## Electronics for Analog Signal Processing - I Prof. K. Radhakrishna Rao Department of Electrical Engineering Indian Institute of Technology - Madras

## Lecture - 14 Cascading of Amplifiers (Contd.)

In the last class, we were discussing cascading of amplifiers. Let us now take an example so as to illustrate the effect of cascading. An amplifier A with R i, the input resistance equal to 10 Kilo ohm and R naught equal to, output resistance of the amplifier, 5 Kilo ohm, has an open loop gain, that is at the output, load is open, open loop gain of 100. It is cascaded with another identical, that means, the other amplifier has the same value of R i, R naught and open loop gain; identical means, same value of R i, R naught and open loop gain; identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical means, same value of R i, R naught and open loop gain, identical amplifier B. You call it B. Full stop.

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The source has a resistance of 4 Kilo ohm. This is the source driving the first amplifier. The load resistance for the amplifier B is 5 Kilo ohm. Determine the overall voltage gain, current gain and power gain. That is the problem. Let us see pictorially, how this cascading looks like. We have the source, with source resistance of 4 K connected to amplifier A; at the input, it has 10 K input resistance. Now, that amplifier has an open loop gain which is 100 times V i 1. Vi 1 is the input to amplifier A. So, 100 times V i 1 and a series resistance output resistance of 5 Kilo ohm. It is connected to another amplifier. The load for this amplifier is going to be the input resistance of the next amplifier. So, that is 10 K. And, it has a gain 100 times V i 2 as the gain from here to here, open loop gain, and a series resistance of 5 K and a load resistance of 5 K. This is the pictorial depiction of the problem.

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So, how do we now solve? What? For the example, we would like to know V naught 2 which is the final output voltage, divided by V s. This is one voltage gain we would like to know. Another voltage gain is V naught 2 divided by V i 1. This is the overall amplifier voltage gain; this we have defined. This is from the transducer, the voltage gain, including the source resistance.

So, then, we would also like to evaluate I naught 2, final output current here, going through the load, divided by I i 1, which is the current gain. I naught 2 divided by I i 1, current gain.

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I have represented this as a current source in shunt with an impedance. I would have had I naught 2 divided by I s; that is another gain corresponding to this. You can leave it like that. Then, we have the power gain P naught divided by, P naught divided by P i, which is... P naught is this voltage at this point, V naught 2 into I naught 2, see, V naught 2 into I naught 2, and P i is going to be V i 1 into I i 1.

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So, it is very clear that if we evaluate the current gain, this one, and the voltage gain, this one, we will get the power gain.



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So, this is just multiplication of what we have here. So basically, we have to evaluate this, this and this. This is self-explanatory. Now, we will do it step by step.

First, let us go from here. V i 1 divided by V s. What is it? V i 1 divided by V s is 10 divided by 10 plus 4. That is already evaluated. Then, let us evaluate, V naught 1 by V i 1. Now, what is V naught 1? You have 100 times V i 1 coming here. So, 100 times V i 1 comes here straightaway; that source is going to have the voltage divided between this 10 K and 5 K. So, we have 10 K divided by 10 K plus 5 – fifteen. So, that is equal to what? V naught 1; which is the same as V i 2.

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V naught 1 is the same as input to output of this; the same as input to the second amplifier. So, we have evaluated this. So, V naught 1 divided by V i 2, V i 1 or V i 1 is going to be equal to 100 into 10 by 15, which is also same as V naught 1 by V i 2. So, this is not yet over.



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We have only partially evaluated the gain from here to here. We have come to this stage. So, V naught, this is V naught 1 by V i 1; and V naught 1 is same as V i 2 divided by V i 1. V naught 1 is same as V i 2.

Now, coming to this stage, 100 times V i 2 comes here, out of which, only a portion appears here, 5 K and 5 K. So, 5 divided by 5 plus 5, which is 10, is going to be equal to V naught 2. Once again, V naught 2 divided by V i 2 is equal to 100 into 5 by 10.

 $\frac{100V_{1} \times 10}{15} = \frac{100V_{2}}{V_{12}} = \frac{100V_{2}}{100V_{2}} = \frac{100V_$ 

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So now, we are in business. We want V naught 2 divided by V i 1 ((as ... Refer Slide Time: 09:20)) which can be written as V naught 2, that is our final thing, divided by V i 2 into V i 2. So, V i 2, V i 2 gets cancelled; divided by V i 1. Is this clear?

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V naught 2 by V i 1 can be written as V naught 2 by V i 2 into V i 2 by V i 1, mathematically. So, this is simply multiplication of these two factors, which we have already evaluated. V naught 2 by V i 2 is given there as 100 into 5 by 10, into, the other portion, V i 2 by V i 1 has already been evaluated as 100 into 10 by 15.

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Now, this is very easy to evaluate. 2, 50, 3. So, 200 into 50, 200 into 50. So, 10,000 by 3.



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So, this is... That is the voltage gain. 3,333 point 3 is the voltage gain, overall voltage gain. And, if you want V naught 2 by V s, we know that V naught 2 by V s is nothing but V naught 2 by V i 1 into V i 1 by V s; so, which we have already evaluated. V i 1 by V s is 10 by 14; and therefore, this is 3333 point 3 into 10 by 14.

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That is the voltage gain. So, the overall gain, V naught 2 divided by V s therefore equals 3,333 point 3 into 10 by 14, how much is it? 2380 point 9. So, that is the overall voltage gain.

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Now, we would like to evaluate the current gain in a similar fashion. This, we will try to evaluate using the information about voltage gain. Let us see. We know that V i 1 is equal to I i 1 into 10 K. So basically, we have I i 1 equal to V i 1 by 10 K and V naught 2 is going to be 10 K, that is, 5 K the load resistance, into I naught 2.

Now, this is plus and minus; therefore, there is going to be negative sign, really speaking. So, I have assumed this as 5 positive and this as negative. If I say I naught 2 is going like this, it will develop a drop here, plus and minus. So, that is the meaning of the negative sign. V naught 2 is this. So, I naught 2 is equal to minus V naught 2 divided by 5 K.

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So, what we want to now evaluate is I naught 2 divided by I i 1. I naught 2 divided by I i 1. That is going to be equal to minus V naught 2 by 5 K. That is I naught 2. And, I i 1 is V i 1 by 10.

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That means, it is minus 2 times the voltage gain, V naught 2 by V i 1, which we have already evaluated as 3,333 point 3, which is, minus 6,666 point 6, with the negative sign indicating, this negative sign is important. Let us understand.

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I i 1 is going in and I i 2 has been assumed to be going in. So, actual current flow is opposite to that of what I have explored. That is the meaning of negative sign. That

means, I naught 2, if it is this sign, the actual current is going to flow in the opposite direction. Therefore, I naught 2 and I i 1 are actually having a phase difference of 180 degrees or a negative sign. So, this is the current gain. And therefore, the power gain is P naught by P i; therefore, is equal to the voltage gain V naught 2 over V i 1, which is 3,333 point 3 into I naught 2 by I i 1, magnitude of this, this into 6,666 point 6.



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This expressed in decibel, because this is a huge quantity; so, let us express this as 10 log to the base 10 of this factor. That is the power gain expressed in decibel. So, P naught over P i in decibel, which is, 10 log natural, that is not natural log, 10 log to the base 10 of this. So, how much is it? 73 point 46. So many decibels is the power gain, which is 10 log of log to the base 10 of this.

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Now, let us try to understand the concept of amplifier more thoroughly, more mathematically, more conjugatively than before. We will now go into linear two port network theory a little bit, so as to understand, how we can link what we have learned in networks with what we are going to do with amplifiers. This, we have already done. We have defined a two port network in the following manner. It has input voltage, input current, output voltage, output current.

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A one port network, linear; it has a voltage and a current.

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Now, some kind of mathematical concepts here. If V is independent variable in a one port network, if V is independent variable, what does it mean practically? If I am going to fix V by ways of a voltage source, a battery, I is going to be determined; or, I is going to become dependent variable. That means, if I fix the voltage here, then, I is dependent upon the property of the device here. Let us call this R; or, let us call this G conductance.

So, G into V is going to be I. That means, if V takes on different values, I is automatically determined by the device that we are connecting. So, this is called independent variable; this is called dependent variable. That means, if I connect a voltage source, the current is always going to be dependent upon the G parameter.

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That is, ((left to me actual, Refer Slide Time: 19:25)) why should I take V as independent variable? I can on the other hand say, I is independent variable. That means, I can force certain current into the element here, one port; then, V is going to be determined as I into R. So, R is going to be 1 over G here. The resistance is 1 over the conductance; so, V is equal to I over, I into R.

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So, I is independent variable; then, V is dependent. This is what we have to understand; what is independent variable and what is dependent variable. This is for the one port. There are two parameters here; two variables, V and I. If one is independent, the other is dependent.

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Let us now consider the two port parameters actually; two port variables. The variables are: V i, I i, I naught, V naught.

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There are four variables out of which any two can be independent variables. There are four variables which are: V i, I i, at the input port, V naught, I naught, at the output port.



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Four variables, out of which any two, if chosen as independent, the other two become dependent variables.

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Just as in the case of one port, if one is taken as independent, the other becomes dependent. In two port network, there are four variables. I can select any two of the four as independent variables; the other two become dependent variables; in which case, there are basically four ways of defining the relationship between these different variables.

There there are two ways; for one port, there are two ways; for two port, there are four ways. And all these four ways were exposed to you in networks. Let us see what is that.

Let us consider the first way. I am considering that V naught and V i are dependent variables. They are dependent upon I naught and I i. That means, they are functions of I naught and I i. This is any function; but we are discussing about linear two port network. That means they are linear functions. This relationship says it can be any functional relation.

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So, V naught is going to be linearly related to I naught and I i; V i is going to be linearly related to I naught and I i. So, in which case, what is it going to be? V naught is equal to I naught, I i into something, plus, I naught into something; factor, linear dependency. What are these factors? This is a voltage and this is a current. Current into impedance. Current into impedance is voltage. So dimensionally, this is impedance. So, impedance is commonly given as Z; so these parameters are called Z parameters.

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So, this will be Z; this also is Z. Let us see what this is. This is relating output voltage with input current. This is relating output voltage with input current. This is relating output voltage with output current. So, this is called Z naught which is really the open circuit output impedance. This is impedance, V naught and I naught; we will see this. V naught and I naught are related by this factor, Z naught. When will you get Z naught? If I i is zero. If I i is zero, then, we can get Z naught as V naught by I naught.

This is therefore called open circuit output impedance. So, these are all called open circuit parameters. That means these parameters can be defined by respectively open circuiting input or output. Why? Because, the dependent variables are V naught and V i; independent variables are I naught and I i; so, those independent variables can be made respectively zero.

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And then, for example, this is an independent variable. If I i is made zero, I can get Z naught. If I naught is made zero, I can get this. What is this? This is giving you output voltage when there is a certain input current. This is what is called the forward transfer current. Something happens at the output because something is happening at the input. So, this is called the forward transfer current. This is called the open circuit output

impedance. This is called forward transfer impedance. So, this f indicates forward transfer, from input to output it is transferring, transfer impedance.

Remember this. We have already discussed this parameter earlier. We called it transfer impedance, forward transfer impedance, or resistance. At that time, we kept it restricted to resistance. Now, it is more general. It is impedance. This is output impedance. Both these things are measured respectively with this. This is measured when output is open. V naught is going to be open circuit voltage and this is measured when I i is zero; that means, input is open.

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In order to measure this, we have to have I naught equal to zero; which means, output is open. In order to measure this, output is open; in order to measure this, input is open.

Now, it is very easy for you to compose the other linear equation; this is for V naught. V i equals, in a similar fashion, something connected with I i, plus I naught, into the impedance. So, this is again Z parameter. So again, let us see whether you people can themselves indicate what suffix we must give to this. Look at this. V i is related to I i; so, this is Z i, totally input port dependent. V i is equal to Z i into I i. So, this is nothing but

input impedance. When do you measure it? When I naught is zero; which means, output open. That is, input impedance with output open.

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Now finally, we have discussed this parameter also earlier. We have discussed this, we have discussed this, we have discussed this, when we discussed the ideal amplifier. Let us see what this is. This is something we had not discussed, but we indicated it here. This is what is developed at the input as a voltage because of a current at output. That means, this is the reverse transfer parameter. From output, it comes to the input.

So, this is the reverse or just like the forward, this is the reverse transfer; or, it is also called inverse transfer parameter; in this case, impedance. All the parameters in this case. How do you measure this? Again, I i has to be zero; which means, input open.

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This is a complete definition of this law in terms of what are called open circuit impedance matrix. Where is the matrix? If I now... This is a linear relationship; therefore, if I represent this as dependent variable and independent variable, then I can represent this in the matrix form, in the following way.

Let us see how this is going to be looking like. This is the port; I can put the matrix here. Z i, Z r, Z f. So, we have to call this Z r, because it is the reverse transfer impedance; and, multiplied by I i, I naught. This is equal to V i V naught; or this is, this could be also written as V column matrix, is nothing but Z; this is the impedance matrix, into I, again a column.

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So, this is the matrix representation of the parameters without really going into anything regarding amplifiers. This is valid for any linear two port network. Now, what I am saying is, let us understand this; and now see what this can lead to in terms of amplifiers. This is the general representation of any two port network, which is linear.



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What is an amplifier? Let us understand. Ideal amplifier. Ideal amplifier is one which is unilateral first. Ideal amplifier as defined by us earlier; we just said it is unilateral. What does it mean?

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There is transfer only in the forward direction; there cannot be transfer in the reverse direction. Then what does it mean? One of the parameters here is definitely zero. What is that? The reverse transfer current. That means, in any ideal amplifier represented in any fashion, in any matrix, the reverse transfer parameter is zero. That means, Z r is zero.

So, I am defining it and then making it zero, in order to illustrate, what is really an amplifier represented in a two port theory. So, the reverse transmission should be going to zero. Now, if reverse transmission is zero, Z r is zero; and Z f alone exists; and what about Z i and Z naught?

In an ideal amplifier, Z i has to be zero and Z naught also has to be zero. So, you will see that an ideal amplifier matrix representation is nothing but zero, zero, zero and Z f. What is that amplifier? This is... look at it. What is Z f? Output voltage in terms of input current.

So, this is the Transimpedance amplifier. It has been earlier called, Transresistance amplifier. So, a Transimpedance amplifier is truly an ideal amplifier. So, this particular amplifier that is defined by Z parameters is a transimpedance amplifier. Is it clear? Transimpedance amplifier.

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Now, you can easily come up with other equivalence of the dual of this. Dual of impedance is admittance; dual of impedance is admittance; which means, when will I get that? When, instead of V i and V naught as dependent variables, I will take I i and I naught as dependent variables; and, instead of I i and I naught as independent variables I will take V i and V naught as independent variables.

So, we will now go towards the other types of amplifiers. We were exposed to Z parameter.

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Now, let us see the dual of that; Y parameters or Admittance parameters. These are called Admittance parameters or Y parameters. Earlier, they were called impedance parameters or Z parameters. Let us see. Again, it is very simple to formulate now. The independent variables are V i and V naught as against I i and I naught earlier.

So, V i and V naught are independent variables and I i and I naught are dependent variables. What it means is, I i and I naught are expressed in terms of V i and V naught. So, that means, I i is some Y into V i; again, Y some other, V naught, Y into... These Ys have to be respectively defined. All these are admittance parameters; dimensionally, they are admittances.

So, I i related to V i; again, it is input, totally. That means, Y i. And, how does it get measured? When V naught is zero. So, either V i should be zero or V naught should be zero in order to measure all these parameters. That means, these are called short circuit parameters as against the earlier parameters which were called open circuit parameters.

So, I make V naught, zero, then I can measure Y i as I i by V i. That is called short circuit admittance, input admittance; so, short circuit input admittance. This is Y i.

Now, I i is related to V naught. That is, when there is something applied at output, that gets translated to input, as I i, when V i is zero. That means, input is shorted and the current flowing in the input, because of the output voltage, is called reverse transfer admittance. So, this is Y r, it is called. This is the reverse transfer admittance.

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Coming to this, again you can see, I naught, output current, is related to input voltage when V naught is zero; output is shorted. That means, output current, when output is shorted; so, relationship is input voltage. That means, this is the forward transfer admittance; therefore, Y f it is called, forward transfer admittance. It is the same as what we have discussed in the amplifier, forward transfer conductance. We took it as conductance; here it is more general, admittance.

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So finally, this is output current, related to output voltage; so, totally output parameter; so Y naught, when input is shorted. So, this is called short circuit output admittance. So, these are all short circuit parameters; and therefore, I matrix can be represented as Y matrix into V matrix. So, that means, I i, I naught, is Y i, Y r, Y f, Y naught, Vi... This is the matrix representation of Y parameters.

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The same equation that we have written, the linear equations, can be represented by matrix this way; and what is this? This is a generalized way of representing any two port network, any two port network, which we have learned in the class. But, we would like to relate it to amplifier. So, an ideal amplifier of this category is one where Y r is necessarily zero; that means it is unilateral.

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And subsequently, we must have the other parameters, Y i and Y naught also equal to zero. The only thing that can exist is Y f, is finite. In such an amplifier, we have called as Transadmittance, which we have earlier called as Transconductance amplifier. As against the Z parameter defined amplifier, which is Transresistance amplifier or Transimpedance amplifier, this is called Transadmittance amplifier.

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This is going to give you primarily a voltage controlled current source. This one is going to give you a current controlled voltage source, ideal. This is ideal. That is it.



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Now, one would say that these two amplifiers are more basic than the other two amplifiers. What are the other two amplifiers? Voltage controlled voltage source and current controlled current source. Why? We can see, this is voltage controlled current

source and that is current controlled voltage source. I can cascade these together. Voltage controlled current source is cascaded to current controlled voltage source. This is a current source feeding a current controlled block; very nicely matched; current source feeding a current controlled block. What it means is current source, a source with infinite impedance feeding on to a short circuit. It is very good, here; and therefore this block now becomes, what? A voltage controlled voltage source.



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So, I can get a voltage controlled voltage source by cascading ideal voltage controlled current source with current controlled voltage source. And, what is the voltage gain; because, this should be the voltage gain. If the... This is voltage controlled current source; this is a transconductance amplifier or transadmittance amplifier. So, this is defined by Y f. This is defined by Z f prime, let us call it, because these are different parameters; here, it is to indicate that Z f is not 1 over Y f, it is different. So this is Y f, this is Z f prime. So, voltage controlled voltage source has a transfer parameter which is Y f into Z f prime ratio.

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This is nothing but your so called K or whatever it is. It is now, this is not all. You can get current controlled current source, ideal, by first putting current controlled voltage source, cascade it with voltage controlled current source. So, look at it. This is a voltage source; that means source resistance is zero; feeding on to a voltage controlled block, which is infinite impedance. So, very nicely matches here; and therefore, this is a current controlled current source, with the same Z f prime into Y f as the current gain now, instead of the voltage gain.

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So, one would say that it is enough if we have these two blocks realized; the other two blocks can be easily realized. Whereas, if we have only the other two blocks, these two blocks cannot be realized. So, these two are more basic than the other two. Even though other two seem to be both popular, these two, if you try to realize, it is possible to realize all the four types of amplifiers. So, it is necessary that these two must be studied and possibility of designing these two optimally should be investigated so as to sort of unnecessarily not waste time on other two designs.

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So, we have voltage controlled voltage source realized this way; current controlled current source realized this way; and therefore, we could say that these two amplifiers, ideal amplifiers, are more basic than the other two, which will be again discussed in terms of ... Now, what are the parameters? You have already taken I i, I naught, V i, V naught; on one side this will come, on other side this will come, interchange.



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Now, we will take I i here and V naught here and vice versa. These are called hybrid, mixed; parameters now will not have the same dimension. They will have mixed dimension. So, these are called hybrid parameters - h parameters or g parameters. These two parameters, we will discuss and see the various types of ideal sources which can be realized using these, in the next class.