Electronics for Analog Signal Processing - I
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Lecture - 15
h & g Parameters

In the last class, we saw two parameters: Z and Y parameters. You can see here, Z parameter, leading to definition for current controlled voltage source or Transimpedance amplifier, it is called, we will call it. Now, in a more general term as Transimpedance amplifier instead of Transresistance amplifier; and Y parameter the dual of that, leading to a voltage controlled current source, we call it as Transadmittance amplifier instead of the earlier definition, Transconductance amplifier.

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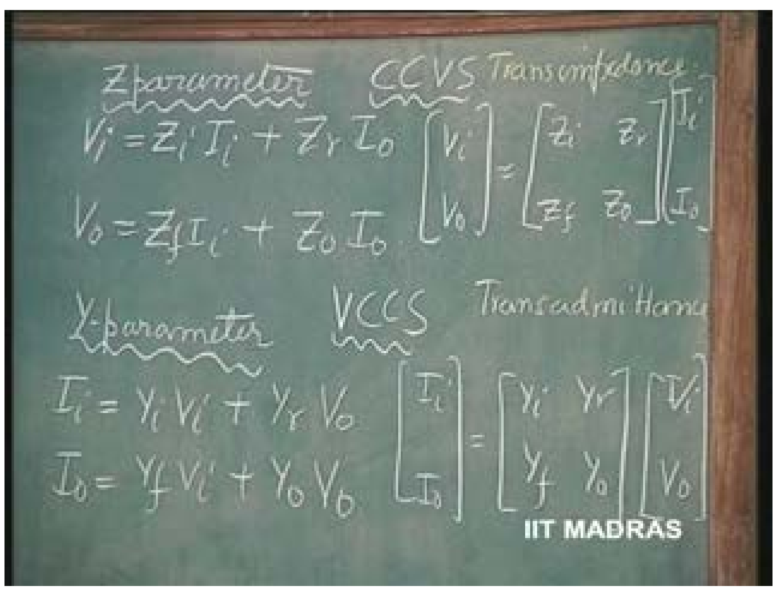

Now, you also understood that these two are more basic than the other two types; and therefore, we can obtain the other two types called current controlled current source and voltage controlled voltage source by simply suitably cascading these two amplifiers. Now, let us come to the other two parameters which are equally important. h and g parameters, we can call. These are going to be having parameters which are dimensionless as well as taking the dimension of impedance and admittance. So, this is normally called hybrid parameter; h is hybrid; g also is dual of h.

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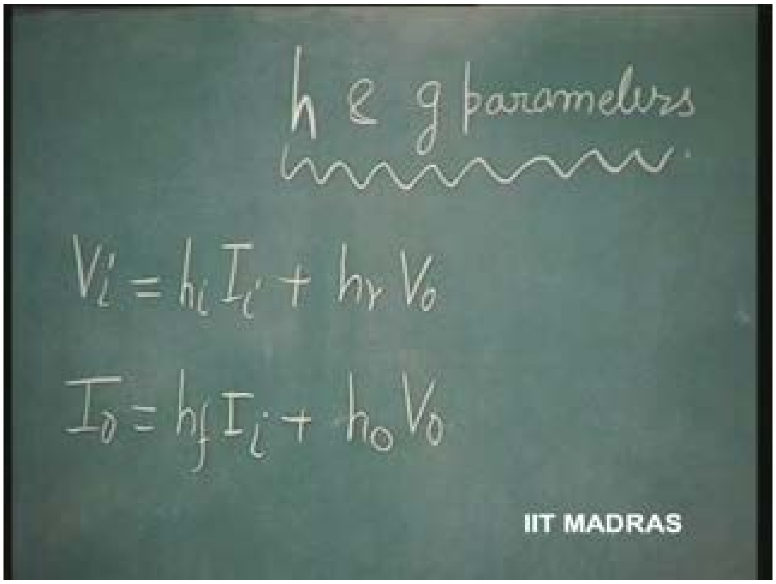

So, consider first, the independent variables I i and V naught. I i and V naught are considered as independent variables and V i and I naught are then the dependent variables. So, in such a situation, you can write linear relationship for these dependent variables; V i as h i into I i. So, this is h parameter; so, suffix is h i, because i is involved with i, totally input parameter.

This is either self-admittance or self-impedance; it is called, self. So, h i is therefore input impedance; dimension is V i by I i, so impedance. When? When h, that is V naught is zero. V naught zero means, output short circuited. So, this is short circuit input impedance, h i.

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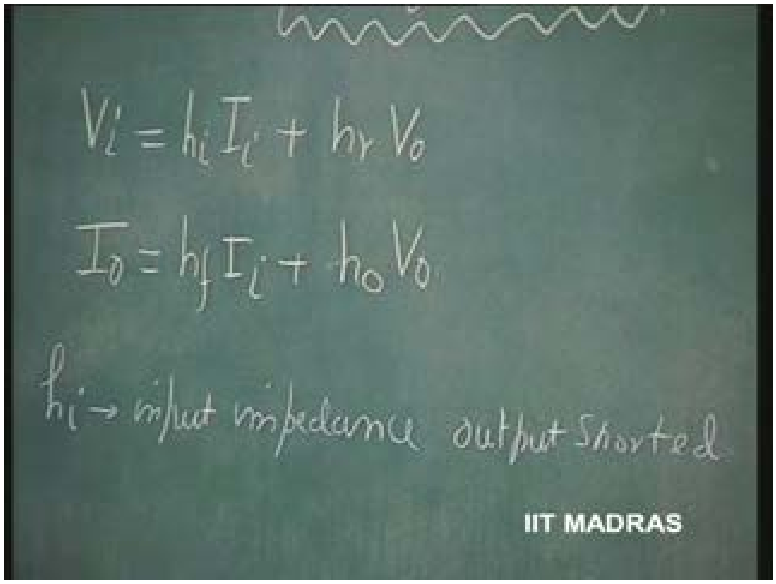

Then, h r; it is relating V i and V naught, when I i is zero; that is, input is open circuited. Then, if I apply a voltage V naught, how much of it is reflected at the input as V i? So, this is the reverse or inverse transfer ratio; actually, it is a ratio, inverse transfer ratio or reverse transfer ratio. So, voltage ratio. And, this is for input open circuited; that is the definition.

Next, h f. Once again, I naught and I i are related; so, this is a ratio. But you can see here; this is obtained when V naught is zero. That means output is short. So, this is a short circuit current gain, so forward. So, this is a forward transfer ratio.

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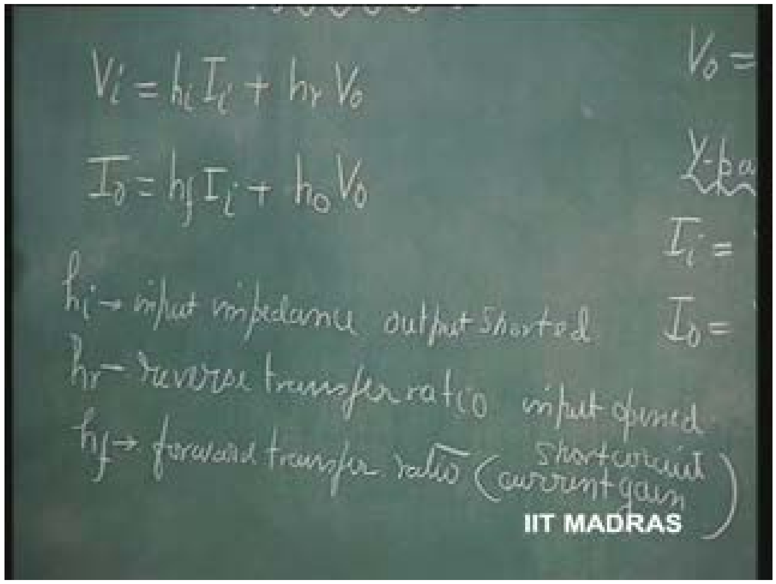

This is called current gain or short circuit current gain, because output is shorted.

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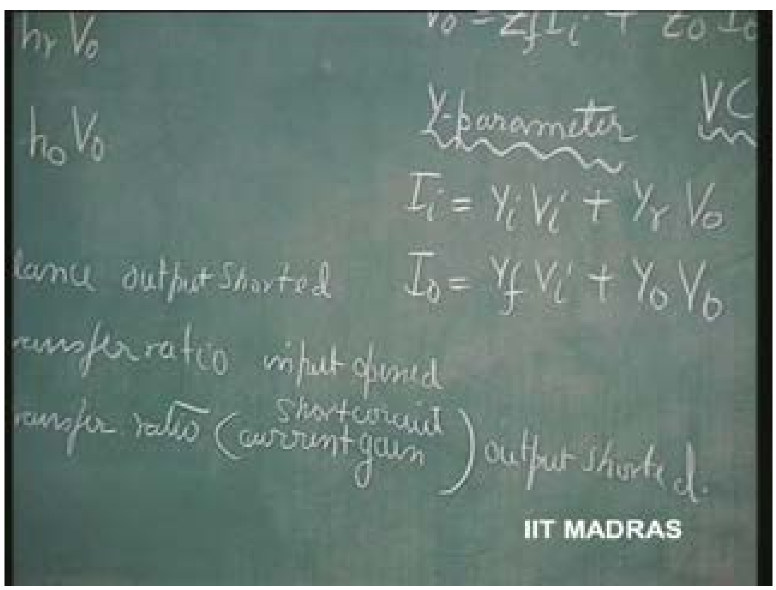

h f is short circuit current gain; I naught over I i. h naught, relating I naught and V naught; it is an admittance parameter. Again, it is a self-admittance parameter, but at the output. So, output admittance, when? I i is zero; that means input open.

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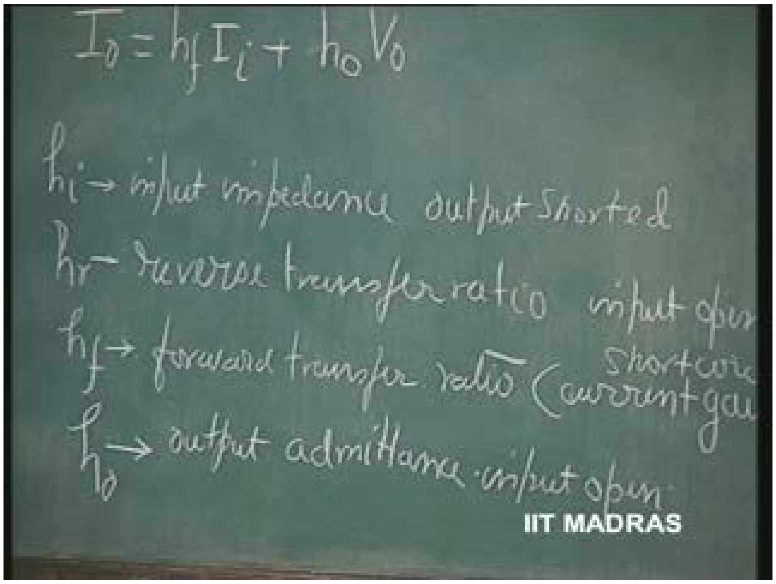

So, these are very important parameters; that is why... Why, they are so important? We are likely to discuss the amplifier which this parameter normally represents. And, what is the amplifier that it normally represents? Obviously, we have seen here; forward transfer ratio is a short circuit current gain. So, this is nothing but current controlled current source; ideal, current controlled current source; which is what is our bipolar junction transistor. Classic example of this category is bipolar junction transistor; I had mentioned about this earlier.

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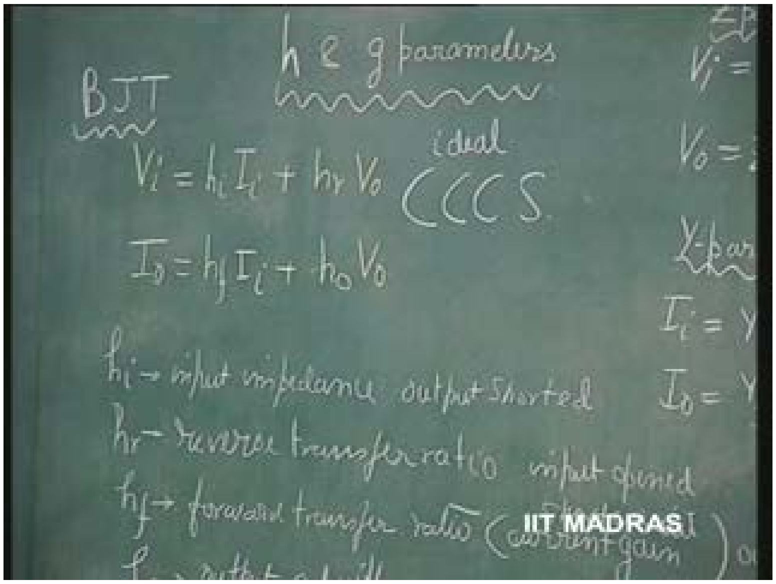

This is very nearly equated to a current controlled current source; and h parameters are commonly adopted in defining the performance of a particular amplifier designed with BJTs.

So, this parameter is quite a useful parameter because of its close association with a device which is closely linked to the ideal source that it is trying to simulate. And other parameter which is also very useful parameter, which is measured normally for any device, is Y parameter; primarily because, these are all short circuit parameters, easily measured.

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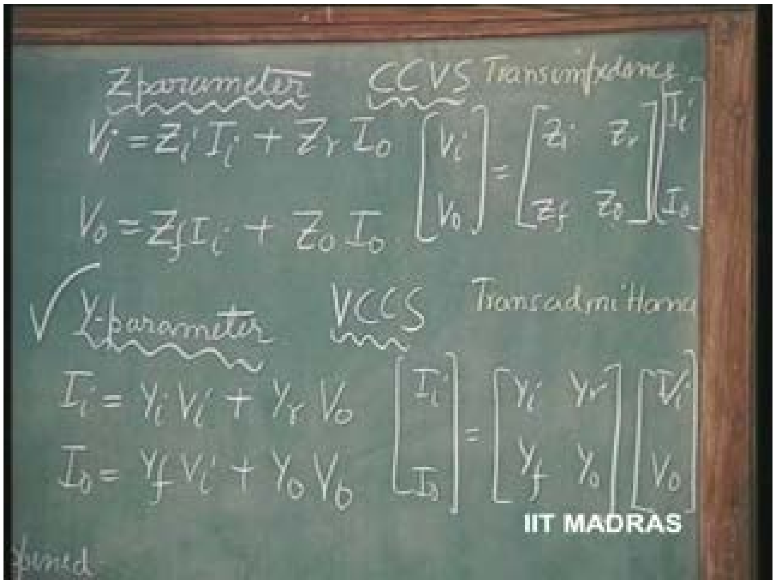

It is very difficult to open circuit a practical device; it is easy to short the device. So, this is a parameter which can be easily measured ((... Refer Slide Time: 9:55)) because these are short circuit parameters. Short circuit parameters are practically easy to measure. And these parameters are nothing but admittance parameters; and capacitors, which are stray capacitors, can be simply adding on to their admittance at j omega C.

So, most of this stray effect is due to capacitors and that can be simply added to this admittance. So, that is why, these parameters are very important parameters in practice which are measured in most of the designs; and there are variety of bridges which can directly measure these parameters for any two port.

Now, what is the ideal current controlled current source? Ideal current controlled current source can be derived from the Z parameters wherein, it is unilateral; so, h r is zero; just as we did in other cases. It can transmit only in forward direction; reverse transmission is zero.

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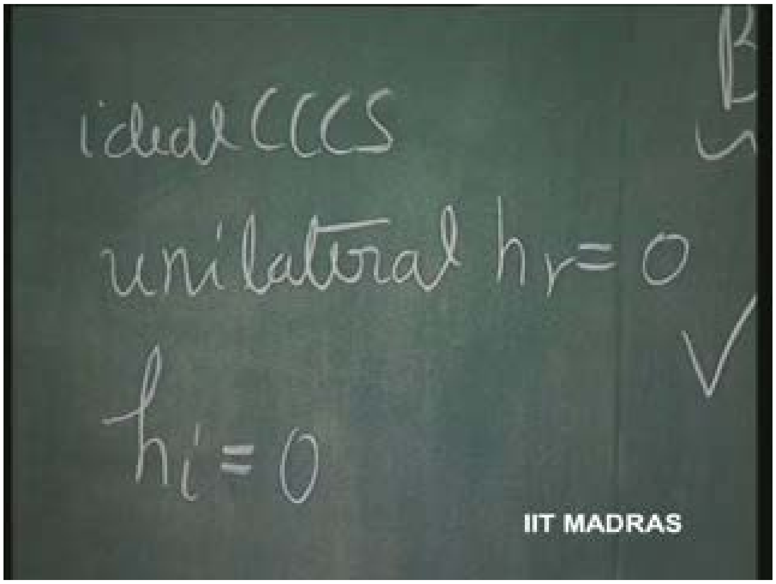

And h i, the input impedance, short circuit input impedance is zero. h, output admittance is zero; because it is a current source output, output admittance is zero. And therefore, the ideal current controlled current source can be represented simply by this.

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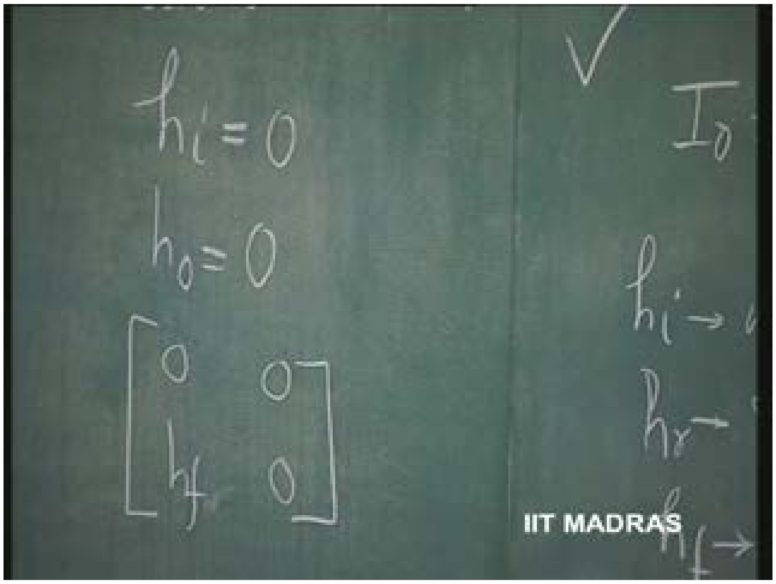

Once again, this is because, this is a hybrid type of parameter. Next type. This is impedance, this is admittance, this is voltage ratio, this is a current ratio. You should not get confused with this. These two are always ratios. In these two types of parameters, h and g, these two will be always ratios.One is a current ratio, the other will be a voltage ratio. This will be impedance, admittance; or otherwise, the other parameter, this will be admittance, this will be impedance.

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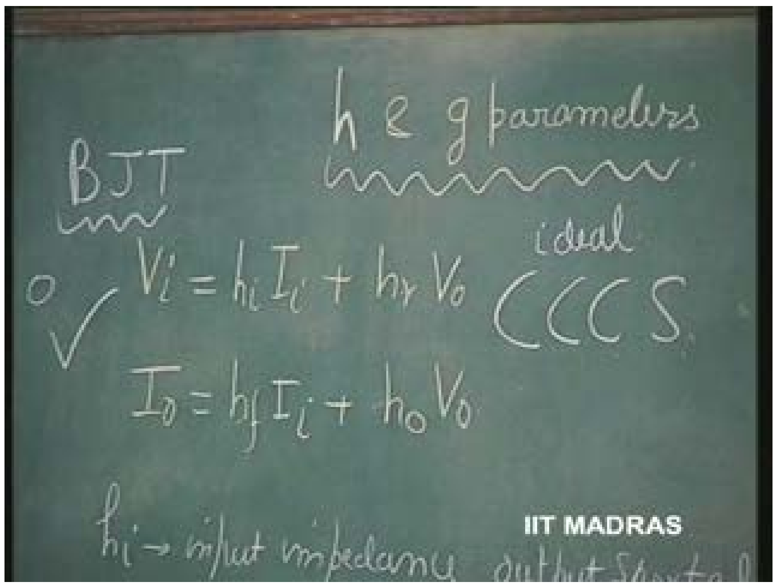

So, let us now discuss the g parameters. There, we had V i and I naught as dependent variables and I i and V naught as independent variables. Here, we will have V i and I naught as independent variables and this will be in turn representing I i and V naught.

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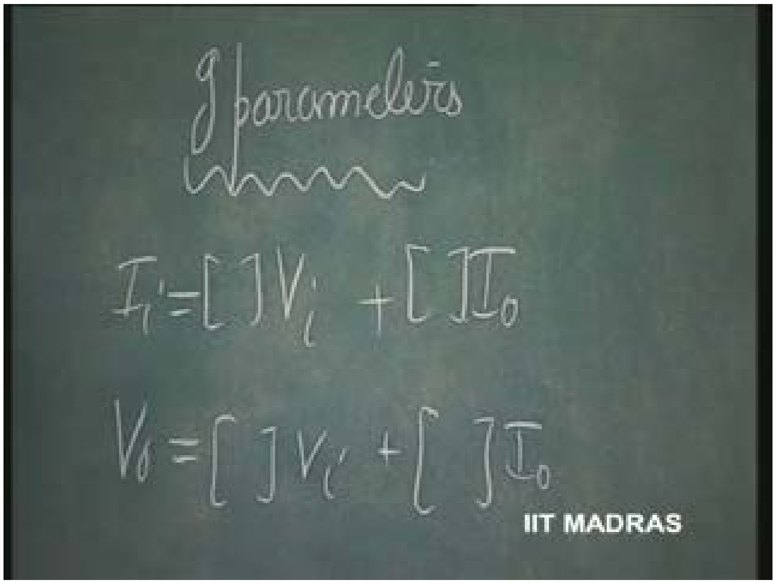

So, V i and I naught are the independent variables and therefore we will express I i and V naught in terms of them, linearly. I i therefore is equal to g, suffix is g i. This game is now very clear to you, right? You can yourself put down those suffixes. So g i, indicating what? Self-admittance, now. So, this is an admittance, input admittance, input admittance, when I naught is zero; that means open circuit output.

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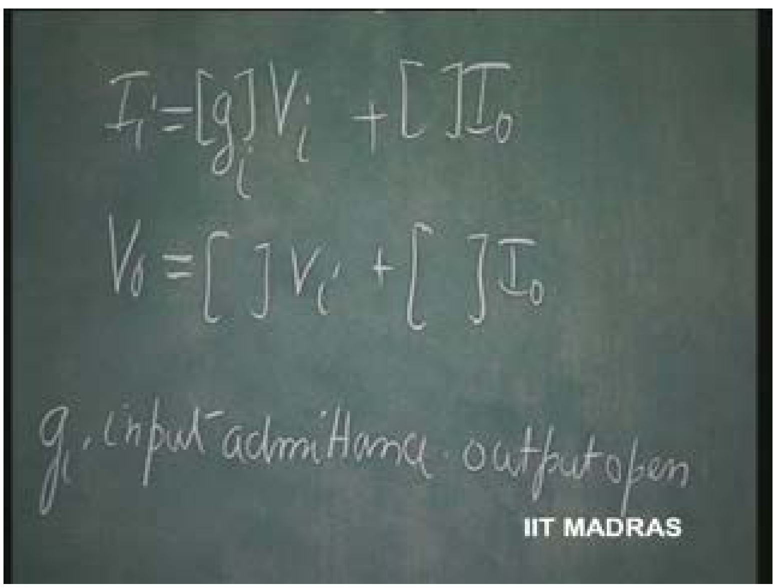

Then, g here. What shall I put? I naught and I i are related; so it is reverse transmission here, g r. So, reverse, what? - transfer ratio, but it is a what ratio? - current ratio, I i by I naught, when V i is zero. That is, input short circuited, g f. V naught, V i; again, it is a voltage ratio, forward transfer ratio; so, forward transfer ratio. What voltage? Again, output is open; I naught is zero.

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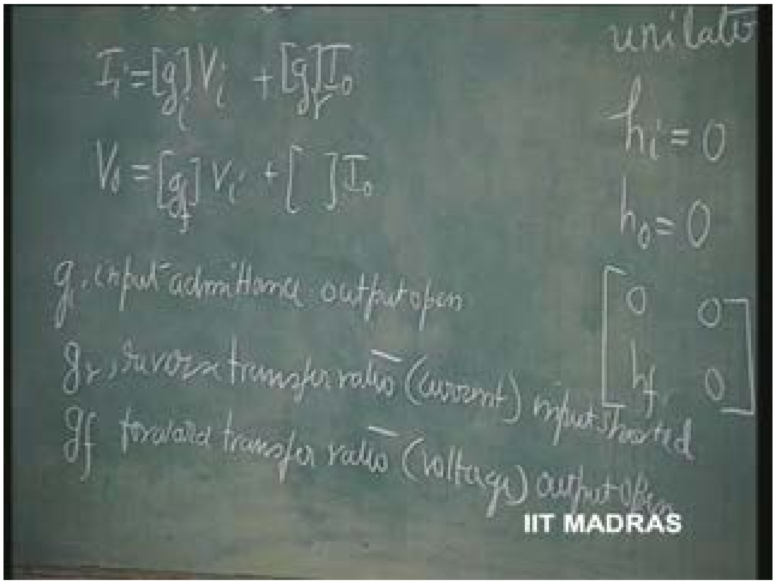

Finally, we have this g; again suffix naught, that means, g naught. Please do not get confused with g being adopted for conductance. These are g parameters; they can be impedances, admittances or ratios. So, g naught is the output impedance, output impedance with V i zero. That means, input short; so it is short circuit output impedance.

So, this is input admittance, but open circuit; reverse transfer ratio, but current; forward transfer ratio, but voltage.

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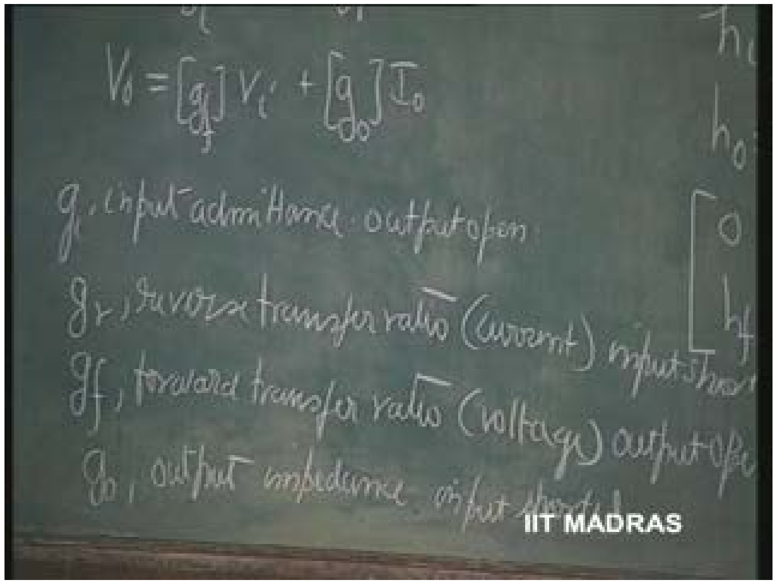

Now, definitions are all over, except that this g parameter is what is normally going to be giving us voltage controlled voltage source. The ideal voltage controlled voltage source results in g i becoming equal to zero. g r - it is unilateral; so g r is zero, g i is zero and g naught is zero, indicating ideal voltage controlled voltage source block; and it is basically defined by zero, zero, zero, g f. This is a voltage ratio.

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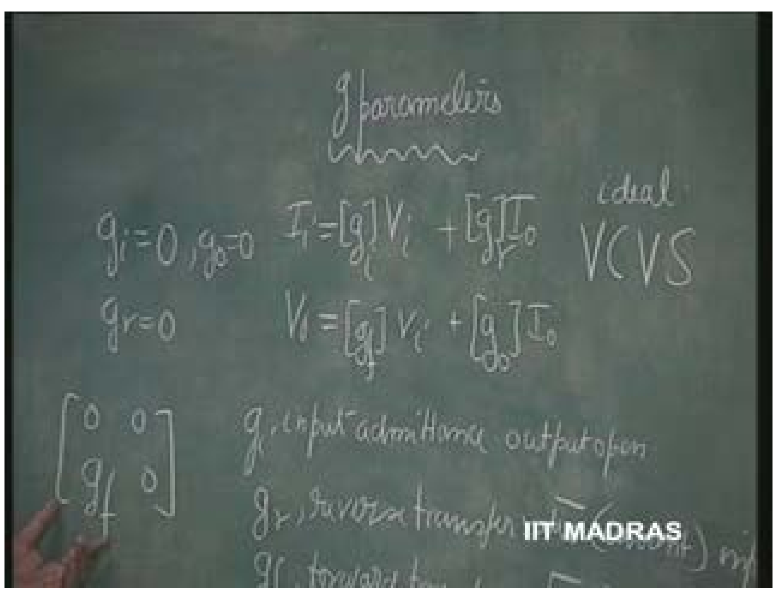

We do not have any device which can really represent this so nicely as BJT represents the current controlled current source. But we can make a combination of these devices and obtain a device which is close to an ideal voltage controlled voltage source.

That is the part of exercise which will be pursuing in most of the lectures in these series. That is, what is our aim in this course of lectures? Basically, we are trying to understand amplifiers and design amplifiers. Ultimately, these amplifiers should resemble the ideal amplifiers that we want; one of those four categories - ideal voltage controlled current source, ideal current controlled voltage source, ideal current controlled current source and ideal voltage controlled voltage source; which is what we should aim for.

And most of our designs, we will be spending time on how to put these various practical devices like transistors available to us, in such a manner, as to design these characteristics for these circuits. So, this is going to be the aim; and in this process, so therefore, we should know first of all, what we are aiming for. So, this is what we are aiming for.

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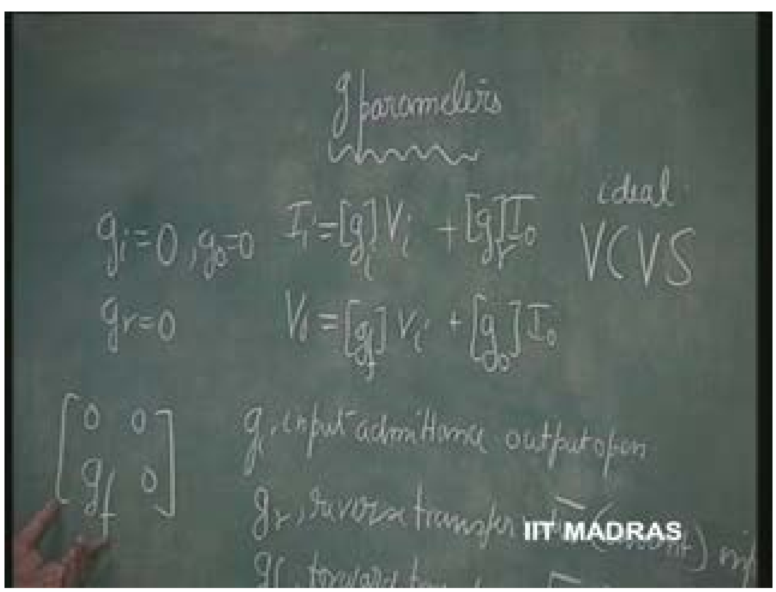

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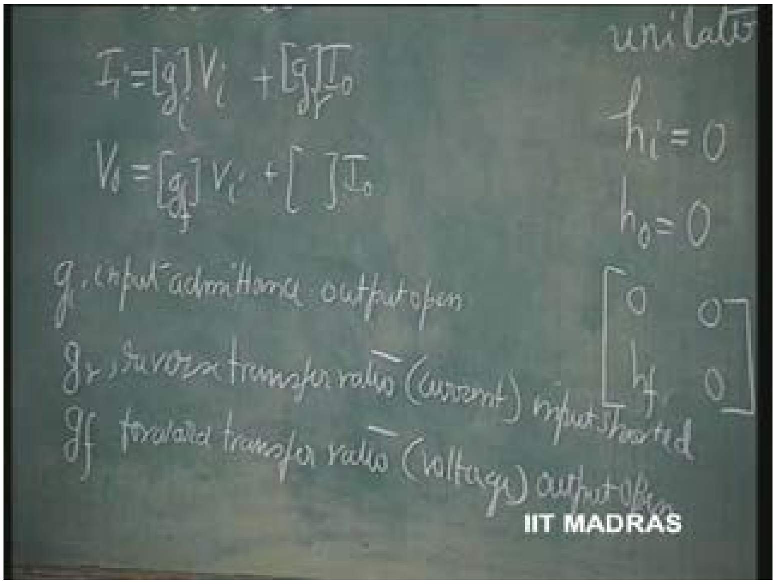

and here, zero, zero, zero, Z f. zero, zero, zero, Y f.

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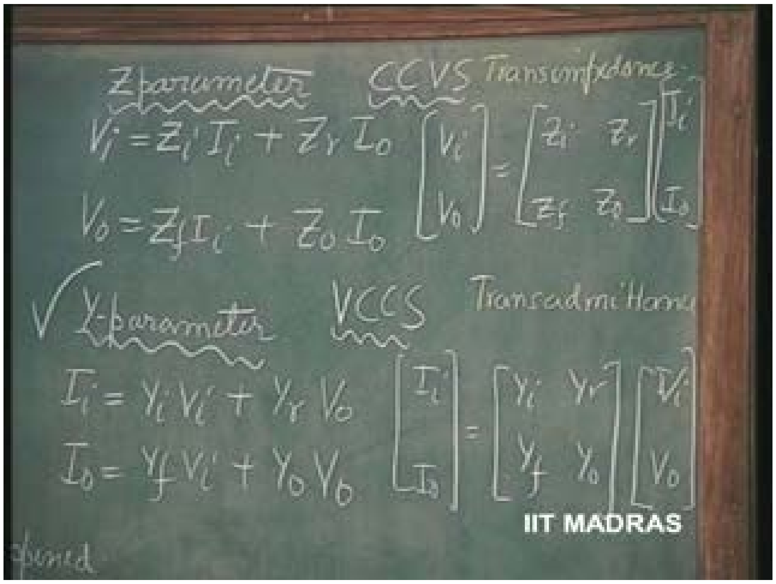

So basically, we are aiming for these amplifiers.

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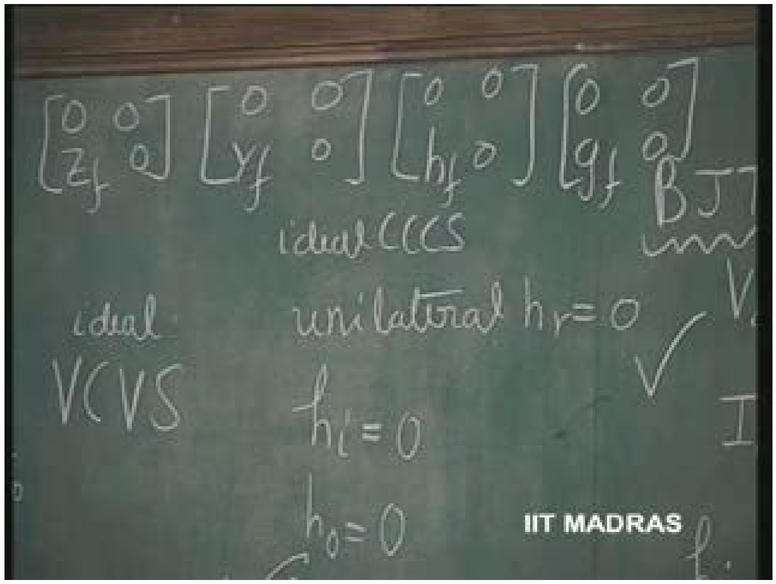

Can we design amplifiers in such a manner that it is close to this, if it is a transimpedance amplifier? Can we design our circuit in such a manner that it can closely follow this; it is a transadmittance amplifier. Can we design an ideal current controlled current source type of amplifier? Or, can we get an ideal voltage controlled voltage source amplifier?

So, this is going to be part of, major part of, what we are going to discuss in our future discussions; how to ((coach Refer Slide Time: 21:02)) these various transistors available to us to come close to these ideas.

Now that we have studied these parameters Z, Y, h and g, we can represent any two port. In this particular case, we are interested in representing our amplifier by what is called an equivalent circuit. This is what an equivalent circuit is. That, this two port can be represented as a circuit block; or, we know that this is V i and this is I i; this is I naught and this is V naught.

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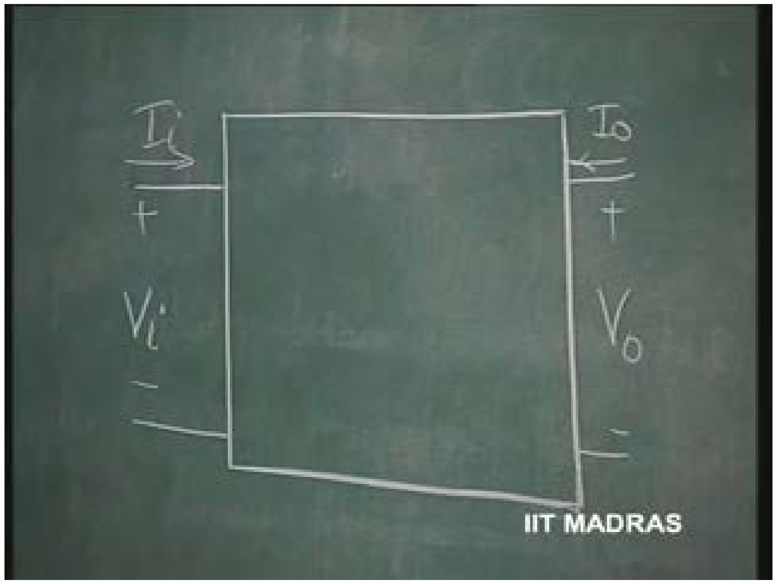

Now, inside. I can represent it as a Z parameter equivalent; just for example, I am taking Z parameter equivalent. Instead of putting those equations, I will put it in a pictorial manner. That is what an equivalent circuit is.

Earlier, we have put down these things in terms of equations; these independent variables, dependent variables. The relationship has a linear relationship in terms of two equations. Instead, now, the same thing will be represented as an equivalent source or an impedance or admittance accordingly.

So, this is an impedance parameter. So, we have here at the input Z i, which is the self-impedance; we have understood this Z i. Let us see what this equation is. V i is equal to I i into Z i. So obviously, V i is a voltage which is totaled as two voltages plus something else, which is going to be I naught into Z r. So, it is equivalent to two voltages. What is that one voltage? I i into Z i; and since I i is flowing through this circuit, it means that it is the drop across Z i. So, this is I i into Z i.

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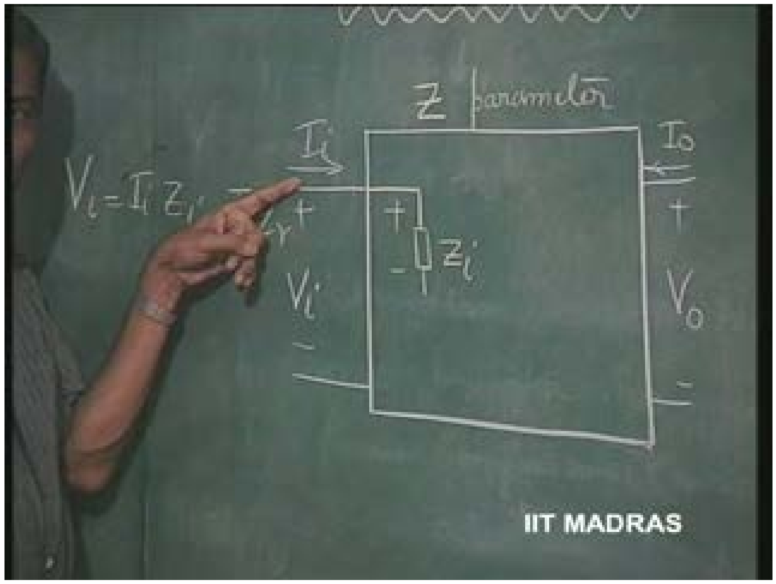

Another voltage source in series which will have an voltage of I naught into Z r; this is related to current flowing there. So, I can only represent it as a dependent voltage source. This voltage source voltage is dependent upon the output current. So, they come in series because voltage is, this voltage plus that voltage.

So, the pictorial representation of this equation is what is called an equivalent circuit. So, V i is summation of two voltages I i into Z i; since I i is the current in the same port, I can represent it by a self-impedance Z i, in series with a voltage which is independent of the current through this circuit. It is dependent upon the current through the other port. So, I have to put it as a voltage source. This is the reverse transfer current; if this is absent, if it is unilateral, there is no voltage source.

So, in the equivalent circuit, if this is absent, if it is unilateral, there will be no voltage source; only this will be there.

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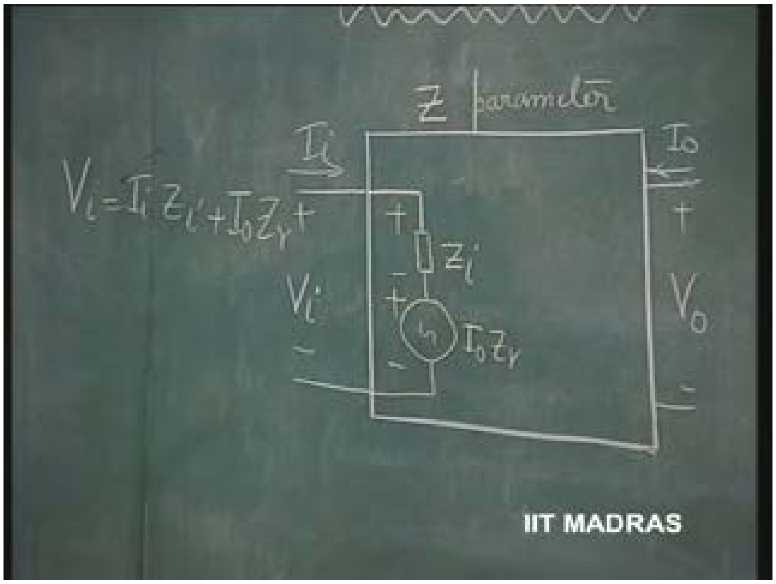

Now you can recollect how we discussed amplifier circuits, etcetera, where the amplifier was unilateral; there was no such source at all in those. At the output, we have the equation which is V naught, was equal to, again I i, into Z f plus I naught into Z naught. Again, summation of two voltages.

So, we have two voltages here; but through one, there is a current flowing and that is I naught into Z naught. The current in this is I naught. So, if I put an impedance Z naught, the drop will be, I naught into Z naught.

Next, I have a voltage here, which is dependent upon input current. So again, it is a source with the voltage equal to I i into Z f. So, this is called an equivalent circuit representation of two port network, any two port, in terms of Z parameters.

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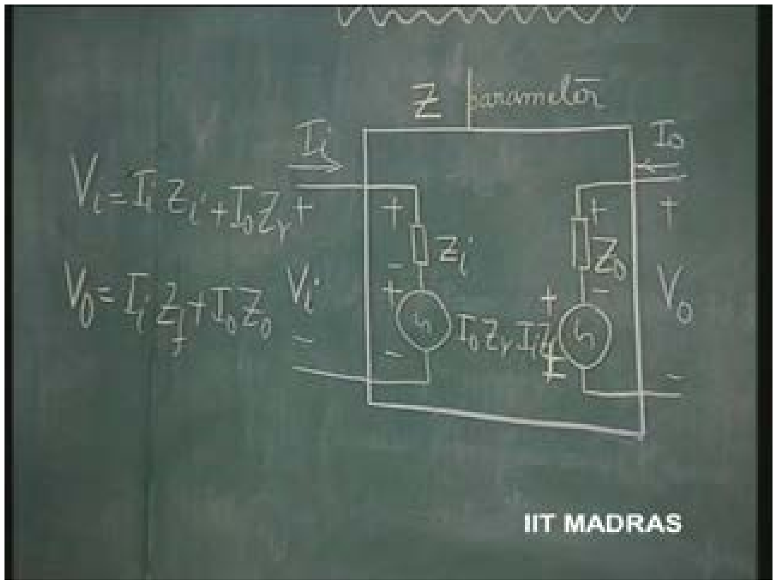

What happens if you do in terms of Y parameters? This is very simple now. I can quickly go through... I do not have to now put down these equations which you have already got in your notes. We will just put down this in terms of the equivalent. Instead of these things coming in series, now, we will have things in shunt. Why? instead of this V i, we have here I i; so I i. I i is summation of two currents. So, the two currents can flow only in two separate paths like this, in shunt. One path will have Y i the self-admittance; another path will have a current source; another path will have a current source, which is going to be equal to something happening here.

So, we are going to represent this in terms of dependent variables which are going to be V i and V naught. So, this will be V naught into Y r. So, I i is equal to V i into Y i plus V naught into Y r; two currents, Kirchhoff’s law. That is the first equation that is represented in equivalent circuit form here at the input.

Similarly, at the output, we will have a self-admittance which is Y naught and a current source which is dependent upon the input voltage which is V i into Y i. So, this is the second equation wherein I naught is equal to V naught Y naught plus V i Y f. Y parameter equivalent.

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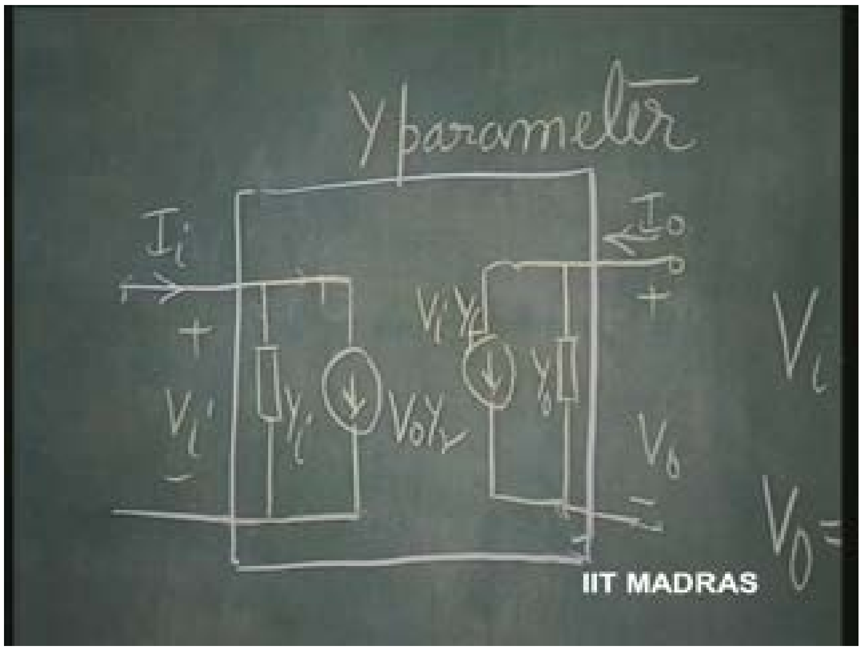

Now, the hybrid variety - we have once again V i and I i at the input represented as two voltage sources; some of one voltage with another voltage; this voltage being h i times I i - dependent upon the input current; and this voltage being h r times V naught - dependent upon output voltage. This is corresponding to h parameter; that is the first equation. We have V i equal to h i I i plus h r V naught.

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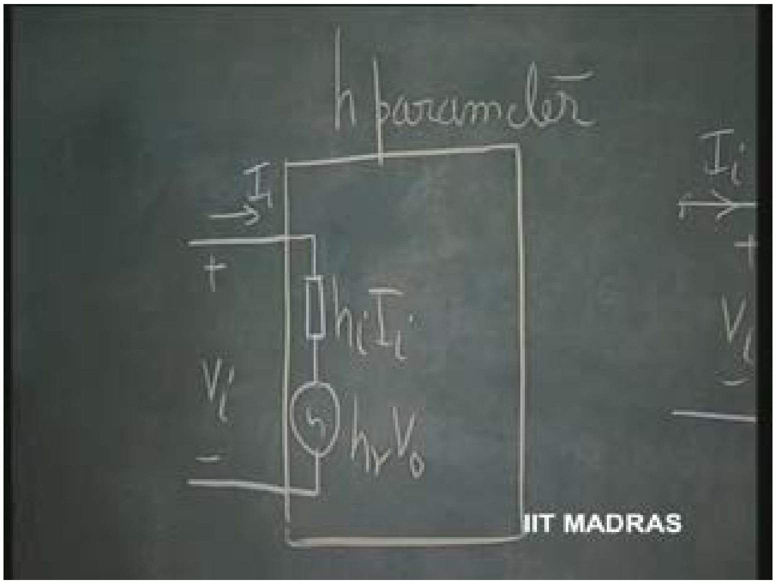

So, I don't have to now put this I i. I can just say that it is an impedance through which I i flows; so, it is h i times... Here, it is going to be now in shunt. This is the hybrid. So here, it is in series; that one, it will be in shunt. So, the current, source current, is going to be h f into I i; and there will be an admittance here, which is going to be dependent upon this voltage, V naught. This is I naught. So, I naught is equal to h f into I i into h naught into V naught. So, this is the admittance h naught. This is the equivalent circuit for h parameter.

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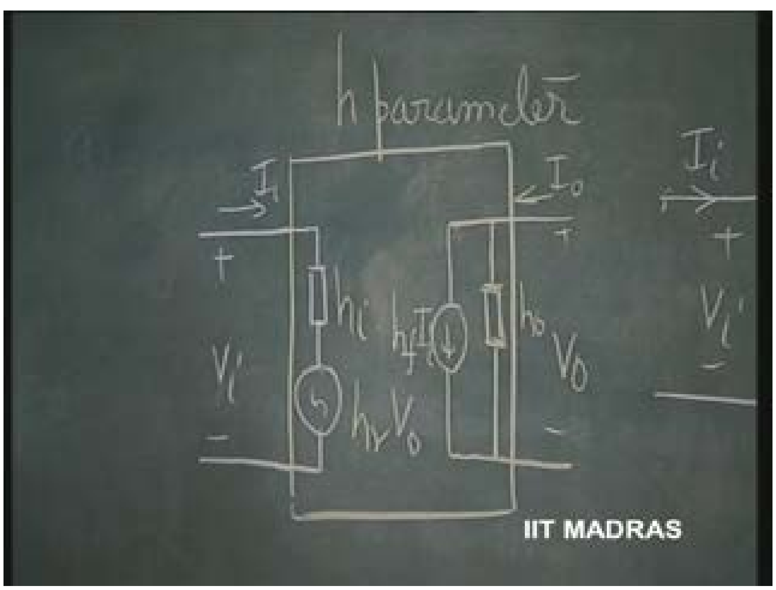

Finally, you can yourself try to draw the other equivalency because it is easy. It will be shunt here, arrangement; and series here. So, let us see. So, it will be expressing I i in terms of V i into g i, V i into g i; so that I do not have to indicate. So, it is a conductance of magnitude g i, with a current source which is dependent upon the output current here, I naught. So, I naught times g r; that is the current source; then, at the output, we have things in series. So, V naught is going to be expressed as a voltage drop across g naught, which is an impedance; and a voltage source, which is going to be g f times V i. So, this is g parameter.

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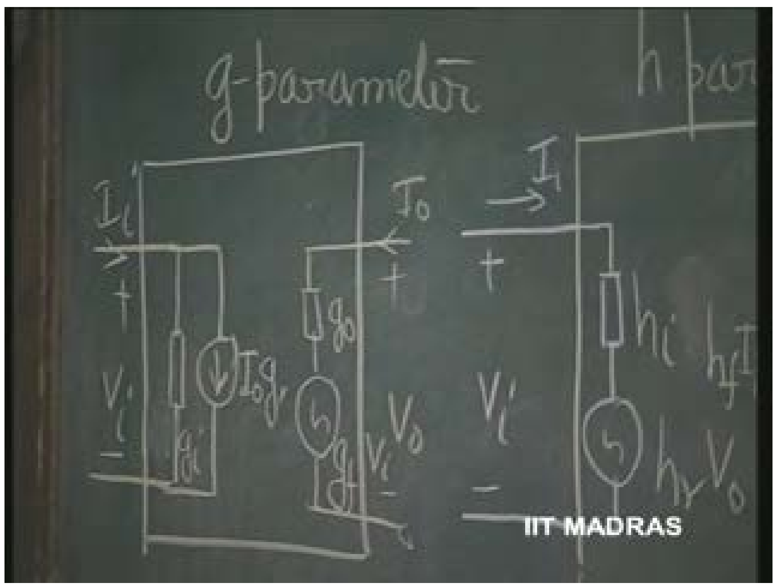

Is this representation there? It is the same set of equations that we are now representing in the network form. In all these cases, if it is resembling an ideal amplifier, you can clearly see; the first thing that has to happen is that all the reverse transmission should go to zero. That means, there will not be any current source, it will be open; there will be no voltage source here, it will be shorted; there will be, again, no current source here, it will be open; there will be no voltage source, it will be shorted. And, if you look back in your earlier notes, you will see; those are the four amplifiers which we had discussed thoroughly without any reverse transmission.

Now that we have learned the equivalent circuit approach for solving any problem on amplifier, let us consider a general amplifier, which is now, for example, represented by Y parameters. We know how to represent any amplifier by any one of the four parameters; as long as the amplifiers are non-ideal, which they are going to be most of the time; and therefore, we can represent these amplifiers by any one of the four parameters.

So, let us now see... So far, we had not included this reverse transmission parameter in our discussion about amplifiers, earlier. So it was very easy. What is this going to do? This is going to give us lot of trouble. This is unnecessary feedback from the output. This is going to interfere with our input.

So, how it is going to interfere with our input, we would like to see. So obviously, the input impedance earlier, without this, was simply determined by Y i, 1 over Y i, if this is not there. But now, because of this, additional current is going to be drawn, which is dependent upon the output circuit. So, this interference from the feedback, which must be kept small in an amplifier, you would like to investigate. How it is going to interfere with the performance of our amplifier, we would like to see, examine.

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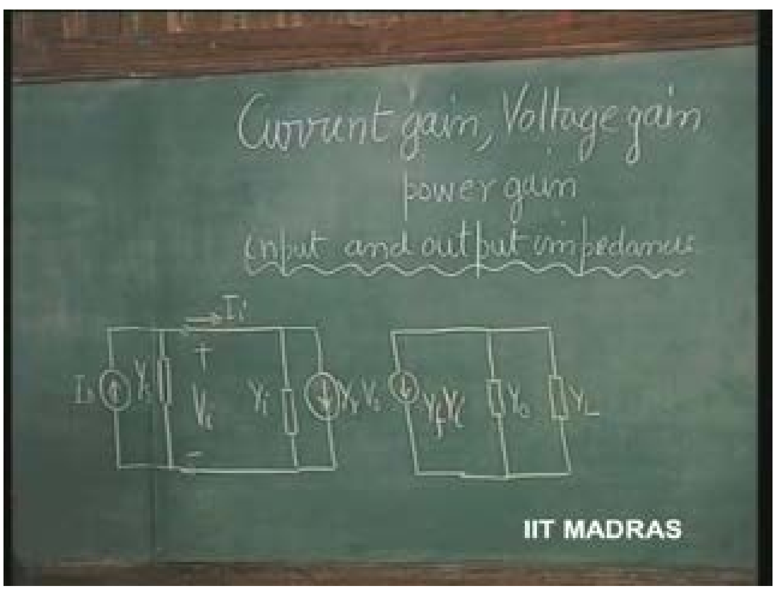

So, Y r is not necessarily zero; but it is kept very small. If, even if it is kept very small how is it going to affect our performance of the amplifier. For that, we can do the analysis now. Same equation is taken. I i is equal to Y i V i plus Y r into V naught.

Now, I would like to sort of indicate to you here that I would like to find out what is the input impedance at the input of the amplifier. What is defined as V i by I i? Or, how much is this? This is the input impedance. How to evaluate this?

So, we have an equation here relating I i with V i. But unfortunately, it has also V naught. This has to be expressed now in terms of V i so that this whole equation is a single equation relating V i and I i. That means, using the other equation, I have to eliminate V naught. So, let us see what the other equation is. Other equation is, in this case, I naught equal to Y f V i plus Y naught. Now, this is the other equation.

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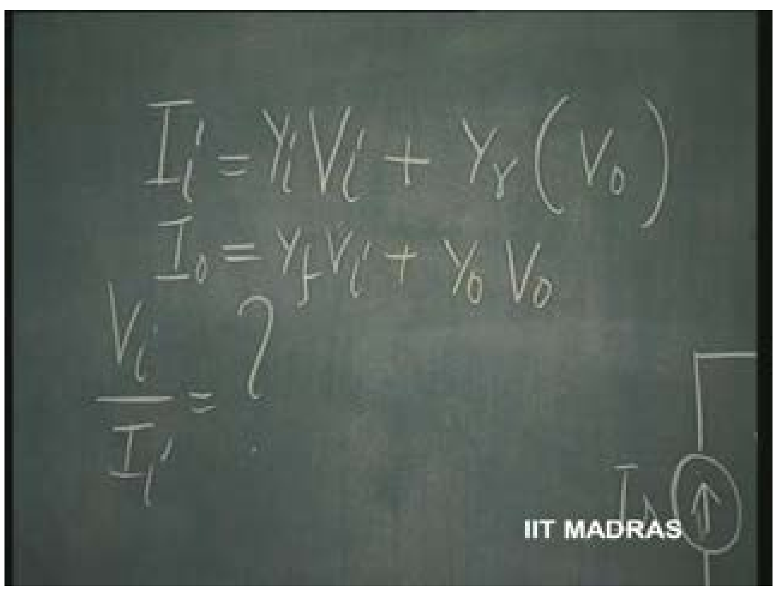

We have another term here; you just see here. Let us investigate circuit Y f. Y f into V i is the current here. I would like to know what the voltage is here, that is V naught. This is going to be V naught; that I know. There are two admittances. The total admittance of this network is Y naught plus Y L. The total admittance of this network is Y naught plus Y L. That is being fed by a current source of magnitude Y f into V i.

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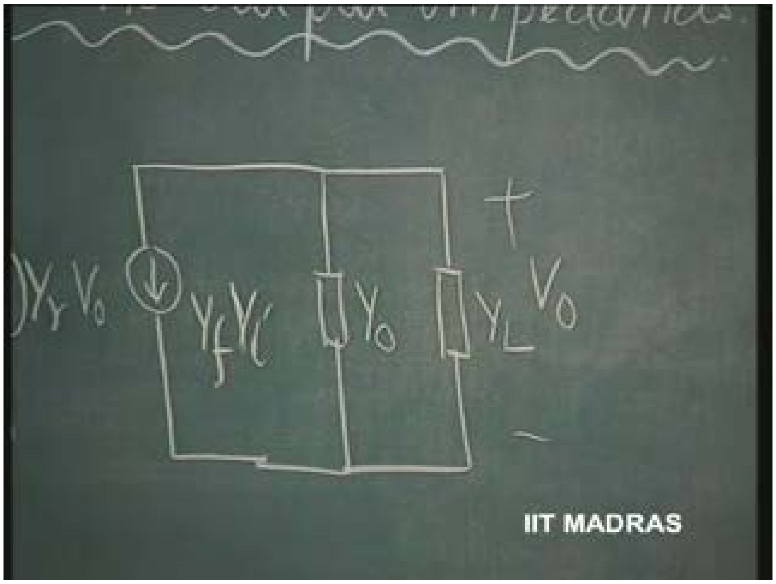

So, what is V naught? V naught is going to be therefore equal to... this current is going to flow in this direction; so, whereas, I have put V naught plus here, and minus here. Therefore, there is a negative sign; because this current is assumed to be flowing this way, this will be plus and that will be minus. So, so, V naught is minus of Y f into V i, current, divided by Y naught plus Y L.

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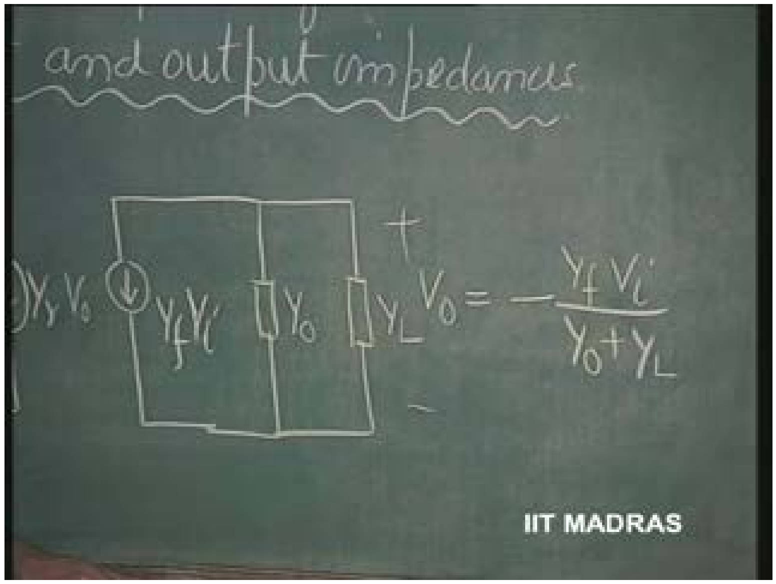

This current, flowing into a total admittance of Y naught plus Y L, will develop a voltage, which is Y f into V i, which is the current, multiplied by the impedance, which is 1 over Y naught plus Y L. Is this clear? So, this is the output voltage. This will straightaway give me what I want. That is, V naught over V i, is always equal to, in terms of Y parameter, minus Y f divided by Y naught plus Y L. Y L is the load admittance. So, this is what we have already evaluated; voltage gain.

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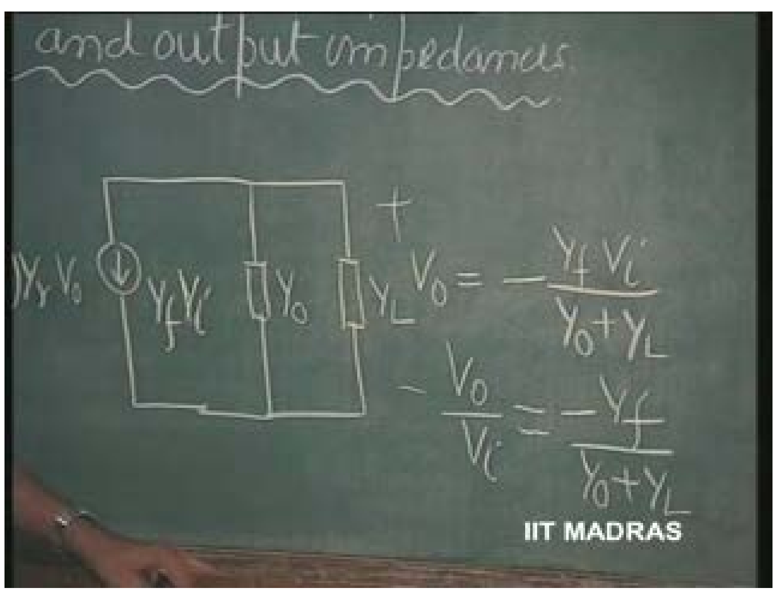

So, we have now V naught in terms of V i; so, that is what we wanted here. We want to get rid of V naught here, in order to find out V i by I i. So, we substitute this here; I i equals Y i into V i, minus, because this is Y r, into Y f divided by Y naught plus Y L into V i; or, I i by V i is equal to Y i minus Y r Y f divided by Y naught plus Y L.

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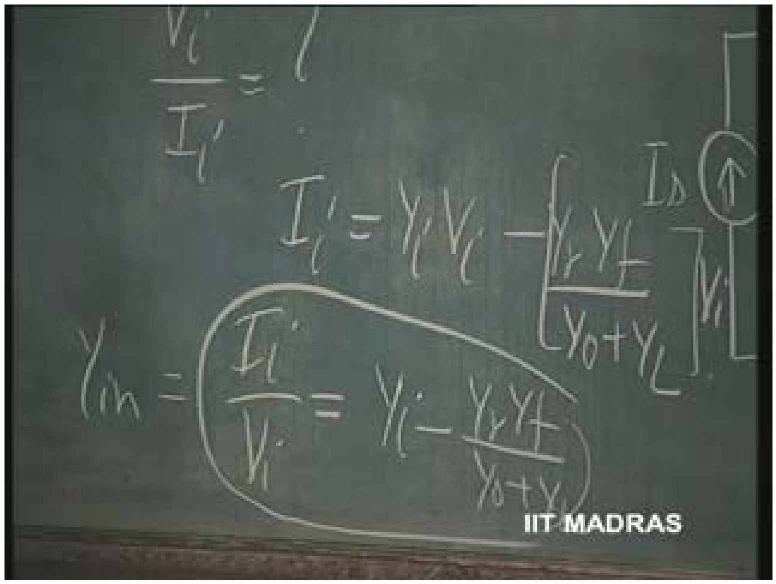

Please remember; this is an important relationship, the admittance. This, you can call it as Y in - input impedance of the amplifier, considering the feedback into account. If Y r is zero, this is Y i itself. So Y in is equal to Y i minus Y r Y f divided by Y naught plus Y L.

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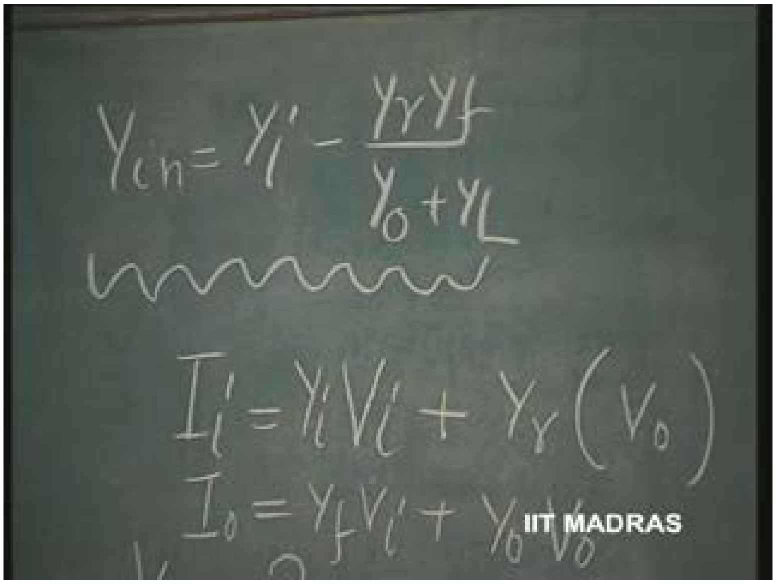

In a similar fashion, you can derive the output impedance. What you should do in such a situation is, you should drive the circuit by at the output, with I s; this is the independent source here. This being removed, I s should be removed; so you are now looking at the output and seeing what is the output impedance.

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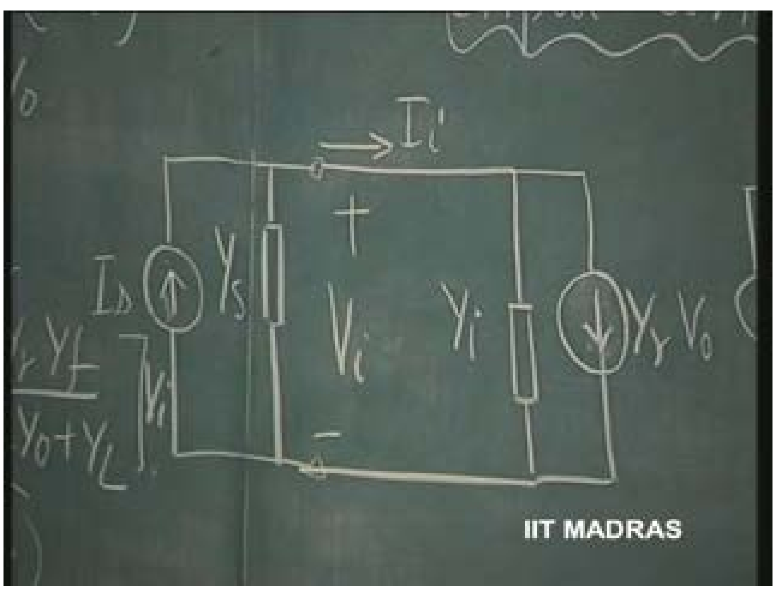

So, what is output impedance? V naught. This is V naught; you can remove this, divided by I naught. So, it is similar; exactly like what we have done here. This is V naught, this is I naught. I am applying a voltage here, removing the load, applying a voltage here, finding out what the current is; then, this should not be there, this is an inverse.

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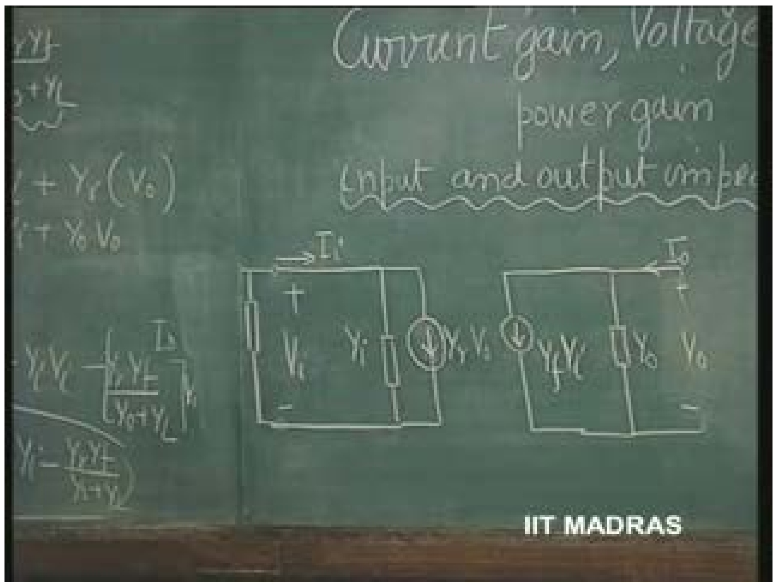

So, I am applying an independent source here. So, while measuring the output impedance, I apply a source here. I will find out the current here. So, output impedance is V naught divided by I naught. This is exactly similar to what we had earlier shown you. Only thing is, I and o simply get interchanged.

So, I would like to leave this as a home work problem for you. Show that I naught by V naught is Y out equals Y naught minus Y r Y f divided by Y i plus Y s.

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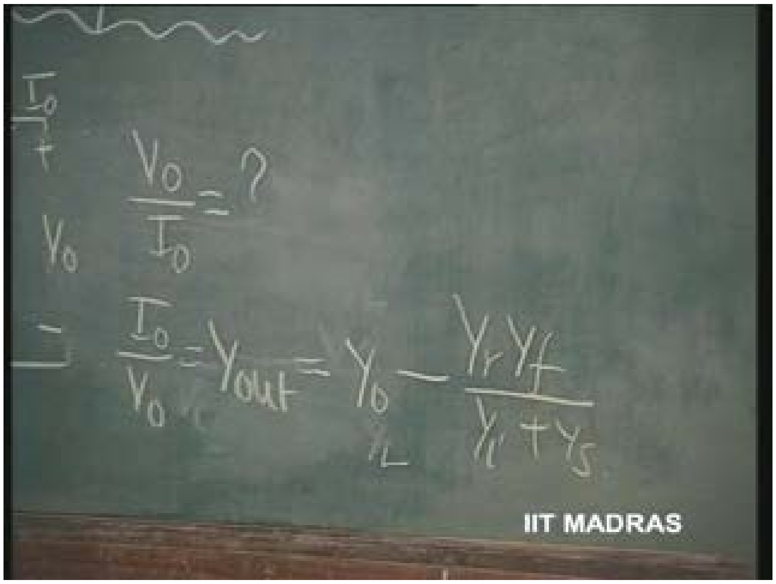

The circuit that you have to analyze is exactly similar to what we did, just previous to this. Only that, I and o simply get interchanged ((s)) L is removed and put, replaced by s. That is all. Instead of load, you have Y s here. That is all that has happened. But...

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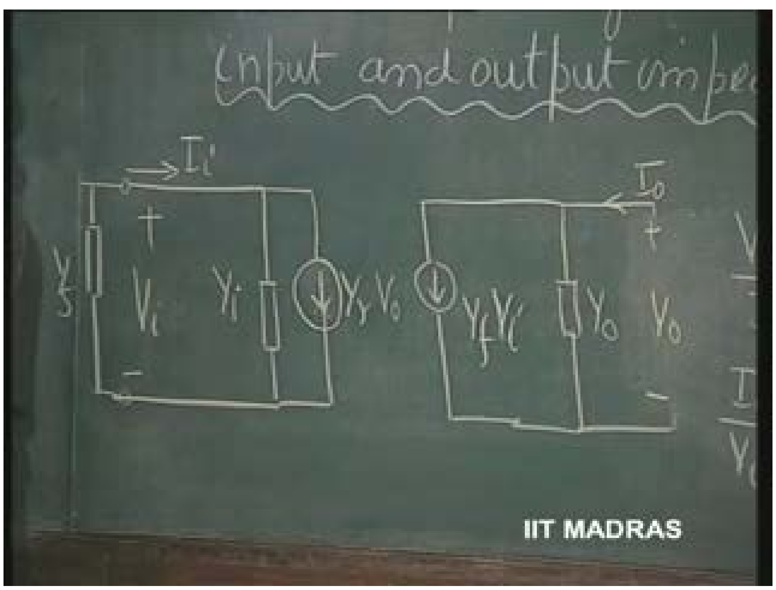

So, you can derive this and you will get the Y ((...Refer Slide Time: 43:20)). We have derived the output impedance; indicated to you how it can be derived. We have shown what the voltage gain is. I would like you to evaluate the current gain, which is I naught by I i, from the equivalent circuits. So, I i is here; I want to know I naught by I i, here.

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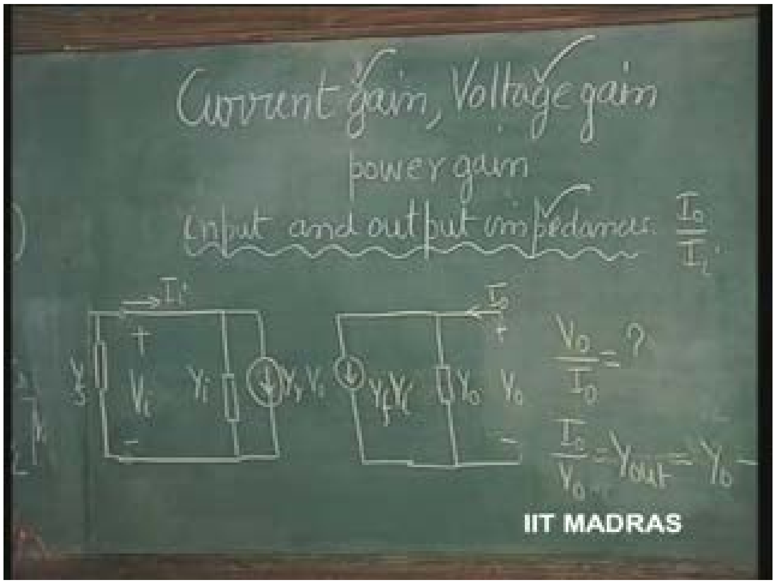

So, please work this out and other problems you will see that whether I adopt Y parameter or Z parameter, h parameter or g parameter, universally immittance, means immaterial what parameter I adopt; this is called immittance parameter.

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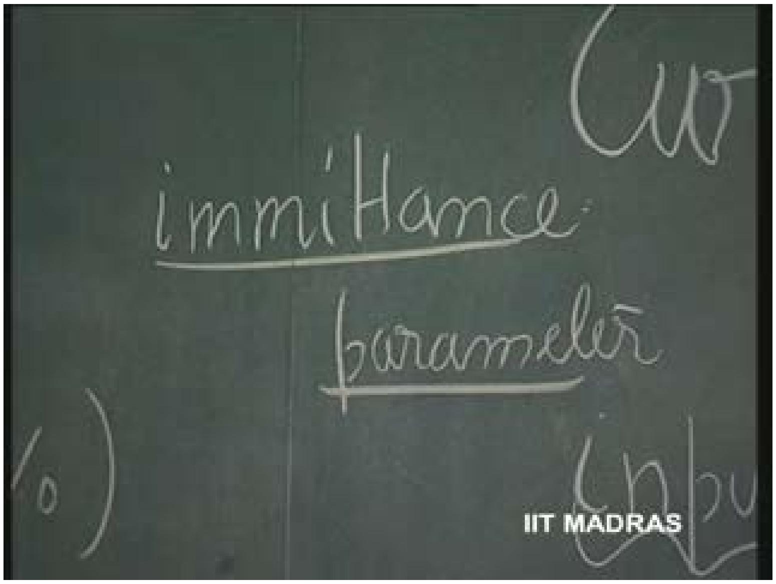

Whether it is h, Y, Z or g, the input immittance, in, is equal to p i minus p r p f divided by p naught plus p L. And output immittance is going to be equal to p naught minus p r p f divided by p i plus p s.

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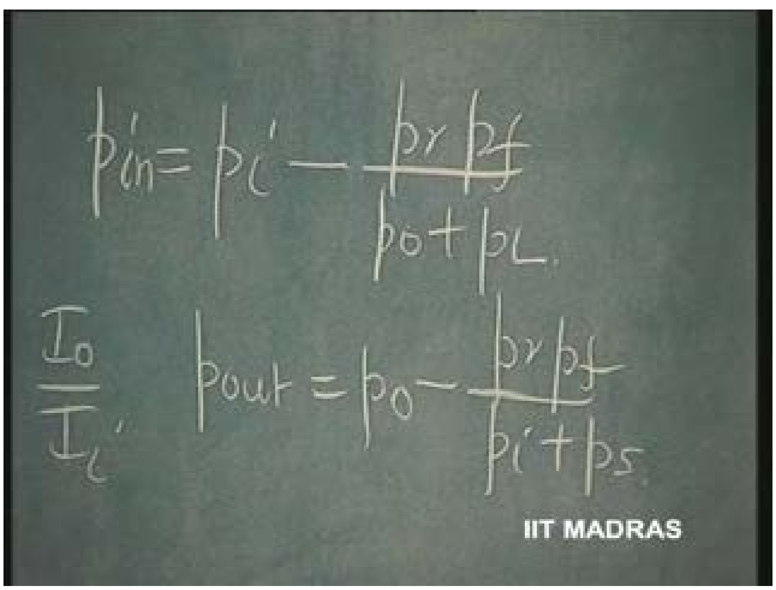

What this means is, if this is Z parameter, this is Z in, Z i, Z r, Z f by Z naught plus Z L; Z out, Z naught minus Z r, Z f by Z i plus Z s.

So, whatever be the parameter you adopt, you will see that the input immittance and output immittance, the nature of this formula, remains the same. This is what I would like you to derive for all the three parameters left out. That is, Z parameter, h parameter and g parameter; show that the input immittance and output immittance are governed by these equations.

That is what can be shown very easily, just as we did it here. These are two sets of equations. You have to appropriately eliminate one of the variables in order to get these different relationships. So, from this we can eliminate V naught; and I naught is already known in terms of V i.

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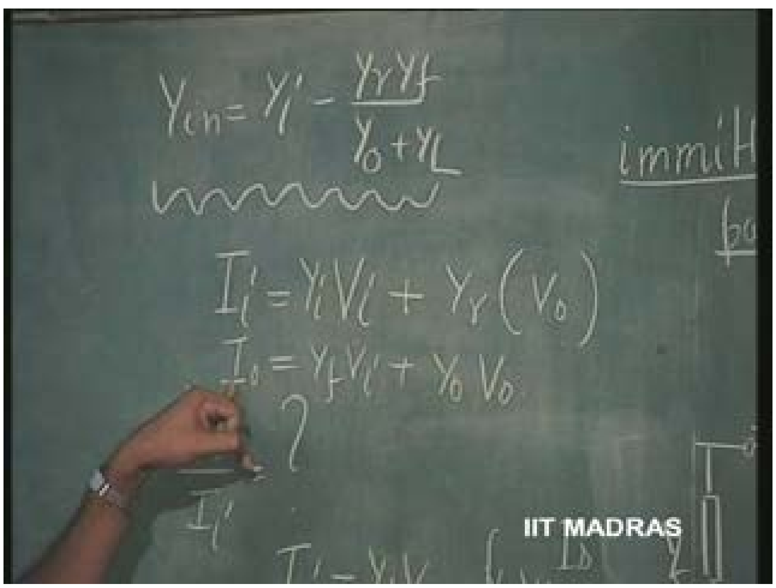

See, for example, here I naught is equal to what? Minus V naught into Y L; because, V naught is this. And, there will be a Y L admittance here. And the current flowing in that is going to be in the opposite direction to what we have assumed. So, this is always going to be minus V naught into Y L.

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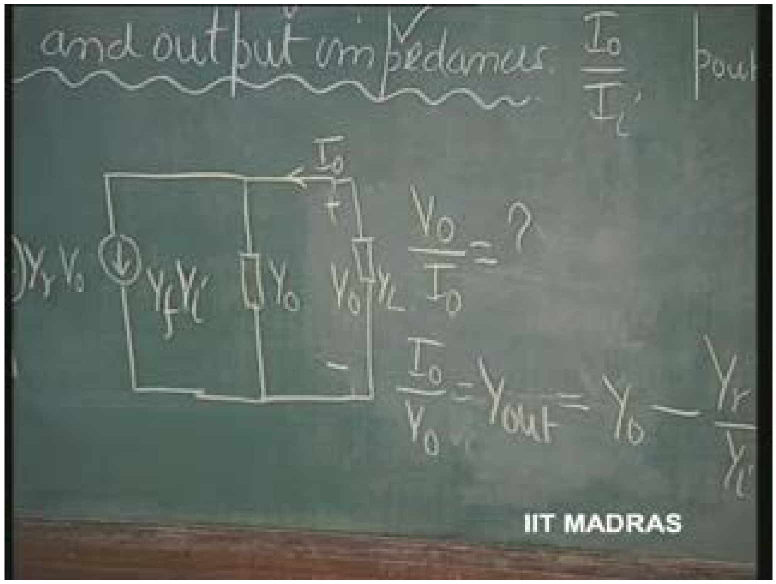

Or, we can now see that this relationship is, zero equals Y f into V i plus Y naught plus Y L into V naught. Or, from this, we can derive, V naught over V i which we have already done earlier, as minus Y f divided by Y naught plus Y L. That is, from using the second equation and the information that this current I naught is equal to minus V naught into Y L, we can get rid of one parameter here, I naught.

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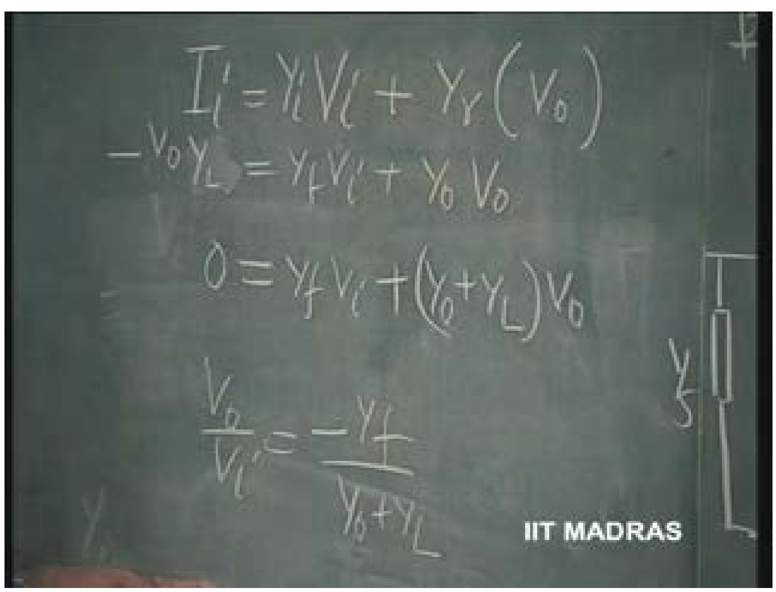

So, we get V naught over V i this way; and that V i, V naught, we substituted here and got the information about Y i as, Y in as, Y i minus Y r Y f by Y naught plus Y L.

Now, as a home work problem, you please try to work out I naught over I i in a similar fashion. You do not have to now get rid of I naught; retain I naught. Express V naught in terms of I naught; and get rid of V naught; and get rid of V i; and you will get a relationship between I naught and I i.

In the next class, I will indicate to you, if you try and do not get, I will indicate to you how we can get I naught over I i, the result; and also how to obtain Y out. I have given you the method of obtaining it. In case you are finding it difficult to get it, we will work it out again, fully.