VLSI Data Conversation Circuits
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Lecture - 39
Integrator Design - 2

This is VLSI Data Conversion Circuits lecture 39, in the last class we looked at various flavours of integrators. Now, let us look another important aspect of these integrators namely device noise.

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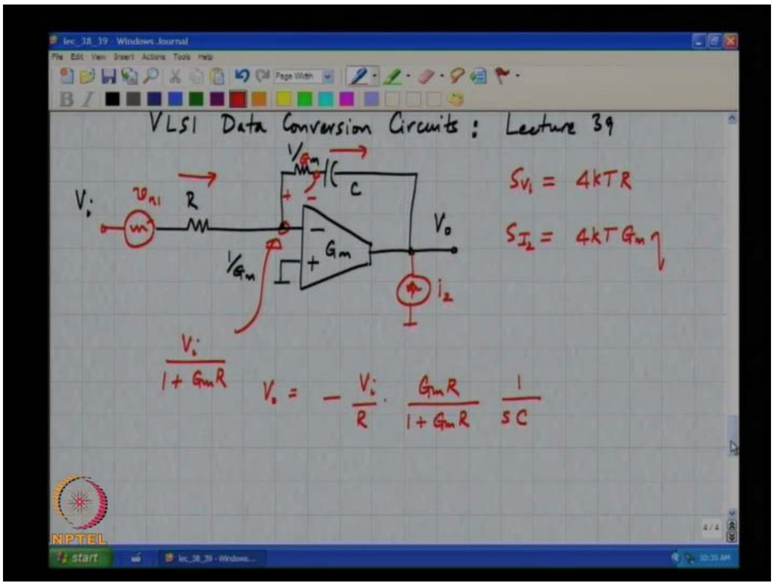

So, if we take the example of the single stage structure, it has several noise sources, one is the resistor itself and the effect of the resistor can noise in the resistor can be thought of as a voltage source in series with the resistor. The spectral density of this noise source is 4 k T R whole square per hertz, the trans conductor on the other hand can be thought of as a noise current being injected at it is output, and the spectral density of this current S I 2 is given by 4 k T times G m, times some factor eta which is called the excess noise factor it depends on the circuit details of the realization of the trans conductor.

But, in principle the trans conductor noise can be modelled by a current source of this, and this makes sense. Because, if I wanted to realize 2 G m what I would do is take two identical trans conductors and put them in parallel right, and the noise current that would be injected at the output, the spectral density would go up by a factor of two. So, it is a self consistent model. now these two noise sources; obviously, cause the output voltage here to be in error all right.

So, to find the effect of these noise sources on the output, we simply use super position, let us also assume that we have the 0 cancelling resistor here all right equal to 1 by G m. So, that the feed forward 0 has gone away, we will ignore the noise contribution of this resistor all right and try and compute the output noise due to both these noise sources. So, before we do that let us find the virtual ground voltage, the exact virtual ground voltage is given by what is the input impedance looking in there.

How will you find the input impedance, you will apply a voltage and find the current, so if you apply a voltage test voltage here what is the current that flows like this G m times v test. So, the input impedance is 1by G m correct, so the input impedance is 1 by G m what would be the exact voltage here, this voltage will be V i times 1 by 1 plus G m R correct. So, the current that flows through the input resistor is given by V i by R times, if the voltage on the right of the resistor is V i by 1 plus G m R and to the left is V i, what is the current that flows through the resistor.

STUDENT: ((Refer Time: 05:54))

It is V i by R times G m R by 1 plus G m R all right that current flows through the integrating capacitor, and what will be the drop across this resistor now.

STUDENT: ((Refer Time: 06:23))

V i by 1 plus G m R, so what will this potential be please note that the virtual ground potential is V i by 1 plus G m R correct, the drop across the 1 by G m resistor is V i by 1 plus G m R, but in which direction, in this direction. So, the potential to the right of that resistor will be 0 is that clear, so the output voltage is simply this current flowing through 1 by s c. So, V o is this, so V o by V i is therefore, minus 1 by SCR times G m R by 1 plus G m and this makes sense because, we know that that resistor was there to cancel out the 0 in the first place all right.

Now, the transfer function from the noise source to the output is the same as that from the input to the output.

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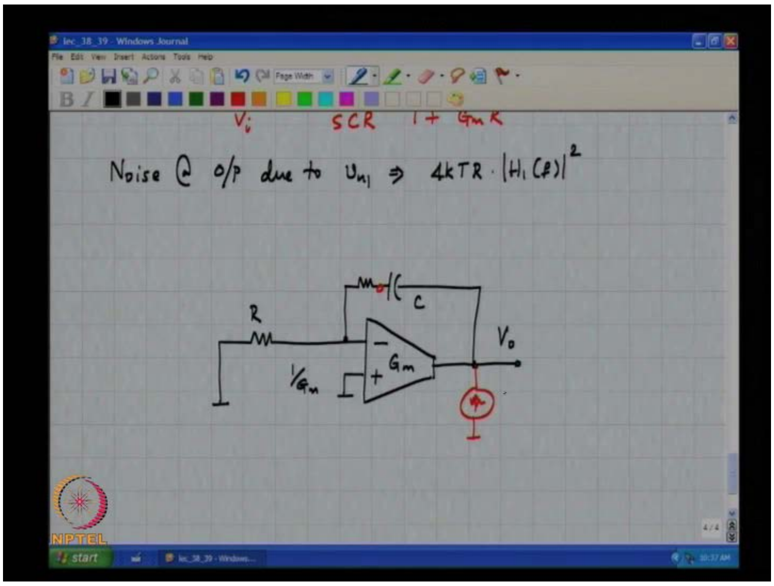

So, the noise spectral density at the output due to the is nothing, but 4 k T R times if I call this H 1 it will be mod H 1 of f the whole square right. So, this noise source we are done with, now we need to worry about what the effect of this noise source will be on the output ((Refer Time: 09:05) all right. So, how would 1 calculate the effect of i n 2 on the output.

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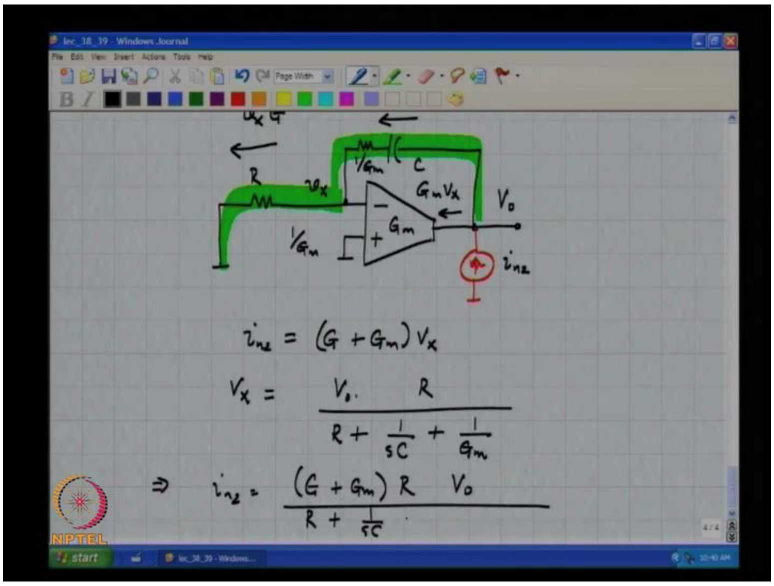

Let us calculate this voltage, let us assume that this voltage is v x correct, so this current will be v x by r which I will denote by G. So, this current will be the same as that correct, so this is G m or rather this is 1 by G m, so this current will be G times v x, this current will also be this will be G m times v x . So, i n 2 therefore, must be G plus G m times V x all right, whatever you are interested in however.

STUDENT: ((Refer Time: 11:48))

We are interested in V o right, so can you help me evaluate V o.

STUDENT: ((Refer Time: 11:56))

V x is nothing, but it is simply V naught into this I mean the same current flows through this branch and this branch right. So, it is nothing, but a voltage divider, so V x is nothing, but V o times R by R plus 1 by s C plus 1 by G m, which means that i n 2 is G plus G m times R times V naught divided by R plus 1 over s C plus 1 over G m.

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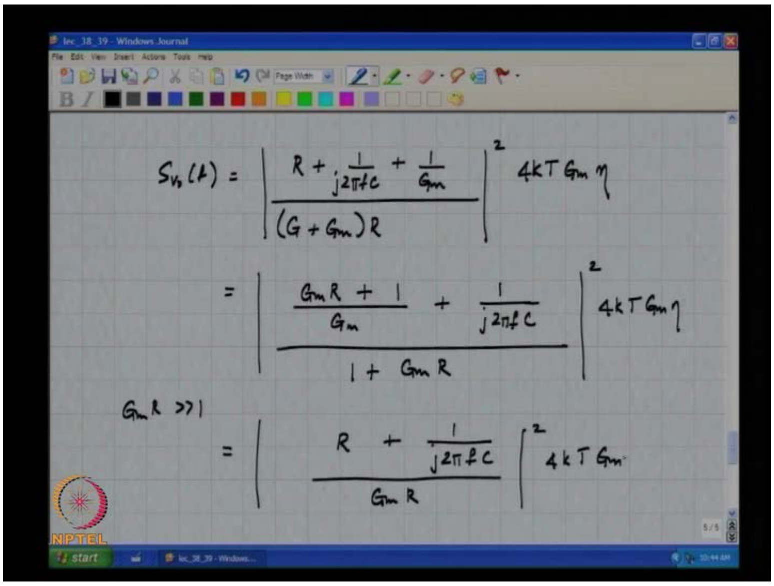

So, V o is nothing, but or S v o output noise spectral density is given by 4 k T G m times eta, which is the noise spectral density of i n 2 multiplied by that square of the transfer function, which is R plus 1 over s C plus 1 over G m divided by G plus G m times r, which can be simplified further for this whole square correct this must be j 2 pi f c since we are interested in spectral density of the function of hertz right.

So, this is nothing, but G m R plus 1 over G m all right plus 1 over j to pi f c divided by 1 plus G m R times 4 k T eta j. This is approximately equal to I mean if we choose G m R to be much, much greater than 1, which 1 would do anyway to make sure that the unity gain frequency of the integrator is close to 1 over R C this spectral density will reduce to 1 over G m plus 1 over j 2 pi f c divided by if G m R is much larger than 1 you can neglect the 1 in relation to the G m R this is R whole square divided by 4 k T eta G m.

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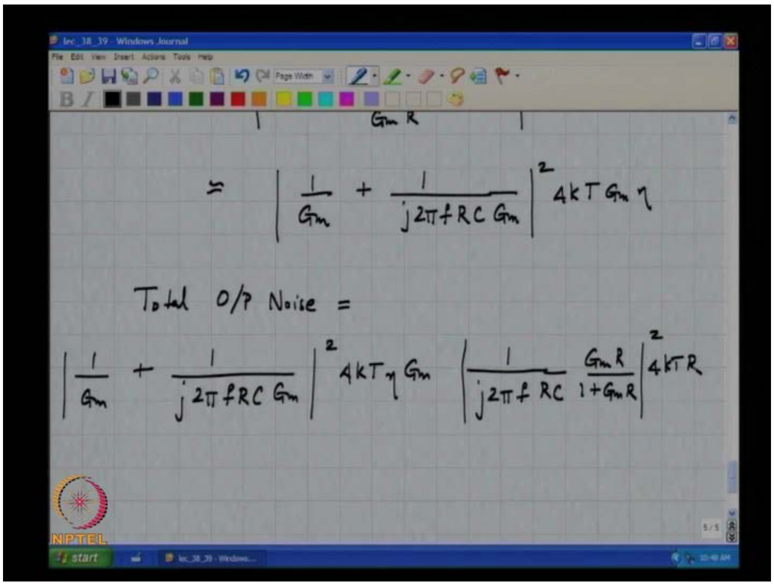

Which can therefore, be written as 1 by G m plus 1 over j 2 pi f R C times G m whole square into 4 k T G m times eta. Let us see the intuition behind the result as frequency tends to infinity what would you expect without doing all the maths the noise voltage here or the transfer I mean is basically this noise spectrum is multiplied by the transfer function the whole square.

The transfer function is nothing, but the output impedance looking into the trans conductor right at very high frequencies what would be the output impedance, at very high frequencies what is happening, the capacitor behaves like a short circuit. So, whatever voltage you apply here. Almost all of it appears at the virtual ground correct because, it is a series divided between this 1 by G m and this R the 1 by G m as you know is very small compared to R clear.

Which means that this voltage will virtually be what you apply here, at very high frequencies, which means that the current drawn by the G m will be G m into whatever test voltage you apply there. So, the output impedance at very frequencies must be close to.

STUDENT: 1 by G m.

1by G m right, at low frequencies of course, the capacitor is an open, so the output impedance is infinite right. And that is precisely what you see here, at high frequencies where f tends to infinity this voltage becomes I mean the impedance becomes simply 1 by g m. Because, this second term tends to 0 and the output noise spectral density simply becomes 1 by G m whole square times 4 k T eta G m all right.

Now, what we are interested in is the total noise spectral density at the output, and that will be 1 over G m plus 1 over j 2 pi f r c times G m the whole square times 4 k T eta G m, this is the noise contribution due to the OTA plus we need to add to this the noise contribution due to the plus the integrator transfer function which is 1 by j 2 pi f R C times G m r by 1 plus g m r the whole square to 4 k T.

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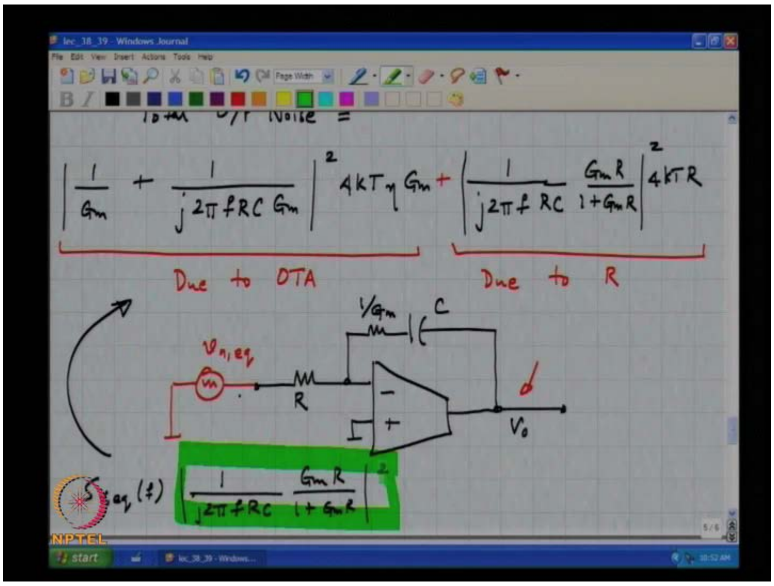

So, this is the contribution due to R and this is the contribution due to the OTA all right, so what one would be interested in practice is to compare, please note that this is the integrator. ((Refer Time:21:07)) And we know what the noise spectral density is here, but what we did like to do is to figure out how this noise spectral density compares in relation to the strength of the input signal itself right. So, in other words we did like to refer all these noise sources to the input.

In other words what we are trying to say is that, let us find the strength of a single noise source at the input, whose effect is to have or whose effect is the same as that of all these multiple noise sources inside the circuit right. So, that is what is called the input referred noise source, so what we call that v n e q, the e q signifies the equivalent single source right whose effect is the same as all the others multiple source noise sources in the circuit all right.

So, if one wants to find the strength of the equivalent noise source, all that 1 needs to do is to observe that the output noise spectrum due to the equivalent noise voltage source we have chosen is simply S v equivalent multiplied by mod H of f the whole square which is 1 by j 2 pi f R C times G m R by 1 plus G m R the whole square times 4 k T R I am sorry this must be equal to this right.

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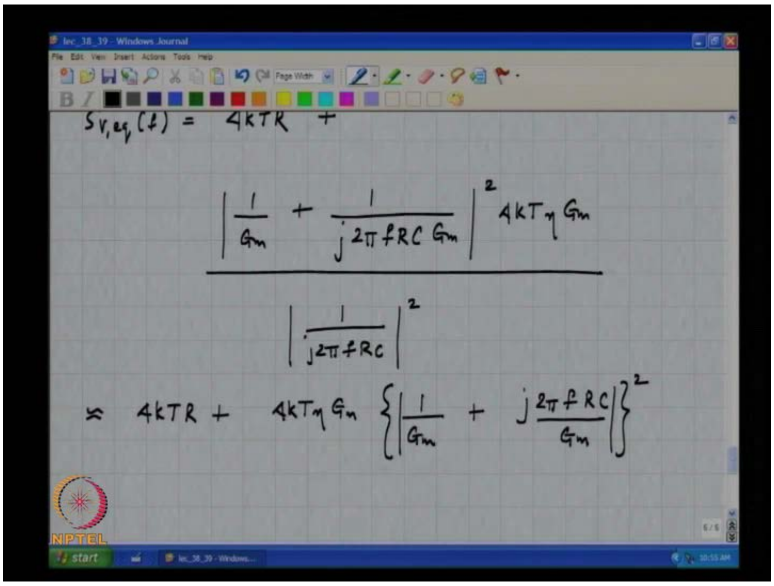

So, clearly S v equivalent of f is nothing, but you divide throughout by this factor, and when you divide this goes away. So, you get 4 k T R this makes sense because, the noise source corresponding to the resistor is at the same place as the equivalent noise source that you are trying to find. So, it is no surprise at all that 4 k T R appears all right, the next thing is this factor divided by ((Refer Time: 25:08)) this.

And again let us make a couple of simplifications assuming that G m R is much larger than 1, I will just simply divide by I will make this approximately 1. Now, what happens

4 k T R plus 4 k T eta by G m times 1 by G m plus j 2 pi by f R C by G m, this the modulus square all right.

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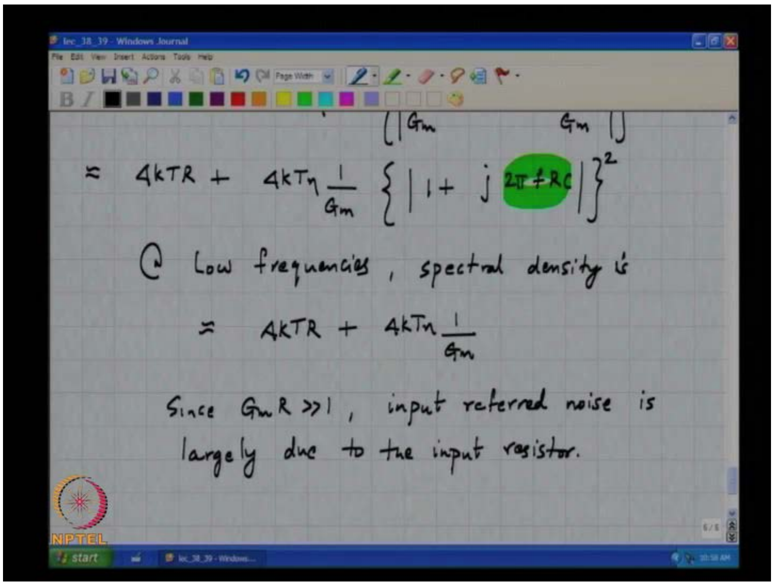

Now, our signal exists only at or rather before I get to that, let me just simplify this by pulling out a 1 by G m common factor and therefore, this becomes 1 plus j 2 pi f RC a whole square our input signal is at low frequencies only right. So, at low frequencies this is going to be much smaller than unity, so the input referred noise spectral density at low frequencies is 4 k T R plus 4 k T eta by G m you understand.

And if G m R is much, much larger than 1, which do you think is a dominant source of noise the resistor is the I mean since G m R is much, much larger than 1. Usually done to make sure that the unity gain frequency of the integrator is 1 by R C, we see that the noise contribution is largely because of the input resistor, does it makes sense all right ((Refer Time: 29:32)) is this clear.

Now, let us see I mean we have now understood what thermal noise does to the integrator, the effect of all these thermal noise sources is a single input referred noise source, whose spectral density is pretty much that of the input resistor, but in our sigma delta modulator we have several integrators.

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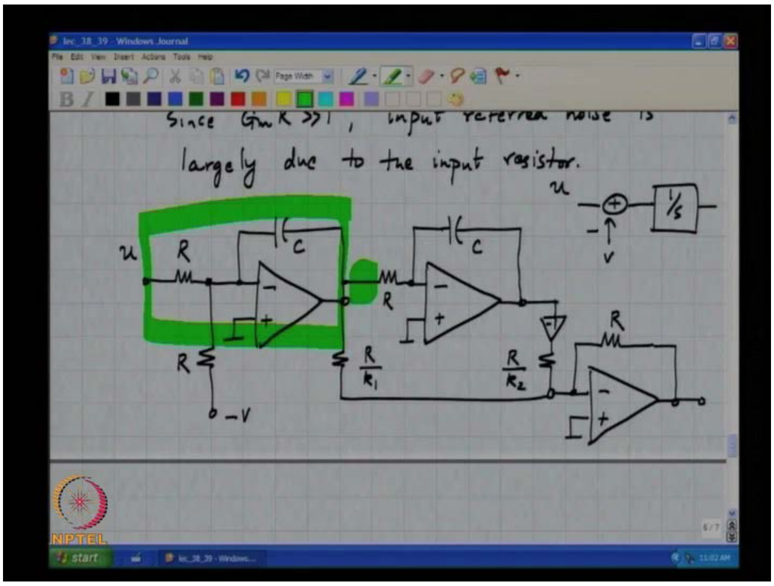

Again let me take the example of CIFF loop, so a loop filter probably looks like this ((Refer Time: 30:50)) all right, and please note that we want recall that the first the input size of the swift modulator looks like u minus v. So, what should we do how do we sum up u and v you can add instead of using a single integrator, I mean you using an integrator you can use a summing integrator by using another resistor like this for instance, this is u and this is minus v, and you need to have I am sorry you need an inversion.

So, all right and once you close the loop around the quantizer the signs must be chosen, so that there is overall negative feedback. you must carefully follow through the inversions and, so on. So, what do you think the input referred noise of this structure would be, what is the input referred noise of this integrator or what is the I mean when referred to it is input approximately 4 k T R.

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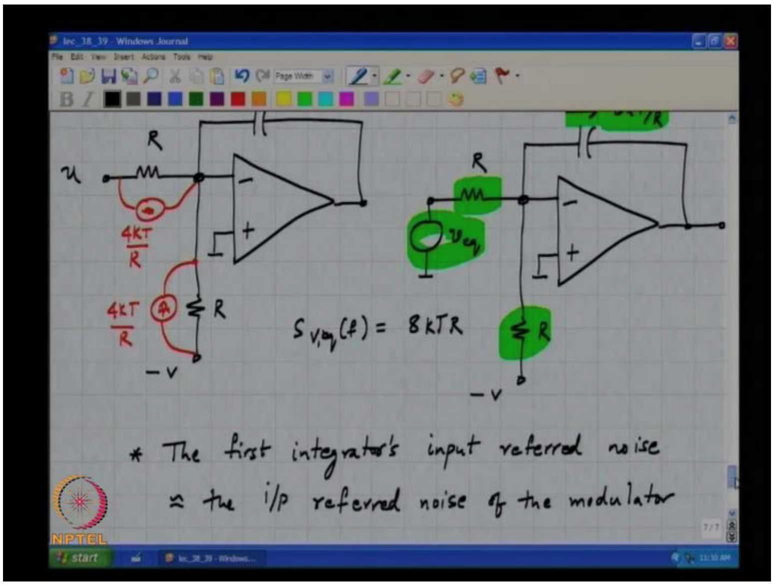

Now, when this noise source is referred to the input of this integrator, what do you think will happen.

STUDENT: ((Refer Time: 34:23))

In other words, what I am saying is that think about it from a block diagram view point like this. You have two integrators k 1 and k 2, and you are actually injecting you can think of this as some noise here all right, so what is the effect of a noise source here when referred to this input here.

STUDENT: ((Refer Time: 35:38))

It is s times noise source, in other words at low frequencies the effect of this noise here is negligible, and why does that makes sense this block is an integrator which is got infinite gain at d c and at and a very large gain at low frequencies. So, any accuracy inaccuracy here, when referred back to the input gets divided down by the high gain or the first integrator. Therefore, the effect of these noise sources that is those added by the second integrator, and subsequently in other integrators or the sum or for that instance.

When referred back to the input of the loop filter become almost 0 right, so this means that the only noise source we have to worry about is that of the first integrator is this clear. And what do you think is the noise input of a noise of this integrator, since we said that the noise of the op amp is negligible let us assume an ideal OPAMP to begin with all right. So, what is the input referred noise of a this integrator it is not 4 k T R I mean the see you cannot say 2 k T R.

Because, without knowing anything right you have taken an without this it was 4 k T R right you have added one more resistor which produces more noise correct. So, the input referred noise cannot be smaller than 4 k T R you understand, if at all it can only be equal to or larger. So, let us see what it is a simple way of doing this because, both the resistors are connected in the parallel as to assume noise current rather than noise voltage.

So, this has a got a strength of 4 k T by R minus current, similarly this resistor also is got a nice current whose strength is...

STUDENT: 4 k T by R.

4 k T by R, so the total noise current flowing through this capacitor is 8 k T by R, if 1 had to have a single voltage source whose effect would be the same as the effect of these multiple noise current sources, at the output the single equivalent voltage source here must be able to produce 8 k T by R into the capacitor correct. And that is only possible when what assume these resistors are now noiseless, what is the transfer function from v e q to this current. The current is v e q, where R is the current through the integrating capacitor.

So, spectral density wise I must divide by 1 by r whole square, so s v e q of f is nothing, but 8 k T into R you understand is this clear. So, to summarize the discussion on noise what dominates the input referred thermal noise of the modulator, the first integrator input referred noise is approximately equal to the input referred noise of the modulator. Now, this can be used to advantage in the sense that, the noise of this integrator and this sum are does not matter. So, one can deliberately make them more noisy, and what is the advantage of making them more noisy.

STUDENT: ((Refer Time: 43:12))

You cannot make I mean how can you make something more I mean noisy and expect better performance what are you gaining.

STUDENT: You can now load design.

A design is not any easier than.

Local ((Refer Time: 43:33))

If have a circuit which is noisy and if you want to get half the noise what will you do I will take to identical circuits and connect them node to node in parallel, and that will reduce the noise by factor of noise spectral density by a factor of 2, while doubling the power dissipation this you are aware of right. So, designing something which is noisy as defining as difficult or easy as designing something which is less noisy, where it is just a matter of copy paste and connect multiple copies in parallel all right.

So, turning this discussion upside down if you double the noise power, and that is done by taking the circuit chopping it up into 2 identical halves, and getting rid of 1 of the half’s. So, the noise is now 3 d b was right whereas, power dissipation is...

STUDENT: 1 half.

Is 1 half of what it would be before, so since we know that the noise of subsequent integrators do not really matter. Because, when referred to the input of the modulator they get divided down by the high gain of the first integrator in the signal band, it is acceptable to make them deliberately noisy right, and gain in the form of power dissipation right. So, in other words if I took this integrator chopped it up into n parts and threw away n minus 1 of those parts, what will happen to this resistor here, it will become n times r what will happen to this capacitance c.

STUDENT: C by n.

C by n, now and what will happen to this current.

STUDENT: It has reduced.

This current has reduced by a factor of n, since this current has reduced by a factor of n, the current that the OPAMP needs to drive has also reduced by a factor of n. And therefore, it makes sense that you can make the OPAMP n times smaller by reducing all bias currents in the OPAMP by a factor of n. So, the OPAMP power dissipation is also reduced by a factor of n right, so it is therefore, common place to choose the impedance levels of subsequent integrators.

And whatever else comes after the first integrator to be much higher than what you did choose for the first integrator. Because, a you save power you get more noise; however, when you refer that back to the input it does not matter you understand, so the noise of the first integrator is a crucial factor that affects the input referred noise of the modulator does makes sense.

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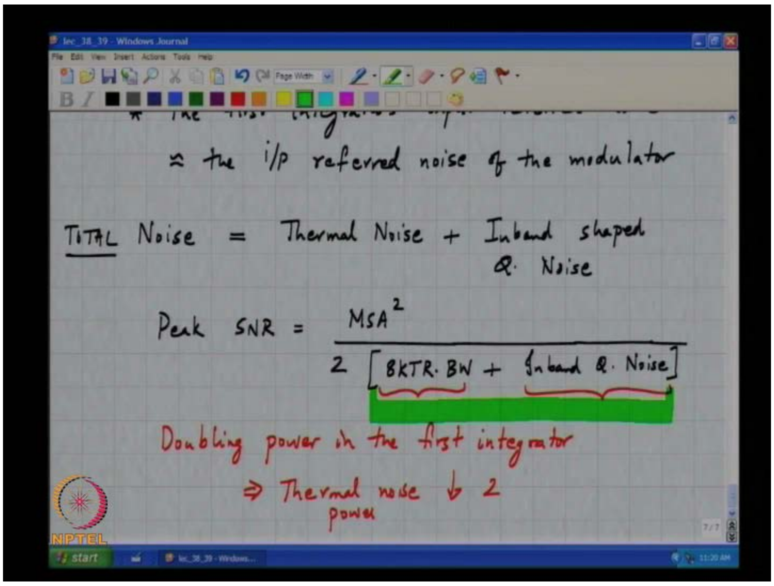

Now, we have several noises to deal with right, the total input referred or the total noise at the output of the modulator consists of 2 components, what are the 2 components.

STUDENT: ((Refer Time: 47:33))

I mean and what else.

STUDENT: One way.

I mean you are taking this and embedding it is inside a sigma delta loop correct, so the total in band noise consists of thermal noise from the loop filter, which as you all say largely comes from the first integrator plus in band shaped quantization noise. And since thermal noise and quantization noise are independent of each other, you can simply add their noise powers you understand all right.

So, the peak SNR of the modulator is therefore, the maximum stable amplitude MSA whole square by 2 and why is this whole square by 2 the sinusoid. And then, the mean square value of the sinusoid is amplitude square by 2 and this divided by the noise power which is thermal noise, which is largely in the example you have taken is 8 k T times R times that is the spectral density to get the power we have to you have to multiply by the bandwidth plus in band quantization noise all right.

Please note that this everything is under our control right, this can be reduced one can keep this the same, and reduce the thermal noise by.

STUDENT: Reducing impedance scale in the...

Impedance scale in the first integrator one must bear in mind that reduction of the impedance level means increase in the power dissipation by the same factor all right. Specifically doubling power in the first integrator will cause the thermal noise to noise power to go down by a factor of 2 all right. One can also keep the thermal noise fixed and reduce the in band.

STUDENT: Quantization.

Quantization noise all right, and how will one do this.

STUDENT: ((Refer Time: 51:16))

There are many ways of doing this you either increase the order or increase the out of band gain or a combination of the above. And therefore, the in band quantization also can be noise can also be varied, now one obvious question is, if I want to shoot for a certain signal to noise ratio is there I mean should I make my modulator be limited by thermal noise or be limited by quantization noise, you understand to make to achieve a certain signal to noise ratio it means that I must achieve a certain.

STUDENT: Noise.

In band noise, the in band noise consists of two components, one is thermal noise, the other is the shaped quantization noise. Thankfully both of them are independent and are can be controlled independently, the thermal noise can be controlled by simply impedance scaling the first integrator. The shaped quantization noise can be controlled by changing parameters of the noise transfer function, in ways which you are all aware of by now.

Now, can we comment on what strategy one would use in practice, do you think I should choose the in band quantization noise to be much larger than thermal noise or should I choose, you understand I want this level of thermal noise of total noise right.

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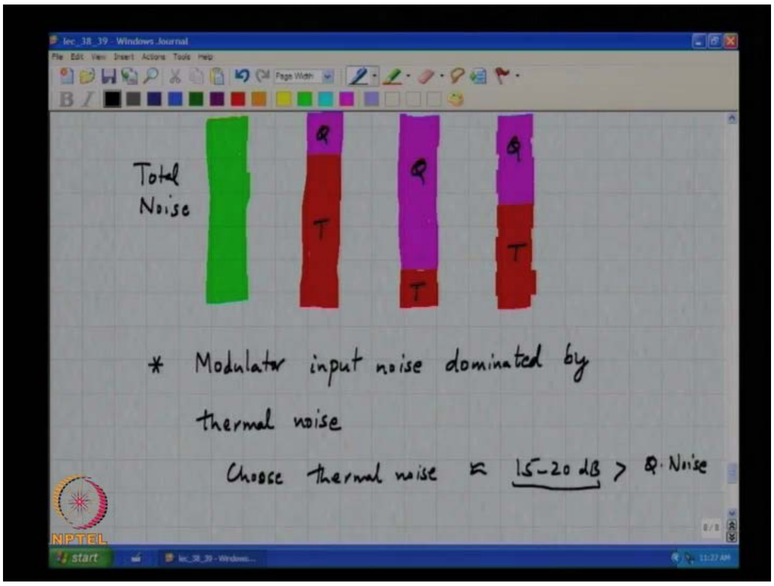

This can be I can think of early 3 scenarios, how you can realize this, this much can be thermal noise, and this much can be quantization noise all right or this much can be quantization noise, and this much can be thermal noise or both of them can be the same Q T let me exaggerate this a little more. So, which do you think makes more sense to do.

STUDENT: More quantization noise.

You want more quantization noise and less thermal noise, why does this makes sense.

STUDENT: If we have less quantization noise means that we end up quantization more difficult quantization noise differential overall modulator design it is difficult ((Refer Time: 54:59))

So, this is the desired total noise, you all understand the question right the total noise is, so much and that is derived from the spec of the SNR. You want that to be some level the question now is does it makes sense to make it all thermal noise or all quantization noise are both of them equal.

STUDENT: No that you people done how much probably we need to relatively how much power we have to spend to reduce quantization noise or how much power you need to spend to reduce some of noise or even that ratio is know you can design.

So, great, so the what he is saying is the following, when you say which what is the best thing to do it; obviously, this has to do with power efficiency correct. Because, to reduce thermal noise there's an implication, if you want to reduce thermal noise you must increase power in the first integrator, I mean the other integrators are you know are total non entities as far as input referred noise of the whole modulator is concerned.

To reduce quantization noise, what should one do I mean do you think there is a power penalty associated with increasing quantization noise.

STUDENT: You are comparative.

No, one can easily increase the reduce the quantization noise by for instance mildly changing the.

STUDENT: Out of band.

The out of band gain of the noise transfer function, you understand of course, one could go and design a more complicated quantizer, but that is not necessary. So, in other words to reduce quantization noise, it is very easy from a power point of view of course, one requires to design a more aggressive or slightly more aggressive noise transfer function. But, there is in principle no power penalty associated with reducing the quantization noise.

However, there is a big power penalty associated with reducing the thermal noise to reduce thermal noise by a factor of 2, you need to double the power dissipation of the first integrator right. Whereas, to reduce quantization noise by a factor of 2 you do not have to do anything, as far as power dissipation is concerned, so what does this mean. Now, that you know this does this make sense to make the input noise largely thermal noise or largely quantization noise, largely.

STUDENT: Thermal noise.

Largely thermal noise right because, if you choose the opposite then.

STUDENT: The power dissipation.

The power dissipation is unnecessarily.

STUDENT: Large.

Large right whereas, it is very easy to get rid of quantization noise by simply changing parameters of the noise transfer function. So, you always want to make sure that the thermal noise dominates the input referred noise of the quantizer of the modulator, so it is common practice to choose thermal noise to be about 15 to 20 d B greater than the shaped quantization noise. So, noise power of the thermal noise is 10 times larger I mean at least I mean here it must be about this means about 30 to 100 times larger in power terms, when compared to the quantization noise.

This has got other benefits in the sense that, the input of the quantizer is now also got a random component due to thermal noise. So, the quantization error is also becomes less periodic because of this a random thermal noise that the device noise adds, you understand. And of course, for now power point of view this makes a lot of sense because, you do not I mean you know that it is very easy to reduce quantization noise power by simply making the noise transfer function more aggressive.

So, this concludes all that I had to say about noise in the integrator, noise in the modulator and how one would choose the levels of thermal and quantization noise.

Thank you.