Electronics for Analog Signal Processing - I Prof. K. Radhakrishna Rao Department of Electrical Engineering Indian Institute of Technology - Madras

Lecture - 13 Cascading of Amplifiers

In the last class, we learned about ideal voltage amplifier which we call as voltage control voltage source, ideal. Ideal current amplifier, then we said, is nothing but current controlled current source. Very simple terminology regarding what is meant by voltage controlled and what is meant by current controlled, we learned, wherein, if it is voltage controlled, obviously, at the input port, I i is zero so that, V i is what is controlling the voltage at the output, as K times V i. This is what is appearing at the output and the source has zero resistance in series. It is a voltage source with zero or naught output resistance and it has a voltage which is K times V i. This is the characteristic of an ideal voltage amplifier.

(Refer Slide Time: 02:37)

IIT MADRAS

Then, we said, if it is current controlled, obviously R i is going to be zero, ideally; and therefore, V i is zero for the port, input port; which means, it is basically controlled by I i

and output current is going to be short circuit current, of course. It is going to be K times I i and the source, current source, which is having K times I i as the current has a shunt resistance of infinity.

IIT MADRAS

(Refer Slide Time: 03:15)

So, this is where we had stopped - the current control current source; and we said, in practice, obviously, we cannot have any of these ideal amplifiers. We have to know, how we can recognize an amplifier as a current amplifier or as a voltage amplifier, in practice.

And then we said, in practice, we have a non-ideal source which can always be represented either as a voltage source in series with the resistance R s or as a current source I s, which is nothing but the short circuit current in the voltage source representation and a shunt resistance of R s. So, this representation, equivalent representation does not mean anything. It is the same, whether you represent it as a voltage source in series with the resistance or a current source in shunt with the resistance.

(Refer Slide Time: 04:18)

Only thing is, ideal independent sources cannot have any equivalents; ideal voltage sources and ideal current sources cannot have equivalence. Non-ideal sources can always be put down as voltage source in series with the resistance or current source in shunt with the resistance. If this is the current controlled current source or current amplifier, obviously, in a practical current amplifier, we will have some input resistance R i and what we want is that R i should be a short circuit. What does it mean? R i should be such that it should take most of the I s, which is possible only when R i is much less than R s; and then, we say that the input port is current controlled.

(Refer Slide Time: 05:34)

So, we can contrast this with voltage control wherein R i was much greater than R s; that was voltage control.

(Refer Slide Time: 05:50)

So, once again, current source output, K times I i, K times I i, is going to be shunted by a finite output impedance of the amplifier.

(Refer Slide Time: 06:22)

This way, you are representing a current source, non-ideal, at the output. This is a dependent, non-ideal current source; so, it is going to be K times I i shunted by R naught. And, if it is a current source, then, R naught is going to be much greater than R L so that most of this current still goes through R L and very little is lost through $($ (its output....Refer Slide Time: 6:55))

So, you can compare this with what we said can be considered as a voltage source, wherein, R s is much less than, that is, R naught is much less than R L.

(Refer Slide Time: 07:20)

So, this is the way we can distinguish between current sources and voltage sources practically and current controlled and voltage controlled source; thereby, declare these as either current controlled or voltage controlled at the input, or voltage source or current source at the output; thereby, distinguish, for a given source combination and a load, the given amplifier can be identified as either a voltage amplifier or as a current amplifier.

This particular aspect can be illustrated just as we did in the case of voltage controlled voltage source type of amplifier or voltage amplifier. Example 2 - An amplifier with R i equal to 1 Kilo ohm is fed by an independent current source with source resistance R s equal to 10 Kilo ohm. Automatically now, source resistance is 10 Kilo ohm and input resistance of the amplifier is 1 Kilo ohm.

(Refer Slide Time: 08:49)

An amplifice with Ki=Ik 6 fed by an indiformidint Resistance RS = 10K. St arount current game **IIT MADRAS**

So, it can be declared as current controlled. If the short circuit current gain is 100, that means, K is given as 100, it is given as 100 and R naught of the dependent current source, this is the R naught, the dependent current source is 100 K, determine I naught over I s, that is, the current gain, for a load resistance of 10 Kilo ohm.

(Refer Slide Time: 09:20)

arrent source with source Resistance Rs = 10K Shiti short are avoint gam (which) Q/η load resistance **IIT MADRAS**

Now again, since the load resistance is 10 Kilo ohm and the current source resistance is 100 Kilo ohm, it can be declared as a current source. So, this is definitely a current amplifier and we can now solve this problem saying that the current I s now, is going to be divided between this and this; very little will go through this because, we have chosen our parameters properly so that it is acting as a current controlled source.

So, I i is going to be, this is 10 K and this is 1 K; this is 10 K and this is 1 K. So 10 divided by 10 plus 1 which is 11, times I s. So, strictly speaking, if it is current controlled, we would have got this very close to 1; this would have been 1.

(Refer Slide Time: 10:30)

So, that is the way to identify a gain. I s, what part of I s is going into the amplifier truly as I i? If it is close to 1, it is declared as current controlled. Again, this I i is multiplied by 100 because of the current gain, K is equal to 100; and R naught is 100 K,

R L is 10 K. This hundred I i is going to divide between 10 K and 100 K as 100 divided by 110, 100 divided by 110. As a result, we have therefore, I naught equal to 10 by 11 square; this is one 10 by 11, this is another 10 by 11, into..., times I s.

(Refer Slide Time: 11:45)

So, we get the value of I naught over I s for this problem as about 100, reduced by factor 10 over 11 because of the input problem, 10 over 11 because of the output problem; so 10 over 11 square into 100.

(Refer Slide Time: 12:12)

Or, if you want to express this as decibel, it will be 20 log, this 10 over 11 whole square into 100, which is, of course, pretty close, this being close to 1, this is about 2 into 20, which is 40 decibels.

(Refer Slide Time: 12:41)

So, this gain now can be more exactly given as 20 log to the base 10 100 minus, so plus, 20 log to the base 10, 10 over 11 whole square.

(Refer Slide Time: 13:03)

This is less than 1. So, this becomes minus and therefore 11 over 10, square. So, this is equal to the same 40, minus, this is evaluated, comes out to be 1 point 66, so many decibels, which we had approximated earlier as 40 decibels.

IIT MADRAS

(Refer Slide Time: 13:30)

Now, coming to the power gain, which is defined as, P naught over P i, which is nothing but V naught I naught divided by V i I i, this can be either represented solely in terms of voltage gain or current gain in the following form. V naught is nothing but R L into I naught, actually, magnitude if you take, because there is $($...sign Refer Slide Time: 14:00)), R L into I naught is V naught, I naught square; and then V i is I i into R i; so, R i into I i square. V naught is R L into I naught; V i is R i into I i.

(Refer Slide Time: 14:27)

Replacing it with this, we know that the power gain is same as current gain square into R L by R i, which is nothing but R L, in our case is 10 K and R i is 1 K; so, that times the current gain, which is already known to be 100 into 10, 100, into 10 by 11. This current gain is the current gain from input to output; not, I naught over I s.

(Refer Slide Time: 15:00)

That is what we had evaluated here. So, I naught over I i is nothing but... the actual short circuit current gain is 100; then it gets divided between R naught and R L; so that factor of 10 by 11, this square.

(Refer Slide Time: 15:22)

So, power gain in decibel is 10 log base 10 of this factor, which is 10, into 100 square 10 by 11 square; which is going to be equal to, 10, log to the base 10, 10 to power 5, minus 10 log to the base 10, 11 by 10 square.

The same factor, log to the base 10, 11 by 10 square has been earlier evaluated and therefore you can use that information. This is very close to 50 decibels. So, please find out how close it is to 50 decibels. You have the same factor evaluated earlier.

(Refer Slide Time: 16:33)

MADRAS

So, this is the power gain. So, you know about current gain, voltage gain and power gain; the three important parameters associated with any amplifier. For any amplifier, we should be able to evaluate these in terms of decibels normally, so that we know the performance of our amplifier. Whether we need one amplifier to fulfill our requirement or two amplifiers, that can be readily found out from how many decibel is your requirement in terms of amplification. If it is 100 decibels in terms of amplification that is required for power, then you know that two such stages must be cascaded. If it is 150, three such stages must be cascaded. We will discuss this later, how to do cascading to improve the power gain of stages.

Now, we have yet to finish our discussion on amplifiers; the types of amplifiers. We have just discussed voltage amplifiers and current amplifiers. We have to now discuss about two other important amplifiers, ideal amplifiers. So, we discussed voltage amplifier, which was an ideal voltage controlled voltage source. We discussed ideal current amplifier, which was ideal current controlled current source.

(Refer Slide Time: 18:50)

Just as ideal voltage source cannot be represented in terms of ideal current source, these voltage source and voltage amplifiers and current amplifiers are unique by themselves. They cannot be represented by the other method; but a non-ideal amplifier on the other hand, can always be represented as a voltage controlled voltage source or current controlled current source. That kind of equivalent circuit can be put. But, what it is really is determined by basically how it is driven by means of the source and how the load is connected.

So now, the other two, obviously, you must have thought of readily, possible amplifiers are when the input port is controlled by voltage, but output port is having, not voltage source, but current source. This is ideal voltage controlled current source wherein R i is infinity, or, I i is zero and this being a current source, R naught is also infinity; and, it is going to be controlled by current output which is, I of that is going to be, transconductance G. This is conductance and it is a transfer from input to output. That is why this is called transconductance. So, this G times V i, R naught is zero; so this is basically called a Transconductor.

(Refer Slide Time: 21:04)

As against voltage amplifier, where the transfer, forward transfer parameter, from input to output was voltage gain here; in this case, it was current gain; in this case it is transconductance. This amplifier is called transconductor amplifier, which is becoming, gaining popularity in present day integrated circuit design.

As against this, we have ideal, instead of voltage control, it would be current controlled at the input port; and at the output port, it would be a voltage source. So, a current controlled voltage source, amplifier. Let us see. Current controlled means, R i of the amplifier is zero and it is current controlled; therefore, we have, just as I i zero here, V i equal to zero. And, it is a voltage source; so, voltage is equal to R resistance, times I i input current. So, this is therefore called, just like this transconductance, this is Transresistance. Just see, transresistance, or this whole thing is called Transresistor amplifier as against Transconductor.

(Refer Slide Time: 22:41)

an Cr **IIT MADRAS**

So, Transresistor, you have an abbreviation, Transistor. That is how transistor derives its name. But it is really not ideal current controlled voltage source. It is close to ideal current controlled current source. We will discuss it later; but transistor derives its name from transfer resistor; so it is one of the ideal amplifiers here. We have here voltage source, so R naught is equal to zero. So, this characterizes the two important amplifiers which have been left out: current controlled voltage source and voltage controlled current source.

(Refer Slide Time: 23:47)

ductance **IIT MADRAS**

So now, let us consider the two different amplifiers which we have discussed; one is transconductor amplifier and transresistor amplifier; when again, the amplifier is not ideal; just as we considered earlier - voltage amplifier and current amplifier.

(Refer Slide Time: 24:23)

How do we recognize that a particular amplifier belongs to a particular category? That is going to be done here also. It is voltage controlled at the input; which means, R i earlier was infinity, in the case of ideal amplifier; here, R i is much greater than R s, that is voltage controlled. It is current source at the output; that means, R naught is much greater than R L.

(Refer Slide Time: 24:58)

So, under this situation, under this drive, source impedance being fixed like this; and load impedance being fixed like this; or, when input impedance is suitably chosen such that it is this; and output impedance of this is like satisfying this condition; then, we call it a transconductor amplifier. And the most important parameter of interest, at that point of time, is the transconductor. However, we can still evaluate the voltage gain and current gain for this circuit, just as we did on power gain, again, for this circuit, in the same manner as we did earlier.

Even though the important parameter associated with this has to be recognized as transconductance, you can just see that a transconductor amplifier is pretty close to a practical field effect transistor amplifier, where, field effect transistors are traditionally voltage controlled devices and the transconductance $gm - small gm$ it is known as $-$ is what is important as parameter for field effect transistor and the output resistance R naught is typically of the order of tens of kilo ohms. And therefore, for resistances of the order of hundreds of Kilo ohms, it can be acting as a voltage source; but resistances of the order of Kilo ohms, it can still act as a current source.

So basically, a field effect transistor belongs to this category and as I told you, a bipolar junction transistor, strictly speaking, the common base amplifier, which we will see is current controlled current source. A common emitter can be pretty close to this amplifier where, R i is not really infinity; R i is, may be, of the order of tens of Kilo ohms. And therefore, for source resistance of the order of Kilo ohms, it can be still considered as voltage controlled, and it is strictly speaking, a good current source, where R naught is of the order of hundreds of Kilo ohms for BJTs, bipolar junction transistor.

So, for load resistances, which are typically of the order of tens of Kilo ohms, the BJT can be still considered as a current source output. So here, it can become voltage controlled if it is a common emitter. If it is a common base on the other hand, it is strictly current controlled at the input and current source at the output and therefore, it is a current controlled current source.

So, we have later, ample opportunity to identify the various devices that we are going to use as amplifiers, as one of these, depending upon the type of source resistance and type of load resistance. This is something that you have to understand very well at this juncture itself so that you are not confused as to what exactly is belonging to what category; or, out of these four. So again, here, this is a current controlled voltage source. This is a voltage controlled current source. So, current controlled means, R i is much less than R s and R naught is much less than R L; dual of that.

(Refer Slide Time: 28:53)

Transresistance, the transfer resistance is what is important here. Still, just as in the other cases, we can evaluate the voltage gain and current gain and subsequently the power gain of this stage. And as I told you, this kind of transfer resistor amplifiers are not all that common, practically. Most of the amplifiers which are being designed in present day, either they are going to be voltage controlled voltage sources or this transconductor type. Very few of current controlled current source type as well as this current controlled voltage source type are in existence; but with the advent of new designs in integrated circuits, the current controlled current source has become popular. And recently, people have attempted to design current controlled voltage sources as well.

So traditionally, we have been taught only about voltage controlled voltage source as amplifier. However, we have now recognized the existence of four types of amplifiers, ideal amplifiers; and these four types of ideal amplifiers cannot be exchanged in terms of... I will represent it in this manner; or, I will represent... These are unique by themselves. When a non-ideal amplifier exists, then, we can represent that non-ideal amplifier by any one of the four categories. Now, what it is going to be out of the four is going to be really defined when R s is fixed and R L is fixed.

So, let us now summarize what we have been discussing so far. There are four types of amplifiers: voltage controlled voltage source which is called voltage amplifier, current controlled current source which is called current amplifier. These are what are called duals. One is the dual of the other. You have done voltage controlled current source, which is called transconductor amplifier; and current controlled voltage source, which is transresistor amplifier. These are, once again, what are called duals.

(Refer Slide Time: 32:00)

1 VCVS voltage amplifier
2 CCCS current amplifier
3 VCCS Transconductor a **IIT MADRAS**

What is a dual? Let us recollect what we have learned in our networks. R, dual of R, is G. Dual of a resistor is a conductor. Dual of L is C. Dual of C is L. Dual of a voltage source is current. Dual of current source is V. These are what are called duals.

(Refer Slide Time: 32:37)

anspeciator **IIT MADRAS**

So now, I told you about cascading. Cascading means what? Connecting one amplifier to another amplifier so as to, $((\text{kept Refer slide Time: } 33:00))$ obviously, increase power amplification. You are not happy with whatever the one amplifier is giving; you would like to connect one amplifier in cascade with another amplifier.

So, let us see now. Cascading. This is amplifier 1, is cascaded to amplifier 2; and then, we are connecting, let us say, the load; and this is the first amplifier driven by the source, non-ideal source. Whether it is voltage source or current source, both are, let us say, nonideal; then, it does not matter which one we use for a $((\dots \text{Refer~Silde Time~} 33:55))$

So, amplifier 1 is said to be cascaded to amplifier 2, forming an overall amplifier, which will give you better performance in terms of forward transfer. As the transfer is concerned, we said, each of them is unilateral. What it means is, transmission is there only from input to output, from this input to this output. Therefore, overall transmission is from this input to this output. There is no connection from here to here, that is back. The reverse transmission is not there.

(Refer Slide Time: 34:34)

So, these are all unilateral circuits. In such a situation, it is very easy to show that the amplification factor is improved. So now, let us try to evaluate, for example, the voltage amplification factor here, and current amplification factor, here as well as here, because these are two different types, where the voltage amplification and current amplification have not directly made an appearance.

So, after that, we will cascade the non-ideal amplifiers together. So, these non-ideal amplifiers again could be represented in only in any of the four and they will be called a particular type depending upon what kind of source and load they have. Now, we can see that, as far as load for this is concerned, it is going to be the input resistance of the amplifier two, which will act as the load for amplifier one.

So, it is the input resistance of this R i 2 which will act as the load for amplifier 1; and also, as far as this amplifier 2 is concerned, look at it, at this point, this is the point where, it is looking at a source, as far as this is concerned, this is the feed for the amplifier 2. So, there, this amplifier 2 can only look at this amplifier 1 as a source. It does not know whether it is an amplifier or a source. It cannot make a distinction. So we can represent this amplifier 1 with the drive there, as a source for this. That means, output resistance of this, is going to be the source resistance for this. So, this can be represented as a voltage source in series with some output impedance which will be really the source resistance for this. So, you have to make a distinction.

Now, when we connect an amplifier 1 to amplifier 2, for amplifier 1, amplifier 2 is the load; for amplifier 1, amplifier 2; for amplifier 1, amplifier 2 is the load now; that means, load is not really necessarily a resistor. It can be an amplifier.

(Refer Slide Time: 37:11)

So, for amplifier 2, amplifier 1 is the source.

(Refer Slide Time: 37:33)

That means, I should be capable of representing amplifier 1 output as a source, voltage source, in series with the resistance; or, current source in shunt with the resistance. That is all.

(Refer Slide Time: 37:56)

Now, you can see here, take any amplifier. This output of the amplifier is already represented as a current source in shunt with this resistance R naught. So, this load can be removed. Now, this will be going to another amplifier, where R L is going to be the input resistance of the next stage. It is already represented as G times V i shunted by R naught; or, if you think that this should be represented as a voltage source, you can represent this. So, I will make the alteration here. G times V i shunted by R naught can be represented as a voltage source in series with the same resistance R naught; but, this voltage is now going to be G times V i into R naught.

(Refer Slide Time: 38:58)

It is nothing but what is defined as open circuit voltage of the source. That is, G times V i times R naught. So now, you see I have represented this as a voltage source with G times V i times R naught; and you will call this G times R naught as the open circuit voltage gain. So now, this was voltage controlled; this has now become a voltage source. And this is, now, G times R naught is what we have earlier defined as K, so this is, K times V i.

So now, we have already evaluated for such a voltage controlled voltage source, the voltage gain and current gain. So, the same expression is valid for voltage gain and current gain except that K has to be replaced by G times R naught. Is this clear?

Next, let us see that. This is already a voltage source; now the voltage here is R times V i I i.

(Refer Slide Time: 40:23)

I would like to represent this in terms of V i. Let us say, this is really the V i of this stage. So, I know that V i is nothing but I i into R i; so I i is nothing but V i by R i.

(Refer Slide Time: 40:57)

This has been already represented as a voltage source. Now, if it is voltage controlled, the voltage here should be this voltage V i, times, some K. So, it is K times V i. So K is now represented as R divided by R i. That is the only difference and now all those expressions for voltage gain and current gain that we have derived, when we discussed current amplifiers and voltage amplifiers, are valid here and the power gain also is, as long as the factor K is replaced by R divided R i.

So, as far as analysis is concerned, for the non-ideal amplifier, it is enough if you do the analysis of one type of amplifier. All those things that are discussed for that can be still made valid, depending upon how you look at the equivalent circuit. So, K in this case has been G times R naught, where G is the transconductance value and R naught is the output resistance of the amplifier; and K here is nothing but R, which is the transresistance divided by input resistance R i.

So, with this, what all we have discussed in our analysis in earlier situation, becomes valid. So, I would like you to work out a problem. The problem is - Problem 1. Determine V naught over V s; or, let us say, I naught over I s, for VCCS and CCVS, when these are non-ideal.

(Refer slide time on 43:45)

dua **IIT MADRAS**

So, please work out this expression. The hint that is given here is that the expression that we have derived earlier for V naught over V s, in the case of voltage controlled voltage source, I naught over I s in the case of current controlled current source, become valid; except that, you have to utilize, in the case of voltage controlled current source, the K by G R naught, and in the case of this current controlled voltage source, K by R. Except that,...

(Refer slide time on 44:28)

So, if you work out this, you will understand that all these amplifiers can be used interchangeably, as long as they are non-ideal; and also, you will understand and appreciate what the load means. Load is always the input impedance of the amplifier, when it is an interface like this between amplifiers, two amplifiers; and the source is the output resistance in series with voltage source, if it is a voltage controlled representation; R naught shunted by a current source, if it is a current controlled representation. They can be used interchangeably.

So this is the information about cascading. Another important, you will say theorem, that we can derive for cascading is, what is I naught over I i? Let us say, you call it as current gain, A i. A v is V naught over V i, for the amplifier.

(Refer Slide Time: 46:03)

This is for the amplifier; amplifier current gain, amplifier voltage gain. Then, this is called, with transducer, that means, I naught over I s. And, V naught over V s, you call this as A I s and A v s. Distinction. This is the short circuit current for the transducer; this is the open circuit voltage for the transducer. Here, we are ignoring the effect of the source impedances. In this case, the source impedance is, let us say, is so high that most of the current goes into the amplifier. In this case, source impedance is so low, most of the voltage comes across the input.

So, this is the basic difference between I naught over I i and I naught over I s V naught over V i and V naught over V s. Always, V naught over V s is going to be greater than V naught over V i by a factor which is going to be determined by R i by R i plus R s. Same thing is $((...$ Refer Slide Time: 47:30)). A I s is going to be greater than A i; that is, A I s into R s divided by R s plus R i is what is going to be the factor involved in both. There is no difference in the factor; factor is the same.

MADRAS

(Refer Slide Time: 47:33)

Now I want to prove an important theorem in cascading. What is it? V naught over V i - I would like to represent it in terms of current gain. So, V naught is nothing but I naught into R L, always, the magnitude; let us say, let us consider magnitude is negative actually. Then, V i is nothing but I i into R i. This, we have used earlier in the example also to obtain the power gain in terms of current gain. So, voltage gain of any amplifier A v is equal to A i current gain of the amplifier, into R L by R i. This is an important statement.

(Refer Slide Time: 49:05)

So, suppose I am cascading. So, if I am cascading at the cascading point, R L, is another amplifier, is equal to R i. Please remember; the load for one amplifier is same as input resistance of the other amplifier. So, R L is equal to R i. Then, it is proven that A v is always equal to A i. This is an important statement.

> oca am **IIT MADRAS**

(Refer Slide Time: 49:48)

While cascading therefore, the voltage gain is same as current gain. That means, if an amplifier is capable of giving voltage gain, but not current gain; or vice versa, if it is giving current gain but not voltage gain; then, it cannot give either voltage gain or current gain, when, it is cascaded.

An amplifier that is cascaded, if they are identical amplifiers, if they are identical amplifiers, it should be capable of giving both voltage gain and current gain in order to give any one gain; because, current gain is same as voltage gain if identical amplifiers are cascaded; because input resistance of one amplifier becomes load of the other amplifier. So, if they are identical, its own input impedance is same as load.

So this is something, common sense thing, that you have to remember; that, when I cascade, if I cascade identical stages, the stage that I use must have both voltage gain and current gain; then only, it can act as power amplifier. This is an important statement, profound in its sort of effect in design.

Therefore, I must not cascade amplifiers which give either voltage gain or current gain, if I am using identical stages. I must use amplifiers which give both voltage gain and current gain, so that, when I cascade, current gain becomes equal to the voltage gain and it is greater than 1; and therefore, subsequently, it will give you power gain.