)} \right)$$

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I am dividing by R_4 . So, $1 + R_3$ over R_4 . This R_2 over R_1 , I have taken out. So, this becomes $1 + R_1$ over R_2 . So that means, if, this is important, if $1 + R_1$ over R_2 equals $1 + R_3$ over R_4 , that co-efficient becomes 1. So, this becomes R_2 over R_1 into V_b minus V_a . A difference amplifier. This is called a difference amplifier.

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Handwritten derivation on a chalkboard showing the simplified output voltage V_o for a difference amplifier. The derivation is as follows:

$$V_o = -\frac{R_2}{R_1} V_a + \frac{V_b R_4}{R_4 + R_3} \left(1 + \frac{R_2}{R_1} \right)$$

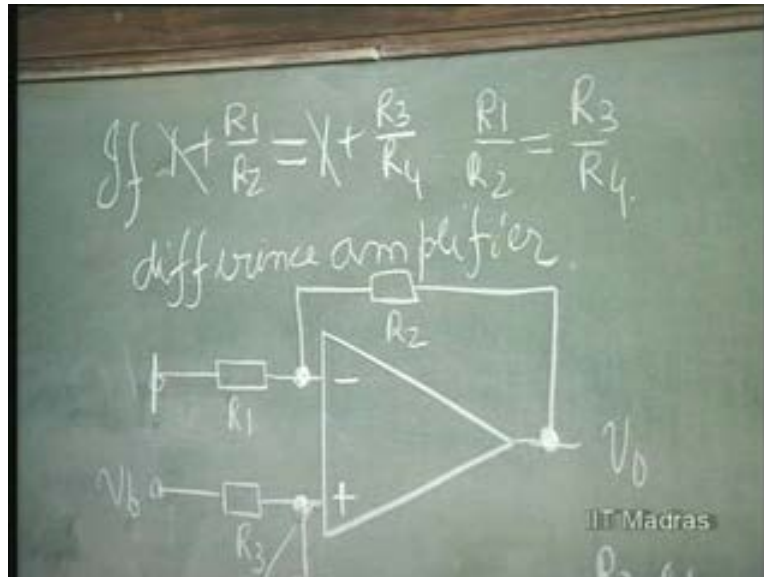
$$= \frac{R_2}{R_1} \left(-V_a + V_b \frac{\left(1 + \frac{R_2}{R_1} \right)}{\left(1 + \frac{R_3}{R_4} \right)} \right)$$

$$= \frac{R_2}{R_1} (V_b - V_a)$$

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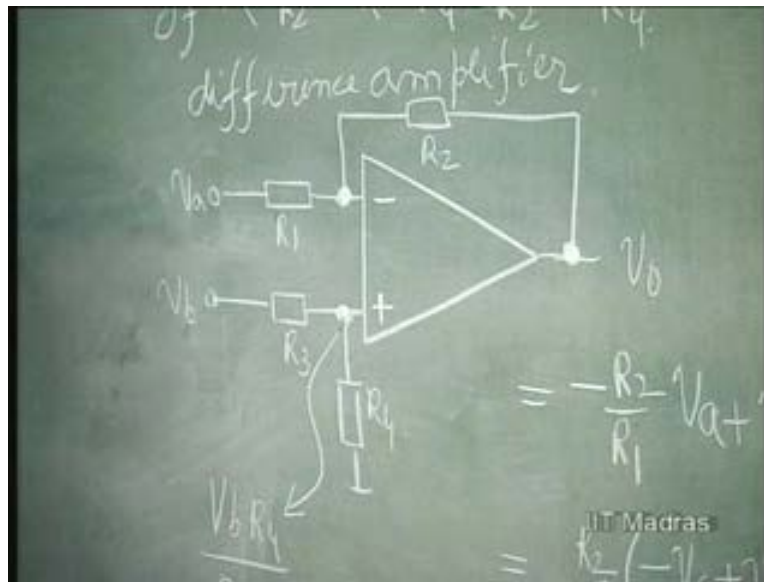
What is the condition? This 1 gets cancelled. So, R_1 by R_2 should be equal to R_3 by R_4 .

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If you make R_1 by R_2 equal to R_3 by R_4 equals 1, then this gain also becomes equal to 1. It is ordinary difference amplifier. Output is V_b minus V_a . Such an amplifier is necessary in all bridge measurements. You want, for example, the bridge voltage comes as a difference voltage V_a minus V_b or V_b minus V_a ; and I want to amplify this by a factor R_2 over R_1 . So, this is a common amplifier configuration which is used along with bridge.

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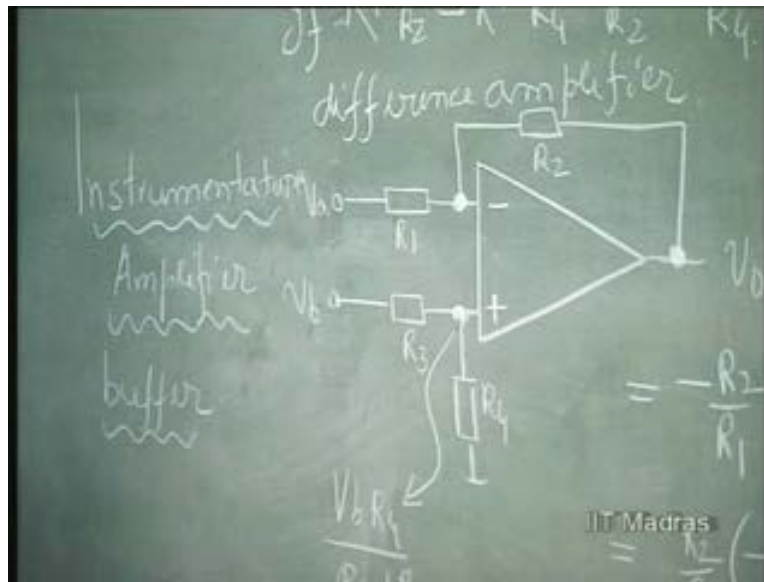


So, where do you use a bridge? May be in resistance measurement, capacitance measurement; in all these measurements, you use bridges. And where do these bridges become necessary? Whenever we have transducers whose resistance varies or capacitance varies or inductance varies, and we want this variation to be measured, we form them into bridges so that the common voltage gets cancelled and only the difference voltage which is the unbalanced voltage comes at the input of the amplifier.

So, this forms a basis of an important amplifier called instrumentation amplifier. Almost all the transducers which are coming as bridge, as bridges, this instrumentation amplifier is the first amplifier which amplifies the different signal component. An instrumentation amplifier is therefore basically a difference amplifier. Now obviously, this instrumentation amplifier might disturb the bridge balance because it is loading the bridge.

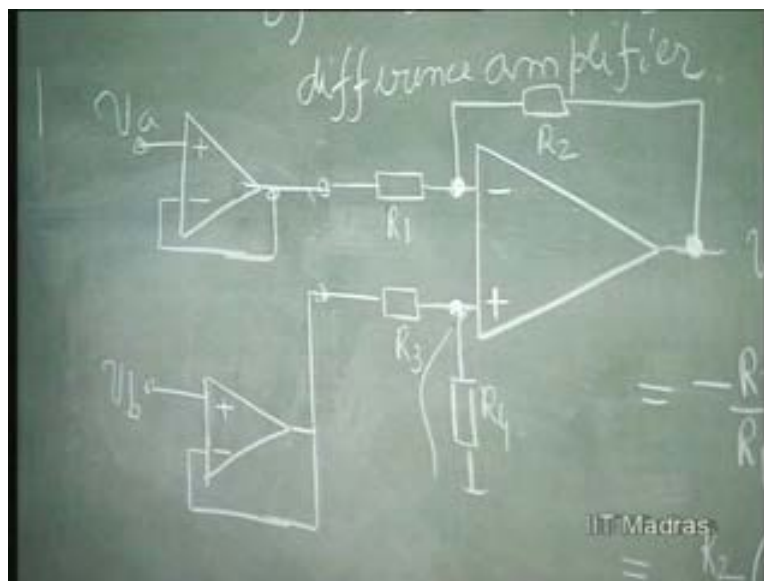
So, if you want a good instrumentation amplifier, you have to buffer it from the bridge. Apart from taking a difference, it should also be acting as a buffer, so that it does not load the bridge.

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Therefore, we will now introduce two buffers here so that this loading actually... Those buffers are already known to us. They are therefore going to be unity gain amplifiers or non-inverting amplifiers. Any of this could be used as buffer stages. V_b which is not inverted and V_a which is inverted there. So, we have the buffer stages just introduced here so that this becomes a good instrumentation amplifier.

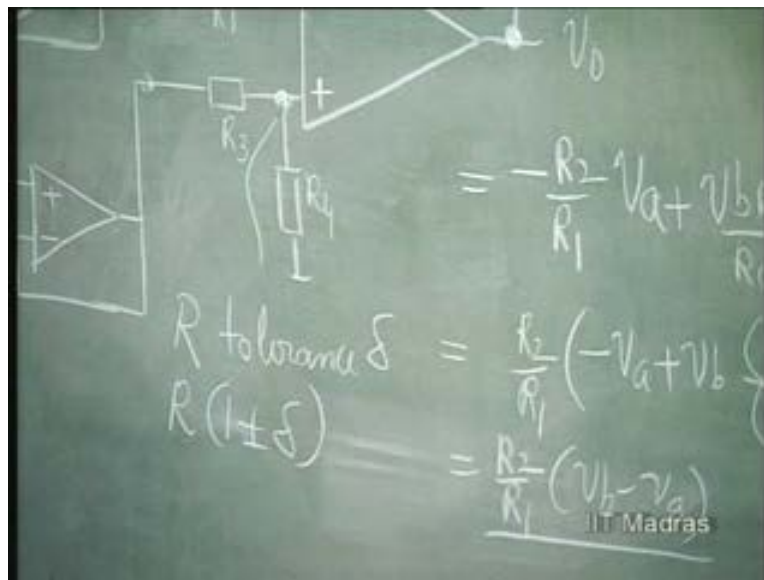
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So now, these three op amp configuration can act as a fairly good instrumentation amplifier with a gain of R_2 over R_1 . But you know; this condition has to be satisfied. R_1 by R_2 should be equal to R_3 by R_4 . Now we say that I have the op amp; almost near ideal op amp; but the resistors I have chosen are just nominal resistances with some amount of tolerance.

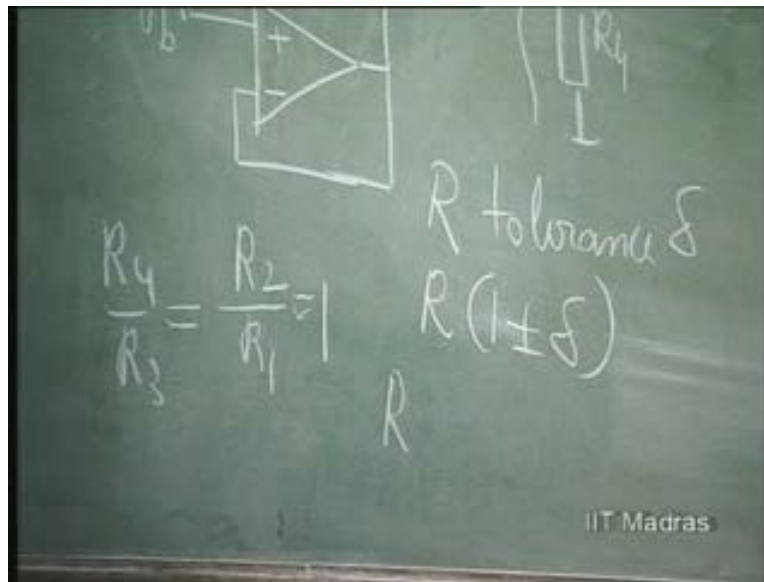
So, let us consider that the resistances that we use have a tolerance of Δ . The resistances that we have have a tolerance of Δ . That means R value is going to be R into 1 plus or minus Δ . That is what it means. Tolerance of Δ means. So, R can be changing from its nominal value by plus minus Δ .

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So, if such resistances are used for fabricating this instrumentation amplifier, this R_1 will be... R_1 , whatever value it requires, and plus minus Δ . So, I want this to merely act as a difference amplifier with unity gain, let us say. R_2 by R_1 is made equal to unity. This is equal to R_4 by R_3 . So, all resistances are made equal to R and all of them have a tolerance of Δ .

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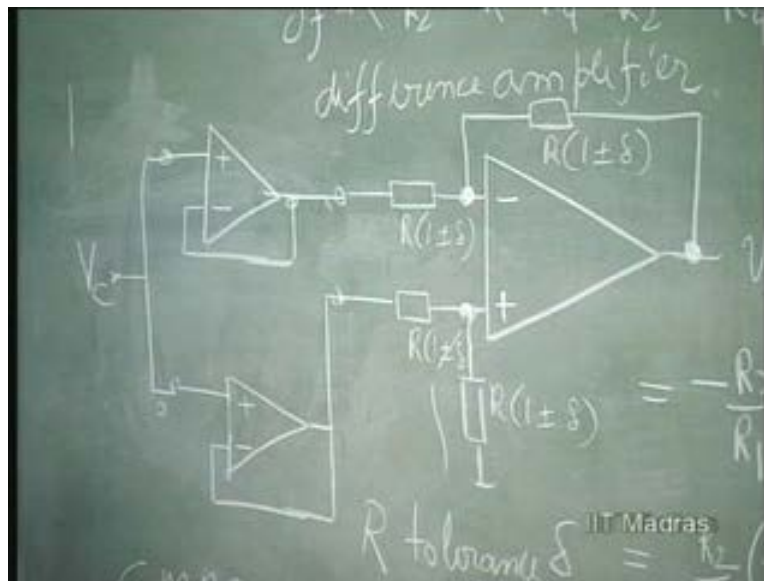
That means this is merely being used as a difference amplifier with unity gain. So, what is the difficulty?

Now please recollect what I discussed about, in terms of differential amplifiers. Any differential amplifier with a differential mode gain also has a common mode gain; and it has a specific common mode rejection ratio. This, I have discussed. What is it? When we want to amplify $V_b - V_a$ or we want to just collect $V_b - V_a$, there will be a common signal between V_b and V_a which is $\frac{V_b + V_a}{2}$.

So, it should be capable of rejecting that common signal. What does it mean? When V_b equals V_a equal to the common mode signal, which is $\frac{V_b + V_a}{2}$, there should not be any output because V_b has been made equal to V_a equal to V_c . There should have been zero output. But, because of the problem with this tolerance, what happens? These resistances are not exactly equal. If this is $1 \pm \Delta$, $R(1 \pm \Delta)$, this also is... The trouble arises not if the error is in the same direction in all. If this is $R(1 + \Delta)$, this also is $R(1 + \Delta)$. The ratio is still 1.

Here also, this is R into $1 + \Delta$. This also is R plus $1 \dots R$ into $1 + \Delta$. The ratio is still equal to 1; and therefore, there is no problem. The gain is going to be... it is going to have no output, as far as common mode signal is concerned. But the problem arises when this is R into $1 + \Delta$, this is R into $1 - \Delta$; or when this is R into $1 + \Delta$, this is R into $1 - \Delta$.

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So, what happens is that this ratio, if instead of being 1, will be deviating from 1. If this is 1, this also should be 1. Then only, this cancellation occurs. Unless this cancellation occurs here and this becomes V_b , just V_b , this cannot be considered as difference amplifier. So, when V_b is equal to V_a , it is going to still give us an output, if these do not get cancelled.

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Handwritten derivation on a chalkboard showing the common mode gain calculation. The equations are as follows:

$$= -\frac{R_2}{R_1}V_a + \frac{V_b R_4}{R_4 + R_3} \left(1 + \frac{R_2}{R_1}\right)$$

$$= \frac{R_2}{R_1} \left(-V_a + V_b \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left(1 + \frac{R_3}{R_4}\right)} \right)$$

$$= \frac{R_2}{R_1} (V_b - V_a)$$

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So, what is the common mode gain under this situation? Worst case common mode gain? So, we will want to find out common mode gain. That is A_c . CMRR is nothing but A_d by A_c , the differential mode gain divided by common mode gain.

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Handwritten notes on a chalkboard. On the left, a circuit diagram of an op-amp configured as a differential amplifier with inputs V_a and V_b . To the right, a resistor is shown with the label $R(1 \pm \delta)$. Below these, the formula for CMRR is written:

$$CMRR = \frac{A_d}{A_c}$$

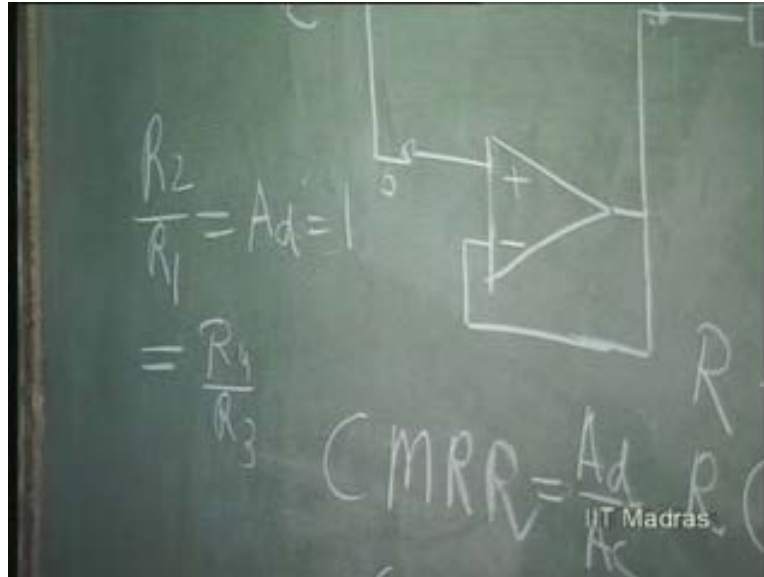
Where R tolerance δ is indicated. Below the formula, it says "Common mode gain $A_c = ?$ ".

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Differential mode gain has been made equal to 1. So, R_2 equal to R_1 . So, R_2 equal to R_1 . A_d is equal to 1. But, we have to also make R_2 equal to R_1 equal to R_4 by R_3 . That

means R 4 also has been made equal to R 3 equal to, nominally, same value of resistance R.

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So, what is the common mode gain? Then V a equal to V b. This is V c, this is V c, now. V a has been made equal to V b equal to V c. So, whatever output I get now... What is the output? We will take R 2 equal to R 1. So, this is made equal to 1, nominally. So, this is having minus V c; let us say, R 2 by R 1. That is a common factor; R 2 by R 1 is very nearly equal to 1. This into minus 1 plus 1 plus R 1 by R 2 divided by 1 plus R 3 by R 4.

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$$\begin{aligned}
 &= \left(-V_c + V_c \frac{\left(1 + \frac{R_1}{R_2}\right)}{\left(1 + \frac{R_3}{R_4}\right)} \right) \\
 &= -V_c \left[\frac{R_2}{R_1} \right] \left[-1 + \frac{1 + \frac{R_1}{R_2}}{1 + \frac{R_3}{R_4}} \right]
 \end{aligned}$$

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What is this? This is equal to this. R_3 by R_4 will go up. 1 will get cancelled. So, you have minus V_c into R_2 by R_1 into minus R_3 by R_4 plus R_1 by R_2 ; from this, this is what you get; divided by $1 + R_3$ by R_4 .

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$$\begin{aligned}
 (1 \pm \delta) &= \frac{-V_c \frac{R_2}{R_1} \left[-\frac{R_3}{R_4} + \frac{R_1}{R_2} \right]}{1 + \frac{R_3}{R_4}} \\
 \delta &= \left(-V_c + V_c \frac{\left(1 + \frac{R_1}{R_2}\right)}{\left(1 + \frac{R_3}{R_4}\right)} \right) \\
 &= -V_c \left[\frac{R_2}{R_1} \right] \left[-1 + \frac{1 + \frac{R_1}{R_2}}{1 + \frac{R_3}{R_4}} \right]
 \end{aligned}$$

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So, I can take this inside. This becomes equal to 1. This is R_2 by R_1 . As before, you see, this is $1 - R_3, R_4, R_3 R_2$ by $R_4 R_1$; $R_3 R_2$ by $R_4 R_1$, which should have been

equal to one, nominally. If this is 1, this 1 minus 1, this is zero. Common mode gain is zero; common mode rejection ratio is infinity.

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The image shows a chalkboard with handwritten mathematical derivations. At the top, there are some faint notes: $V_0 = -\frac{R_2}{R_1}$. The main derivation starts with an expression for V_0 in terms of V_c and various resistors. The expression is simplified step-by-step, involving terms like $\frac{R_3 R_2}{R_1 R_1} + 1$ in the numerator and $1 + \frac{R_3}{R_4}$ in the denominator. The final result shown is $V_0 = -V_c \left[\frac{R_2}{R_1} \right] \left[\frac{1 + \frac{R_1}{R_2}}{1 + \frac{R_3}{R_4}} \right]$. The text "IIT Madras" is visible in the bottom right corner of the chalkboard image.

In this particular case, what is the worst case value for this? When all the nominal values are R , this will be R square, this will be R square; that gets cancelled. Nominal value is 1. Inside the bracket, you will have R square into 1 plus Δ into 1 plus Δ . So, 1 plus 2 Δ plus Δ square. Here, you have to consider the worst case situation; 1 minus Δ into 1 minus Δ . So, R square into 1 minus 2 Δ plus Δ square.

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Handwritten equations on a chalkboard:

$$V_o = \frac{R^2(1+2\delta+\delta^2)}{R^2(1-2\delta+\delta^2)} V_i$$

$$V_o = -V_c \frac{[-\frac{R_3 R_2}{R_4 R_1} + 1]}{1 + \frac{R_3}{R_4}}$$

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If you neglect Delta square because Delta itself is very small, this is the value of... So, the so called 1 or how much is it? Different 1 plus Delta by 1 minus Delta is approximately equal to 1 plus 4 Delta.

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Handwritten equations on a chalkboard:

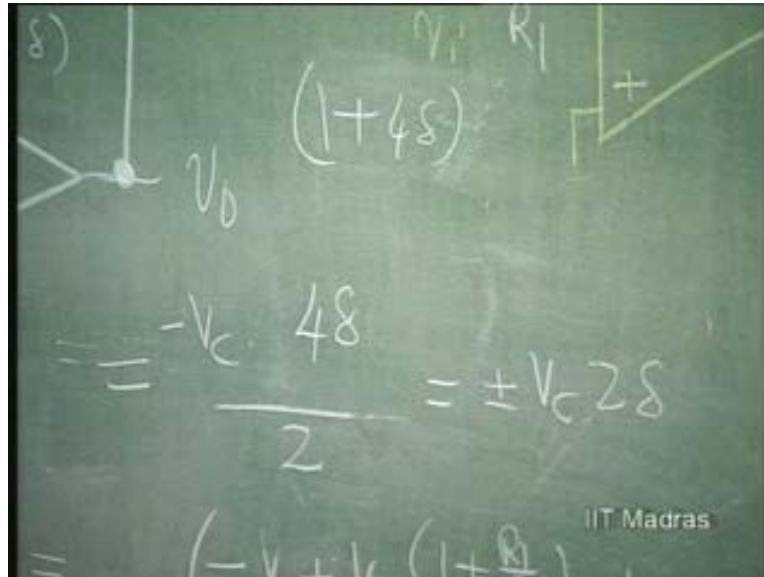
$$V_o = (1+4\delta) V_i$$

$$V_o = -V_c \frac{[-\frac{R_3 R_2}{R_4 R_1} + 1]}{1 + \frac{R_3}{R_4}}$$

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So, this becomes 1 plus 4 Delta. So 1, 1 gets cancelled. The whole thing will give you a contribution of only 4 Delta and the denominator R 3 is equal to R 4. This is going to be 2, nominally. So, this is going to be minus V c into 2 Delta; plus or minus, of course.

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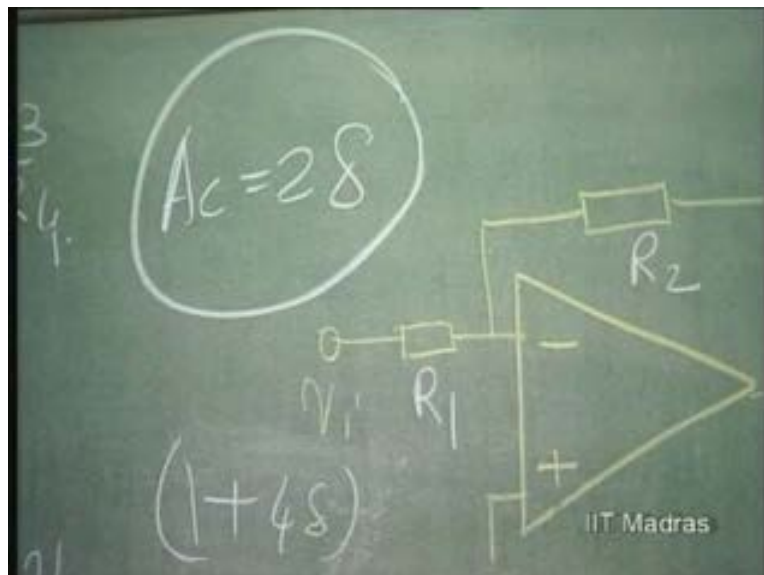


$$V_0 = -V_c \cdot \frac{4s}{2} = -V_c \cdot 2s$$

$$A_c = 2s$$

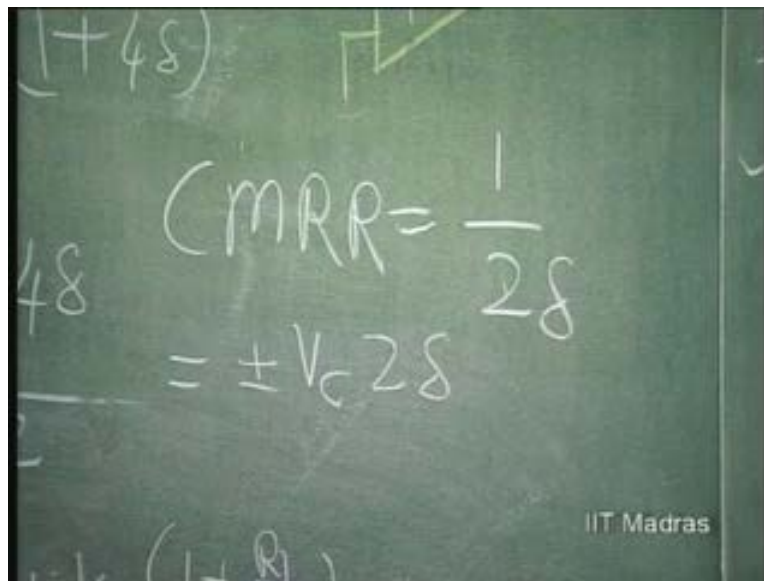
So, the gain A C... V naught is equal to plus or minus V c into 2 Delta. So, A C is equal to 2 Delta.

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This is important in case of information which is needed by any designer of an instrumentation amplifier. If the instrumentation amplifier is designed using resistance of tolerance Δ , the common mode gain is 2Δ . Worst case, that is. The best case value is zero. So, the worst case value is 2Δ . That means common mode rejection ratio for this simple difference amplifier with gain equal to 1 is $1/2\Delta$.

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The image shows a chalkboard with the following handwritten text:

$$CMRR = \frac{1}{2\Delta}$$

$$= \pm \frac{V_c}{2\Delta}$$

There are also some other faint markings on the board, including $(1+4\Delta)$ and 4Δ on the left side, and $(1+R)$ at the bottom. The IIT Madras logo is visible in the bottom right corner.

Consider ordinary resistances of 5 percent or so. Δ is 5 by 100. So, common mode rejection ratio is going to be $1/2\Delta$ into 5 by 100. That is only 10. Its ability to distinguish between common mode voltage and differential voltage is only a factor of 10, which is a very poor value and therefore you have to use precision resistors here in order to get good common mode rejection ratio.

So, we are now confronted with a very serious problem in instrumentation amplifier design. That, if I have to use actual resistors here and come out with huge number of instrumentation amplifiers... Now, please be careful. If I adjust these resistances in such a manner that this ratio becomes satisfied, then I can always make A_C equal to zero.

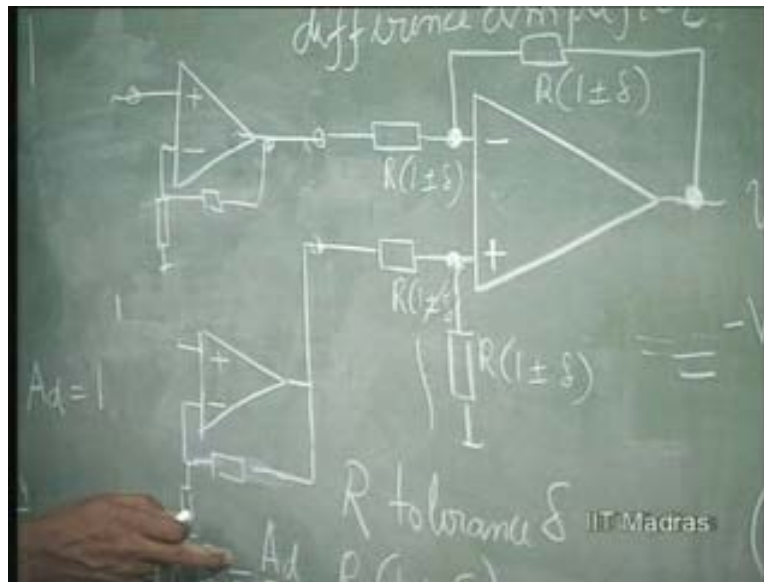
That means you can always make an instrumentation amplifier with infinite common mode rejection ratio; but practically, what happens is, we cannot keep on adjusting these resistors for each instrumentation amplifier because for the test alone you will have to spend lot of money. So, the price for the instrumentation amplifier will go up. When the mass manufacture of a unit is being made, you would like to simply connect this and expect it to work satisfactorily.

In such a situation, tolerance of resistance becomes important. The resistances used must have good tolerance so that all of these units that are made will have good C M R R. And the units can be sourced straightaway without being tested. Then, they can be sold cheaply.

So, in order to make the instrumentation amplifier be sold at low rates, we must use off the shelf components without further testing the instrumentation amplifier. In such a situation, we must accept certain tolerance for this resistance. So, how to improve the common mode rejection ratio? This is important. How to use co-tolerance resistors here and still get better common mode rejection ratio is one thing; and how to buffer - that we have already discussed - is going to be the topic for the next class.

Therefore, we will now convert this into, let us say, instead of buffer, we will have non-inverting amplifier with the same gain. If you do this, V_a is going to be amplified; V_b also is going to be amplified by the same factor hopefully, because same factor is important. Otherwise, again, this will contribute to common mode.

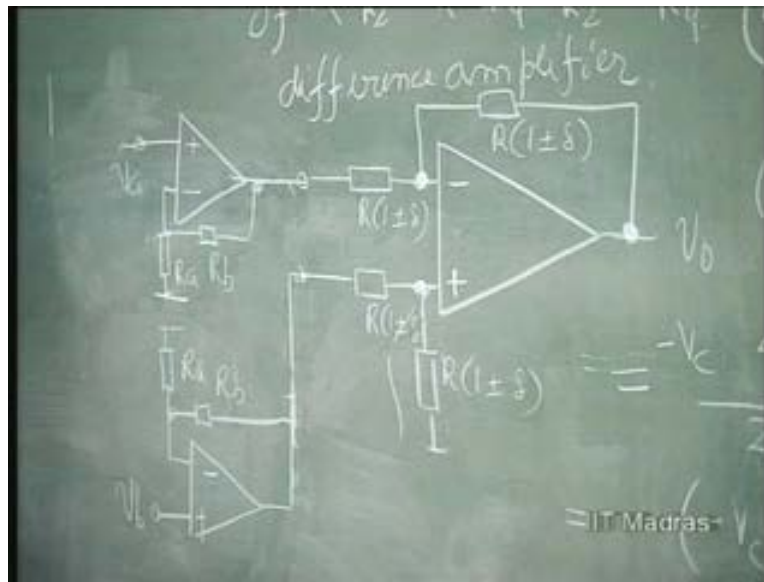
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So, if these are amplifying by the same factor, V_a and V_b , by the same factor, then overall gain can be attributed by these gain factors. This can be R/R ; but this is not going to solve our $CMRR$. $CMRR$ remains as bad as ever. In fact, it gets worsened because these tolerances also come into picture.

I have redrawn the circuit; same circuit, with these resistances put on this side as same as this side; but both of these amplifiers are now giving a gain of $1 + R_b/R_a$, if these resistances are identical.

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Now, there is only one simple trick in order to get over the problem of tolerance, but still give gain. So, this grounding is responsible for the current in this to be made V_a by R_a , current in this to be made V_b by R_a . The current should be same V_b by...that is determined by the same resistance; but these resistance values, if they differ, the currents will be determined by different value resistors.

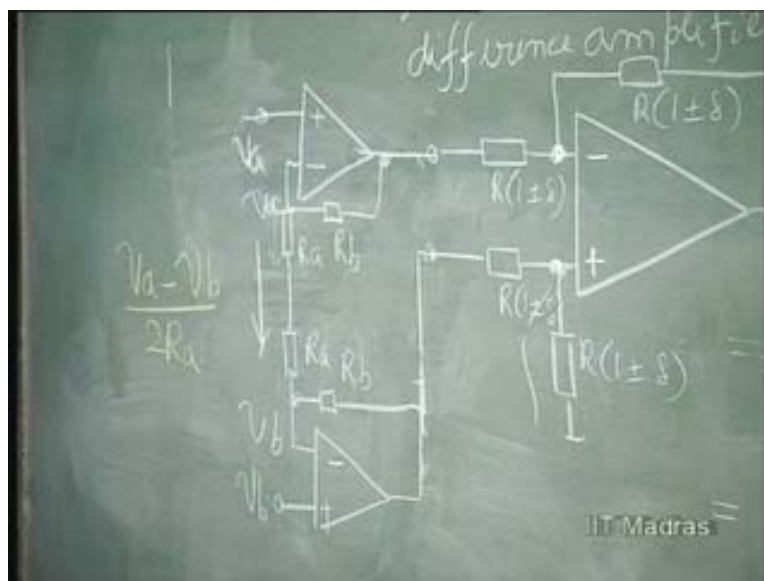
That can be made same by simply lifting this above ground. Look at this. Both these resistances are lifted above ground and connected together. Now, it simply loses the common mode problem.

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Only the difference mode voltage will have any say in fixing the current in R_a . Let us see how it is. If this is V_a , this is V_a . If this is V_b , this is V_b . That we know. Nullor. So, the current in this is now uniquely determined by V_a minus V_b divided by twice R_a .

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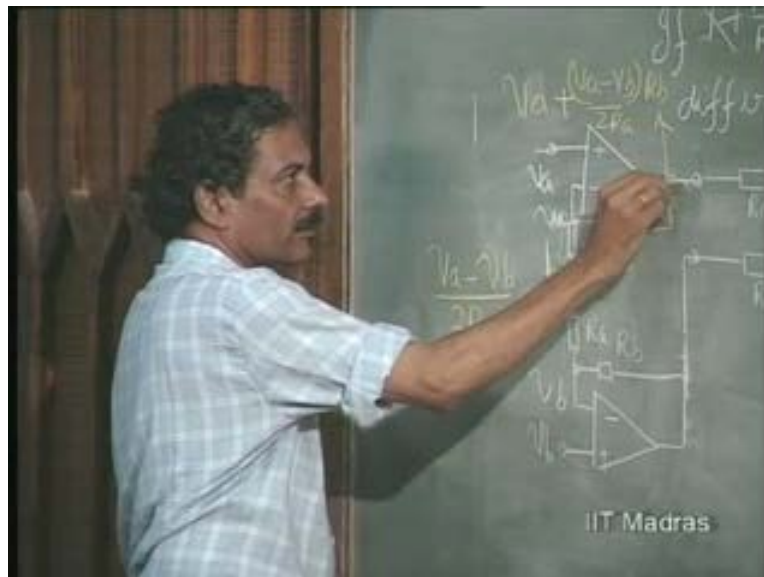


So, the difference is simply taken because of the fact that I have removed the ground connection here. If I had had the ground connection, this current would have been V_a by R_a . This current would have been V_b by R_a and those R_a 's would have been different. Here, this resistance determines V_a minus V_b by twice R_a .

So, the difference is automatically taken here without the influence of any resistor coming into picture. So, I can make this R_a as small as I please and make this difference current as high as I please; and therefore make... I can make the differential mode gain alone very high. Have you understood this point? V_a minus V_b is automatically the difference voltage coming across a common resistance of $2 R_a$.

So, V_a minus V_b by $2 R_a$ can be made as high as you please; but, when V_a is equal to V_b , current is still equal to zero. So, it does not amplify the common mode voltage. It amplifies only the differential mode voltage. This is the trick in the so called three op amp instrumentation amplifier. So, if this current is this, this current will flow through this and develop a voltage here which is V_a . This is V_a . This will be plus; this will be plus; so, plus V_a minus V_b by $2 R_a$ into R_b ; that is the potential here.

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And this potential here, this current is also going to flow through this like this, develop a drop here. So, V_b which is this, minus... plus, minus; so, minus V_a minus V_b [Noise] divided by twice R_a into R_b .

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