VLSI Data Conversion Circuits
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Lecture - 35
Effect of Clock Jitter on CTDSMs – 2

This VLSI Data Conversion Circuits lecture 35.

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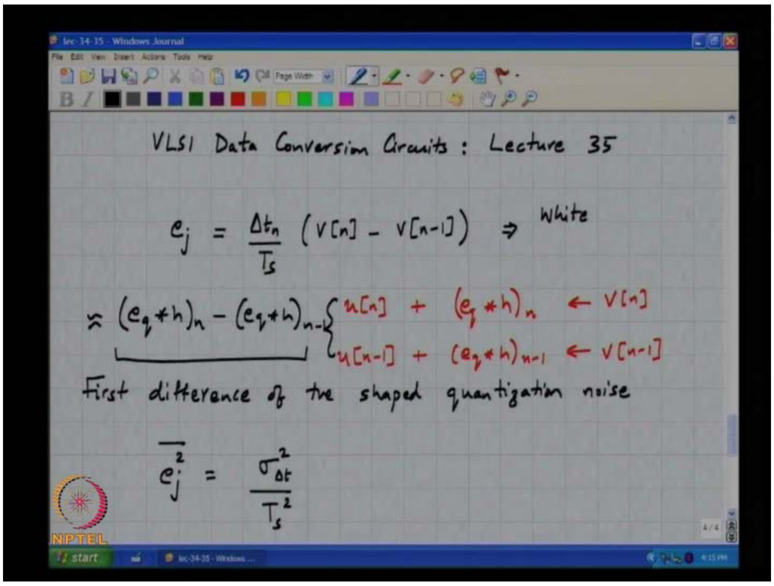

In the last class, we were discussing the effects of clock jitter on the performance of a delta sigma converter loop. And we saw that, clock jitter affects both the sampling instant of the A to D converter as well as the time at which the output of the D to A converter changes value. And we were discussing specifically the NRZ DAC and what we found was that, even though the sampling instant of the A to D converter is altered, the error does induced, does not really matter as far as the closed loop is concerned.

Since this error occurs in parallel with the quantization noise, which as we know is shaped out, therefore the jitter affecting the A to D converter is not really a concern. On the other hand, DAC jitter acts directly at the input and is therefore, detrimental to the SNR that one would otherwise obtained. So, in the last class we said that, the effect of jitter is equivalent to an error sequence at the output of the modulator, which is delta t n by T s times V of n minus V of n minus 1 and this is a white sequence, simply because we assumed delta t n to be a white sequence.

So, what is the mean square value of the sequence, yes I mean, why am I interested in finding the mean square value of the sequence...

Student: ((Refer Time: 02:44))

I know that the sequence is white, if I calculate it is mean square value, that value is spread over 0 to pi and I am more interested in that amount of noise, which occurs in 0 to pi by OSR. So, calculating the mean square value of the sequence will give me the inband noise power. So, the mean square value of this is simply, let me call this e j is this, so e j square is nothing but, mean square value of delta t sub n, which is sigma square of delta t divided by T s square times the mean square value of V n minus V n minus 1. Now, what is V n, V n is nothing but, the output sequence which is u...

Student: ((Refer Time: 03:54))

Plus quantization noise e q convolved with h of n, where h of n corresponds to the impulse response of the noise transfer function. So, what is V n minus 1, so this is u of n plus e q convolved with h in the n sample of that and V of n, V of n minus 1 is u of n minus 1 plus e q convolved with h n minus 1. What we are interested in is the difference between the two of these, which will be approximately e q. So, that will be the successive difference of the output sequence.

And that make sense, because we saw that there is error only when the output of the DAC changes and the output of the modulator consist of two parts, one is the input, which is varying very very slowly and the other one is the shaped quantization noise. So, if you take successive differences of samples, the difference between the successive samples of the input is very small and can be neglected what remains is the...

Student: ((Refer Time: 06:10))

The first difference of the shaped quantization noise sequence, so this is the first difference of the shaped quantization noise. So, what do you think is the mean square value of this sequence, we are interested in finding the mean square value of V n minus V n minus 1. V n minus V n minus 1 can be thought of as taking the shape quantization noise and passing it through a filter, which is 1 minus z inverse. So, the mean square value of that sequence is therefore, I mean, however you are getting the shaped quantization noise to begin with.

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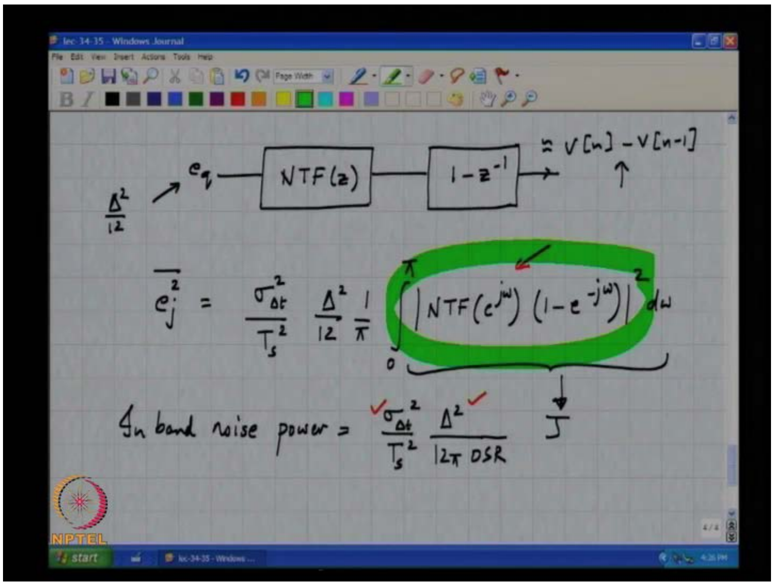

You can think of it as taking e q, which is the white sequence with mean square value delta square by 12 passing it through a filter...

Student: ((Refer Time: 07:53))

Which is the NTF and now what you want is the first difference and this is going to give you approximately V of n minus V of n minus 1. So, what is the mean square value of this quantity, you know that the mean square value of this is...

Student: Delta square by 12

Delta square by 12.

Student: Into NTF square

So, the mean square value of the input is delta square by 12.

Student: ((Refer Time: 08:48))

Then, we can do this in the frequency domain by using Parseval, so it is 1 by pi integral 0 to pi NTF of e to the j omega times 1 minus e to the minus j omega...

Student: Mod square

Mod square...

Student: D omega

D omega, so this corresponds to the mean square value of this sequence which then means that, as far as the inband part of this is concerned, all that we need to do is, multiply this by I mean, this extends all the way from.

Student: 0 to pi

0 to...

Student: Pi

Pi, we are interested in only that part which extends from 0 to pi by OSR, so the inband noise is nothing but, a noise power rather, is nothing but.

Student: ((Refer Time: 10:16))

You divide this by...

Student: OSR

OSR, so sigma delta t square by T s square times delta square by 12 pi OSR times this quantity j, which is just a short hand notation for writing this big integral. So, in the math is all right, a lot of these terms make sense, this we know where it comes from. Can you give me the intuition on, why you think the inband noise power increases when the LSB size increases.

Student: ((Refer Time: 11:28))

So, if you use a low resolution quantizer then, the step size is larger which means that, we saw that the error due to DAC I mean, clock jitter occurs when the DAC change a state. And clearly when the step size is large, the change is large which means that, all else being the same increasing the step size of the DAC will result in a larger sensitivity of the modulator to clock jitter. So, do you think as far as jitter noise is concerned, is better to use a 4 bit quantizer or a 1 bit quantizer.

Student: 4 bit

You want to use a 4 bit quantizer, in other words single bit quantizers are very sensitive to clock jitter, because the feedback signal is jumping rail to rail.

Student: When we subtracting V of n minus V of n minus 1, the u of n and V of n means, we have assumed approximately.

Yes and that make sense, because u of n is varying very slowly, so u of n minus u of n minus 1 will be very very small.

Student: Sir, if you multiply a white sequence it is ((Refer Time: 13:06)), so if the resultant be white.

Yes, because you are trying to find the average value of I mean, if these two are independent sequences, one is white and other can be anything, as long as the two are not themselves correlated, the average value of successive samples is a average value of e 1, say e 2 times. So, we understand therefore, why this make sense, you understand why this make sense and now let us see, why this make sense. If you have a more aggressive noise transfer function, do you think there will be more jitter noise or less jitter noise and what is the intuition behind this, what do we mean by an aggressive noise transfer function?

Student: More inband rejection

Pardon.

Student: More in-band rejection

More inband rejection, which is equivalent to...

Student: Higher out of band

Higher...

Student: Out of band gain.

Out of band gain, so if you have a higher out of band gain, in the domain time how does it look like?

Student: ((Refer Time: 14:30))

It will be beginning a lot more than a NTF with a low out of band gain, so if you have an aggressive noise transfer function, because the out of band gain must be large by bode sensitivity. It follows that, in the time domain, successive output samples of the DAC will be jumping up and down by a lot more. And we know that, the error due to clock jitter is dependent on the successive...

Student: ((Refer Time: 15:05))

Jumps between successive samples, therefore the error would be large, so you can see that, most of this contribution to this integral is coming around where...

Student: Pi

It is coming at high frequencies, because 1 minus e to the minus j omega starts of at 0 and goes to 2 at omega equal to pi. The noise transfer function itself is starting off at 0 at a low frequencies and going to out of band gain at omega equal to pi. So, most of this contribution is coming from high frequencies, that make sense. Because, this is telling you that, only if the sequence wiggles, will that affect the inband noise with I mean, by way of clock jitter degrading the performance.

Student: Sir

Yes.

Student: This V n and V n minus 1.

Yes.

Student: These are the modulator outputs in presents of jitter, right?

Yes.

So, will it be simply equal to s T p u plus ((Refer Time: 16:17))

It will be I mean, the actual this thing will be I mean, you must understand that, this is being, this whole thing is a feedback system. So, if V n and V n I mean, V n was the sequence of the input of the modulator, due to jitter you can think of it as being this sequence e j added at the, you can refer it back to the input by this sigma delta t by T s and so on. So, the actual output will be.

Student: ((Refer Time: 17:01))

I mean, see the validity of all this is assuming that, the loop is behaving like a...

Student: Linear system

Like a linear system, so quantization noise is additive, jitter noise is also additive I mean, in the presence of jitter, the real samples that the I mean, the A to D converter will sample and what is going into the D to A converter will be different. But, this is only an average statement, because we were assuming that, the quantizer itself behaves like an additive noise source, do you understand. So, this is this, now the next thing that I wanted to draw your attention to is the following.

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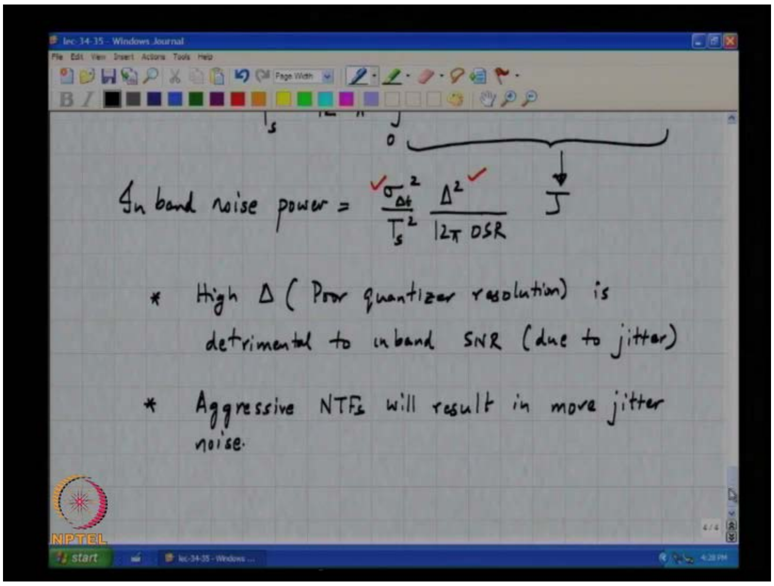

So, high delta which means, poor quantizer resolution is detrimental to inband SNR due to jitter. In a similar fashion, aggressive noise transfer functions will result in more jitter noise. And obviously, as the variance of the jitter increases, you will get more noise, you understand. Now, let us also ponder over, what the DAC pulse shape does to the jitter, so this is, so the discussion so far had to do with NRZ DAC’s.

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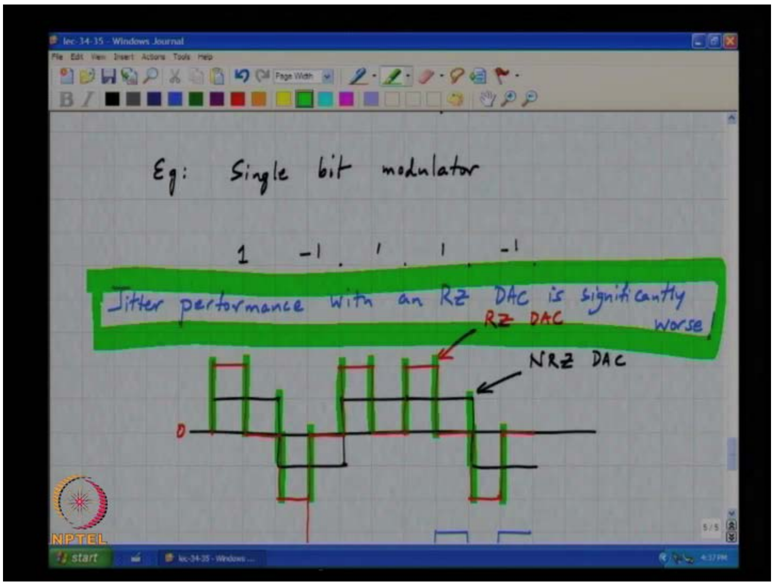

So, the question is, what happens when the DAC pulse shape changes, so various pulse shapes we considered, where the NRZ DAC and the RZ DAC. So, let us take example of a single bit modulator, let us say the sequence was 1 minus 1 1 1 minus 1. So, the output waveform of an NRZ modulator will look like this, what do you think will I mean, how do think the output of a return to 0 DAC will look like for the same input sequence, how will it look like.

Student: ((Refer Time: 22:10))

It will be twice as high and come down to 0, then again it will go down to minus 2, come back to 0. So, this is NRZ DAC waveform and this is the RZ DAC waveform, now it is jitter, what do you think happens, so what is the error. In other words, what is the error that occurs when user returned to 0 DAC.

Student: ((Refer Time: 23:54))

That I mean, you have to notice two things with respect to the return to 0 DAC, the first is that every transition is now.

Student: ((Refer Time: 24:03))

Twice is high when compare to a non return to 0 DAC further there are...

Student: ((Refer Time: 24:13))

There are transitions every bit period, whereas when you had a non return to 0 DAC, if two bits happen to be the same value, there is no transition at all in the output waveform which means that, there is no error due to jitter. On the other hand, with an I mean, with a return to 0 DAC, not only is there a transition, in fact there are two transitions every bit period and each of these transitions is.

Student: Twice as large

Twice as large, so without getting into the math, what can you say about the jitter performance when use an RZ DAC versus an NRZ DAC.

Student: Noise is not find

So, the jitter performance with an RZ DAC is significantly worse, when you use a multi bit modulator, this only gets worse and worse for the return to 0 DAC.

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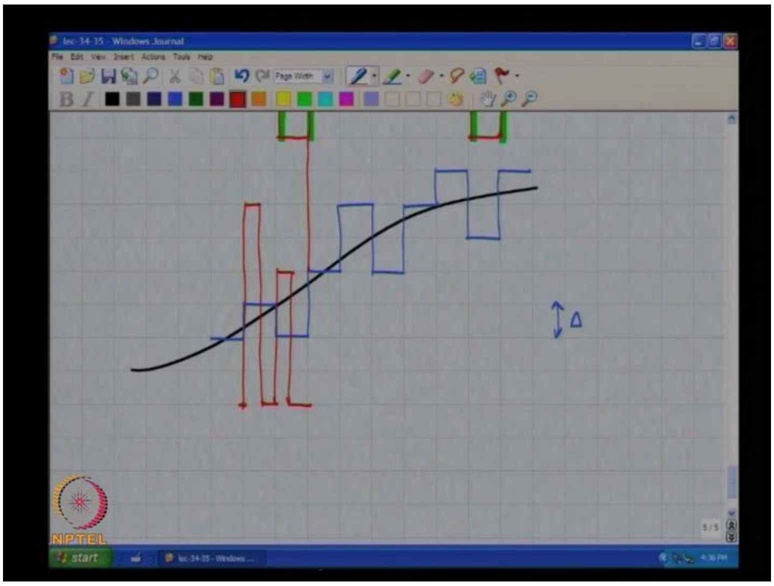

Because, if for a instance, let us say you had a waveform like this, a non return to 0 DAC would do, perhaps this is the sigma delta modulated sequence, I am just drawing what I think might be something reasonable, the step size is delta. And as you can see, the feedback DAC waveform is roughly tracking the input on the average, that is what the delta sigma modulator does. The output sequence is not the same as the input, but is only so on the average.

Now, if you had a return to 0 DAC, what do you think will happen, the waveform would look, it will start from 0, it will go to twice the height, come back then, go to twice this height and come back equal to twice the height and come back and so on. So, with a multi bit modulator, can you now comment on the jitter of, when you use a return to 0 DAC versus a non return to 0 DAC...

Student: ((Refer Time: 28:00))

The return to 0 DAC is very bad compare to the non return to 0 modulator, so that only strengthens this statement, which is the jitter performance with a return to 0 DAC is significantly worse, which is why you find that, many many implementations simply stick to...

Student: An NRZ

NRZ DAC.

Student: Sir, if you compare single bit modulator with multi bit, it forms better performance.

So, the comment made was, if you compare a single bit modulator with a multi bit modulator, as we discussed before, if delta becomes smaller and you deal with an NRZ DAC then...

Student: ((Refer Time: 29:15))

If you have a larger number of bits in the quantizer, you will do better with respect to jitter noise. So, it must naturally follow that, if you had a single bit modulator, you will have...

Student: Mod deltas

No, much larger step size, therefore the inband noise due to clock jitter will be much high. A third pulse shape which is also used in practice, is what is called the exponentially decaying pulse.

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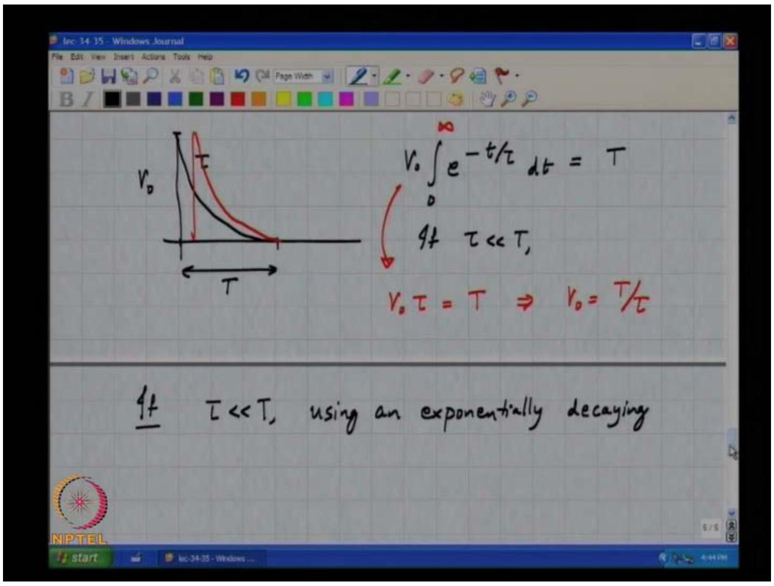

So, in other words, you have a DAC which goes like this, so when compared to a non return to 0 DAC, if you want the same dc gain for the DAC in all cases then, what must happen. When we went from NRZ to RZ, we wanted the same dc gain for both DACs, so the pulse height was double, because the pulse width was one half. So, the area must be the...

Student: Same

Same, now if you have an exponentially decaying pulse then, what do you think will happen?

Student: ((Refer Time: 31:17))

I mean, you should make sure that, this initial value V naught for example and this is say T and the time constant of decay is tau. You want to make sure that, integral e to the minus t by tau dt 0 to T must be equal to.

Student: 1

1 if we say that, it should be equal to T, because if you assume that the non return to 0 is 1 for a period T, this is what one should have. And if this time constant is much much smaller than T then, it does not really matter, whether this limit of integration is z I

mean, T or infinity. So, let us say this is infinite, which is an approximation of course, in which case this becomes V naught into.

Student: ((Refer Time: 32:40))

V naught times tau, this must be equal to T which means that, V naught becomes T by tau. This make sense, because if you want the exponential would decay very rapidly, meaning tau is very small. If you want to get that area, you better have a very big peak initially. So, at this point we will not worry about how one may generate such a DAC pulse, but I mean can you make a guess.

Student: Capacitor discharge

If you take a capacitor and discharge it, you charge the capacitor upto a voltage and you discharge it through a resistor R then, the current waveform through the capacitor will be this exponentially decaying pulse. So, what I want to point out at this juncture is the property of this kind of pulse shape with regards to clock jitter. So, if the edge jitters, what happens?

Student: only one of the edge is...

Yeah.

So, let us say the clock jitter like this, if the time constant tau is much smaller than T, what happens?

Student: ((Refer Time: 34:33))

It will still end up at 0 at the end of the...

Student: Clock cycle

Clock cycle and what does that mean as far as the amount of charge that has gone into the loop filter is concerned.

Student: It remains same

It remain the...

Student: Same

Same, so this is in big contrast to both the NRZ DAC pulse as well as the return to 0 DAC pulse.

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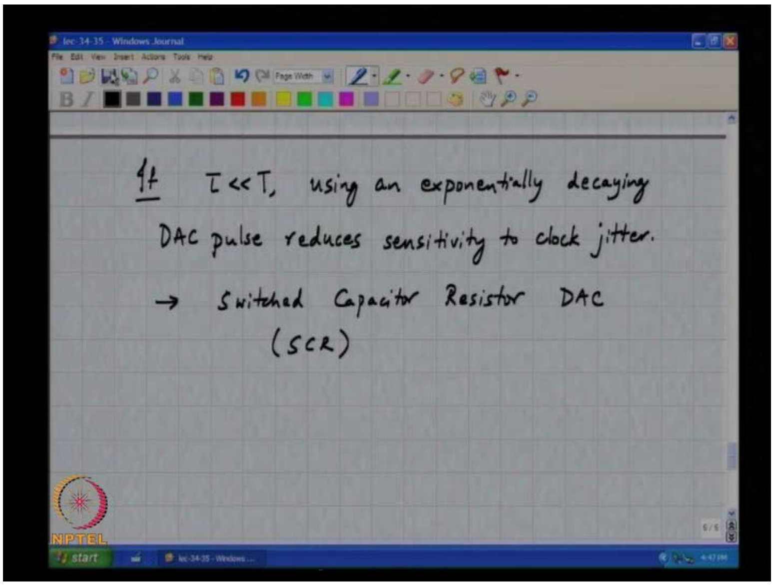

So if, that is a big if, tau is much much smaller than T then, using an exponentially decaying DAC pulse reduces sensitivity to clock jitter. And just for information, this is realized using a capacitor discharging through a resister and is often what is called a switched capacitor resistor or SCR DAC. So, one disadvantage of this, however is that, if you want to reduce sensitivity to jitter, that initial value V naught must be...

Student: Very large

Very large and you know that, any practical loop filter is made with transistors, which are non linear. So, if we excite a loop filter with signals, which have a very huge peak value, even though they may have a very small average, you will make the modulator quite non linear. Because, the large signals will drive transistors into non linear regions of operation and so on. So, that is one disadvantage of having to deal with signals, where this is general principle.

If you have an active circuit, which is processing signals with large peak to average ratios, you can expect distortion problems, that way the most benign waveform is the NRZ waveform. Because, the peak to average is 1 for the pulse, whereas for the exponentially decaying pulse, the peak to average is very very large quantity and that quantity depend is inversely proportional to tau. So, that concludes the discussion on the effects of clock jitter.

Student: Sir

Yes.

Student: In switch case...

Yes.

Student: ((Refer Time: 38:37)) decaying pulse, it also comes back to 0, right.

The assumption is that, if tau is sufficiently large, it will come back to 0.

Student: So, the problems on RZ which means, they are destructive.

Student: Amplitude will be

No, amplitude is still V naught, see the problem with the RZ is that, if the edges jitter, the net area changes, which does not happen here. I mean, it does not mean that, this is free of clock jitter, the math is somewhat involved, which is why I do not want to bring it into this class. But, you can I mean, at least you can see that, the area is the same regardless of when the edge starts as long as that tau is much much smaller than T.

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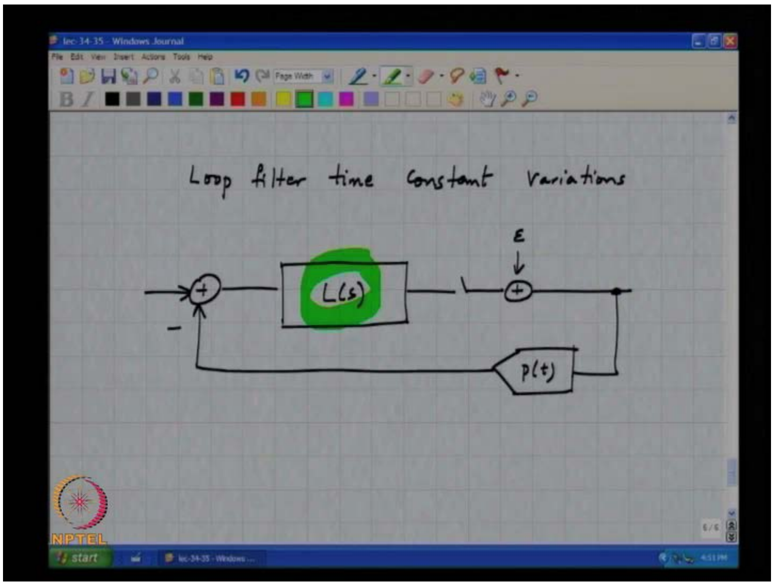

The next non ideality that I wish to talk about is loop filter time constant variations, so ideally this again, let me take this special case of a shift type loop. Ideally, we have gone and calculated L of s such that, if you bracket with P of T and sample the outputs at multiples of 1 second we get exactly...

Student: ((Refer Time: 40:55))

The L of z, which happens to be 1 by NTF of z minus 1, now this is a continuous time filter. So, the time constants will be of the form some r into some c or some g m divided by some c and so on. And in practice, capacitance will vary with temperature, resistance will vary with temperature and you cannot expect that, this variation cancels out that variation, because resistors and capacitors are unlike elements.

So, if all time constants for instance, change by some factor, this can happen for example, if all the resistors change by the same factor or all the capacitors change by the same factor, this is at least plausible. Because, all resistors are of the same kind and all capacitors are of the...

Student: Same kind

Same kind, So, what do you think will happen to L of s, how can you model the variation of time constants in L of s. Let us say, all time constants decrease by a factor k.

Student: You can replace omega by e omega by f s.

So, I mean if all time constants in a network change by the same factor then, the shape of the frequency response will not change, it is only the I mean, it is like stretching the X axis, the frequency axis.

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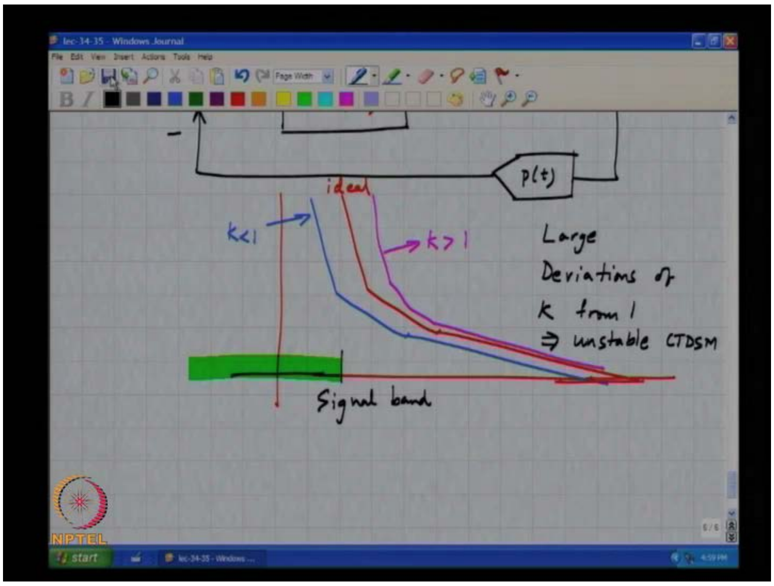

So, in other words, one can model the variation of time constants in the loop filter by this factor, say L of s by k. If k is greater than 1 it means that, the band width of the loop filter has gone up. So, in other words, if I plot the magnitude of the loop filter on a bode diagram like this, this is ideal. So, this must be k less than 1 and a similar diagram, if k greater than 1, please notice that the shape stays the same, which only that the and where is the signal band, this is the signal band.

So, can we now comment on what you think will happen if k changes, if k equal to 1, we get the ideal L of s which means that, we get the ideal noise transfer function. Now, if k is greater than 1, what do you think will happen to the noise transfer function, can we make any comment about this at all.

Student: ((Refer Time: 46:02))

Pardon.

Student: ((Refer Time: 46:05))

What is the meaning of NTF will decrease, where?

Student: Out band

So, for inband signals, if k is greater than 1 it means, that the gain of the loop filter in the signal band is increased which means that, the inband signal to quantization noise ratio will increase. But, if you have better inband performance you must expect...

Student: ((Refer Time: 46:38))

OSR out of band performance, so while it is true that the inband quantization noise is smaller, there will be peaking in the out of band frequencies. So, what can you say about the max maximum stable amplitude, you should expect it to come down. On the other hand, if k becomes smaller than 1, the inband signal to quantization ratio will...

Student: ((Refer Time: 47:15))

Will reduce or the inband quantization noise will increase why, because the gain of the loop filter within the signal band is now smaller which means that, the noise transfer function within the signal band has to be higher. Because, it is 1 by 1 plus loop gain or within this signal band, the noise transfer function is approximately 1 by loop gain. So, the inband quantization noise will increase.

And you can generally expect that, if this happens, the out of band gain must or rather the area above the 0 dB line must reduce, which in general will be accompanied by an increase in the maximum stable amplitude. So, and if the k deviates too much from 1 then, the modulator can become...

Student: Unstable

Unstable, you understand, so large deviations of k from 1 will result in an unstable modulator, does make sense. So, there must be some mechanisms which must ensure that, this k in other words, the time constants do not vary too much from their nominal values. For example, let us say, you could for instance have a variable resistor and a variable capacitor to implement an R and C, for instance inside the loop filter. Somehow if we knew, if we are able to measure the actual resistance and the actual capacitance and if we had something, which is variable. If we measure that, I mean rather in other words, if one was able to measure the time constants some way, we know what we want. Once you measure the time constant, you are able to figure out instantly, whether it is too high or too low. Based on this information, you could in principle go and tweak...

Student: Time constants

The time constants, by either changing the resistors or the capacitors or both in a clean manner so that, the deviation of the time constants from what we want are as small as possible, does it make sense. So, these cover the important non idealities in a continuous time delta sigma modulator that is, excess delay, clock jitter and time constant variations and now you know, how one deals with it. In the next class, we will discuss some practical issues, which are, I have a continuous time delta sigma modulator, how do I simulate it.

For discrete time modulators, we can write the difference equations governing the loop filter and then, it is a bunch of different equations. Now, we have a system, where part of the loop is continuous time, part of it is discrete time. So, we need to understand, how to simulate the loop and we will also see, how one can implement the loop filter. We already know that, it is a bunch of integrators, we will have to see how to implement them in practice, given the common active elements namely the opamps and trans conductors and the like.

So, that will then complete the discussions on continuous time delta sigma modulation. As far as the loop filter and non idealities are concerned, we still have not looked at, how one can implement the quantizer itself and which is the ADC and the DAC. So, we will do that as we go along, great.

Thanks.