VLSI Data Conversion Circuits
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Lecture - 59
Dynamic Element Matching – 1

This is VLSI data conversion circuits lecture fifty nine. In the last class, we were investigating ways of making the performance of D/A converter which is embedded inside a delta sigma loop to be free of distortion, and the basic reason for the increase in the noise floor and harmonic components when the elements or the unit elements inside the DAC are not matched properly is that the individual bits that drive the unit elements are non-linear mapping of the input sequence.

Conventionally, what is done is you take the input sequence decompose it in a thermometer fashion and each one of these lines goes and drives a unit element and you add the outputs of the unit elements. Now, what happens in practice is that since the unit elements are all not properly matched the when there are individual a when the individual drive wave forms have distortion, and a noise flow will increase which is bound to happen simply because these individual drive wave forms are derived from the input sequence through some non-linear transformation.

So, last time we saw that for instance the this the a lowest bit of the thermometer code remains on pretty much most of the time, whereas the highest bit in the thermometer code only goes high when the input wave form touches the peak correct. So, these are since these a transformations that is from the input sequence to the individual drive wave forms of the thermometer code these are fundamentally non-linear, which means that if you take the input spectrum which is noise shaped that is the input sequence that being fed to the DAC.

And, that you can assume to be if you assumed to be a nice noise shaped sequence with a very high in band SNR the moment you take this sequence and process it nonlinearly to get each one of these thermometer drive bits. The spectrum of this each one of these drive sequences will have an elevated noise floor and a lot of distortion there is nothing wrong with this is what you should get because these transformations are non-linear. However, since the input sequence is being decomposed you know exactly into these thermometer sequences by construction we know that the some of these thermometer drive sequences is equal to V which means however, non-linear these individual thermometer drive sequences are when you add them together the spectrum must become must have a very high in band SNR because that is the way we have constructed it correct.

Now, if the weights of the unit elements were all exactly equal to 1, in other words if there was no mismatch even though you add up all these horrible wave forms with lots of harmonics and noise floor in the signal band, when you add them all up those harmonics all go away because they cancel and similarly the high noise floor also goes away, because of this magic combination of these wave forms which when added together must in principle give you the same spectrum as the noise shaped sequence. Unfortunately if these unit elements are mismatched the cancellation is not perfect, so whatever effects you see in the individual wave forms namely a raise in the distortion as well as raise in the noise floor will leak to the output by some factor.

Because, the cancellation is not perfect right clearly as the cancellation I mean as the matching becomes better and better, in other words when the mismatch becomes smaller and smaller the cancellation becomes more and more perfect, and you will in the limiting case that mismatch is zero you will get something which tends to the ideal linear spectrum I mean the spectrum of the input sequence that you are indeed looking for does it make sense.

So, the culprit therefore, is the fact that the individual wave forms that drive the unit elements are derived from the input sequence in a non-linear manner. So, a fix therefore, would be to say I am not going to derive these individual drive wave forms nonlinearly if I derived them in a linear way then by definition if you derived them if they are linearly related to the input there will not be any harmonics. So, when I combine them I should be able to eliminate distortion and hopefully noise floor understand. So, the first attempt in that direction is simply randomization which is what we discussed in the last class if this is the conventional thermometer decoded DAC we set this is a 4 level DAC.

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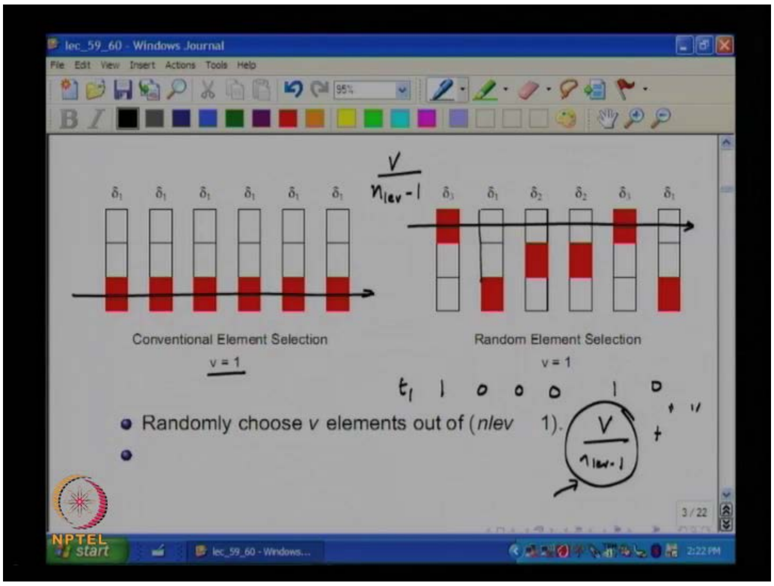

So, there are three unit elements if V was equal to 1 conventionally you would always see that the lowest the I mean these elements should always be selected on the other hand when you do random element selection you find that on an average each element is selected at 2 time over 6 or in the more general case the average duty cycle of each element is the same and will be equal to whatever V by number of levels minus 1 correct. So, the mean of each of these elements right will simply be equal to V divided by N level minus 1. And then we saw last time that if you find the auto correlations sequence it will give you I mean the auto correlation of each of these drive sequences it will give you something which is the auto correlation of the input sequence V divided by n right plus white noise.

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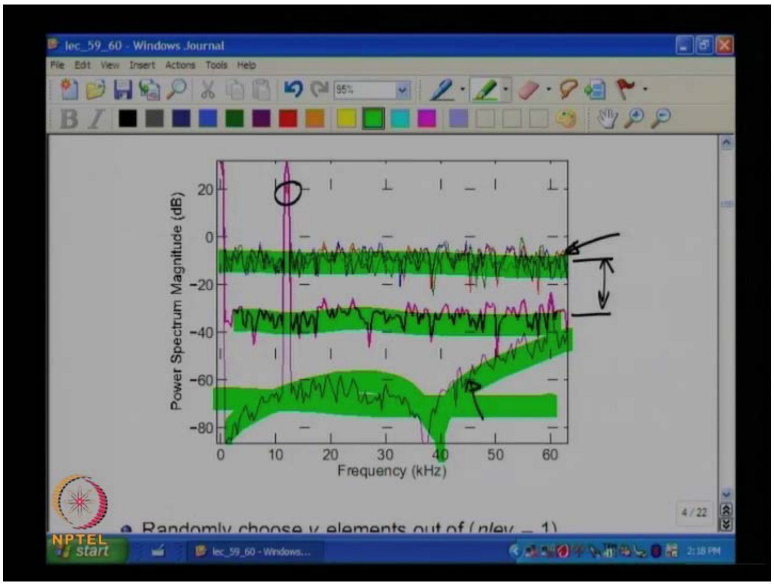

So, needless to say when you find the spectra of the individual drive wave forms we see that they all consist of the signal in the same amounts. This makes sense because on the average every unit element is contributing equally to the output correct. This is in contrast with a thermometer decoded version where some unit elements are contributing more and others are contributing less you understand.

So, when you add all these up without any mismatch this is what you should get and this is a spectrum of the underlying V 30 of supplied to the DAC. Unfortunately if there is mismatch you will see that the cancellation is only partial and therefore, the un-cancelled part will bear the same spectral shape as this noise does that make sense. So, the total in band noise is a combination of 2 things, 1 is the un-cancelled noise due to mismatch plus the quantization noise and clearly if the quantization noise is way below this mismatched noise you will be limited by mismatch, and as you can see the penalty is actually quite a lot. So, if you say that this is the average kind of noise floor in the signal band you see that with mismatch even after randomization you are about 30 d b worse that is 5 bits worse all right.

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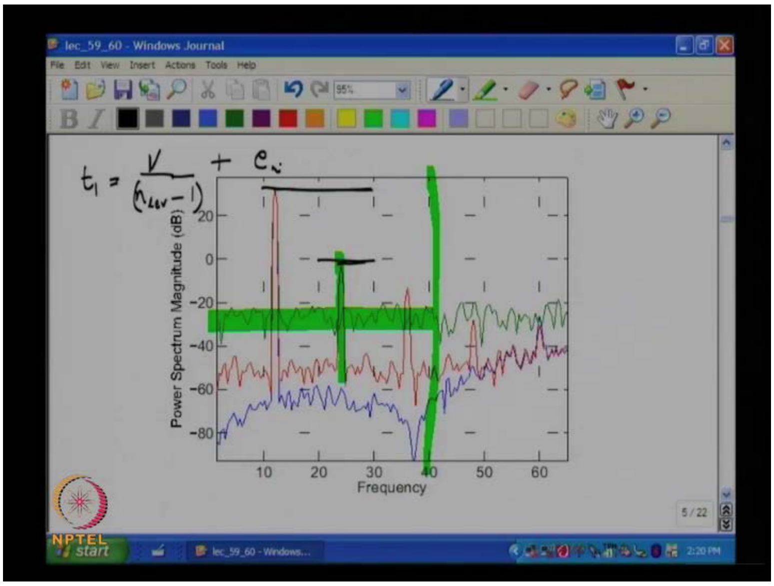

However what we saw was that when compared to not doing anything at all we are better of randomizing because the distortion components are gone away correct. And I mean please note that since this is a log scale if you find the SNDR in this signal band for instance, you will find that you will be for the case where you do not do anything at all you will be limited by distortion because this peak is about whatever on the average some 60-80 d b above the quantization noise floor.

So, pretty much the SNDR will be the ratio of those 2 because the rest of it all this is right. So, small compared to this spike that I mean they do not really effectively add to anything in the noise plus distortion does it make sense? So, with randomization you have greatly I mean you have greatly improved the situation with respect to not doing anything at all, now you start to get greedy and say all I am doing is generating each thermometer drive wave form to be 1 by or V by n level minus 1 plus some other error sequence correct you understand, please note that each thermometer drive wave form, now is a 2 level wave form for example, here it is t 1 is 1 0 0 0 1 0, it is a 2 level wave form whose average value is the same as V by n level minus 1 right apart from this average value it has got some other wave form right. And that other wave form happens to have a white spectrum.

So, in the presence of mismatch a part of that white spectrum comes through, so now this the idea what if this other sequence we of course for each thermometer drive wave form, we want this V by n level minus 1 correct, but and you know once you understand this it seems a pretty straight forward thing to say can I make I mean the problem with the solution. We have as for now is that we are getting rid of the distortion as we want, however the in band noise floor is white and you know is large. So, the question now is if I make this, so called extra stuff right have a spectrum which is noise consumption or which has very little frequency content at low frequencies, then even if there is mismatch it should not matter too much or it should matter to a lot less degree than with simply pure randomization does this make sense all right.

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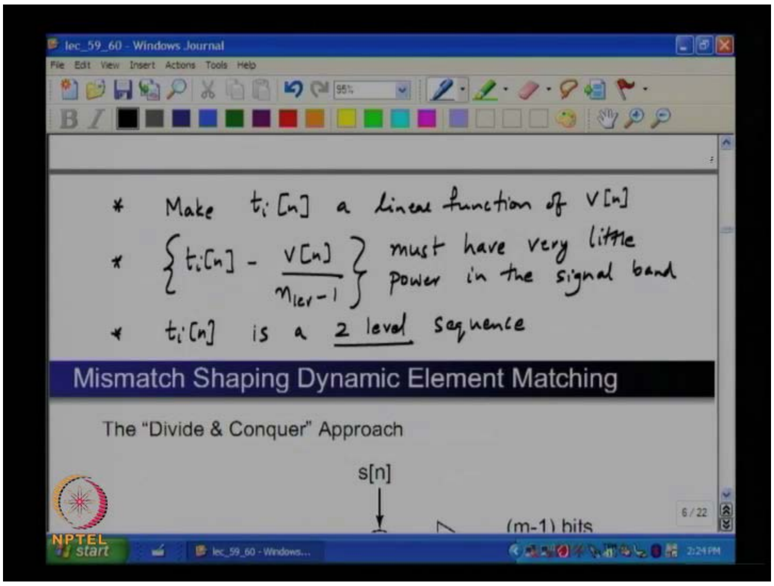

So, one way of doing this the aim therefore, is to make each thermometer drive wave form a linear function of V all right. And t i of n minus V of n by n level minus 1, how must this look like? must have very little power in the signal band. Signal band in our case the signal band happens to be at low frequencies all right. So, and please note I mean this might sound cliché, but t i of n is a what kind of sequence is it an any arbitrary sequence or a are there some restrictions on t i of n? It can be take 2 values. Two values is at 2 level sequence this is a very important all right. So, now in other words we want a 2 level sequence.

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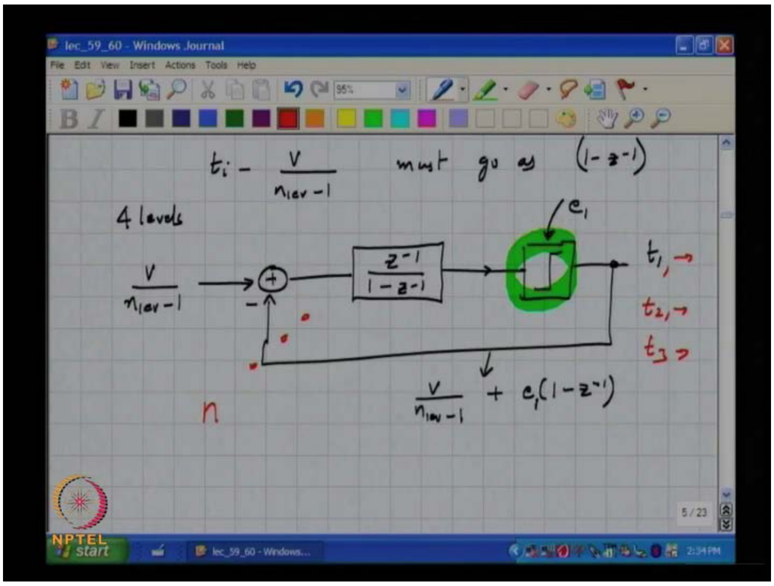

Whose low frequency value is the same as at low frequencies this must resemble?

Student: Pardon. Make sure.

No.

Student: It should resemble V itself.

V by N level minus 1 correct all right, and must have actually and the rest of it, so in other words t i minus V by n level minus 1 as we just said let say for argument sake must go as, the spectrum must be shaped by say 1 minus z inverse this is the simplest thing you can think of where there is no spectrum at there is no power at dc at low frequencies it is small, isn't it. So, can you think of a how do you think will achieve this?

Student: Pardon.

First order.

Student: a first order delta sigma.

A first order delta sigma converter, so this is V by n level minus 1 correct. And, then if I want 1 minus z inverse I mean I need a 2 level sequence right. So, what do you do you take the 2 level sequence you I mean you compare it with V by n level minus 1 you subtract the 2 and you integrate this at low frequencies and use that to drive the quantizer correct. So, this should give you on the average V by N level minus 1 plus some shaped quantization noise. Let me call this some e 1 times 1 minus z inverse. This becomes t 1 this e 1 is the, so called is the nothing, but the difference between t 1 and the output of the accumulator does it make sense all right. Now, if I what should I do to get t 2.

Student: Stating the skew.

Well, so, and t 3 and t 4 and t 5 what should I do. Well, so the suggestion is that if you have in our example if we have 4 levels you will have 3 unit elements. So, 1 thing you can say is I am going to make 3 such loops like this right and get t 1 t 2 and t 3 and apply them to the three unit elements at three unit elements. So, is this or.

Student: Sir then they should have different at a time.

No no no. What is the problem?

Student: It will generate the same sequence at the output.

Yes. So, what? Out of them, so and the, what he is pointing out is that hey why do you want to have 3 different delta sigma modulators. Feed the same thing to all of them. Mow what happens what is that becoming.

Student: I mean to satisfy the condition that you want to…

No no no no what is wrong with what so, far everything seem logical right and all of you are nodding your heads without disagreement what is wrong with this?

Student: Pardon.

Yeah. So, what? So, the average value of t 1 is V by n level minus 1 right, if you take 3 of them the average of the output will be V.

Student: First of all is this insensitive to mismatch or is it sensitive to mismatch?

Insensitive.

Student: Insensitive.

Insensitive to mismatch and why do you think it is insensitive to mismatch?

Student: Error is random.

Because we want p to p they are linear functions of… So, if you did not know all of that can you look at it straight away and say this got to be insensitive to mismatch.

Student: ((Refer Time: 21:56)) It is a relative feed evaluate would be.

No if t 1 t 2 and t 3 are all identical right then the output wave form of the DAC will be all either 1 or it will either be n or it will be only a 2 level sequence.

Student: ((Refer Time: 22:16)).

Correct, you understand and what is that if you have a 2 level sequence? what is that becoming what is the equivalent digital 2 level DAC, It is becoming a two level DAC it is becoming a single bit DAC right which we know by definition is insensitive to mismatch, but what is the problem?

Student: ((Refer Time: 22:36)).

No we have to satisfy to I mean. No no no no do not jump ahead what I am asking is what is the problem with making it a single bit DAC?

Student ADC is an multi level this thing and feedback is missed one.

Yeah that is perfectly fine right, you just through the many levels of ADC and then you have only.

Student: Are we going to run the DAC pulse at the same clocking at the same frequency.

Apparently right ((Refer Time: 23:11)).

Student: ((Refer Time: 23:11)).

No, if you take a multi bit modulator remove the multi bit quantizer and put a single bit quantizer what effect will it have on the output SNR?

Student: ((Refer Time: 23:11)).

Definitely the quantization noise will increase.

The quantization noise will increase. ((Refer Time: 23:42)) difference right you understand. So, it is true that if you do this the output of the modulator will be devoid of harmonics at least in principle because each one of these wave forms is a linear representation of the input unfortunately the in band quantization noise will rise significantly, because it is now as if you are operating the whole multi-bit I mean you have converted a multi bit delta sigma modulator into a single bit one. So, it is not just enough to have an array of what you call delta sigma loops like this right even though each one of these thermometer drive wave forms has you know is the linear function of the input and the rest of it is shaped and so on, does it make sense.

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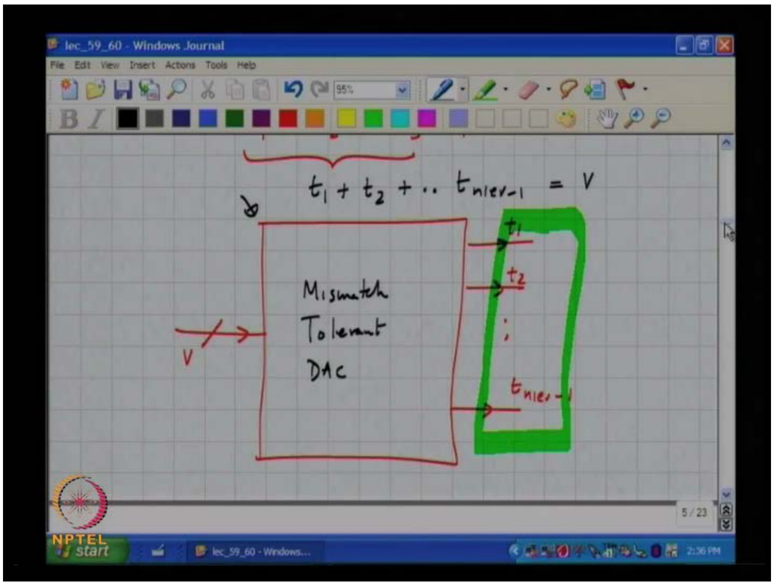

So, I mean one more observation to make as he pointed out is that, the if you look at in this particular case t 1 plus t 2 plus t 3 all right. This will not at all times b equal to V of n you understand it will only be equal to V of n on the average. So, in other words what you are doing is that if you put this whole thing inside a box, you are sending in V and you are getting out t 1 t 2 through t n level minus 1. One important thing to make sure is that just like in the randomized case what was the sum of this t's at any time?

Student: V c minus V t.

Right like we saw in the randomized case. So, this is the mismatch tolerant DAC right it is not only important to make sure that these are linear functions of V it is also important to make sure that at any instant of time t 1 plus t 2 plus t n level minus 1 must be must be equal to V right. So, that in the absence of mismatch this must simply look like for example, if there was no mismatch at all then I mean you must get the what you call regardless of what algorithm you use inside you must get the the ideal spectrum that you want does it make sense now. So, one approach to do this is a something that we are we are aware of from many context right is you take a take a complicated problem and make it into a less complicated problem.

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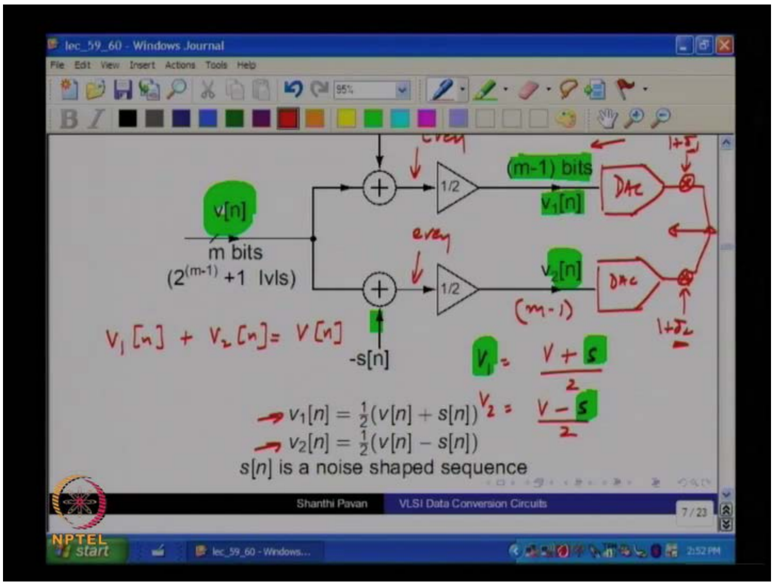

And then you apply this recursively and then you eventually end up with I mean whatever the the simplest thing you can solve and and then you are done you understand for example, search if you want to search for something in a big heap you divide it into 2 heaps right and and then you search a you divide each of these heaps into 2 smaller heaps and then even smaller heaps and. So, on until you eventually find what you are looking for isn't it. So, in the same way assume you have a 2 to the m minus 1 plus 1 levels this is a this is a technique which will work right only if number of unit elements is a power of two. So, if the number of unit elements is a power of 2 the number of levels must be.

Student: One.

One sir.

Student: It must be an integer.

Student: Yeah I mean of course, it must be an integer right ah. If the number of unit elements is the power of 2 how many levels must we have.

Power of 2 plus1.

Student: A power of 2.

Plus 1.

Student: Plus 1 how many bits do you need what is the smallest number of bits you need to be able to represent. So, many levels 2 to the m minus 1 right plus 1 level.

What is the minimum number of bits you you have to use to represent.

Student: M bits.

You need m bits correct of course, you I mean it is almost a m minus 1 bit number, but since you have 1 extra you do need 1 extra bit does it make sense. So, let us assume that you have a 2 to the m minus 1 plus 1 levels. So, that and the motivation for this is that the number of unit elements is a power of 2 and why I mean what is the motivation to have unit elements being a power of 2 why do you think.

Student: Because when you square it.

Yeah. So, you are taking something dividing into 2 simpler problems and each of those becomes 2 simpler problems say eventually the number of you know simplest possible dacs will all have to be multiples of will have to be a power of two. So, if you want to have an even number of a unit elements it may follows that the main DAC must have a power of 2 plus 1 bits ok.

So, let us a do 1 thing we need to split this let us say we want to split this into 2 sequences which are 1 bit smaller correct. So, in other words V 1 and V 2 are 2 sequences which are m minus 1 bits long and the sum of V 1 and V 2 is exactly equal to V all right why does this make sense why do you think this constraint make sense.

Student: Because we got it.

Why do you think this makes this constraint make sense.

Student: Sir during the m minus 1 or the sum should be in the V of n.

Pardon no forget about what is there inside this box right now what I am trying to do is to split this sequence V of n into.

Student: Two sequences.

Two sequences V 1 and V 2 each of these sequences V 1 and V 2 has occupies.

Student: M minus one.

M minus 1 bits all right you understand and the reason for doing this attempted to do this is that in the next step I will take V 1 which is got m minus 1 bits and split it into 2 dacs 2 I mean sequences which are m minus 2 bits and. So, on eventually until I get.

Student: 1 bit.

1 bit.

Student: One bit right. So, this 1 bit sequence can be used to drive a unit element you understand, so at every stage if I allow for.

1 by 2 plus…

Student: If I allow for.

Not only that these 2 sequences must be m minus 1 bits, but if I make sure that the sum of V 1 and V two.

Student: Is the same as V then what will happen eventually.

Finally when we add all the…

Student: Finally when you add all the thermometer t 1 through t n level minus 1 that by construction will be the same as.

V you understand does it make sense all right. So, so if I say that I am going to split V of n into 2 sequences V 1 and V 2 where V 1 and V 2 have 1 less bit than V and the sum of them is exactly equal to. V all right. So, that is equivalent to saying that finally, if V 1 and V 2 add up to give V of n then you can represent I mean you can represent V 1 as V by 2 plus something and V by 2 minus something correct. So, which is also equivalent to saying V plus something by 2 and V minus something by 2 I call that something s does it make sense. So, now, by construction V 1 plus V 2 is equal to V and I need to find some sequence s such that.

What all I satisfy. So, the basic idea is to split V into 2 sequences V 1 and V 2 where V 1 and V 2 have 1 less split all right which means that V 1 and with the additional constraint that V 1 plus V 2 is exactly equal to V of n which means that V 1 can be written as V plus some s by 2 and V 2 can also be written can be written as V minus s by 2 this way by construction the sum of these 2 is equal to V we need to find therefore, some sequence s such that these resulting sequences are 1 bit smaller I mean if you add any arbitrary sequence s the sum will be.

Student: Smaller.

V, where we need to find a special sequence s right such that these V 1 and V 2 are.

Student: M minus.

M minus one.

Student: M minus 1 bits long ok.

Sir that is the last split of s should be will it be same as v. So, that add or subtract.

Student: Yeah I will come to that right. So, we need. So, clearly V 1 V 2 and V are all integers correct. So, if this has to be an integer it must follow that this must be.

Multiple of…

Student: If you take something and you divide by 2 and you get an integer what must be the input.

It must be…

Student: It must be.

Multiple.

Student: Multiple.

It must be even right. So, regardless of what V is after you add and subtract s the result must be even. So, that when you divide by 2 you will get a sequence which is m 1 bit smaller and.

Student: Integer.

An integer. And is an integer now since V 1 and V 2 are of this form half V plus minus s of n 1 thing you must observe is that these are linear functions of V plus some sequence. So, if I gave this to an ideal m ideal m minus 1 bit DAC and this to another m minus 1 bit DAC and if I added these 2 together will this be insensitive to mismatch between gains of this DAC and this dac.

Student: sir it is sensitivity.

Pardon.

Student: Sensitive in mismatch between gains.

Think carefully what I am saying is let say I had 2 ideal DACS.

Student: minus.

Being driven by V 1 and V 2 let us say these 2 DACS are exactly identical the only difference is that this is 1 plus delta 1 this is 1 plus delta 2 and I add the 2 can you comment on the sensitivity to delta 1 and delta 2 of the combined output.

Student: This is the multiplier or...

It is a I am sorry it is a multiplier.

Student: so...

Yeah you will have some s of ten sitting there the only.

Student: Ok.

Depending on the gains.

Student: Ok.

Depending on the (( )).

Student: So if I mean do you all follow that if there is gain mismatch between these 2 dacs right the output spectrum will be the spectrum of V plus.

Residue (( )).

Student: A small residue of which whose shape is the same as that of.

S of n.

Student: S of n.

S of n.

Student: S of n.

S of n is that clear.

Student: Yeah.

So, if you want that shape to be to have very low content in the signal band which is at low frequencies what must you say about s what can you say about s of n.

Student: S of n should be output of s.

What must you say about the spectral shape of s of n. It must have.

Student: Shape noise.

Very low frequency I mean very low power at.

Student: Low.

Low frequencies.

Student: Low frequencies.

Correct. So, in other words if you want the noise or with the mismatch between the gains of these 2 DACS right to be shaped out of the signal band right you must have s of n whose spectrum also has.

Student: Same noise.

Same noise shapes. Has a the the same kind of a noise shaping in this signal band does it make sense. So, what exactly are we looking for as far as s of n is concerned right s of n should be we we concluded that.

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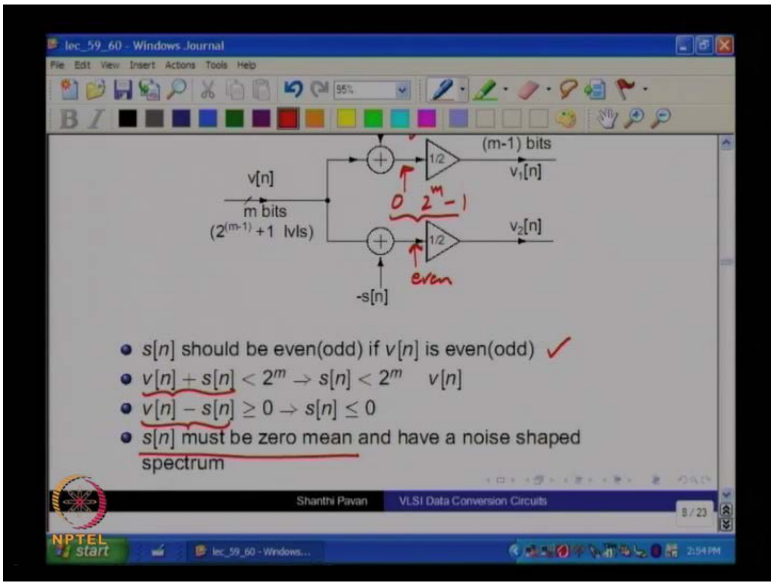

These 2 must always be even regardless of what.

Student: V of n is.

V of n is which means that s of n should be even if V of n is even and odd if.

Student: V of n.

V of n is odd this way when you divide by 2 you will have I mean no remainder is that clear and if you want to make sure that these are exactly m minus 1 bits you must make sure that V plus s must be less than.

Student: Two.

Two raise to m and similarly V minus s must be. Greater than or equal to zero. So, these 2 must lie between 0 and 2 to the m minus 1 I am sorry this must lie.

Student: 2 power m.

Two to the power m minus 1.

Student: Minus one.

That is these numbers must lie between 0 and 2 to the m minus one. So, when you divide down by 2 you will get something which lies between 0 and 2 to the m minus 1 minus 1 does it make sense and on top of this s of n must be 0 mean and must have a.

Student: Noise shape.

Noise shape it is right is this clear. So, so before we understand how to generate this special of s s of n we seems to have an full out of constraints correct

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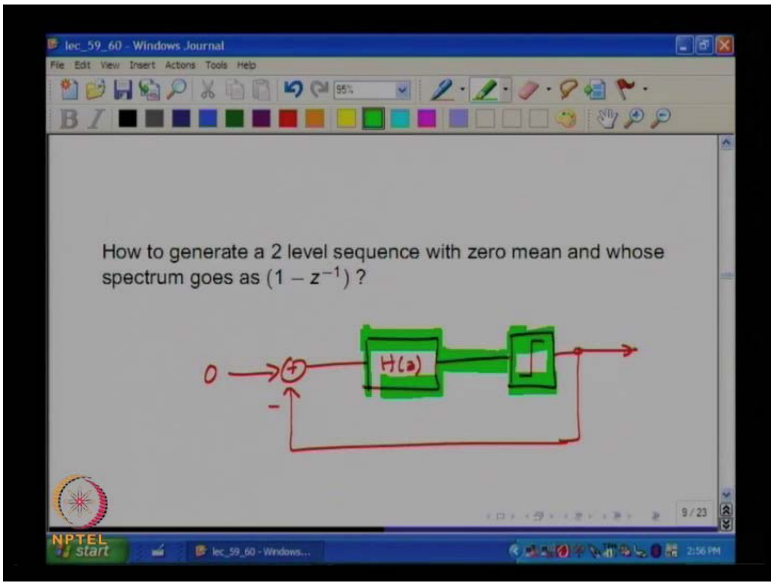

So, the first thing as a prelude to this how do you generate a 2 level sequence with 0 mean and whose spectrum go goes as 1 minus z inverse.

Student: I mean take as first order sigma delta modulator feed the 0 input.

Yes.

Student: So; that means, the output V g of it also will be zero.

So you have a 2 level sequence if I have.

Student: Z inverse by ah.

Yeah h of z in general and do this this will give you a sequence with 0 mean correct and.

Student: Noise shape problem.

Noise shape reliably 2 levels because the quantizer is enforcing the 2 levels and it will have a spectrum which goes as 1 by 1 plus h of z is that clear. So, if h of z is z inverse by 1 minus z inverse then.

Student: The output will be the.

The spectrum at the output will be.

Student: 1 minus.

1 minus.

Student: Z inverse.

Z inverse times some quantization error is this clear. So, before we get into more details I just want you to step take a step back and see what we have done here right all I said was we need a 2 level sequence which has some specified behavior at low frequencies. So, the first thing is to say I am going to have a I i am going to have a generator which generates.

From 2.

2 levels right then I compare that 2 level sequence at low frequencies with what I want which happens to be 0 in this case right and because I am only interested in low frequencies I multiply it up by some h of z which has got a high gain at low frequencies and if this loop is stable that is a very important if all right if this loop is stable the output sequence will.

Student: 0.

Will have a spectral shape which is of the form 1 minus z inverse does it make sense now the next thing is to say all right now how will you generate. So, let us assume that I am trying to generate s and for argument sake I am just going to say s is a 3 level sequence which either takes values 1 minus 1 or 0 I mean it could in general be a I mean more than 3 level sequence because all that you need to do make sure is that you do not over flow the I mean V plus s

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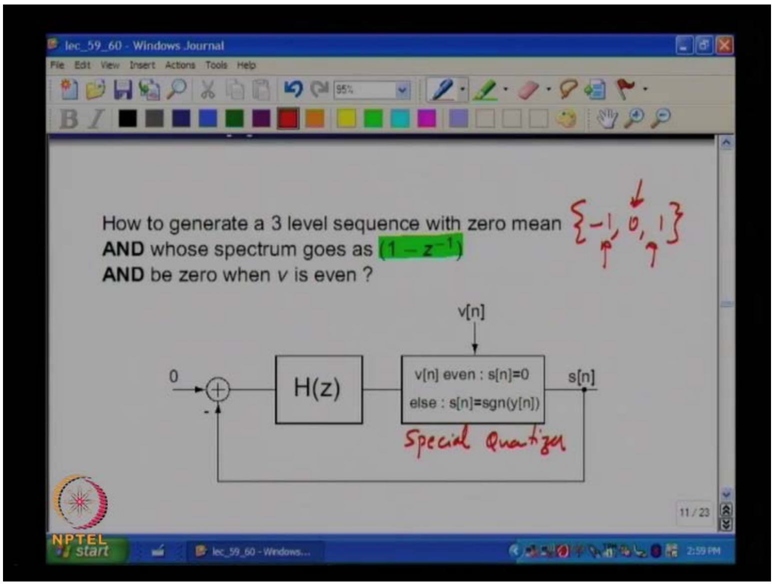

And V minus s should not become I mean should not go beyond the range 0 to 2 to the m minus 1 correct, but the simplest case perhaps is to say I am going to assume an s of n which is 3 level minus 1 one and.

Student: 0.

0. So, I need to generate a 3 level sequence with 0 mean and whose spectrum goes as for instance 1 minus z inverse and it must also be.

Student: Minus.

What 0 are the properties of s.

Student: I mean when we add with V it should be…

When V is even s should be even when V is odd.

Student: S should be odd.

S should be odd correct here thankfully if you assume that this only takes on levels of minus 1 0 and 1 when V is even the output must be.

Student: Even.

Is even.

Student: Even and the only even integer here is 0 correct when V is odd it should be.

Minus.

Student: Or it can be either 1 or minus 1 and you want it is spectrum to be.

1.

Student: 1 minus.

1 minus z inverse at low frequencies so, you I mean. So, this is some special kind of quantizer right you impose more conditions on the quantization that is all. So, you first have a block which satisfies all your conditions right this is exactly analogous to what we did earlier we have a block

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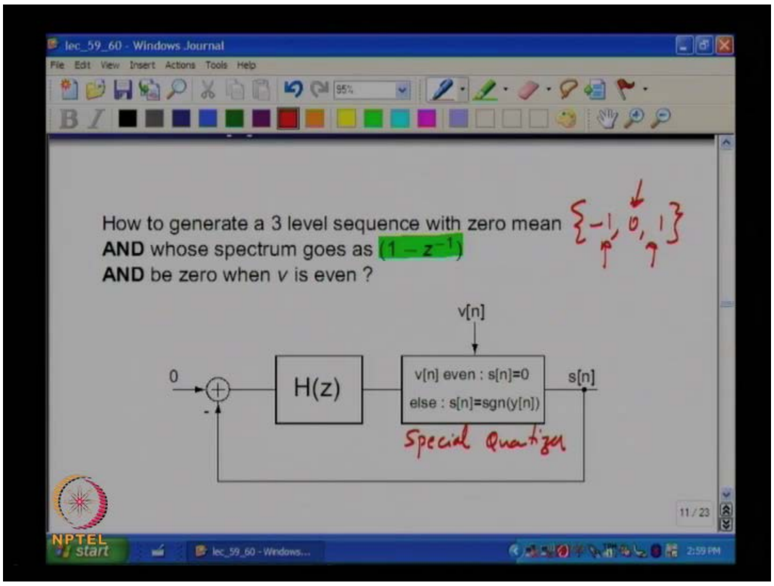

Which satisfies our conditions our conditions were only 1 that is you want this sequence to be a 2 level sequence right the only difference departure from that is now that I have a lot more conditions on my sequence right it should be 3 level it should be odd when something is happening right I mean you could also impose a condition that this sequence should be exactly 1 during [FL] correct and you know and that is still you understand all right as long as this loop is stable and you do this.

Student: This sequence will have.

0 mean.

Student: 0 mean and a low frequency behavior which goes as 1 by 1 plus h of z is this clear all right. So, now, you put this all together this is the sequence generator it generates s of n by definition therefore, or by construction this will be 0 mean and will have a shape which goes as 1 minus.

Z inverse.

Student: Z inverse you add and subtract to V divide by 2

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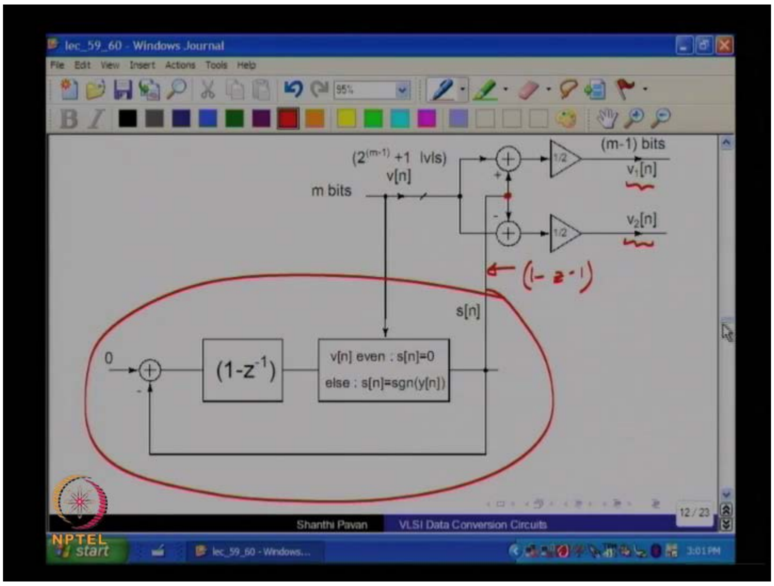

So, you will get V 1 and V 2 which are.

Student: M minus 1.

M minus 1.

Student: M minus 1 bits long. So, now, we have done this. So, what do you think we should go and do going forward.

Is like split the same thing.

Student: Copy and paste except that m now becomes m minus.

1.

Student: 1 and when you do that you get this

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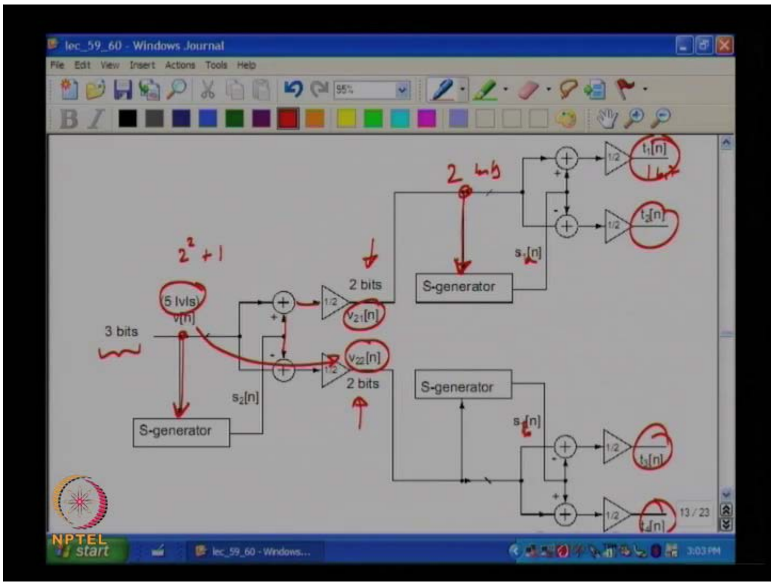

So, let us take an example where there are 5 levels which is 2 to the 2 plus one. So, how many unit elements must be there finally, if you have 5 levels how many unit elements must be there.

Student: 4.

Four correct so, but the 5 levels have to be represented by 3 bits correct and therefore, this is after the first time you generate s all right this s generator must look at V because it is doing this even odd business all right and it generates V 2 1 and V 2 two which are 2 bits long the sum of these 2 must exactly be equal to V by construction now I take V 2 1 and V 2 two and again have an sequence generator which generates another s let me call this s 1 a and s 1 b all right and what if there are 2 bits here this must be 1 bit. So, these are t 1 t 2 t 3 and t four. So, in the next class we will take a look at the spector of each one of these thermometer drive signals and see what happens to the overall spectrum.