VLSI Data Conversion Circuits
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Lecture - 37
Simulation of CTDSMs

This is VLSI Data Conversion Circuits lecture 37. In the last class, we looked at dynamic range scaling of the loop filter so that, the integrators in a loop, all kind of saturate it about the same input level. This way, not one single integrator prematurely limits the performance of the loop. The next subtopic that I wish to discuss is the practical issue, how one goes about simulating a delta sigma loop.

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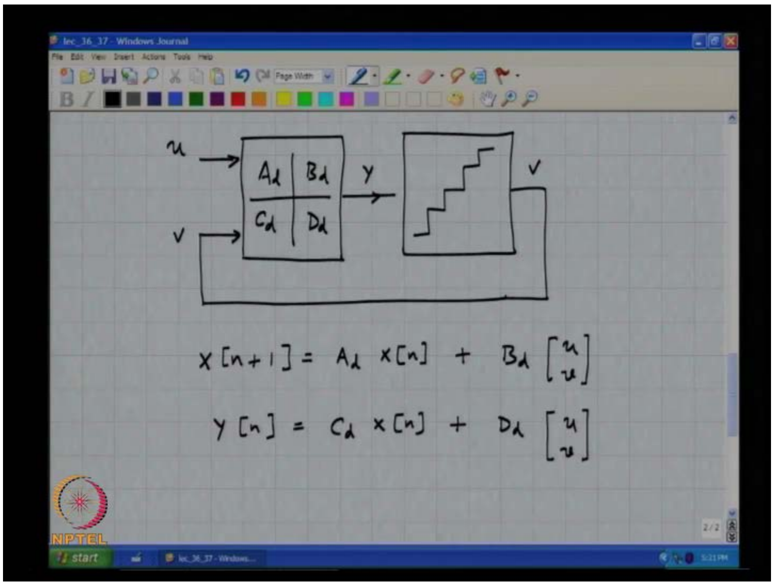

So, with a discrete time modulator, one can think of the loop filter as being a discrete time system with two inputs and one output, and the output is quantized and fed back. So, this is a linear filter and can therefore, be represented by the state equations, the difference equations which can be recast in states ways form and the state matrices are given by A d B d c d and D sub d. And the state formulation is that, the set of states X, please note that X is a column vector at instant n plus 1, is linearly related to the set of states at instant n plus B d times the inputs to this linear system, which are.

Student: U and v

U and v, so this is a two row one column and Y of n is C X plus D times u and v.

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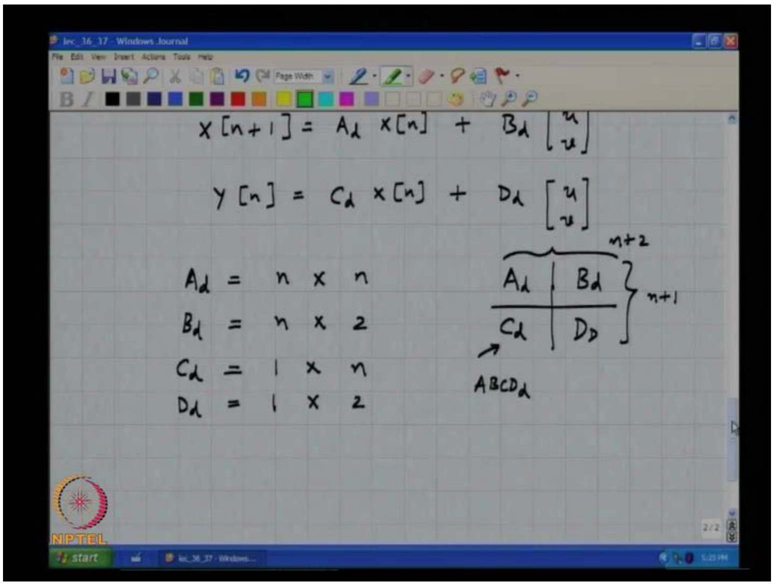

So, in general, the A d matrix will be an n by n matrix, just it is a square matrix, B d in our case will be a, what will be the dimensions of B d...

Student: ((Refer Time: 03:56))

N cross 2 C d

Student: ((Refer Time: 04:07))

N cross

Student: ((Refer Time: 04:12))

1 cross

Student: 1 cross n

So, you have only one output and D d...

Student: 1cross 1

1 cross

Student: 2

2, and so if you concatenate these matrices together, it will form

Student: ((Refer Time: 04:54))

How many columns are there

Student: ((Refer Time: 05:00))

If I concatenate A d and B d

Student: 1 plus 2 columns

How many columns are there

Student: ((Refer Time: 05:08))

N plus 2 columns and how many rows are there, if I concatenate B d and D d

Student: 1 plus 1

B d has got

Student: ((Refer Time: 05:25))

N rows

Student: N plus 1

N plus

Student: N plus 1

1, so if you put A d B d c d and D d in this fashion, you will get a big matrix which is...

Student: N plus 1

N plus

Student: 1 cross 1

1 cross

Student: N plus 2

N plus 2 and if you know the order of the system, given this composite matrix A d B d C d and D d or if I call this ABCD sub d and if I know the order of the system then, I can find everything, you understand. So, this is an useful way of passing A B C and D as one matrix, rather than four matrices, any way this is an aside. Now, you see the simulation is quite straight forward, once you know the present state and you know A B C and D, and you know the input, you can find the next state Y that is, quantized and that becomes the next input.

And as you have all played around with the tool box, if normally implemented in MATLAB, this would require a lot of

Student: Time

No, for statements, because I mean, this is the feedback system, so you cannot run it parallely, at least in a state forward way. So, what the tool box does is, implements this loop in C so that, the computation is very fast and interfaces with MATLAB. And therefore, this simulate DSM routine, is actually a very fast and very functional, where you specify the loop filter in state space form for instance, give a certain random initial conditions for the loop state variables to begin with and it does the rest.

It will also give you the maxima of the state variables and what are the utility of the maxima of the state variables.

Student: ((Refer Time: 08:17))

I mean, this is the simulate DSM routine only for simulates a linear loop filter, but the maxima of the state variables can be used to advantage to scale the coefficients inside the loop filter so that, the outputs of the each of the integrators...

Student: ((Refer Time: 08:37))

Are confined to some limit, so now the problem that we are trying to address is, how does one simulate a continuous time loop filter.

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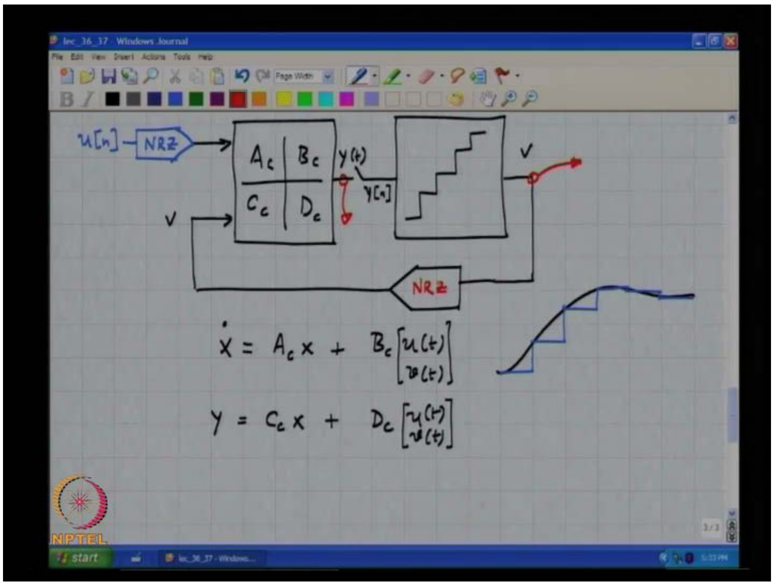

The situation is actually quite similar in the sense that, this is a function of time and this is sample. The loop filter now is a linear system and can therefore, be described by a state space formulation and what is the state space formulation X dot is...

Student: ((Refer Time: 10:08))

A c times X plus B c times u and v, and Y is nothing but, C x plus v and u, now the remarks we made with respect to the dimensions of A B C and D hold here also, but one thing we need to be aware of is that, the u and the v here are now functions of...

Student: t

T, so to make that explicit let me, one thing to observe however is that, you are really only interested in the samples of the loop filter output, because that is what determines the quantized version of that, is what the output sequence is. We had be interested in finding what the output discrete time sequence is for a given continuous time input u. So, we like to find out therefore, only the outputs of the loop filter at...

Student: Discrete

Discrete instances of time, we are not really interested in the output of the loop filter for all time. So, the question now is, can this be exploited to simplify the simulation of the loop, do you understand. Normally, we have what we would have to do otherwise, would be given u of t and given v of t, we have to integrate these set of differential equations which are linear, because the system is linear and generate Y of t.

Student: ((Refer Time: 12:47))

And then...

Student: Sample it

Sample it and use that sampled value, quantize it and feed it back, where it seems like an awful waste of effort, because you are computing the output of the loop filter for all time, I am going to get the samples. So, the question is, can I be smart about it and simplify things if I am only interested in samples of the output to begin with. So, what one should observe or a couple of things, one of course is that, v of t is v of n pass through a pulse shape P of t.

And for most practical purposes, these pulse shapes are either the NRZ pulse shape or the RZ pulse shape or the exponentially decaying one. I show you the method for an NRZ pulse shape and the rest of it is actually quite straight forward and works in a similar way. The input on the other hand, what we say the input is slowly varying, because the system is an oversampled one. So, the input can always be modeled as approximately a first order held waveform, so the input being slowly varying.

So, as far as the input is concerned, because it is slowly varying, since we are dealing with an oversampled converter, one can think of it as a sampled version u of n driving the pulse shape, which is actually an...

Student: NRZ

NRZ pulse

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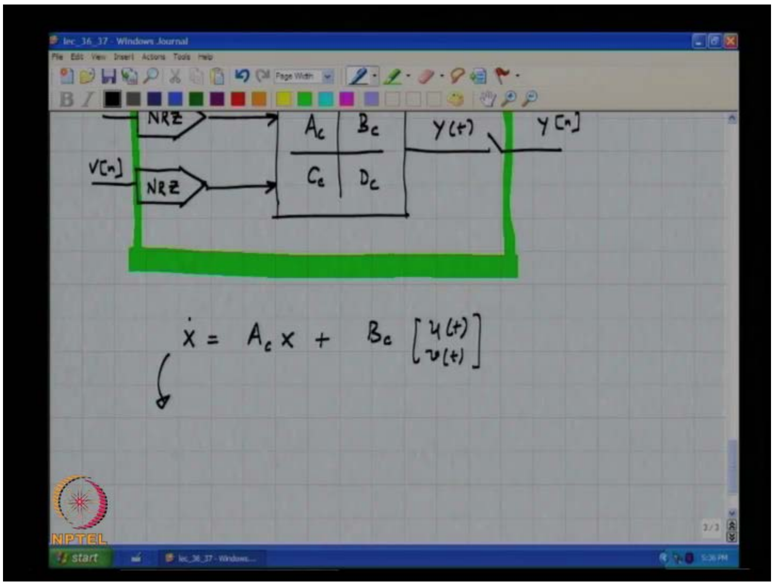

Therefore, if this pulse is also an NRZ pulse, you now have a linear system, whose two inputs are basically u of n and v of n with NRZ pulse shapes and this is Y of t, we are actually interested in Y of n. Now, let us try and see, what the output I mean, please note that as far as this system is concerned, the one drawn in the green box, this is still a...

Student: Discrete

Discrete time system, so the inputs are discrete time sequences, the output is a discrete time sequence. So, if you are able to find, therefore the equivalent discrete time system, you understand. If you think about this green box, two discrete time inputs, one discrete time output, the box is linear, so one should in principle be able to derive...

Student: Discrete

A discrete time equivalent or this continuous time state space system and once we do that, it is just a matter of using the delta sigma delta tool box to simulate the continuous time modulator. So, if you have an input, so X dot is A c times X plus B c times u and v or u of n v of t, but now the u of t and v of t are slightly different in the sense that, they are held, both u and v are both held constant between in a adjacent sampling outputs. So, if this is a set of differential equations, what is the solution to this.

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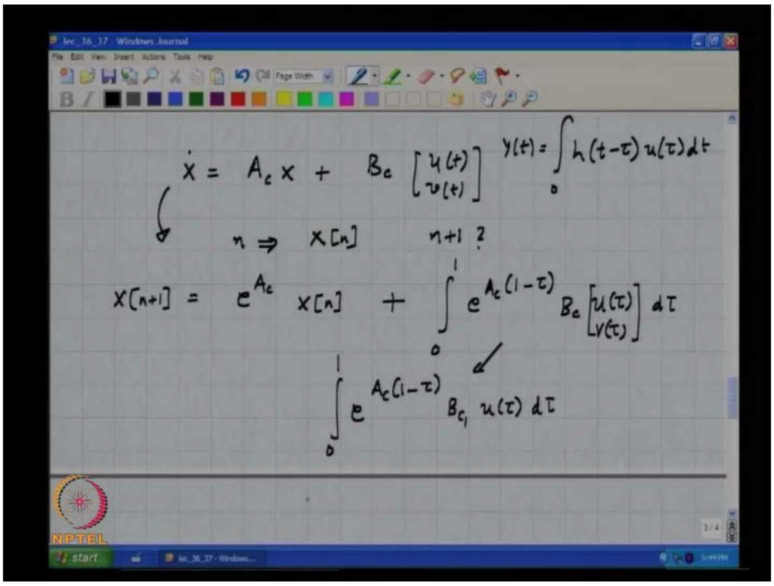

Let us say, the initial condition at time n was some X naught, we need to find what the states will be. Solving the differential equation is nothing but, finding the states and how do we derive the state. I mean, what are the state equations of the discrete time system here after, the state formulation is telling you, how the next state relates to the...

Student: Previous

Previous state, that is how you get A d and B d, so in other words, as far as the continuous time system is concerned, given an initial set of states X naught and given the input to the system, we like to find what the output or what the states would do after 1 second, 1 second is, I have assume that to be the sampling rate. So, in other words, the loop filter which is continuous time, has an initial set of states X naught, I had like to find what the state will be at...

Student: N plus 1

N plus 1, rather than call it X naught, as I suppose it is better thing to call it X of n, so what will be the state variables at n plus 1, there are two things happening here. If the input was 0, you would get the 0 input response and what would that do. So, it turns out that, the impulse response of this set of differential equations is e to the A c times t. So, if you had an initial condition X of n and no input at all, the states would follow the trajectory e to the A c times t times X of n.

So, at t equal to n plus 1 what will happen, in other words if you, what I meant was, so t minus e to the A c times t minus n. So, at t equal to n plus 1, what do you think will happen?

Student: ((Refer Time: 22:34))

The states will have changed by e to the A c times n plus 1 minus n, which is e to the A c, so X of n plus 1 will consist of two components, one which is the natural response of the system of the transient response, where the states decay with I mean, as e to the A c times t minus n. We all familiar with this from basic circuits, let us say you have an R C circuit, the capacitor is charged to some voltage v, v naught and then, you let the circuit go what will happen, the capacitor will attempt to discharge through...

Student: Resistive

The resistive with the time constant tau, so after time of 1 second, it will be the initial voltage multiplied by e to the minus t and it turns out minus t by R C. And if you do the state phase formulation, you will get exactly the same thing, but there is also a forced response, because the inputs are trying to...

Student: Excite

I mean, excite the system, so the forced response will be, you have a bunch of inputs and you have a set of impulse responses. So, once you know the input and the impulse responses, the output is nothing but, the convolution of

Student: The input and impulse response

Input and the impulse response, and what we are interested in is, this convolution integral, because we assume that, for all practical purposes, whether you start at n and go find the output at n plus 1 or start from 0 and go to 1, it is the same thing. So, this is a convolution, which is integral 0 to tau, the impulse response e to the A c t minus tau. Please recall that, h of Y of t is nothing but, h of t minus tau u of tau d tau integral 0 to t, this is the standard convolution integral.

So, you want the output at the after 1 second, so e to the A c 1 minus tau B c times, say u of t v of t d tau

Student: U of tau

U of tau, but within this interval, what are u and v

Student: ((Refer Time: 26:01))

Where u of n times constant 1, similarly this is v of n times

Student: Constant

A constant 1

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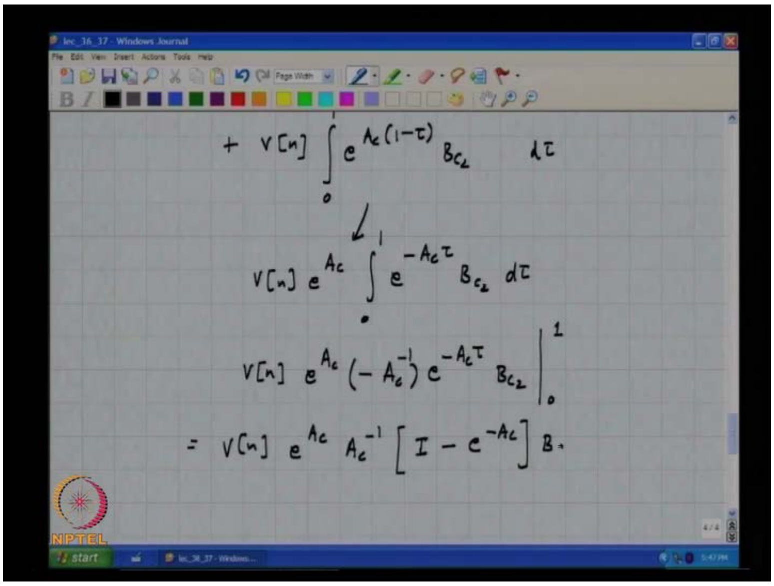

So, if you think of this as, you can also write this as 1 minus tau B c 1 into u of tau d tau plus integral 0 to 1 e to the A c to 1 minus tau B c 2 v of tau d tau, where B c is nothing but, B c 1 B c 2 and u of tau and v of tau are both I mean, you can think of them as u of n times 1 and similarly, this is v of n times 1. Now, both these integrals are of the same form, except for change of variable names. So, let me just evaluate this character, this is v of n e to the A c integral 0 to 1 e to the minus A c tau B c 2 d tau and please note, the v of n is a scalar, this is e to the A c times...

Student: ((Refer Time: 28:37))

It is minus A c inverse e to the minus A c

Student: Times

Times tau times B c 2 and this has to be evaluated from

Student: ((Refer Time: 29:03))

0 to 1, which is v of n e to the A c A c inverse and this becomes

Student: 1minus

I, no 1, it is identity matrix minus

Student: ((Refer Time: 29:35))

e power Minus A c into B c 2.

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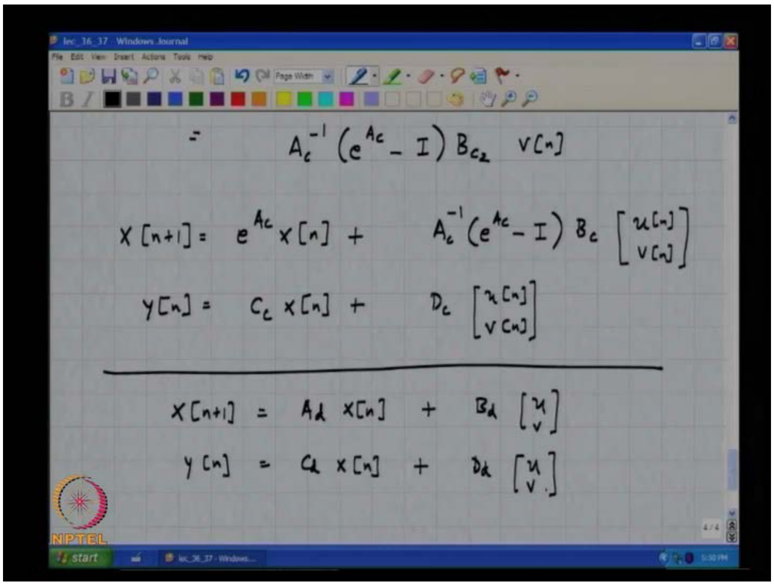

And because, A c inverse and e to the A c convolute, I can flip the order of multiplication and push e to the A c inside and I will get A c inverse times e to the A c minus I into B c 2 times v n. So therefore, X of n plus 1 is e to the A c X of n plus A c inverse e to the A c minus I times B c times u of n v of n, does it make sense. And thankfully, Y of n is simply, Y of t is nothing but a linear combination of the states, plus a linear combination of the inputs which means that, you must also follow that, the samples of Y must simply be the same linear combination of the samples of the states plus samples of the inputs.

So, this will simply be C c X of n plus D c u of n v of n, so we will now compare this with the standard formulation for a discrete time system, which is X of n plus 1 is A d X of n plus B d times u and v and Y of n is C d X of n plus D d u and v. So, if one wants to find the state space representation of a discrete time system, which is being mimicked by the continuous time loop filter as long as the two inputs can be modeled by...

Student: First order waveform

First order held waveforms

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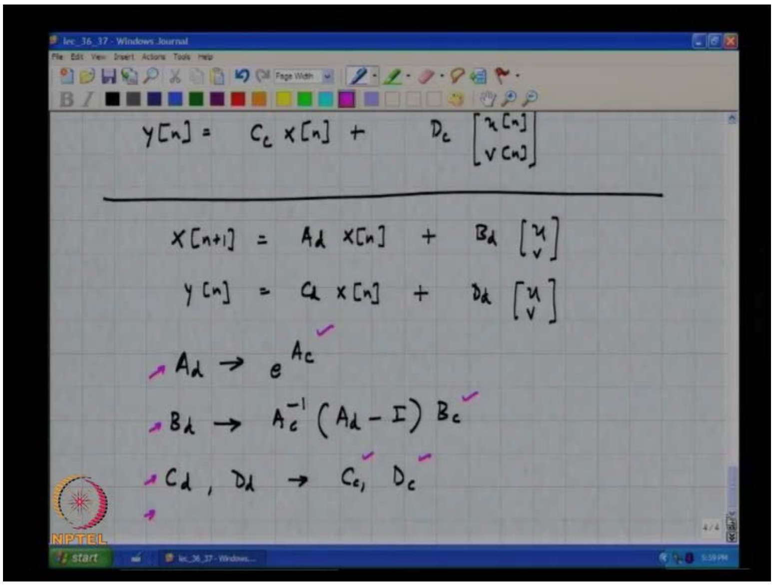

One can simply replace or the equivalent discrete time A matrix can be obtained from the continuous time A matrix by

Student: E minus A c

E to the A c, similarly B d is

Student: ((Refer Time: 33:44))

A c inverse e to the A c is nothing but, A d minus I times B c, C d and D d are simply C c and D c. So, the poles of the loop filter are nothing but, the given A c

Student: Eigen values

They are nothing but, the Eigen values of the

Student: A matrix

A matrix, so when we were discussing this in the context of loop filter design we said that, if a discrete time loop filter prototype has a pole at p 1, the continuous time loop filter will must have a

Student: ((Refer Time: 34:50))

The continuous time filter must have a pole at

Student: ((Refer Time: 34:56))

Ln of

Student: Peak

Peak and then, that is being reflected here also, because the Eigen values of A d and the Eigen values of A c are exponentially related, because A d is exponential of e to the A c, you understand. So, once I mean, given A c B c C c and D c, the corresponding discrete time state description can be found. So, one can therefore, use a...

Student: Discrete

Discrete time simulator to run all your simulations, of course this one thing I had like to bring to your attention is the fact that, you must make sure that, the assumption of input being...

Student: Modeled

Modeled well by a

Student: ((Refer Time: 36:18))

First order held waveform is valid, now what if you want to I mean, after all, one of the advantages of continuous time delta sigma modulation as we mentioned in the very beginning is the fact that, there is implicit anti aliasing. So, what if one wants to check this out in simulation, using this formulation can one put a sinusoid at f s

Student: ((Refer Time: 37:04))

I mean, ideally if you take a real continuous time sigma delta modulator and put in a sinusoid at f s, what should the output sequence be

Student: Should be 0

It should be

Student: 0

0, but if you emulate the sinusoidal behavior I mean, the continuous time behavior by an equivalent discrete time system, the underlying assumption behind that emulation is that, the input is varying sufficiently slowly that it can be...

Student: Modeled

Modeled fairly accurately by a first order held waveform, so this simulation technique that we have just discussed will not be accurate for excitations, which are high frequency excitations. I mean, high frequency inputs are perfectly legal inputs to have in a continuous time delta sigma, but this simulation technique is not good enough for that. I mean, one way of doing this is to deliberately assume that, the loop filter sampling at a much higher sampling rate then, the quantizer.

So, instead of assuming that, the loop filter samples are needed only at 1 hertz, if you say I will compute the loop filter samples not at 1 hertz, but at say 32 hertz, which is still better than computing the loop filter output for all time and only taking one sample. If the loop filter is sampled at a much higher rate than the sampling rate then, at least intuitively you can think that, for inputs as high as about 16 hertz, the output I mean, at least whatever the, you should be able to recover the input from the samples.

In other words, there is no aliasing in the simulation, but that is something that I will not cover in this class. But, as far as low frequency inputs are concerned, this is a easy way of implementing the entire simulation. So, maximum stable amplitude, stability, all these stuffs can be simply gotten from a simulator intended for discrete time modulator design.

Student: ((Refer Time: 39:54))

Again I mean, it depends on what accuracy you want I mean, you already know what the error is, the error is the difference between the continuous time waveform and the...

Student: First order waveform

First order held approximation.

So, you know for oversampling ratios of about 32 or so, I think this still visually you can see, that it is kind of so ok. But, I mean it particularly fails when you are trying to simulate really high frequency inputs into the modulator, which is a very common scenario for CTDSM operation. The fact that you do not need an explicit antialias filter means that now, somebody will say, why do you just remove the filter and then, I am going to put in an input, which consist of all sorts of orbit tones but then, you modulate and I know has got free anti aliasing.

So, none of that stuff will alias into my signal band and stuff which does not alias into my signal band will get knocked out by the dissemination filter in the digital domain, which you need anyway, you understand. So, you know the input content into a modulator like this can be very hostile, it need not necessarily be confined to the signal band as would be the case for a

Student: Discrete time

Discrete time delta sigma modulator, so as a final point, you can go through the similar analysis. Let us say that, the DAC was not a non returned to 0 DAC, but only a return to 0 DAC, what would change in this approach

Student: ((Refer Time: 42:10)) v of s

What do you think will happen to this guy

Student: Means

That will remain the same.

Student: That will remain the same

Because, the poles cannot change with the input, the only thing that is changing is the nature of the input. Now, as far as the input is concerned, we can still modulate as a first order held waveform. So, the B c 1 will still remain an expression, which look similar to this. The only thing that will change is B c 2 I mean, B d 2 and the change will be, instead of integrating this from 0 to 1

Student: 0 to 1

You integrate from

Student: 0 to 1

0 to

Student: Half

Half, that is all and you will get a different expression, but that is about the only change, if you have an arbitrary pulse shape, of course you have to compute the integral, you may I mean, you may not be lucky to get a nice clean close form expression like this. So now, let us start getting into implementation details I mean, now we have seen all possible I mean, all the issues relevant to the within code system design of the loop.

We know how to given a noise transfer function, we know how to design the loop filter, how to get it is coefficients, what to do when there is a excess delay, what to do when there is jitter, what to do when the time constants vary. We also know how to scale the outputs such that, none of the integrators prematurely limit the performance of the modulator and we know, how to simulate the loop given the loop filter properties. So, the next thing to do is.

Student: ((Refer Time: 44:27))

Start looking at how one might implement some of these circuits, so today we will take a very top level view of implementing the integrators. So, there are several possible integrators, I am only going to discuss macro model type approaches for in your analog integrated circuits class, you will learn or you have learnt how to design a good opamp.

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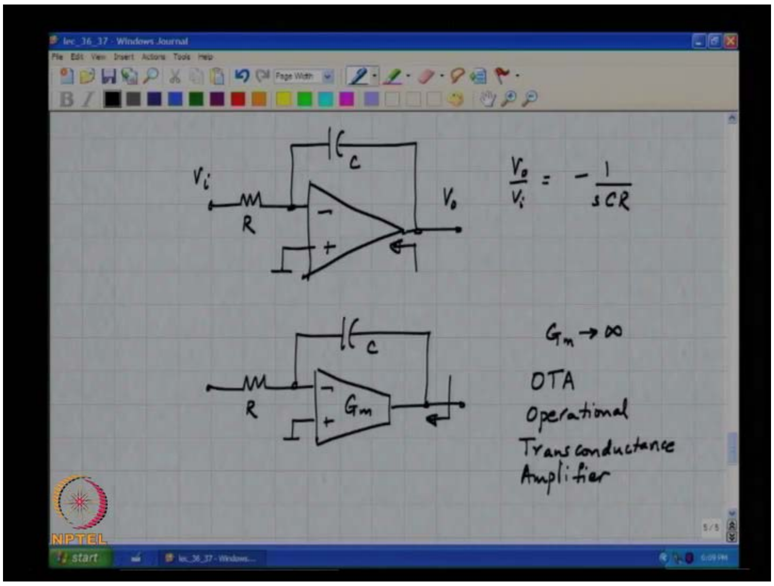

So, the opamp is the workhorse of lot of analog engineering, so from your earlier classes you probably seen, this is an example of an integrator. So, if the opamp is ideal V o by V i is nothing but, minus 1 by

Student: SCR

SCR, now clearly new opamp is ideal, so in practice, there are several ways of implementing the the opamp. One can realize an opamp as a, one way of realizing the opamp is to have think of it as a voltage controlled voltage source with a very large gain A. Unfortunately it turns out that, in C MOS technology, if you try to realize an opamp as a voltage controlled voltage source, the signal swing of the opamp can support at it is output is severely constraint.

I mean, have you all be through two stage opamp design at this point, in analog ICs, yes no...

Student: Yesterday one introduction class

So, it turns out, therefore that if you have I mean, if you make the opamp a voltage controlled voltage source, by definition the output impedance has to be...

Student: Small

Small, if the output impedance is small it means that, internally what must you have, what kind of signal transistor stage gives you a small output impedance

Student: ((Refer Time: 47:19))

You need to put a voltage controlled voltage source inside, which is basically a source follower and it turns out that, when you have a source follower, you know that there is a output voltage is one threshold voltage plus one overdrive below the input which means that, on the lower side, the output voltage can go almost

Student: Minimum

To the negative real, but on the upper side, you will be constrained, because not from this transistor, but the transistor

Student: Driving

Driving this, switch is most likely a P MOS device and this can go or I mean, a little below supply, 1 over drive below supply which means that, this voltage can go only 2 over drives plus a V t below supply, you understand. So, instead of thinking of the opamp as an ideal voltage controlled voltage source, where the gain tends to infinity, that is what one normally thinks of an opamp as. As the voltage controlled voltage source with the gain, which tends to

Student: Infinity

Infinity, one can also think of an opamp as a voltage controlled current source, where the G m tends to infinity. So, many advantage of this as I said is that, it does not have the problem of losing voltage swing, because you need a voltage controlled voltage source in the form of a source for our insight. Now, if you had a buffer, if you had a voltage controlled current source then, one way of realizing an integrator is to chose this to be G m and please note as I said again, that the G m must tend to infinity.

So, this such a three terminal element where the transconductance tends to infinity, is what is called OTA, which stands for Operational Transconductance Amplifier. And of course, it is not possible to get an infinite G m, in practice G m will be finite which means that, the integrator will not give you, if G m tends to infinity, the input output

Student: ((Refer Time: 50:52))

Relationship will be minus 1 by SCR again, if this G m tends to infinity, the transfer function will tend to minus 1 by SCR and what will be the output impedance...

Student: ((Refer Time: 51:15))

If this is an opamp, voltage controlled voltage source, what will be the output impedance

Student: 0

0, now if I use an OTA instead of the opamp, what will be the output impedance be

Student: ((Refer Time: 51:36))

Please think about it, we will continue in the next class.