

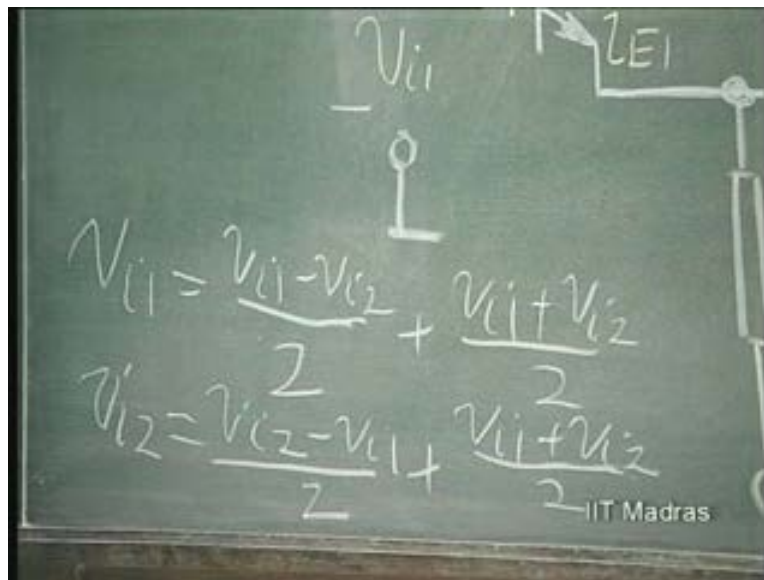
Electronics for Analog Signal Processing - I
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Lecture - 32
Differential Amplifiers

In the last class, we saw how this differential amplifier came into being, a sort of story; and then we also understood how the signal V_{i1} , V_{i2} , general differential signal, can be split into two components.

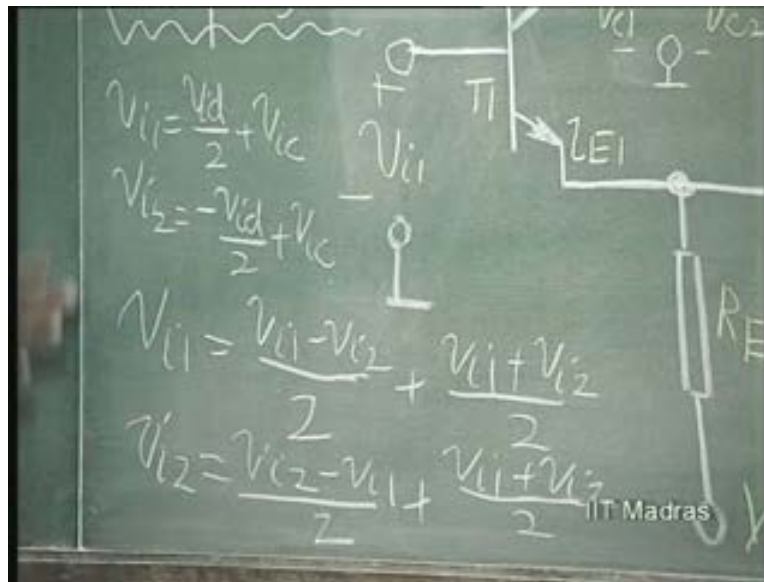
One - the differential component divided by 2 plus the common component; another - the same differential component by 2, but with a negative sign, plus the common component.

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So, what happens is V_{i1} is, strictly speaking, equal to $V_{\text{differential}}$ divided by 2 plus V_{common} and V_{i2} is minus $V_{\text{differential}}$ divided by 2 and V_{common} .

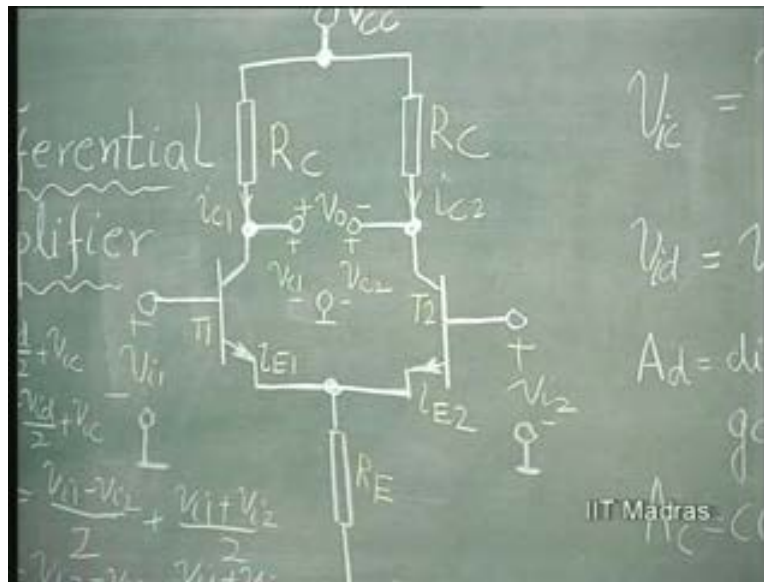
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So, any two general signals can be treated this way. That means for any two signals, general signal, there is a common component and there is a differential component. So, we can analyze the differential amplifier for the common component first; this should not be amplified at all; and then, the differential component which should be amplified; and then superimpose; one output over the other, and get the total output.

Yesterday, we did the analysis by considering V_{i1} is equal to V_{ic} , V_{i2} equal to V_{ic} . Then, this becomes symmetric and then we saw it can be split into two separate circuits, each containing $2R_E$, $2R_E$, in series with emitter. And then we concluded that the common mode gain is minus R_C divided by small r_e plus $2R_E$; and you can make it very low by making R_E very large. This was the conclusion.

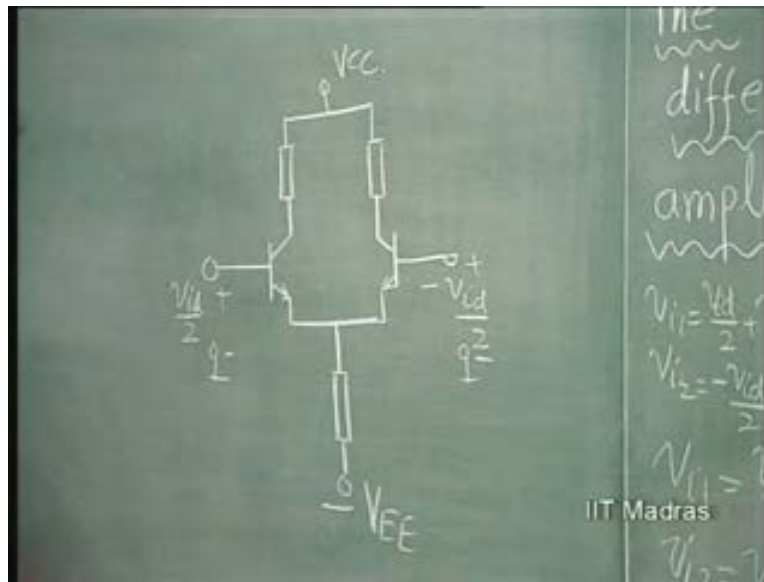
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Today, we will see what happens when we apply only differential signal without any common mode signal. That means we have to apply $V_{id}/2$ and minus $V_{id}/2$ to the two inputs V_{i1} and V_{i2} . So, let us do that. So now, the circuit is going to be considered with only differential mode signal existing and common mode signal being zero.

So, what happens to the circuit? This is an important concept; and therefore, let us see what happens exactly. When we have here V_{i1} equal to $V_{id}/2$; and this one, we have minus $V_{id}/2$. ((Apply it. Refer Slide Time: 04:25)).

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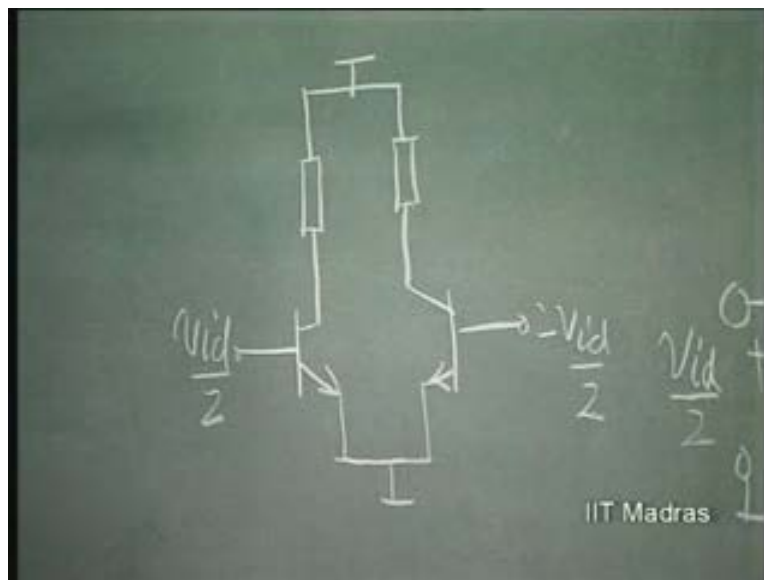
There is no common mode signal at all. Now again, this is a symmetric circuit. As far as this part of it is concerned, it is still symmetric. To one end, you are applying certain positive voltage; to the other end, same value but negative. So, what should happen to a common terminal? You are applying in a symmetric circuit. If you apply a positive voltage, suppose this changes by ΔV , because of only that positive voltage. If you apply a negative voltage to the other end, what will happen? This will decrease by ΔV . So, the effective voltage change here, if you apply positive at one end and negative at the other end to any symmetric circuit corresponding to a common point here, is going to be zero.

That means this will remain at the same state at which it was originally existing without having anything applied at all. That means this will remain at the quiescent state. Is this clear? So, this is the property of symmetric circuit that we are going to use. What does it mean? Actually speaking, therefore, if there is an increase in voltage here, that increase in voltage should be contained within this itself; and the decrease in voltage by the same amount should be contained within this itself; and therefore, nothing happens here. This remains at the same state at which it was existing earlier.

So, this will continue to have whatever current it had before applying the signal. So, there will not be any change in current through this resistance. Is this clear? There will not be any change in current through this resistance. What is this point therefore equivalent to A C wise? This is an A C signal. This is remaining at a constant voltage. So, this is equivalent to ground potential.

So, since this has remained at a constant potential, whatever this varies, this will vary by the same amount in the opposite direction. Then, this will remain at quiescent point; and therefore, this has no variation. That means no variation with respect to ground. That means this is at ground potential, A C wise, signal wise. So, signal wise, picture is going to be simply that we have V_{id} by 2, ground. This is minus V_{id} by 2, ground. This is the signal picture, as far as differential signal is concerned. This potential is going to remain unchanged.

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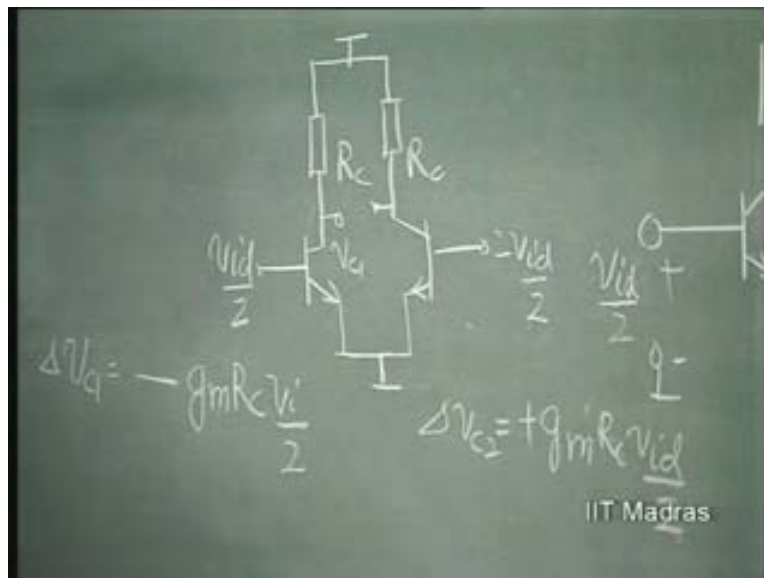
So, once again, let me tell you that in a symmetric structure like this, if I excite one end of this input by some voltage and some change is caused here; whatever be the change, depending on it, the opposite polarity change will be caused by applying opposite voltage by the same amount of magnitude.

So, this change occurring at this point is going to be zero. No change will occur; which means this is going to be imperfect; equal to ground being at ground potential. So, we therefore see that even though I have physically connected a resistance R_E , this R_E has no effect on the differential signal.

This R_E has effect only on common mode signal. So, you now have, as far as differential signal is concerned, $V_{id}/2$ and minus $V_{id}/2$ applied here. This is grounded. So, these are equivalent to two separate common emitter amplifiers. So, you can analyze differential amplifier as two separate common emitter amplifiers and then find out the outputs of the individual amplifiers.

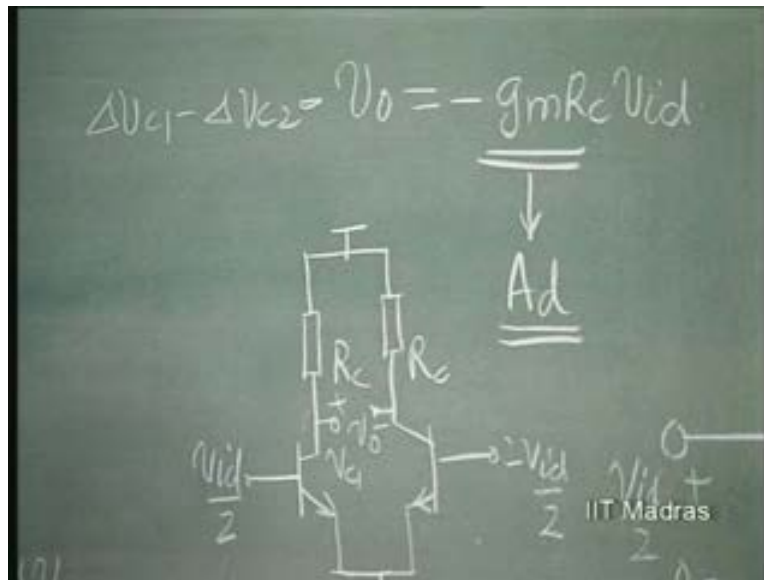
So, we know that the output of this corresponds to minus g_m into R_C into $V_{id}/2$. It is a phase shift of 180 degrees; minus g_m into R_C . And this is minus $V_{id}/2$. So, this output will correspond to...this is V_{C1} , ΔV_{C1} will correspond to this; and ΔV_{C2} will correspond to $V_{id}/2$ into minus g_m into R_C . So plus g_m , v into, R_C into $V_{id}/2$.

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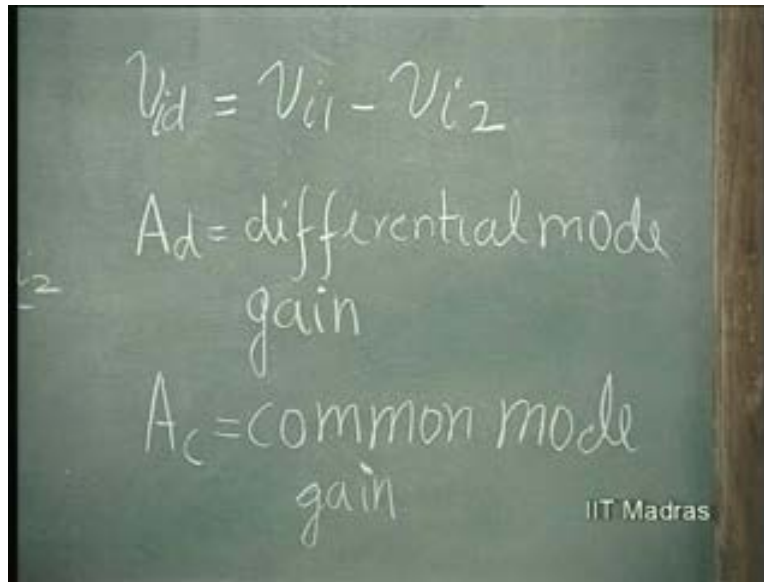
So, you get the same value of output voltage but with a phase shift of 180 degrees; and therefore, if you take $\Delta V_{C1} - \Delta V_{C2} = V_o$, which is nothing but V_{naught} , as marked there, this is V_{naught} , it is going to be how much? - minus $g_m R_C$ into V_{id} . This is what I told you earlier also; that the gain of a differential amplifier, the differential mode gain of the differential amplifier - this is nothing but what is this equal to - A_d , differential mode gain.

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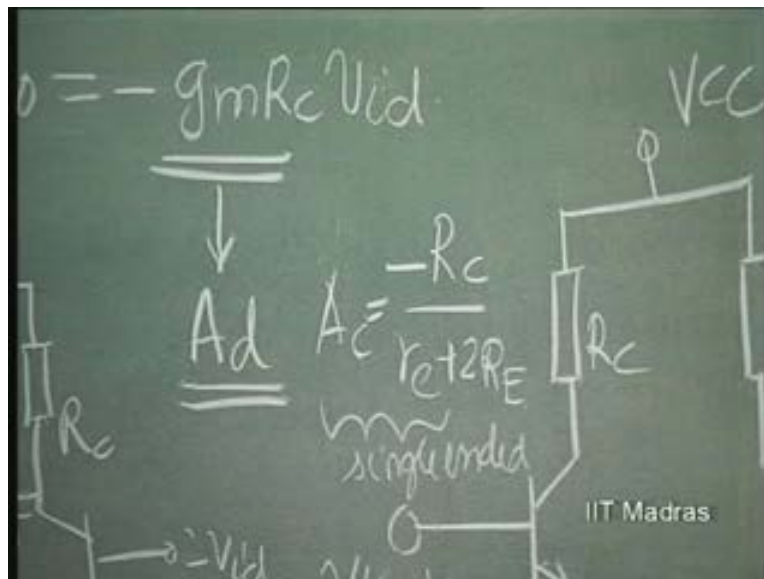
A_d is defined as the differential mode gain. A_c is common mode gain; that we have already evaluated in the last class.

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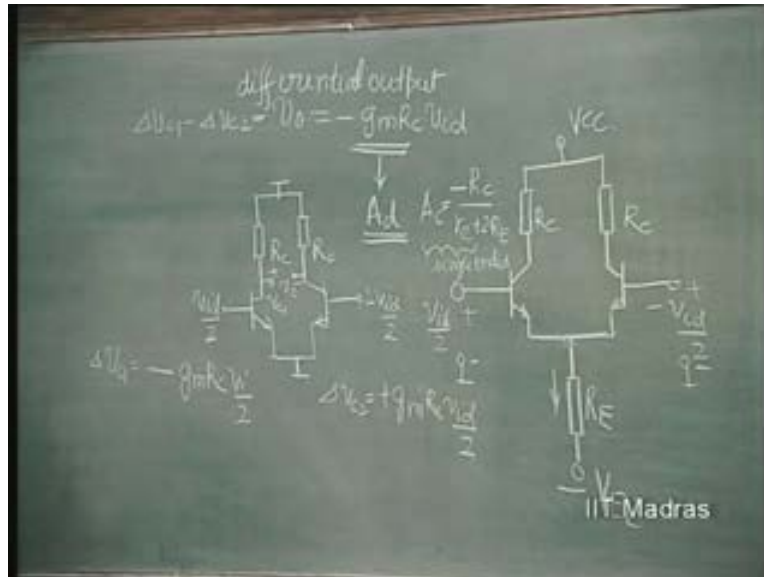
So, A_c is equal to... How much is it? minus R_C divided by r_e plus twice capital R_E . Now, this is for what? - single ended output. Do you remember that? Because differential output, it is still zero, by symmetry. A_c is zero.

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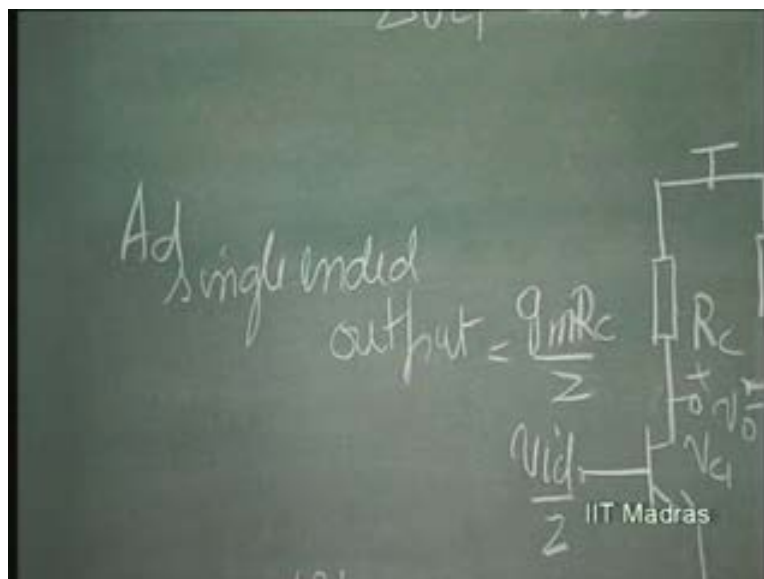
But **but** for single ended output, minus R_C divided by r_e plus $2 R_E$. For differential output... This is called differential output. Therefore, the gain is g_m into R_C .

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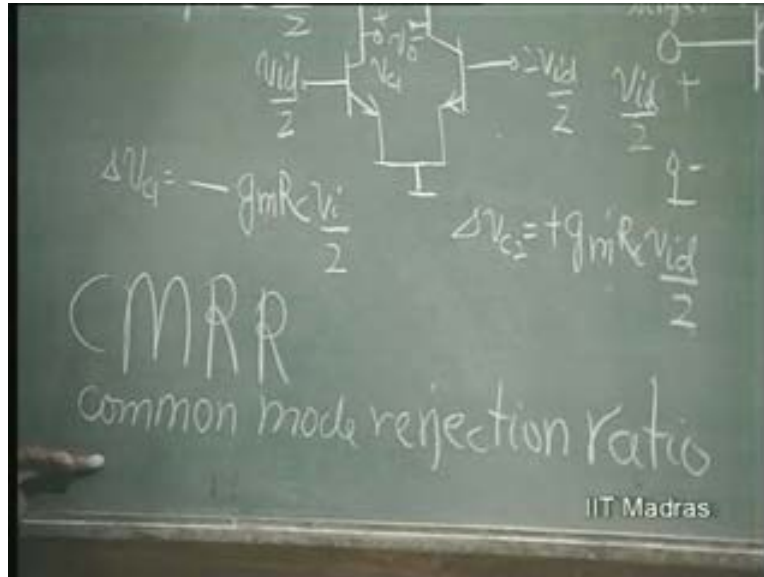
What is the gain for single ended output, A_d for single ended output? That corresponds to $g_m R_C$ by 2, whether it is here or here.

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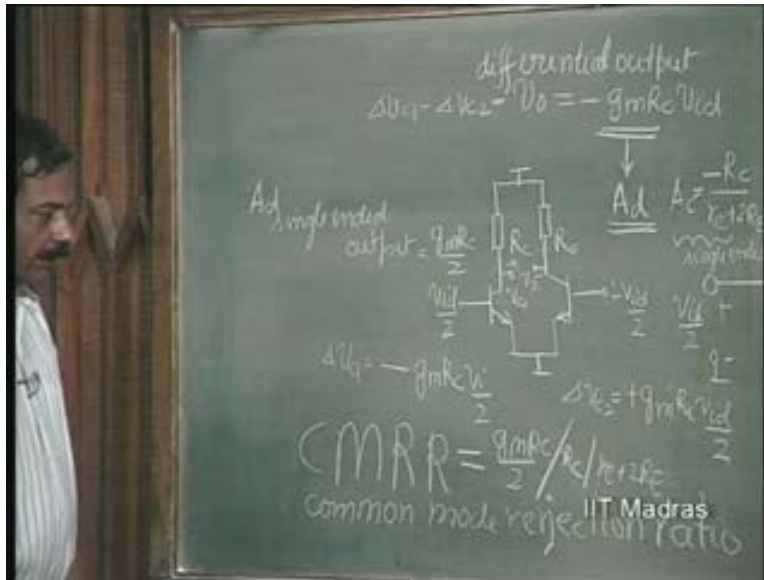
And CMRR, what is this? Common Mode...Common Mode Rejection Ratio. Common Mode Rejection Ratio. CMRR, henceforth; very important parameter associated with the differential amplifier.

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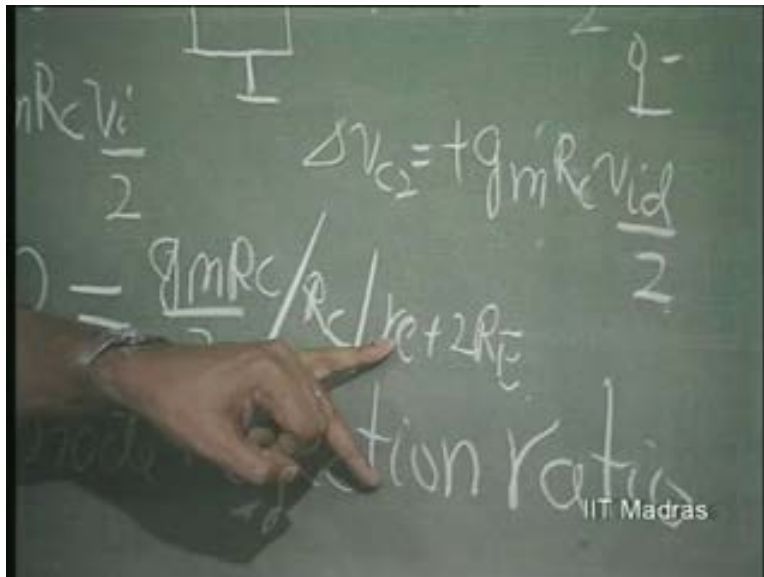
Its ability to, what? - distinguish between common mode and differential mode signal. That is a measure of its ability to, what? - dissociate itself from the common mode signal; and associate itself only with the differential mode signal, and that in this case is, $g_m R_C$ by 2, $g_m R_C$ by 2 divided by R_C slash r_e plus $2 R_E$.

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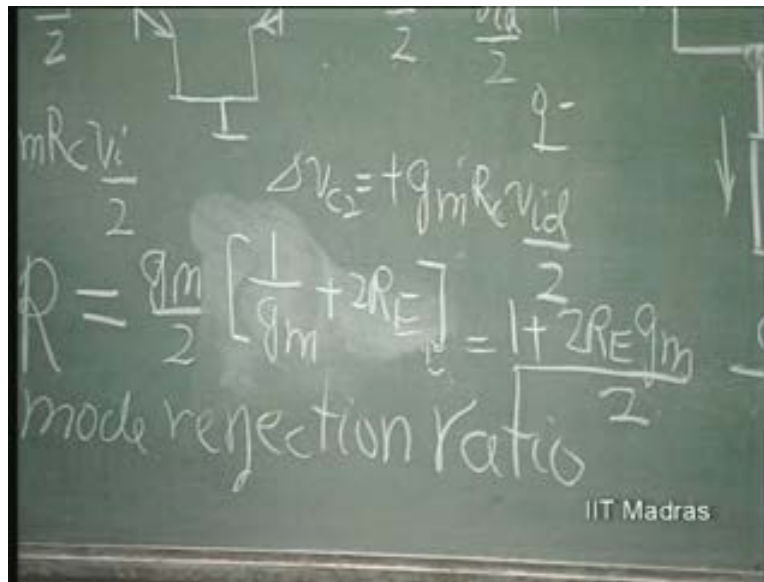
This r_e is equal to $1/g_m$.

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So, you see that it is independent of what? – R_c . It is independent of R_c and it is equal to g_m into...this is equal to how much is it? – g_m into this goes up, $1/g_m + 2R_E$, divided by 2. This therefore normally is going to be equal to $1 + 2R_E/g_m$ divided by 2. This is the common mode rejection ratio of any differential amplifier.

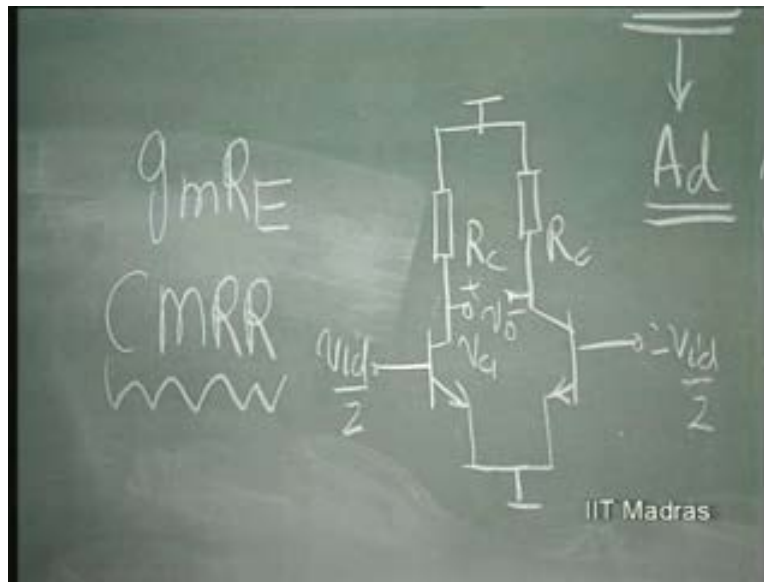
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So, it is an important thing; one you can neglect. So, it is equal to g_m into R_E . If it is a good common...what is it? - a differential amplifier, obviously, you want this ratio to be very high and therefore this one can be neglected. So, it is, strictly speaking, g_m into R_E .

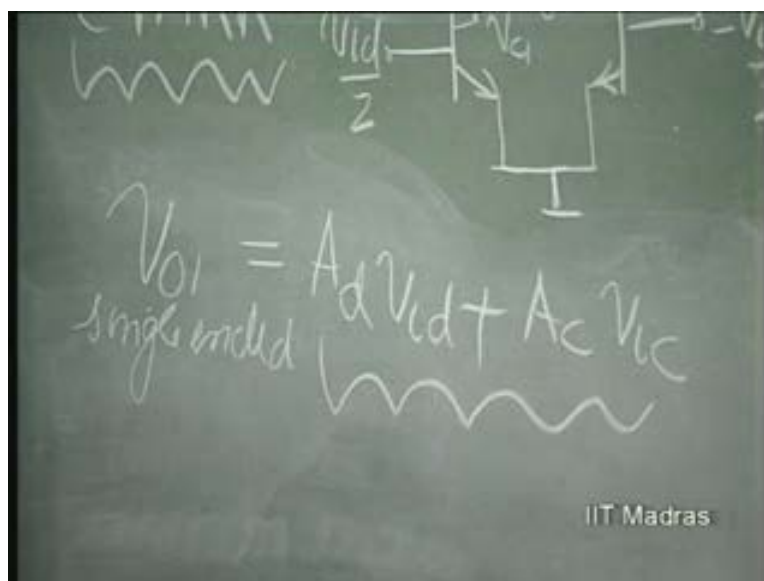
So, please remember this. The CMRR of any differential amplifier which is designed properly will correspond to g_m into R_E ; the gain, differential mode gain, corresponds to g_m into R_C . So, these are the things which we can quickly remember and assess without doing much of calculation, etcetera. So, g_m into R_E is the CMRR.

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So, this is the evaluation of what is called small signal performance of your op amp, of your differential amplifier. What is the effective output? Effective output corresponds to, let us say, single ended, single ended, is equal to the differential mode gain into differential mode signal plus what? - common mode gain into common mode signal. This is the total output, now.

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The effect due to that analysis plus the effect due to this analysis now will be the combination of the effect due to differential output, effect due to the common mode. Or, what we really want in a good differential amplifier is only this. What is it? A_d into V_{id} . The rest of it is the error; 1 plus, this I am taking out. So, this becomes A_c divided by A_d , into V_{ic} divided by V_{id} . This is the error due to finite CMRR because A_d over A_c is defined as CMRR.

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The image shows a chalkboard with handwritten mathematical equations. At the top, there is a small schematic diagram of a differential amplifier with two inputs and two outputs. Below the diagram, the following equations are written:

$$v_{\text{out}} = A_d v_{id} + A_c v_{ic}$$

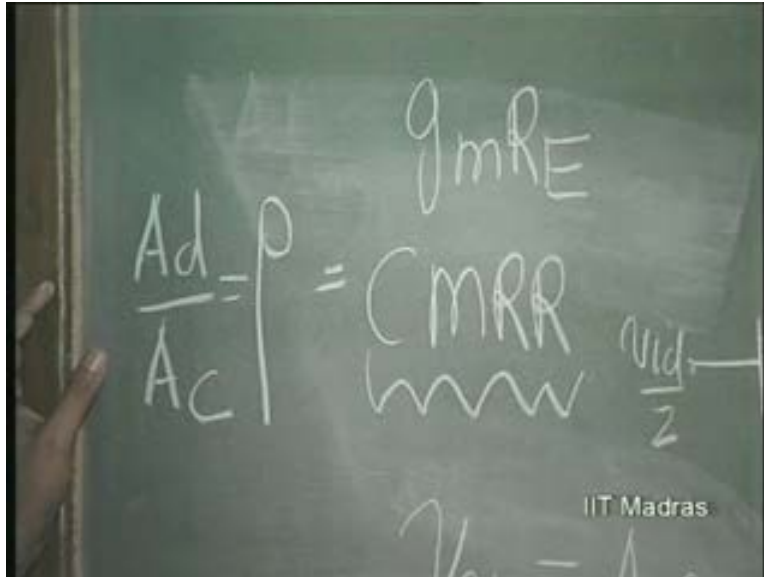
The second term, $A_c v_{ic}$, is underlined with a wavy line. An arrow points from this term to the right. Below this, the equation is rearranged to show the common-mode term as a correction factor to the differential gain:

$$= A_d v_{id} \left[1 + \frac{A_c}{A_d} \times \frac{v_{ic}}{v_{id}} \right]$$

The text "IIT Madras" is visible in the bottom right corner of the chalkboard image.

This is normally assigned this symbol rho. So, A_d over A_c . This is nothing but A_d over A_c .

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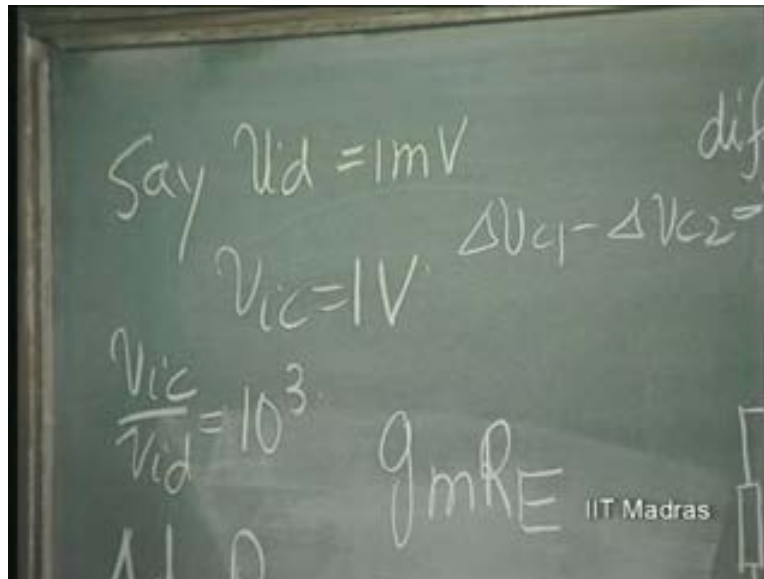
Therefore, we see that this error is 1 over rho. Now you can see; the significance of this differential amplifier analysis lies in estimating this error.

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Let us assume that V_{id} is of the order of 1 millivolt. I have differential voltage of the order 1 millivolt superimposed over a common mode voltage of 1 volt. So, this is 1 millivolt. This is 1 volt. That means this factor, V_{ic} by V_{id} is already of the order of 10 to power 3. So, as an example, V_{id} is 1 millivolt, V_{ic} is 1 volt. So, V_{ic} by V_{id} is equal to 10 to power 3.

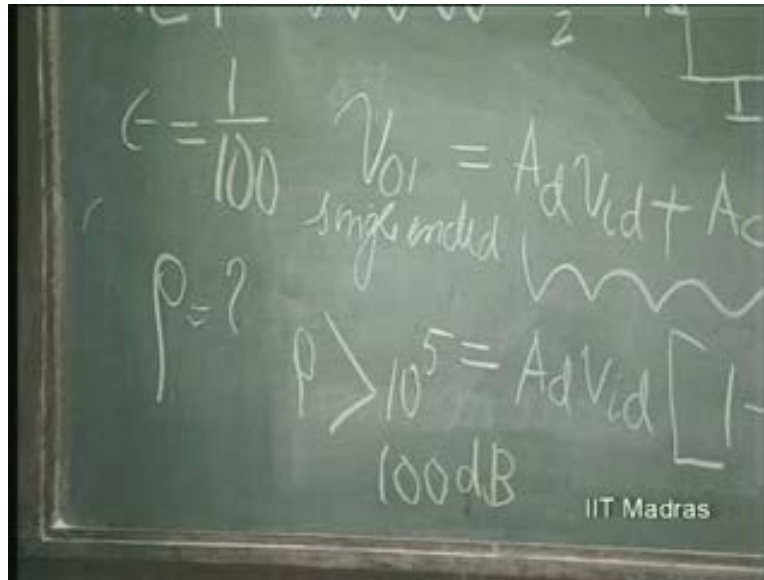
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And I say that the error should be less than 1 percent for my differential amplifier. So, error epsilon should be equal to 1 over 100. Error should be less than 1 percent, I say. Then, epsilon should be 1 over 100. So, what should be the rho in order that error is 1 percent for a signal which is going to be worst case situation only, when the differential signal is 1 millivolt, the common mode signal is 1 volt.

How much is that now? So, what should be the rho? Rho should be greater than 10 to power 5; or, in terms of decibels, how many decibels is it? It should be greater than 20 log 10 to power 5, which is 100 decibels. Is this clear? 20 log 10 to power 5; or, it should be greater than 100 decibels.

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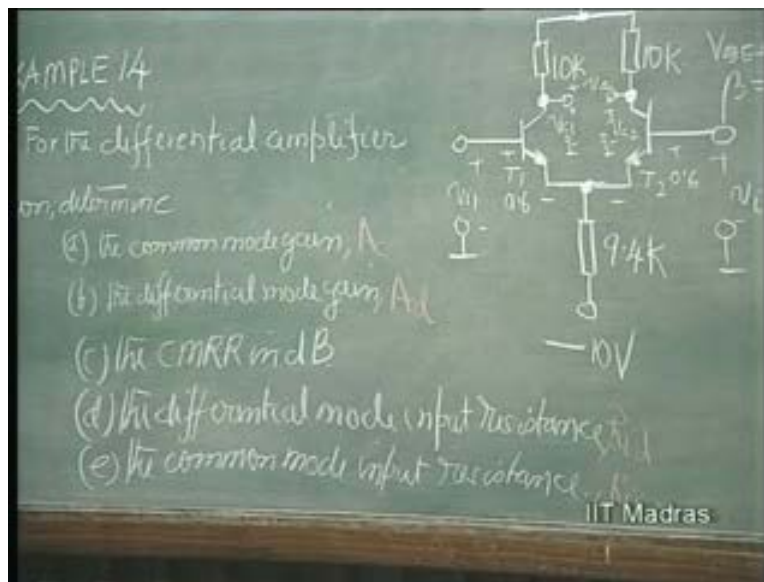


So, if you are told, design a differential amplifier with CMRR greater than 100 decibels, then you know, that is, how to go about designing it; and what kind of signal it can handle also you know, what kind of error will occur depending upon the signal.

So, please remember that you should always consider this effect. For example, a practical, commonly encountered problem is your problem of measuring differential voltage in the case of cathode ray oscilloscope. If you want to make measurement, then, if you are making measurements of the order of 1 millivolts or so, superimposed over 1 volt, which is a typical situation in the case of cathode ray oscilloscope, what kind of common mode rejection ratio your cathode ray oscilloscope differential amplifier which is placed inside should have, in order that error is less than 1 percent, is easily computed from this kind of discussion.

In order to see whether we have assimilated what has been discussed so far, let us see this Example 14. For the differential amplifier shown as indicated in the figure, determine the common mode gain which we have been calling as A_c , the differential mode gain which we have been calling as A_d , the common mode rejection ratio in decibels, the differential mode input resistance, the common mode input resistance. We will call these things as $R_{i\text{ differential}}$ and $R_{i\text{ common}}$. So, this is something that we have not yet studied, but we will see that.

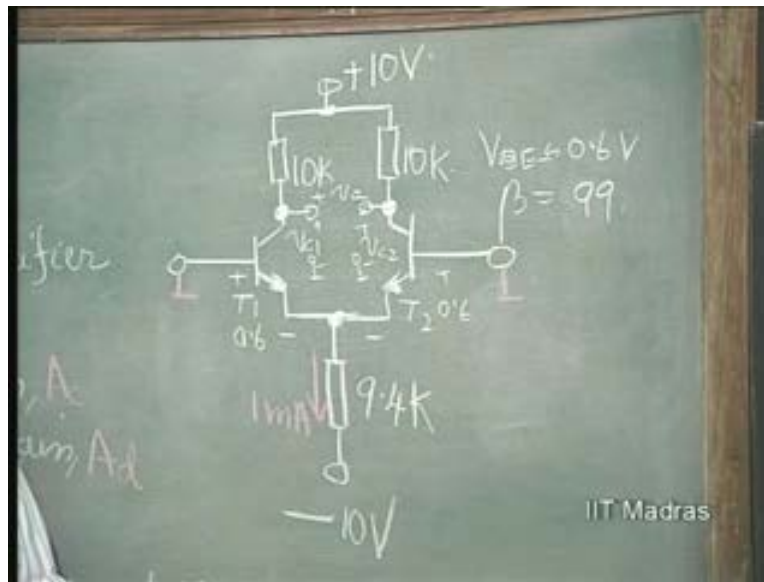
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We know the technique of splitting this pair into 2 separate circuits; either for common mode signal or for differential mode signal. And then we know how to evaluate input impedance, etcetera. So, the same technique is therefore adopted now.

So, let us now consider this circuit. This is grounded; let us say, this is grounded, quiescent wise; and this is point 6 volts; and this is minus 10 volts. So, the drop across 9 point 4 K is 9 point 4 volts; and therefore, the current in this is 1 milliampere. Is this clear?

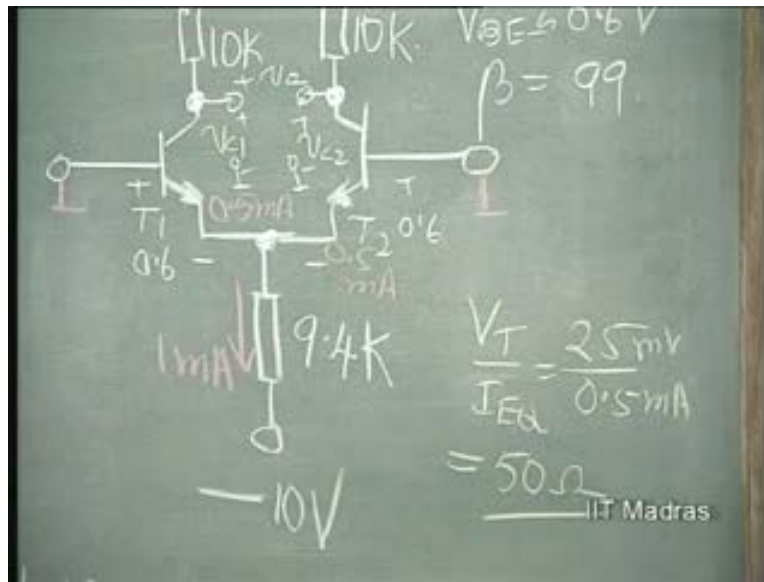
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So, this is point 6; this is minus 10 volts; the drop across 9 point 4 K is 9 point 4 volts; and therefore, the current in this is 1 milliampere. And therefore, currents in these two transistors will be, because it is symmetric, point 5 milliamperes each. This we have understood; that the quiescent current of each of these transistors will be point 5 milliamperes. So, the r_e for the transistors will be, instead of 26, henceforth we will use 25, for computational ease.

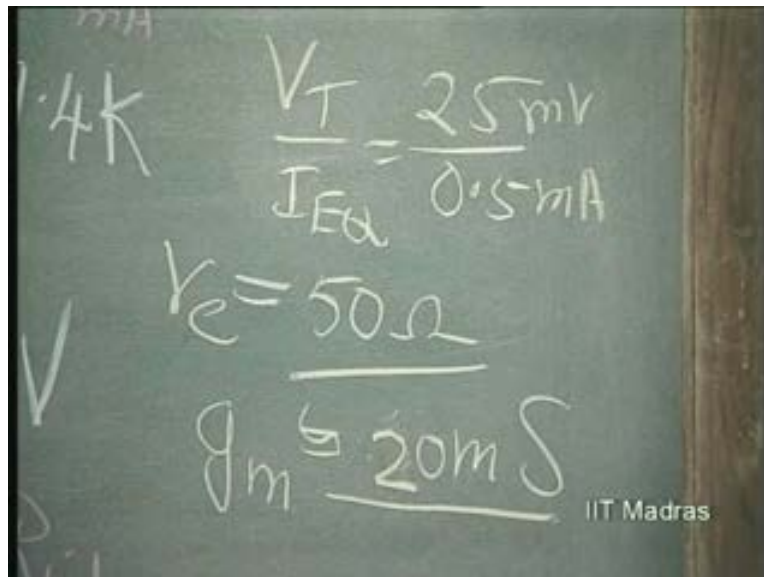
So, 25 divided by point 5 is equal to r_e . So, V_T by I_{EQ} ; we will take this as 25 divided by point 5 milliamperes; 25 millivolts divided by point 5 milliamperes, which is 50 ohms.

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This is equal to r_e of... So, g_m is roughly equal to $1/r_e$ which is equal to, approximately equal to, 40 millisiemens; or sorry, it is equal to 20 millisiemens, because it is 50 ohms; for 1 milliampere, it is 40; for point 5 milliamperes, it is 20.

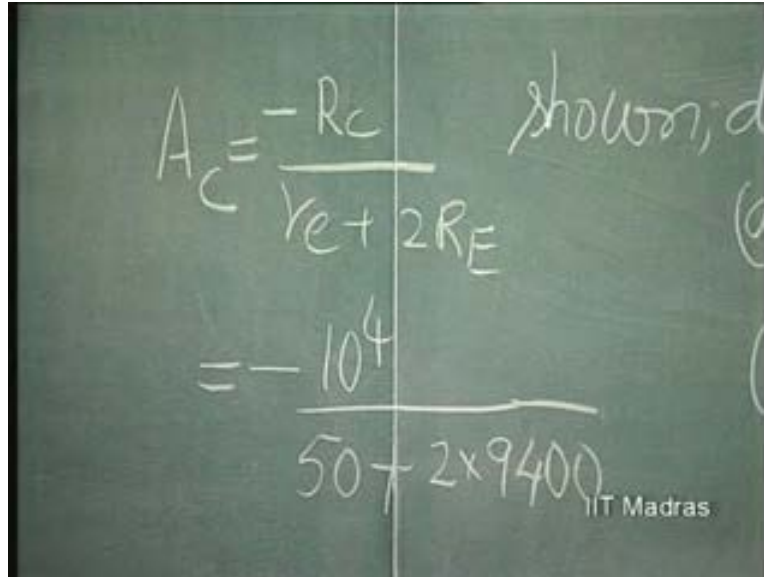
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Now that we know g_m , we know r_e , we are in business. So, let us first find out the common mode gain. What is the common mode gain? We have seen that common mode

gain A_c equals minus R_C divided by r_e plus $2 R_E$. In this case, it is... How much is it?
 10 K divided by r_e plus 2 times R_E , 2 into 9400 .

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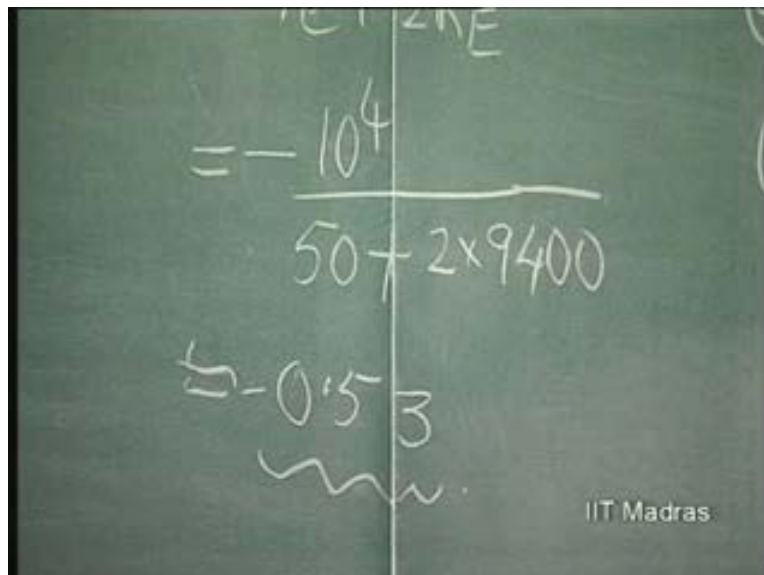
The image shows a chalkboard with the following handwritten text:

$$A_c = \frac{-R_C}{r_e + 2R_E}$$
$$= \frac{-10^4}{50 + 2 \times 9400}$$

There is a faint "shown, d" written to the right of the first equation. The IIT Madras logo is visible in the bottom right corner.

So, strictly speaking, this is roughly equal to half; point 5... How much is it exactly? point 53. That is the common mode gain, minus of course; the sign indicating there is an inversion. So, this is point 53.

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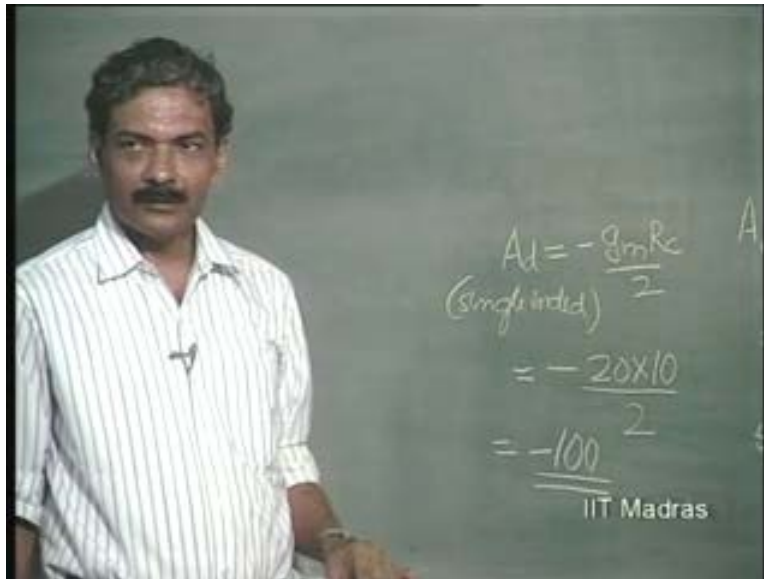
The image shows a chalkboard with the following handwritten text:

$$= \frac{-10^4}{50 + 2 \times 9400}$$
$$\approx -0.53$$

The result -0.53 is underlined with a wavy line. The IIT Madras logo is visible in the bottom right corner.

The differential mode gain is equal to...now this depends, whether it is single ended or differential. Since nothing is asked, we will give the single ended output. So, single ended. So, minus g_m into R_C by 2 is equal to minus 20 millisiemens into 10 K divided by 2, which is going to be minus 100. So, the common mode gain is minus 100.

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Then, common mode rejection ratio equals A_d by A_c , which is 100 by point 53, which is around 200. How much is it? 1, 188, which is a pretty low common mode rejection ratio.

In terms of decibels, it is 20 log to the base 10 of 188. So, please find out this; log to the base 10 of 188, which is going to be 45 point 5 decibels.

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$$A_c \text{ (single ended)} = \frac{-100}{0.53} = -188$$
$$20 \log_{10} 188 = 45.5 \text{ dB}$$

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Then, we would like to find out the differential mode input resistance, $R_{i d}$. In the differential mode operation, we said, emitter is grounded. So, emitter is grounded; we have concluded. Therefore, these are 2 different common emitter amplifiers; and therefore, common emitter amplifier input impedance is r_e into Beta plus 1 because you are looking at the input from the base; so, which is 100 into 50, which is 5 K.

This...this is 100, actually, let us put it this way - this is 50 into 100 which is 5 K. This is the case for one input, $V_{i d}$ by 2.

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Handwritten equations on a chalkboard:

$$R_{id} = r_e(\beta + 1)$$
$$= 50 \times 100$$
$$= \underline{5k}$$

Below the equations, the text "A_C = -R_C" is written, with "IIT Madras" visible in the bottom right corner.

But, we would like to have it for...this is for V_{id} by 2. V_{id} by 2, the input impedance is 5 K. For V_{id} , how much is input impedance? So, V_{id} therefore, the input impedance is 2 times 5 K. V_{id} by 2 - the input impedance is 5 K. What we want is...differential signal is V_{id} ; not V_{id} by 2. So, this is 10 K, ...common emitter.

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Handwritten equations on a chalkboard:

for $\frac{V_{id}}{2}$ it is eq. C.E.

$$R_{id} = r_e(\beta + 1)$$
$$= 50 \times 100$$
$$= \underline{5k}$$
$$V_{id} = 2 \times 5k$$
$$= \underline{10k}$$
$$A_d = -\frac{g_m R_c}{2}$$
$$A_c = \frac{-R_c}{r_e + 2R_E}$$

The text "IIT Madras" is visible in the bottom right corner.

Next, for V_{ic} , it is again two individual stages with r_e plus $2R_E$ in the emitter lead. So, input impedance is going to be... How much is it? r_e plus $2R_E$ into $\beta + 1$. This is for V_{ic} . Is this clear? That means one amplifier is going to feel this much input impedance; another amplifier is also going to feel the same input impedance.

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for V_{ic}

$$R_{ic} = (r_e + 2R_E)(\beta + 1)$$

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Since V_{ic} is supplying to two amplifiers, what will be the effect of input impedance? One amplifier is seeing this much input impedance; another amplifier which is connected to the same voltage is also seeing the same impedance. Now, as far as V_{ic} is concerned, there are two such impedances connected to the same amplifier. So, what is the effective impedance seen? This impedance divided by 2. Is this clear?

So, effective R_{ic} is going to be r_e plus $2R_E$ into $\beta + 1$ divided by 2. Normally, this r_e is very small compared to $2R_E$. Therefore, effective, this is R_E into $\beta + 1$.

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$$R_{ic} = (r_e + 2R_E)(\beta + 1)$$
$$R_{ic} = \frac{(r_e + 2R_E)(\beta + 1)}{2}$$
$$s R_E(\beta + 1) CMRR = \frac{A_d}{A_c}$$

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Once again, let us go through this. V_{id} by 2 was seeing an impedance of r_e into $\beta + 1$. Then, how much V_{id} we will see? $2R_E$ into $\beta + 1$. Here, V_{ic} was seeing two such impedances connected in parallel.

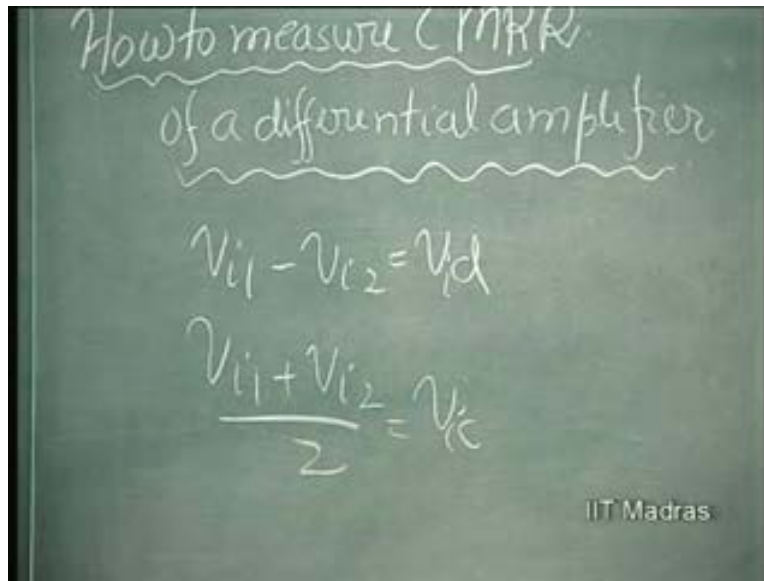
So, how much V_{ic} we will effectively see? This impedance divided by 2. And therefore, this is...this therefore, it has a common mode impedance component as well as differential mode impedance component. The differential mode impedance components even though might be low; the common mode impedance component should be very high.

Strictly speaking, therefore, common mode rejection ratio also is estimated by simply taking the ratio of the impedances. This R_E divided by this impedance is going to give us the common mode rejection ratio. This impedance is $2r_e$ into $\beta + 1$. This is r_e into $\beta + 1$. So, it is r_e by $2R_E$. Again, it is going to be your g_m into R_C by 2; g_m into R_E by 2.

So, this factor also gives you a measure of what? - your common mode rejection ratio. Is this clear? So, this completes the problem.

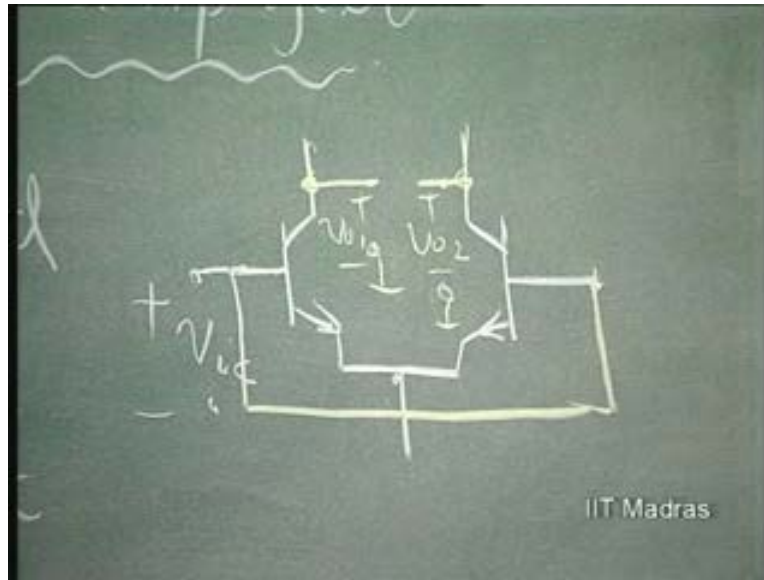
Now, let us see how in a laboratory, we can measure different common mode rejection ratio of a differential amplifier. Now, why I am telling this is, we actually came up with an idea of splitting any two given signals V_{i1} and V_{i2} as differential mode signal and common mode signal.

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So, if I have to apply a common mode signal to a differential amplifier, it is easy. I will not complete the thing. We will say we are applying V_{ic} ; and V_{ic} , what does it mean? The same voltage is applied. So, this is what is done. The same voltage is applied to both the bases and the output is measured; V_{o1} and V_{o2} .

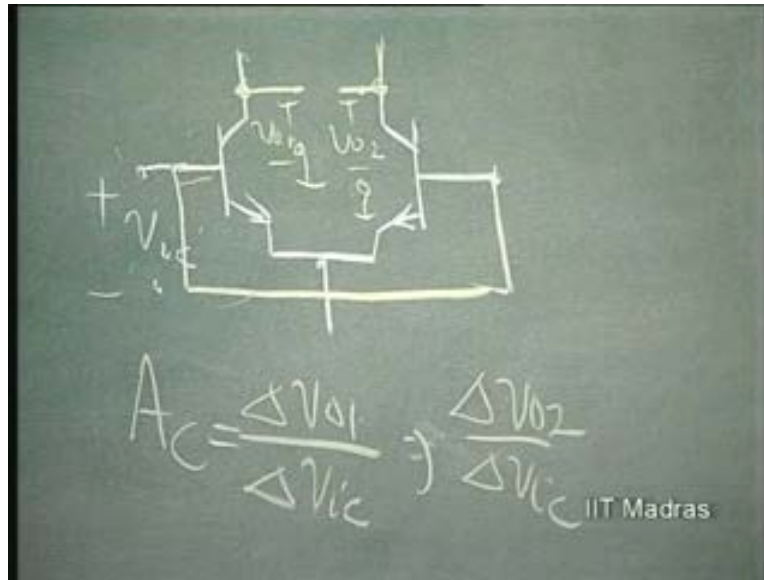
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So, same output voltage is, I mean, same input is applied to both the bases. The both the bases are connected together and same voltage is applied. That is why we actually had a current which is this current plus this current. So, this voltage, $V_{i c}$ divided by these two currents which are one and the same, resulted in half the effective output, input impedance.

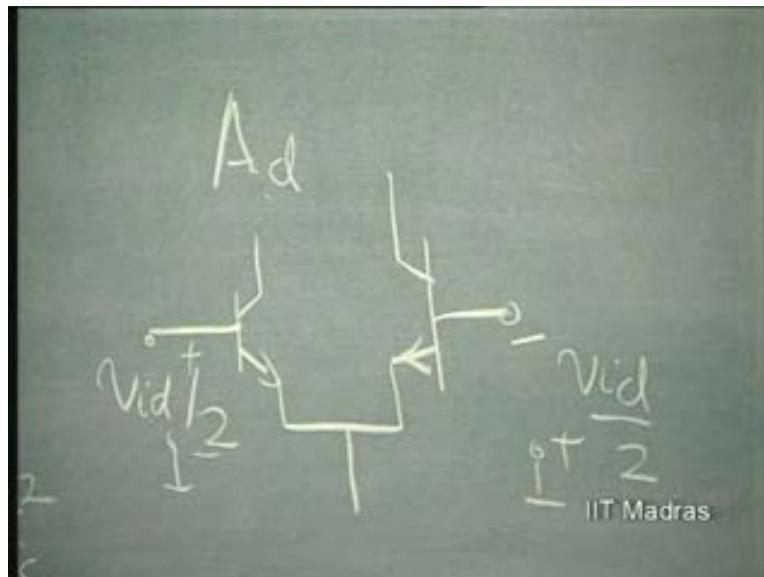
Now, measurement is easy as far as this is concerned. If I want to measure A_c , I measure $V_{o 1}$, change in $V_{o 1}$, $\Delta V_{o 1}$ by R , that is, $\Delta V_{i c}$. Or, this is also going to be the same as $\Delta V_{o 2}$ by $\Delta V_{i c}$. If these are identical; these two should be the same. I can therefore either measure this or this. These two will be exactly same. This is simple enough. There is no problem.

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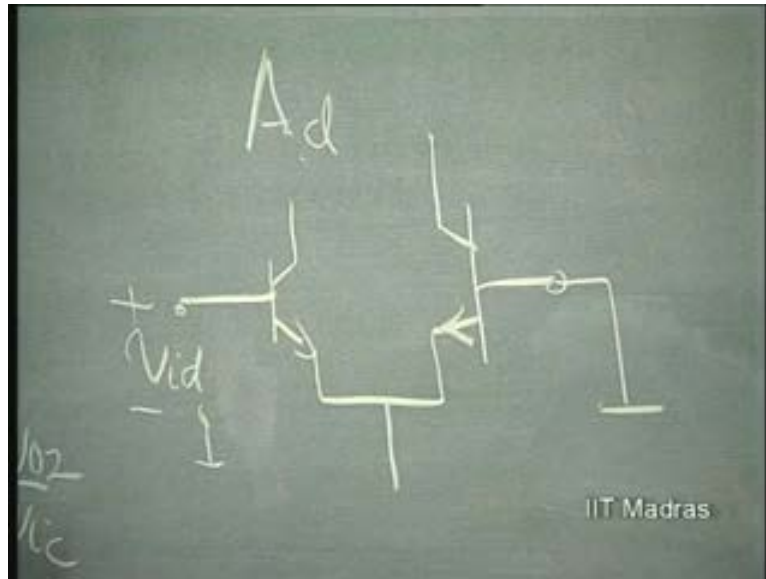
As far as differential mode gain is concerned, I have to have a situation here of this type. This should be v_{id} by 2 and this should be having a phase shift of minus, 180 degree; that is, this should be minus v_{id} by 2. How do I get it?

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It is not very simple. I should use a transformer or generate a phase shift of 180 degree and then earth it. This is not necessary. That is why I put it... This is not necessary. What I do is I apply simply V_{id} to one end and ground this end. This is not strictly differential mode. This voltage is zero and this is V_{id} .

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So, what happens now? The output is A_d times V_{id} plus A_c times V_{ic} . This is actually the output here. This is not zero. V_{ic} is not zero. What is V_{ic} ? V_{ic} is zero plus V_{id} by 2. So, this is going to be A_d into V_{id} plus A_c into V_{id} by 2.

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$$V_o = A_d V_{id} + A_c V_{ic}$$
$$= A_d V_{id} + A_c \frac{V_{id}}{2}$$

A_d

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So, this is what is going to be measured in this. If I measure the output here or here, it will be the same in magnitude, but opposite in sign. So, whether it is this or this, it is going to be the same. So, ΔV_{n1} or ΔV_{n2} , ΔV_{n2} ; magnitude wise will be the same. But this is therefore is equal to A_d into V_{id} which is wanted, into $1 + 1/\rho$. So, the error you are committing is extremely small if the differential amplifier is having good CMRR.

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$$|V_{o1}| = A_d V_{id} + A_c V_{ic}$$
$$= A_d V_{id} + A_c \frac{V_{id}}{2}$$
$$|V_{o2}| = A_d V_{id} + A_c \frac{V_{id}}{2}$$
$$= A_d V_{id} \left[1 + \frac{1}{\rho} \frac{1}{2} \right]$$

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So normally, we are sensible enough to design good CMRR of differential amplifiers and therefore this situation, this particular thing, is always going towards zero; and therefore I can still claim to measure A_d accurately, even though this has a common mode voltage, which is V_{ic} by 2. Is this clear? So, this is the method of measurement without using the phase splitting arrangement, or a transformer, which is good enough for accurately measuring A_d in a good differential amplifier circuit.

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The chalkboard shows the following derivation:

$$v_d + A_c v_{ic}$$

$$= \frac{v_d + A_c v_{ic}}{2}$$

$$= \underline{A_d v_{id}} \left[1 + \frac{1}{\rho} \frac{1}{2} \right]$$

There is a small circuit diagram on the left side of the board showing a differential pair with a common mode input v_{ic} and a differential mode input v_{id} . An arrow points from the text above to the common mode input node. The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

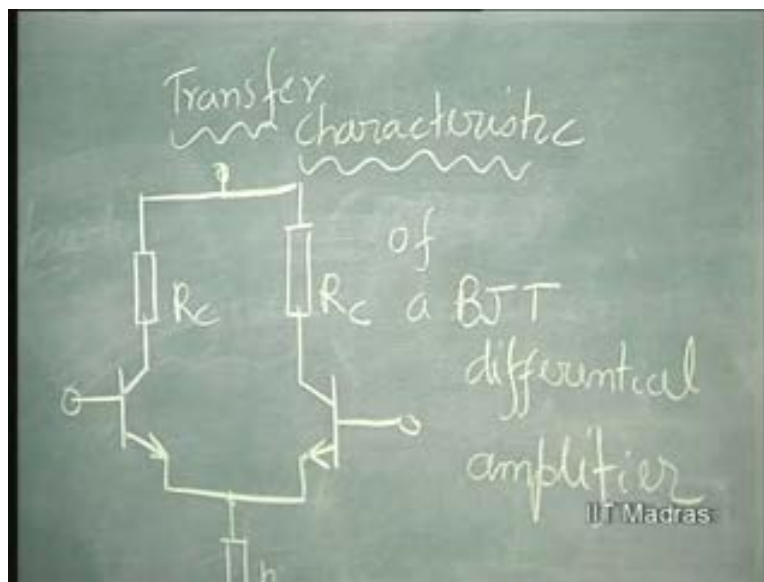
So, this is all I wanted to indicate, that in a good differential amplifier circuit, the input impedance also will correspond to only what? - V_{id} ; that is the differential mode signal. And the common mode signal effect is going to be very low. Therefore, if you apply a single ended input, it is as though it is differential mode. You can consider as though it is a differential mode operation in a good what? - differential amplifier.

This is emphasized in practical situation. You do not have to therefore apply the differential mode signal to a differential amplifier. You can apply input only to one end and still achieve good differential properties.

Now, we are going to consider the differential amplifier, in general, for large signal. I am not going to assume any small signal model. We have assumed the model in the earlier situation; that the common emitter amplifier can be replaced by its model which is V_{BE} , ΔV_{BE} into g_m as the current source at the output.

Now, we are not going to assume such models. Therefore, in general, for a differential amplifier, let us see how output versus input characteristic...that is, this time we would like to know the output versus input characteristic or transfer characteristic of a differential amplifier; how it looks like. This we had seen for common emitter amplifier, etcetera. Now, we would like to see the same thing for differential amplifier; transfer characteristic of a BJT differential amplifier.

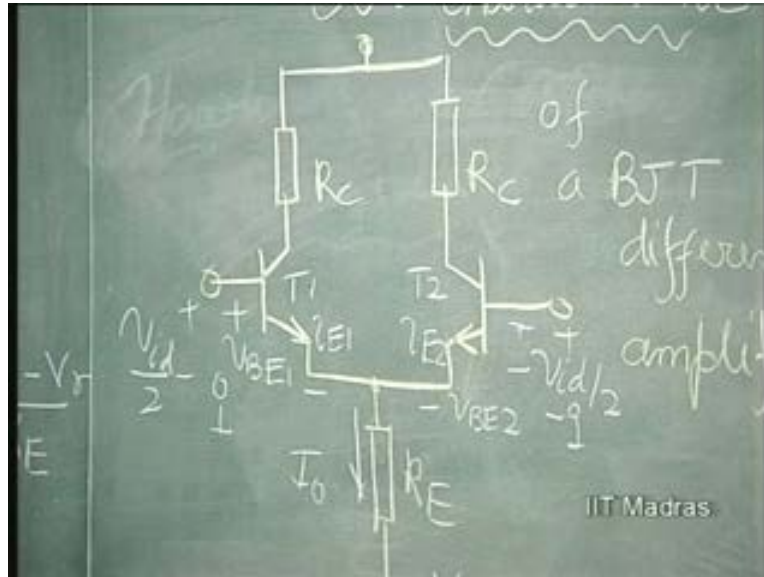
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Now, let us consider the input V_{i1} and V_{i2} . Strictly speaking, we are now considering only the differential mode signals. So, I will put it as $V_{id}/2$ and minus $V_{id}/2$. This is equally well applicable, I told you, when it is V_{id} here and zero there. So, if this is $V_{id}/2$ and this is minus $V_{id}/2$, what happens, I would like to know. The current in this remains constant; that is important here, at let us say, I naught.

What is the value of I_{E1} ? $V_{EE} - V_{BE1}$ divided by R_E . So, this I_{E1} is going to be $V_{EE} - V_{BE1}$ divided by R_E . Now, let us call this I_{E1} , this as I_{E2} , this as V_{BE1} , T_1 , this is T_2 ; and therefore this voltage is V_{BE2} .

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So, we know that V_{id} equals, V_{id} which is, V_{id} , this voltage between these two is V_{id} , equals $V_{BE1} - V_{BE2}$. At any instant of time, V_{BE1} and V_{BE2} being instantaneous voltages across the base emitter junctions of the two transistors.

At any given time, therefore, this is $V_{BE1} - V_{BE2}$, always. There is no assumption made whether the signal is small or large. It does not matter. This is always valid. Clear? So, V_{id} is equal to $V_{BE1} - V_{BE2}$. Next, $i_{E1} + i_{E2}$, instantaneous values of the two emitter currents, will always equal what? $-I_{E}$. This is also always valid; Kirchhoff's law.

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A chalkboard with two equations written in white chalk. The first equation is $V_{id} = V_{BE1} - V_{BE2}$. The second equation is $i_{E1} + i_{E2} = I_0$. In the bottom right corner, the text "IIT Madras" is visible.

Now, we also know that i_{E1} , according to the diode relationship is, I E naught, approximately, exponent V_{BE1} by V_T . That minus 1, I am ignoring; the diode relationship. Then, i_{E2} is approximately equal to what? - I E naught exponent V_{BE2} by V_T . This also we know; at any instant of time, of no approximation involved, except that it is made that minus 1 is ignored.

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A chalkboard with three equations written in white chalk. The first equation is $i_{E1} = I_{E0} \left(e^{\frac{V_{BE1}}{V_T}} - 1 \right)$. The second equation is $i_{E2} = I_{E0} \left(e^{\frac{V_{BE2}}{V_T}} - 1 \right)$. The third equation is $I_0 = \frac{V_{EE} - V_{BE}}{R_E}$. In the bottom right corner, the text "IIT Madras" is visible.

Now therefore, this is an important relationship in integrated circuit. What is it? i_{E1} at any instant of time by i_{E2} for two transistors which are identical, identical in the sense they have the same i_{E} naught, always equal to exponent V_{BE1} minus V_{BE2} by V_T . This is an important relationship; the most important relationship I can ever think of in integrated circuit, because, just because, the two transistors are identical, the ratio of the two currents of the transistors will be an absolute constant, which is independent of the parameters of the transistors; only equal to differential input V_{BE1} minus V_{BE2} divided by V_T , exponential of this there. So, what is V_{BE1} minus V_{BE2} , in this case? It is nothing but V_{id} .

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The image shows a chalkboard with a handwritten equation:
$$\frac{i_{E1}}{i_{E2}} = \exp\left(\frac{V_{BE1} - V_{BE2}}{V_T}\right)$$
 To the right of the equation, the text $V_{id} =$ is written, indicating that the difference in base-emitter voltages is the differential input voltage. The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

So in this case, this relationship is valid, always in general. Let those transistors be anywhere in a circuit. Let those transistors be anywhere in a circuit. The ratio of the two currents of the transistors will always be equal to V_{BE1} , exponent V_{BE1} minus V_{BE2} by V_T . This is a general relationship.

In the case of a differential amplifier, we know that $V_{BE1} - V_{BE2}$ is equal to V_{id} . In that case, it simplifies to exponent V_{id} by V_T .

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The image shows a chalkboard with the following handwritten equations:

$$\frac{i_{E1}}{i_{E2}} = \left(-\frac{v_{id}}{V_T} \right)$$

$$i_{E1} = I_{E0} \left(-\frac{v_{id}}{V_T} \right)$$

$$i_{E2} = I_{E0} \left(-\frac{v_{id}}{V_T} \right)$$

The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

So, in a differential amplifier, this is always valid. The ratio of two currents i_{E1} by i_{E2} which is same as what? $-i_{c1}$ by i_{c2} , because Alpha times, Alpha times i_{E1} by Alpha times i_{E2} will be i_{c1} by i_{c2} is exponent V_{id} by V_T .

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The image shows a chalkboard with the following handwritten equations:

$$\frac{i_{c1}}{i_{c2}} = \left(-\frac{v_{id}}{V_T} \right)$$

$$\frac{i_{E1}}{i_{E2}} = \left(-\frac{v_{id}}{V_T} \right)$$

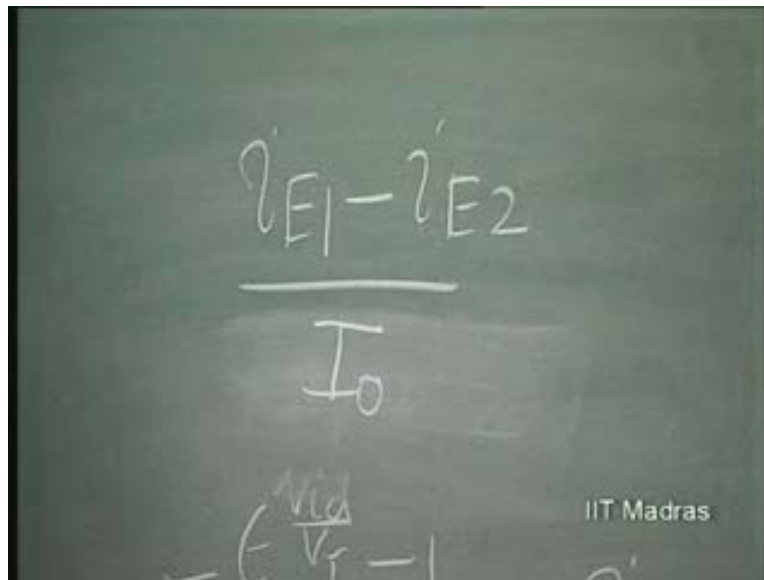
The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

This is valid for several decades of variation of the current. That means i_{E1} by i_{E2} may be a huge value or a very small value, this relationship is valid. There is no other device which satisfies such fantastic relationship up till now in electronics. I would therefore like to emphasize the fact that this is the most important relationship about to occur in electronics.

Now, we will have therefore...if a by b equal to R , a minus b by a plus b is equal to R minus 1 by R plus 1. This is the ratio relationship given. If a by b is equal to a ratio R , a minus b by a plus b is equal to R minus 1 by R plus 1.

So, using that, i_{E1} minus i_{E2} by i_{E1} plus i_{E2} equals exponent V_d by V_T minus 1 by exponent V_d by V_T plus 1. What is i_{E1} plus i_{E2} at all times? It is a constant current, I_0 . So, we will replace that. i_{E1} plus i_{E2} equals I_0 . Beautiful.

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In this equality now, i_{E1} plus i_{E2} , summation of the instantaneous values of these 2 currents, i_{E1} and i_{E2} , will always be equal to I_0 ; and i_{E1} minus i_{E2} is equal to i_{C1} minus i_{C2} by α . And, look at this... V_{C1} minus V_{C2} is V_0 ; and V_0 is equal to V_{C1} minus V_{C2} and V_{C1} is equal to V_{CC} minus $i_{C1} R_C$ and v_c

2 is equal to, again, $V_{CC} - i_{c2} R_C$, which will tell us that, this is equal to $i_{c1} R_C - i_{c2} R_C$.

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$$\frac{i_{E1} - i_{E2}}{i_{E1} + i_{E2}} = \frac{\frac{v_{id}}{V_T} - 1}{\frac{v_{id}}{V_T} + 1}$$

$$i_{E1} + i_{E2} = I_0$$

$$i_{E1} - i_{E2} = \frac{i_{C1} - i_{C2}}{\alpha}$$

$$v_o = v_{C1} - v_{C2} = V_{CC} - i_{C1} R_C - (V_{CC} - i_{C2} R_C) = (i_{C2} - i_{C1}) R_C$$

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And therefore, we can rewrite this equation as $i_{E1} - i_{E2}$ is $i_{C1} - i_{C2}$ by α into I_0 ; $i_{E1} + i_{E2}$ being equal to I_0 . This is equal to the exponent $v_{id} / V_T - 1$ by exponent $v_{id} / V_T + 1$.

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$$\frac{(i_{C1} - i_{C2}) R_C}{\alpha I_0} = \frac{\left(\frac{v_{id}}{V_T} - 1\right) R_C}{\left(\frac{v_{id}}{V_T} + 1\right)}$$

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If I multiply this now by $R C$, this also we can multiply by $R C$; this factor - i c 1 minus i c 2 into $R C$ from this is nothing but minus V naught. So, this is... So, we get V naught equal to minus βI naught $R C$ exponent V i d by $V T$ minus 1 by exponent V i d by $V T$ plus 1. So, this is an important relationship between the output and input of the differential amplifier.

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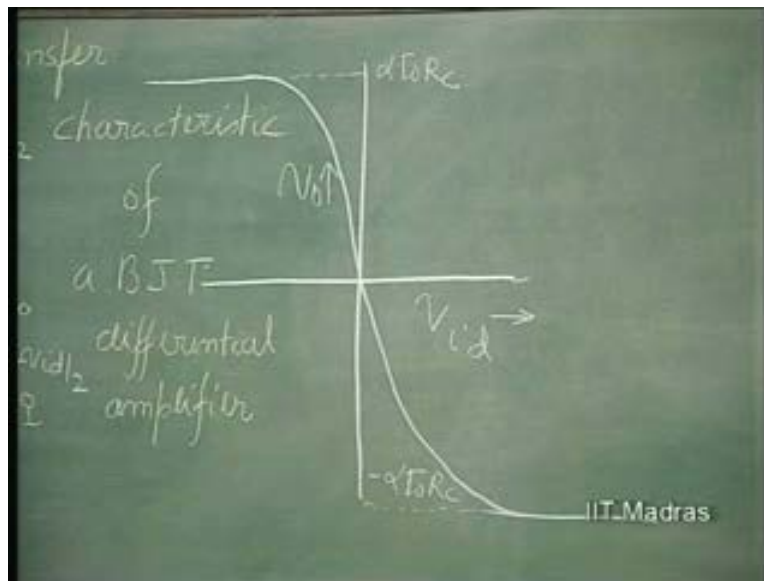
$$V_o = -\beta I_b R_c \left[\frac{e^{-\frac{V_i d}{V_T}} - 1}{e^{-\frac{V_i d}{V_T}} + 1} \right]$$

Now, let us look at it. When V i d is large negative, this factor will go towards zero. This factor also goes towards zero. So, this whole thing becomes minus 1. So, it will...minus into minus; so, plus βI naught $R C$.

So, that means then, we can plot this here. V naught versus V i - it will start with βI naught $R C$ when V i d is large negative. Now, when V i d is large positive, this is very large compared to 1; this also is very large compared to 1; and this whole factor will go towards 1.

And therefore, this V_{out} will go towards minus $\alpha I_{bias} R_C$. That means the other asymptotic approach is $\alpha I_{bias} R_C$ when V_{id} is large positive. In between, what is it? - when V_{id} is very small or zero, this is going through zero because this is 1; this is 1 minus 1, so zero. So, it is passing through zero and that plot, actually speaking, is going to be something like this. This is a well-known plot.

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This, we can write in the following manner. $\alpha I_{bias} R_C \frac{V_{id}}{2V_T}$. So, I can write it as... multiply both sides by exponent $\frac{V_{id}}{2V_T}$, minus $\frac{V_{id}}{2V_T}$ by $2V_T$, numerator as well as denominator. So, what do you get? This will become $\frac{V_{id}}{2V_T}$. This becomes minus $\frac{V_{id}}{2V_T}$. This will become this and this will become exponent minus $\frac{V_{id}}{2V_T}$, which is going to be nothing but... This is nothing but hyperbolic tan function. Minus $\alpha I_{bias} R_C \tan$ hyperbolic $\frac{V_{id}}{2V_T}$.

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The chalkboard shows the derivation of the output voltage V_o for a differential pair. At the top, the expression $e^{-v_{id}/2V_T} + 1$ is written. Below it, the equation is boxed as follows:

$$V_o = -\alpha I_0 R_c \left[\frac{e^{-\frac{v_{id}}{2V_T}} - \frac{v_{id}}{2V_T}}{e^{-\frac{v_{id}}{2V_T}} + \frac{v_{id}}{2V_T}} \right]$$

Below the boxed equation, the result is simplified to:

$$V_o = -\alpha I_0 R_c \tanh \frac{v_{id}}{2V_T}$$

The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

So, this function, which is going from Alpha I naught RC to minus Alpha I naught RC is nothing but, strictly speaking, minus Alpha I naught R C tan hyperbolic exponent V i d by 2 V T; that is, the exponent is removed; V i d by 2 V T. Is this clear?

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