VLSI Data Conversion Circuits

Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 04
Nonidealities in Samplers

So, welcome to lecture 4. In the last lecture, we were looking at what nonidealities, there exist in a 2 way sample and hold. And what the effect of those nonlinearities is on the performance of the sample and hold. Last time we also saw the, I mean we saw that using a time interleaved sample and hold has its problems. The advantage of being able to sample at 1 half the rate of the overall system comes with some disadvantages. For instance, we saw that if there is offset in each of these channels when you stitch the sequence together. Finally, there is an artifact in the form of a tone at f s by 2 in addition to a DC offset that you now only expect.

Further because of the way the sample and hold is designed it is not possible to have exactly the same gain in both the parts. And as we were seeing last time around if there is a gain error or a gain mismatch between the 2 parts that introduces artifacts in the spectrum. And that appears as a multiplicative effect where you take the original sequence and multiplied by a sequence which can be written as a constant plus 1 which has a component at f s by 2 or at omega equal to pi. And we went through the map the last time.

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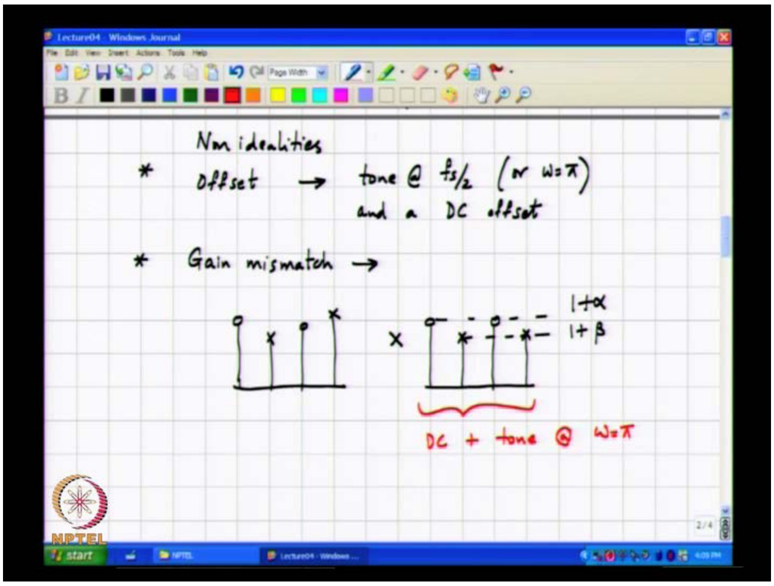

And let me just me just quickly summarize what we dealt with the last time around. So, if there is offset between the 2 channels what happens? What are the effects there will be a tone at f s by 2 or omega equal to pi and a DC offset? If there is a gain error and a gain mismatch, what would we expect the last time around? We said that if there is a gain mismatch between the 2 channels. It is like taking the original sequence, the ideal sequence that you are supposed to get out, perhaps it was something like this. And it is like multiplying it with a sequence which is 1 plus alpha, 1 plus beta, 1 plus alpha, 1 plus beta and so on. Does it make sense? And this sequence as w so, this is 1 plus alpha and this is 1 plus beta and this sequence as we saw the last time can be represented as DC plus a tone at. So, it is like taking the ideal sequence and modulating it with with a tone at omega equal to pi or f s by 2.

So, if you modulate or multiply a signal with a sign wave you will see you will see side bands. So, if the input was a tone at f in you will see in the output spectrum tones at tones coresponding to f s by 2 plus minus f a in the discrete time domain you will see tones at pi plus minus f in by f s times 2 pi correct. So, if you are trying to infer what the input signal was by looking at the sequence at the output of the sample and hold. And if you are not aware that there is a gain error then you would make the mistake of assuming that the input tone consists of not only f in. but also f s by 2 minus f n. You understand all, that I wanted to appreciate this is that this is an error and that is happening, because of the time-interleaved nature of the system.

Now, if one went further and said I am going to do a 3 way interleaving you can go through the math it just becomes messier. And there you will see that each channel is now sampling at f s by 3 and there are 3 channels. So, when you when there will be significant aliasing in all the 3 channels. But if things are just when you add all the outputs together 2 of those images get cancelled off. And you are left with you know the ideal expression that you would expect for a simple for a single channel sample and hold opertaing at f s, does it make sense?

And please note that this I mean the output becoming magically equivalent to a single channel sample and hold opertaing at f s is being achieved by cancellation of the aliased images in all the 3 channels. And any technique which is based on cancellation will always be sensitive to well, everything is sensitive to nonidealities. But every anywhere you are expecting to get 0 by subtracting 2 quantities you are banking on the fact that the 2 quantities are exactly matched. So, if one of them is slightly larger than the other, the cancellation is not perfect which will lead to artifacts which we have seen mathematically for the 2 channel case. The last ideality last deviation from ideality I want to consider is what is called timing skew.

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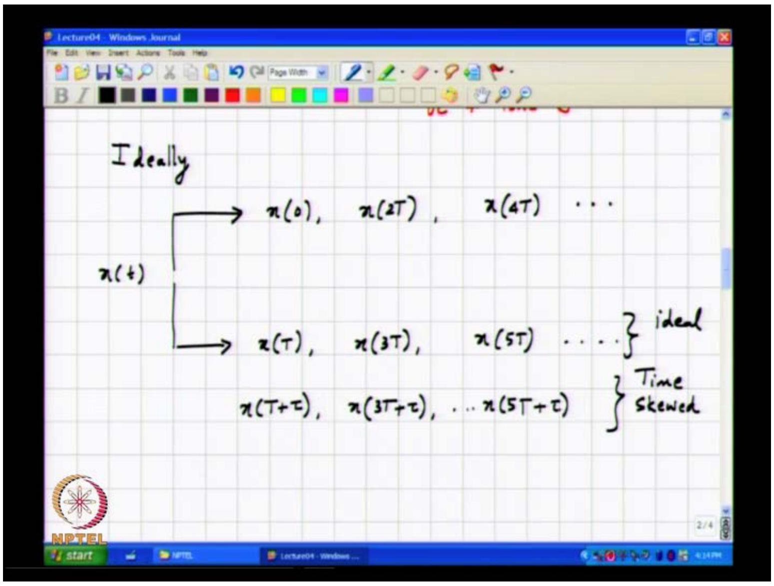

So, ideally the samples we expect to get on the top channel are x of 0 x of 2 T x of 4 T and so on. And the samples one expects at the in the lower channel are x of T x of 3 T x of 5 T and so on. So, what do you mean when we say there is a timing skew between the

2 channels. Obviously, there is already a timing skew in the sense that 1 channel is sampling at a time instant which is 1 which is T skewed from the sampling instance of the other channel. So, when we mean timing skew it means that what does that mean? Shifting on the other channel is not sampling exactly at at time T apart it is sampling at a time say T plus tau. So, this is ideal this is time skewed and that is x of say T plus tau x of 3 T plus tau x of 5 T plus tau and so on. You understand?

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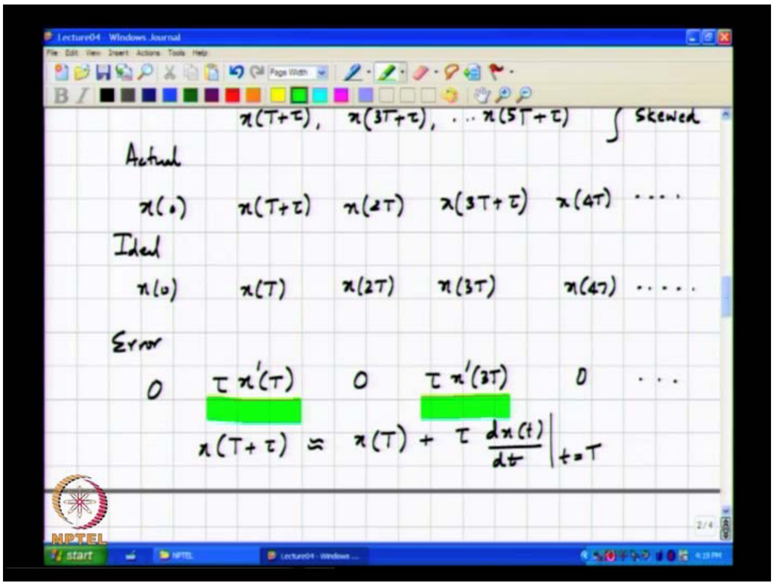

So, the, when you stitch the sequences together the actual sequence you get will be x of 0 x of T plus tau x of T I am sorry x of 2 T x of 3 T plus tau x of 4 T and so on. Clearly this is not the same as the the ideal I mean not the same as the spectrum that not the same as the sequence that you would expect it to get if the timing skew was 0. So, can you tell me a way in which you can determine the spectrum or relate the spectrum of this discrete time sequence to the spectrum of the continuous time signal assuming tau small. So, this is the actual sequence; this is the ideal sequence what is the ideal sequence? x of 0 x of T x of 2 T x of 3 T x of 4 T and so on. So, what is the error? So, the error is basically nothing, but the top sequence minus the I mean the actual sequence minus the ideal sequence.

So, this becomes 0 and for small tau what can you approximate the error as J minus tau into tau I mean by taylor series x of T plus tau is approximately Sir T plus tau X of T plus tau.d x by d times by d T evaluated at equal x equal to d t T equal to T capital T correct. So, the error sequence is simply tau d x of T by d T evaluated at T equal to capital T which I will abbreviate using the following notation that is x prime of T correct. The next error sequence sample is 0, the next sequnce is tau times x prime of 3 T 0 and so on. Does it make sense? So, one thing you notice is that if the input signal is varying slowly, what do you think the error sequence is do you think it will be small or large.

Small small

It will be small why does that make sense?

Because between 2 adjacent values the x of T would.

The input is varying slowly by definition it means that if you sample it a little off the ideal point. It is not changed by very much correct which means that the error sequence will be small. In other words you are going to really get hit by timing skew when.

When the input (( )) is constant.

When the input frequency is high and that makes sense also from this expression here you see that the error sequence has got terms which are a proportional to tau, the timing skew that make sense. Because if you deviate widely from the ideal timing instance you can expect to make a lot bigger error. Further you also see that it is dependent on the derivative of the input signal which means that the input which is varying fast is likely to be sampled with a bigger error in the presence of timing skew. Both these are intuitively apparent I will leave the job of finding the actual spectrum as an exercise for you in your homeworks. You can proceed along the same lines as we did earlier. One way of attacking the problem is to a either try and find the spectrum of this sequence. This is nothing but when what is the spectrum of this sequence?

No x dash of 2 is.

Yes.

J omega j omega j omega.

Great. So, the you know the spectrum

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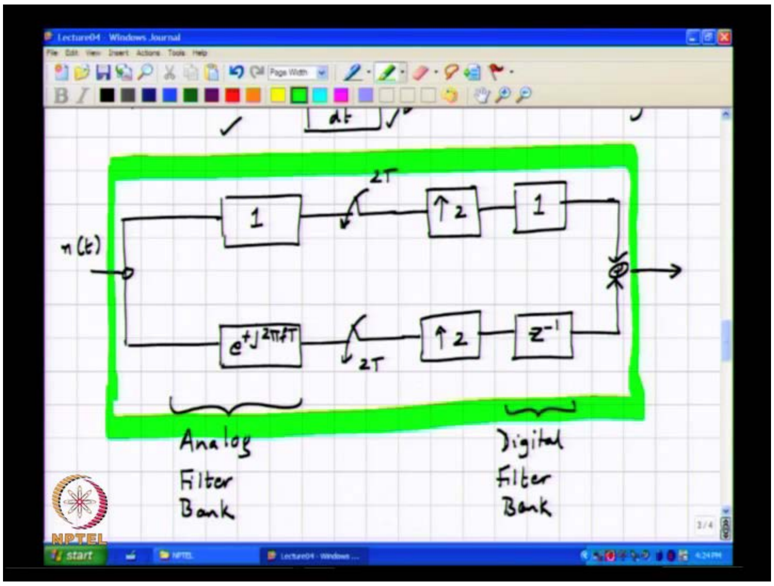

So, this is like the error sequence is like taking x of T passing it through a differentiator correct. So, that you get d by d T sampling this at at a sampling rate of 1 by I mean sampling it every 2 T seconds. And I am skewing it appropriately and finding the spectrum, you know this spectrum; you know this spectrum. This will be a fully transformed of the signal there is simply j 2 pi f times x of f. Then you know what happens when you take a continuous time signal and sample it at a given rate and and so on. So, you can directly find the input spectrum I mean the spectrum at the output. So, this is the expression for the error between the ideal spectrum you expect in the discrete time domain and the actual 1 you get we call the timing skew is this clear.

So, the last point as far as the fourier transform part is concerned that I wish to bring to attention is the following. If you think about it what we have take done is taken conceptually taken this signal, the input signal passed it through 2 filters. One filter is has got a gain of 1 the other, one has got a transfer function of e to the plus j 2 pi f T correct. You have sampled both these at the rate f s by 2 or 1 by 2 T hertz correct. And then what have we done? We have inserted now you have 2 discrete time sequences each with a rate of f s by 2 samples per second each of these channels results in aliasing. Now, what we have done is up sampled both these sequences and passed them through in principle a digital filter on each channel on the channel on top. The digital filter also has a trans function of 1 and in the lower channel what did we do?

Digital.

So, we had I mean we had all did we had was a delay, but you can think of a delay as a digital filter with a transfer function.

J inverse.

Z inverse and then we added the 2 outputs. So, this is an analog filter bank; this is a digital filter bank. And as far as this entire box is concerned it looks like a sample and hold operating at f s all.

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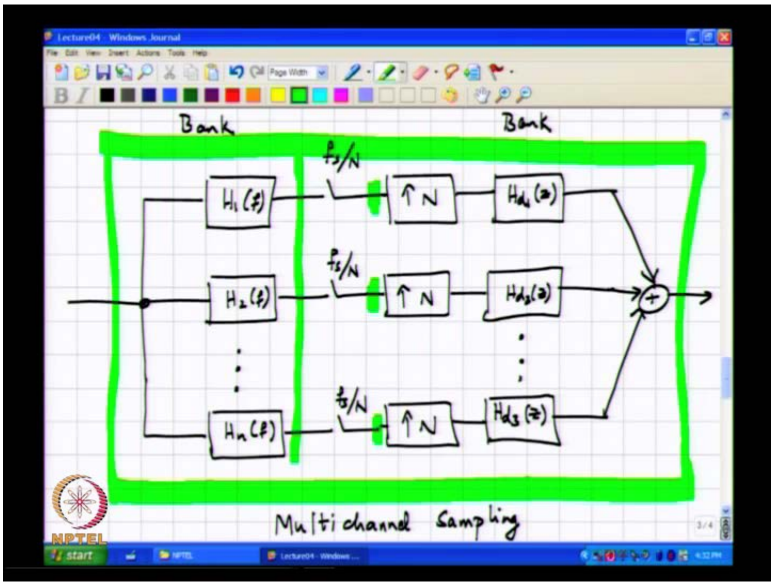

So, it turns out that this is the special case of a general principle called multi channeled sampling. So, you can extend this to N channels where the general idea is like this, if you have N samples if you have N channels you can sample at.

15 into t N.

So, you sample each of these channels at f s by N . So, at the output of each of these channels there will be a lot of aliasing. However, if you up sample each one of these up by a fact of N. And then pass them through digital filters. So, if you pack all of this inside a box And you chose these analog filter bank transfer functions properly and the digital filter bank transfer functions also properly. It is possible to stitch all these outputs together to make the whole system look like a single system operating at f s.

In other words you have taken each channel and made the apparent sampling rate higher by a factor of N by having an N channel system. So, this is what is called multichannel sampling all. And depending on the choice of the the analog and the digital filters you will have different degrees of aliasing along each channel.

But when you stitch them all together the aliases will all cancel just like what we saw in the 2 channel case where this I mean these analog filters are called the the analysis filters. And the digital filters are called the synthesis filters. So, in our special case of 2 channel sampling the analysis filters are very simple; one is simply a time delay, the other one is simply 1. And the digital filters are also correspondingly very simple 1 is 1 and the other one is simply a delay element j inverse, does it make sense all? So, p is not that this is a very dependent on the nature of the transfer functions of the the filters. For instance you cannot say I am going to use the same I mean I am going to say I am going to cheat by using the same H 1 in all the analysis filter using them the system will work. Pardon.

You would not get the sample.

What does the meaning of you would not get the sample.

Aliasing, reconstructing.

Yeah I know, but why?

We are using the same sampling.

I mean.

Completed.

If you cheat and say I am going to have the same filters in all the arms clearly I am not getting I am getting the same sequences at the output of every channels. Obviously that is not helping I mean you understand this is I mean; this is a concept that we are all very familiar within in daily life. You know in India there is this this is institution called arranged marriage you understand. So, you know you find some bridegroom and some some bride and you know enquiries begin you understand. So, the bride’s party will ask you know whom all they can find who is even remotely associated with the bridegroom.

You understand the idea is to be able to sample the bridegroom's; you know x y z T as a function of time with as high a resolution as possible.

Obviously, you cannot keep tagging along with the bridegroom like 24 7 . So, what do you do? You will ask his neighbour; you will ask his boss; you will ask his teacher; you will ask his friends. So, each one of this guys is is a filter which is being exposed to the true signal you want to sample correct you understand. And you look at the outputs of each of these filters and attempt to reconstruct what the true x of T is you know. So, if one of his friends tells you he gets drunk every night in a bar. You know regardless of what everybody else tells you I had basically means that there is a problem you know. Or there is no problem at all you know as the case may be you understand. So, I mean multichannel sampling is a is something we all you know common sensically. We are all used to this of course, it does not make sense to ask his father; you ask his mother; you ask his brother; you ask his sister. It does not make any sense at all why they are most likely to say the same thing.

So, you need to get as many independent opinions as as possible and then after that you know you need the synthesis filter is here where you take all these inputs and try. And reconstruct the original signal which you have never seen; you are only seen these outputs correct. And many times you only have one sample from that you are trying to reconstruct what x is in the same way. The they must be some degree of independence with respect to these analysis filter transfer functions they cannot be all the same or related to each other you understand. So, now you might be wondering where this kind of approach is used at all, it turns out that in in in some very high speed scopes or in very high speed wave form digitizers.

It is just simply impossible to get the the kind of what you call sampling speed that you would want to achieve. So, one approach is of course, to try and do time-interleaved sampling. Like the way we discussed, but one disadvantage of time interleaved sampling is that even though you are sampling at a rate which is much lower than the over all sampling rate. The signal at the input of each switch is varying just as fast for all the sample and holds is that am I clear. For example, look at this even though there are 2 channels and the output of each channel is being sampled at f s by 2. We see that the signal here at the output of the first channel varies just as fast or rapidly as the signal here. The same thing will happen if you have many N sample and holds skewed by N clock cycles.

So, then I mean the signal is varying very fast it turns out as we will see going forward that sampling can become a challenge one thing which is challenging to begin with is to sample at such a high rate. We have tried to avoid that by having many sample and holds operating slowly. However, each the input bandwidth seen by each sampler is still the same which can become problematic when there is real transistors and real circuits which is why sometimes an approach to use is to have a more complicated set of filters one straight forward. But non trivial example is to say let us say I have a bank of band pass filters each one with a bandwidth of 1 n-th the signal. Let us say the signal varies from has a bandwidth f s by 2 f s is the overall sampling rate.

The signal bandwidth is 0 to f s by 2. Now, if I pass this input signal through a bank of N filters each of which has a bandwidth of f s by 2 N. Then what would be the outputs of each of these? What would be the bandwidths at the output of each of these filters? The output of each of these filters would be narrow band signals with the bandwidth of.

F s by 2.

F s by 2 N correct. Now, if the bandwidth is f s by 2 N then it means that the signal is narrow band which means that from cycle to cycle give or take a carrier the. There is significant corelation between adjacent samples which makes presumably makes the sampling easier you understand. So; however, the moment you have a complicated bunch of analysis and synthesis filters. As we discussed with the time-interleaved case this aliasing which is happening in all channels is getting cancelled out. And resulting in the nice equivalent sampling of at a rate of f s. And every time you cancel; you are dependent on this matching I mean something matching with something somewhere else.

And most often you will find that the cancellation is not perfect which means that you will see a whole bunch of artifacts in the spectrum. The analysis of those artifacts is likely to be a lot more messy when you have general filters in the analysis bank all. So, anyway this is a something to just be borne in mind that if you have a signal; you can sample it at 1 n-th the rate. If you have N channels where the analysis filters are reasonably independent. Then it is possible to reconstruct perfectly a signal even though on each channel there is significant aliasing.

Sir the issue.

Yes.

There is the only the bandwidth or like the last 1 H N of f is the frequency of operation is very high frequency .

The the centre frequency is high.

High.

But the bandwidth is low.

So, the sampler has if you only the bandwidth or.

Well in principle you can say that if the signal is narrow band it is much easier to design a sampler than if the signal is broad band, you understand? In principle you know that I mean if you have a narrow band signal, what does it mean? It means that between successive samples there is significant corelation that is what it means? Is not it? If it is a band, if it is a band pass signal with narrow bandwidth you know that it is almost a sin wave at the centre frequency. If just minor variations around that sin wave correct. So, it is while it is not easy to do. It is certainly easier to do than trying to sample a system where the bandwidth is very high all. So, this covers all that I had to say about continuous time to discrete time conversion as as far as relating the transforms when the 2 sequences are concerned. The next thing is to actually see what circuits you can how you can design circuits to implement the basic building block of all these structures which is the sampling and holding.

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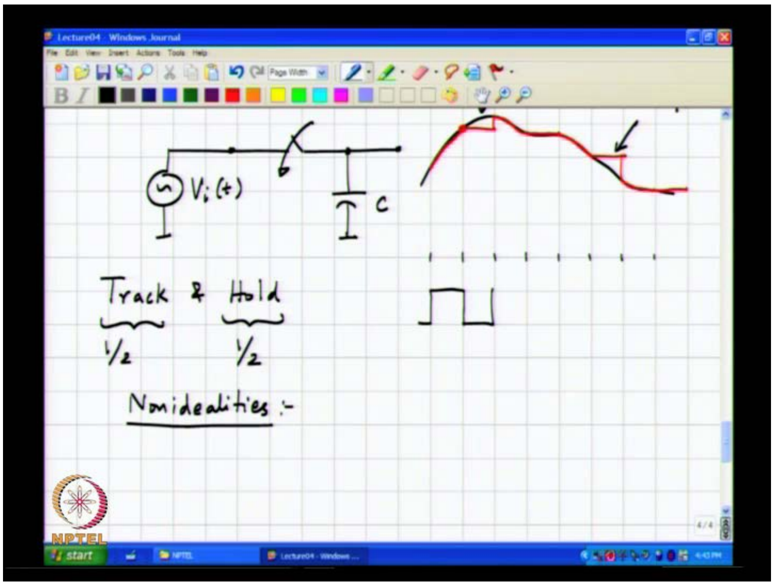

So, what is. So, conceptually what are the simplest sample and hold that you can think of.

Switch.

You need.

A switch.

You need a switch.

Capacitor.

And you need a capacitor which is some device which holds charge. So, all so, when the switch is closed, how will the output wave form look like? let us say the input was like this, how will the output wave form look like. Let us say the switch is closed for 1 half the clock period which I denote like this and is open for the other half of the clock period. So, how will you think, what do you think the output wave form looks like during 1 half? It will if the switch is ideal.

Follow.

It will.

Follow or track the input during the other half it will.

Holds holds.

Hold then what will happen.

If the switch is ideal it will jump up immediately to the input wave form again during the next track phase hold track hold and so on, the black is the input. So, sometimes circuits like these are also what are called track and hold where the tracking operation happens for a part of the clock cycle. And the holding operation is there for the rest of the clock cycle. In this particular example I have chosen the track period to be half the clock period and the whole period also to be half. So, simple enough. Now, let us see what all non idealities there are in this system? What is the and what is the only problematic part in the whole thing?

Switch.

Anyway switch which can be controlled by some signal and make it turn either on or off. So, without getting into the details of the implementation of the switch, what do you think is the basic non ideality that you can attribute to a switch?

Setup box you have a on resistance.

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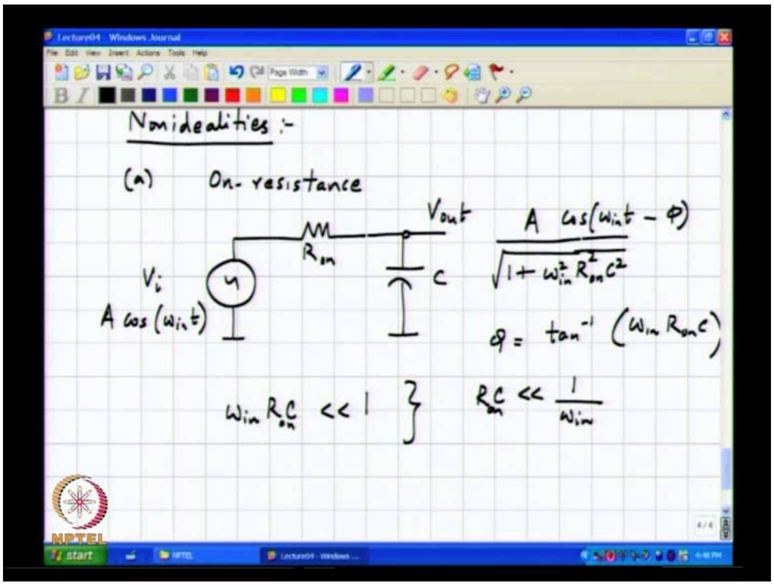

One the first thing I can think of is on resistance that is we have assumed that the switch behaves like an ideal one. So, when you close it it behaves like a short circuit, quite intuitively or rather it is not very surprising that the real switch will have some.

Resistivity.

On resistance. So, now, let us see what happens when there is on resistance . So, if this is v i let us say this is A cos omega N t during the track phase; this is the equivalent circuit what do you think of v out will be.

1 minus e power minus 3 c.

Please note that the input is a sinusoid, what will be the time domain wave form at the output in steady state?

Magnitude.

There will be a change in amplitude there will be a change in phase, what will be the amplitude at the output? A divided by.

Divided by.

Omega 1 square.

Square root of 1 plus.

Omega (( )) square.

Omega square.

R 1 square.

R 1 square c square and cos omega N T minus some phi where phi is tan inverse.

Omega.

Omega N times R on times C, does it make sense? If the track fails this is simply a first order R C filter. So, if you want the output to track the input properly what is the design consideration? For a given c how will you choose R.

Very less should be very very less than.

Pardon.

R r omega smaller when compared to 1.

So, if you want the output amplitude to be a closely or you want it to be a good replica of the input. Then the best you can do is to make sure that omega in R C is much much smaller than 1 in english. This means that the time period of the input sinusoid is much greater or much lesser than the R C time constants.

Much high sir, greater than.

No; do not look at the equation think and tell me if you want the output to follow the input properly. Do you want the input period to be very large or very small compared to the time constant of the R C circuit.

Input very large sir.

The input time period must be.

Very high.

Many many many time constants of the.

Time constant.

R C circuit correct which is what this is telling you correct; this is telling you that R C must be much much smaller than 1 by omega in 1 by omega in is related to the period. So, in other words to have a good tracking bandwidth 1 by R C is nothing but the bandwidth during the tracking phase to have a good tracking bandwidth you must make sure that the 1 by R 1 times c time constant is very very small. So, the tracking bandwidth determines the R C time constant that is how you choose the the R C time constant, is that clear?

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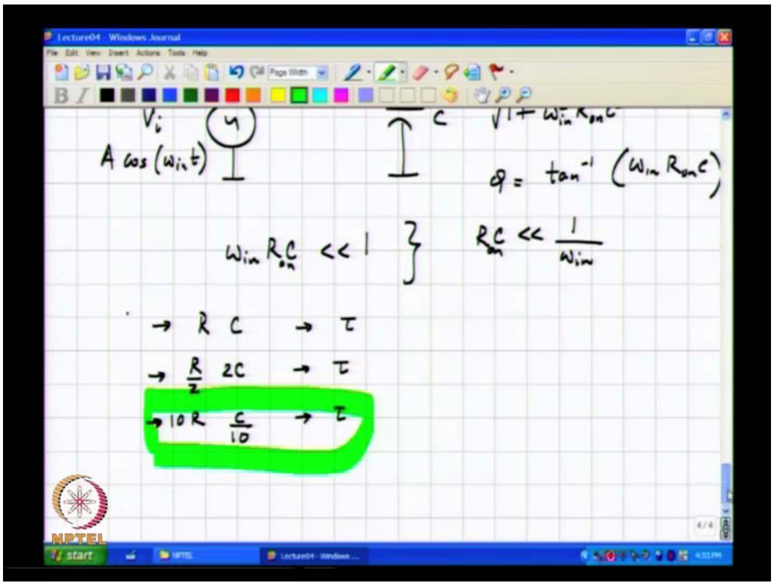

The next question is there are many ways of choosing R and C which result in the same time constant. For example, you know R and C result in tau r by 2, 2 c also results in a time constant of tau 10 R and C by 10 also results in the same time constant tau. So, logical question to ask is there are many choices of R C which give me the same time constant during the track phase which is equivalent to saying that they will all result in the same bandwidth during the track phase. So, the question is is there a reason to prefer 1 set of R C's over the other or does it not matter we can choose anything.

The smaller the best

I mean see there seems to be an advantage to doing this or how would you argue that this is a we did not know anything else how would you argue about this being a better choice.

Constant more resistance.

I mean I need here is a very small capacitance and.

A small source.

And a large switch resistance I mean to get a given time constant I can use either a small switch resistance and a large capacitance or a large switch resistance and a.

A large switch resistance and the small capacitance because this means an individual bad switch and get away correct. So, any comments in that one.

Leakage is there.

During.

Right one problem is that.

It no.

Please note the signal is being held as charge on a capacitor.

Growth.

Smaller you make the capacitor.

Leakage is more.

I mean the more the probability of it getting disturbed.

By all sorts of experience.

So, choosing a large switch resistance and a small capacitance while it seems from the point of view of time constant one thing that I would think about is you say hey our information. Tthe how crucial input signal is being stored on a capacitor as charge now if I have a very small capacitance. It means that the amount of charge on that capacitors is also small which means that in 1 way or the other it is going to be easily susceptable to extraneous disturbances 1 such thing is noise. So, now, we will take a closer look at what happens during the sampling phase.

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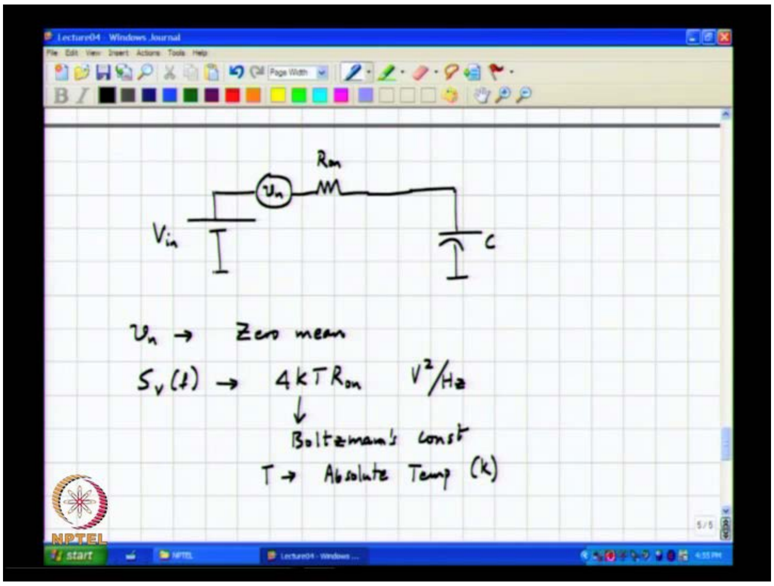

So, let us say there is a constant input voltage v n during the track phase there is a resistor which models the switch unfortunately every resistor as you know is also associated with a noise source all of you aware of aware of this thermal noise correct. So, we model this noise source as a voltage in series with the resistor. And here is the capacitor all now nothing is lost if I move this noise source out here. So, this is the model for a resistor in thermal equilibrium with its surroundings the spectral density of v N. In other words v n is a noise source with 0 mean has a spectral density has a spectral density 4 K T r on what are the units?

Volt square per Hertz.

Volt square per Hertz k is the Boltzman's constant, T is the absolute temperature in Kelvin. And R is the value of the resistor what does this physically mean? What does it mean when we say the spectral density or the noise spectral density of a resistor is 4 K T R volt square per hertz.

It should be very less.

What does it mean?

Only temperature has frequency.

Pardon.

Given a temperature increase in frequency the amount of noise.

No that is that is a that is how I am looking for what is it I mean what does this statement mean? The spectral density of the noise voltage in series with the resistor is 4 K T R on volt square per hertz.

Constant.

It is constant; obviously, with as a function of frequency I mean I mean that is you are restating the formula in words. That is not what I am looking for, what is what does it mean? Physically I mean if you say you know x y z weighs 40 kilograms. What does it mean? Physically you take him put him on a balance and then it will read you know on a weighing machine and for it will read forty kilos that is what it means . So, by the say same token if I say the spectral density of this noise source is 4 K T R on what does this mean?

The energy the energy associated with a noise is 4 K T R.

What is the meaning of I mean first of all this has got dimensions of volt square per hertz correct.

When you pass this voltage source to the band pass filter with a 1 Hertz bandwidth. The output the spectral density will be I mean the mean square value of that voltage will be 4 K T R on.

Yes.

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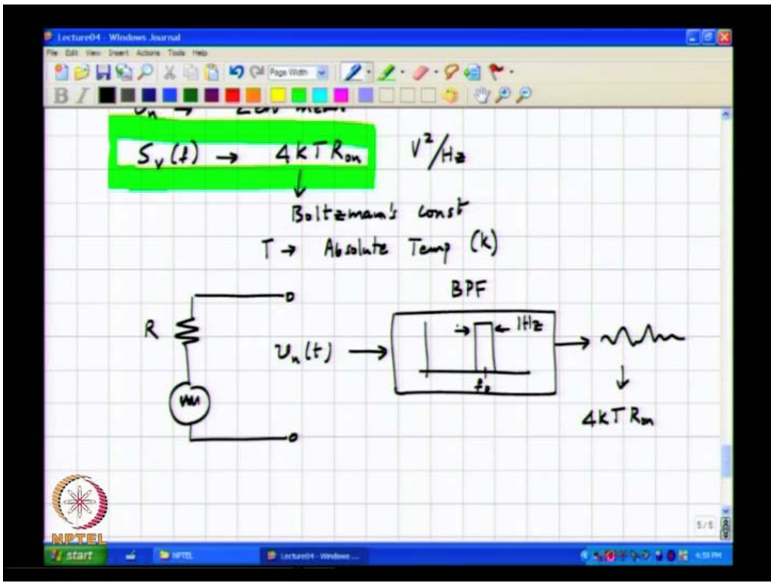

So, what this means is that if I took this resistor which when in thermal equilibrium with it is surroundings has a noise source in series with it. If I monitor this voltage; this voltage will simply be v n of T correct. If I just put if I record the voltage across those 2 terminals, it will be some v n of T. Now, if I took this wave form pass this through a band pass filter centered at some frequency f naught. But has a bandwidth of 1 hertz what do you think will happen to the output of the band pass filter? What will you see? Will you see anything or will you not see anything at all.

Will not see.

I mean clearly you are taking a filter and driving it with some noisy wave form. So, at the output of the filter you will see something the frequency of that something will be the frequency content will be bat around f naught. But with the bandwidth of only 1 Hertz, because I have put a band pass filter ideal band pass filter with a bandwidth of 1 Hertz. Now, this is some wave form here correct I can always find it is mean square value. So, the mean square value happens to be 4 K T times R on, what will be the dimensions?

Volt square.

The input is a voltage band pass filter the output is also voltage the mean square must have dimensions of volt square. So, we will continue with this after the break.