VLSI Data Conversion Circuits
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Lecture - 33
ELD Compensation

This is VLSI Data Conversion Circuits lecture 33. In the last class we saw what excess loop delay does to the noise transfer function of a first order continuous time delta sigma modulator. We saw that stability is compromised, and we saw how one can fix this problem by adding a feed forward path around the loop filter which is equivalent to having a direct path around the quantizer.

Now, let us see what happens when you have a high order loop filter, and like last time we are more interested in fixing the problem. Rather than understanding what happens to the noise transfer function qualitatively I mean quantitatively, qualitatively we know that the stability of the feedback system will be compromised because, of this excess delay right. What we are interested in is how do I tweak the coefficients, in order to get the noise transfer function back to it is original value, and without loss of generality I assume a CIFF loop filter.

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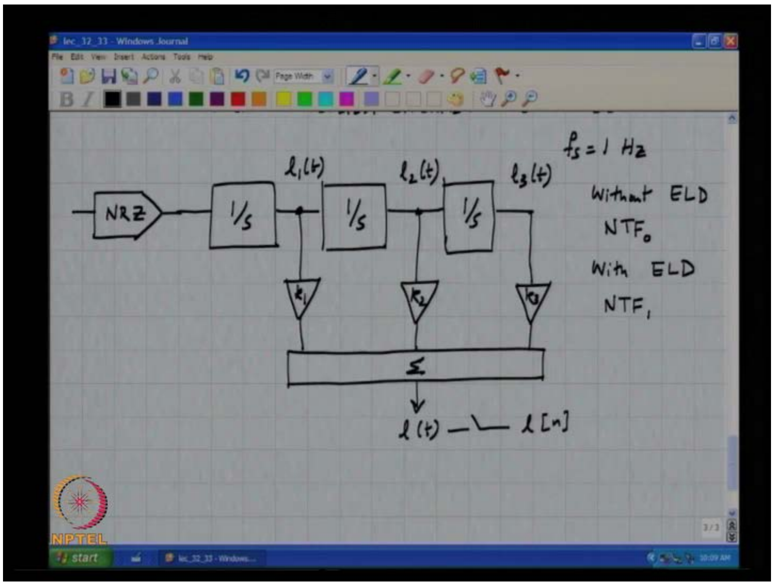

So, the loop filter can be thought of as being a cascade of integrators, let me take a third order example it is sufficiently complicated to be useful. And as at the same time is not, so complicated that we get lost in notation, that companies n integrators in cascade, so the sampling rate as usual is 1 hertz, we assume that there is an N R z pulse, the loop filter output is simply the weighted summation of the three integrator outputs.

So, this is l of t this is l 1 of t, l 2 of t, l 3 of t correct, and the output of this loop filter is sampled resulting in l of n. Let us first assume that we have already calculated k 1, k 2 and k 3, so that the noise transfer function is what we want, so without delay we have some noise transfer function, let me call that NTF 0. Now, with ELD it is something else, and that will be say NTF 1, now the question is what should I do with the loop filter in order to get back NTF 0 all right.

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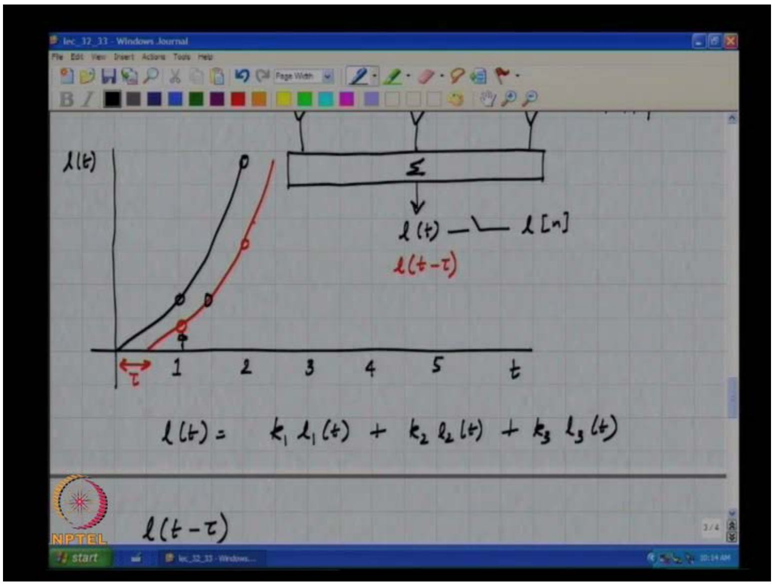

So, if the loop delay is tau and tau is less than 1, the ideal output of the loop filter is probably something like this, and why does this make sense ((Refer Time: 04:52)) this is the pulse response of the ideal loop filter and why does this make sense.

Student: No 2 c s differs third integrate output had been come.

So, the first integrated output will be a ramp which then saturates to 1 multiplied by k 1, the second integrated output goes as t, the third integrator go output eventually goes as t square. So, in general the output of the loop filter will do something like this, now these are the samples of the loop filter output, with excess delay what do you think happens this curve is simply shifted to the right in this fashion right. And the samples as you can see, this samples are in error because, of this term.

In other words ideally we were expecting l of t to the output at the output, and what we are actually getting is l of t minus tau, but you sample l of t minus tau at multiples of 1 second, the samples are off is this clear. Let us see what l of t is or a I just want to remind you that l of t is k 1 times l 1 of t what is the l 1 of t, the output of the first integrator plus k 2 times l 2 of t plus k 3 times l 3 of t.

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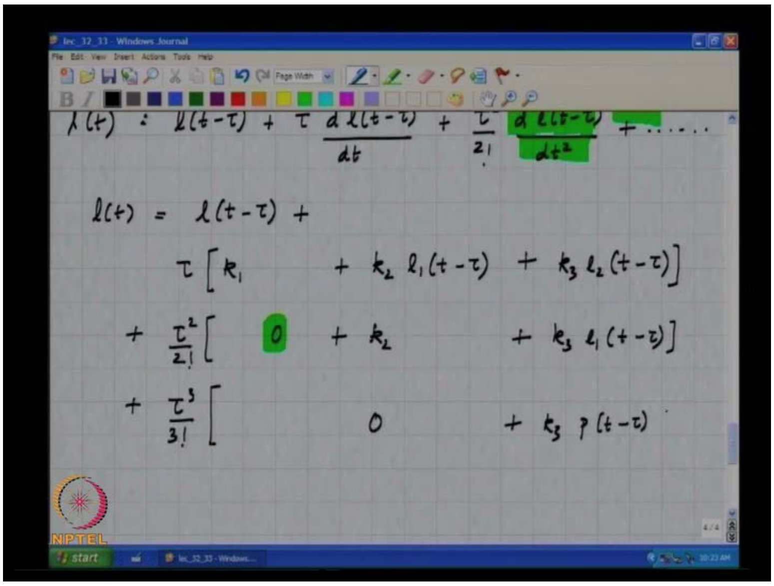

But, what we have is l of t minus tau correct, if we could somehow generate this samples of l of t from the waveform l of t minus tau. Then our job would be done correct, in other words even though we have only l of t minus tau, if we were somehow able to generate the samples that would have resulted from l of t. Then in principle I have fixed the loop filter correct, do you think this is possible to do or this is not possible to do.

In principle this is possible correct because, I know tau right and I know since I know tau I know that I am actually here. Whereas, the correct sample would be what should I have what will actually occur a little later, you understand these are the correct samples right. And since I know tau, and if I know that I am here I should be able to predict what I will be at a time tau later correct, and that is indeed the basic idea. So, l of t can therefore, be written as l of t minus tau plus tau correct.

And this can be written as l of t minus tau, please note that we only have access to l of t minus tau, and we need to figure out the samples of l of t given l of t minus tau all right. So, if you are given this waveform, how will you figure out the samples of l of t this the samples of l of t are the samples of l of t minus tau right at I am sorry if you know l of t minus tau at t equal to 1 for instance, what you want is l of t minus tau at 1 plus tau that would have corresponded to the sample of l of t at t equal to 1.

So, if you know you have a function and if you know it is value here, how can you find the value elsewhere use the Taylor series. So, l of t therefore, will be l of t minus tau plus tau times d l of t minus tau by d t plus tau square by 2 factorial d square l of t minus tau by d t square and so on does it make sense. But, what is l of t minus tau if l of t is given by this expression l of t minus tau will be k 1 l 1 of t minus tau plus k 2 l of t minus tau plus k 3 l 3 of t minus tau correct.

So, l of t from the Taylor's series expression is given by l of t minus tau plus, I mean in English what is this telling us. Since if I know l of t minus tau right and all the slopes at that point I should be able to go and figure out l of t minus tau, tau later which is the same as l of t is this clear right. So, I know the value and if I know all the slopes, I should be able to find what l of t minus tau would be a time tau later, which is the same as what l of t would have been, if there was no excess d this is clear.

So, plus tau times d l of tau l of t minus tau by d t, so if I differentiate, so let me just, so how do I find the derivative of l of t minus tau I mean to have access to it or not.

Yes why

Student: l axis t minus tau.

So, we need I mean we understand that l of t or l of t minus tau is the sum of the outputs of 3 integrators correct. So, if you want the derivative you need to take the inputs of the 3 integrators correct and that is what the math says, so it is tau times d l of t minus tau by d t which is given by k 1 into l 1 prime of t minus tau and what is that.

Student: Which is the p p of t minus 1

It is please note that what is exciting the loop filter is p of t minus tau correct, so k 1 into p of t minus tau plus k 2 the derivative of l 2 is l 1 plus k 3 l 2 of t minus tau right plus tau square by 2 times the second derivative of l of t minus tau, which is the first derivative of this animal here correct. So, what is the derivative of p of t minus tau at multiples of the sampling period.

Please note that p of t minus tau is an N R z pulse shifted by tau, but if we are only interested in samples of this at multiples of 1 second the derivatives are.

Student: 0.

0 correct, so this becomes 0 plus k 2 times the derivative of l 1 of t minus tau which is.

Student: P of t minus.

P of t minus tau correct plus.

Student: K 3 number of ((Refer Time: 16:58))

K 3 l 1 of t minus tau right, please note that this is 0 because, we are talking about an.

Student: Samples.

Samples and what.

Student: No that around that sampling point the p of t is not changing at all it is.

It is flat that is because, this is an n R z pulse you understand, so if p of t is an N R z pulse then p of t minus tau at the sampling period will be 1, you understand. Similarly this will be 1 correct and plus tau cube by 3 factorial times the derivative of what we have here which is therefore, 0 correct plus k 3 l sorry p of t minus tau, which is the same as 1. So, this is what we have this is clear plus higher order derivatives will all go to 0 you understand.

So, therefore, in other words if you want to find the samples of l of t at integer multiples of 1 second given l of t minus tau all that you need to do is to add.

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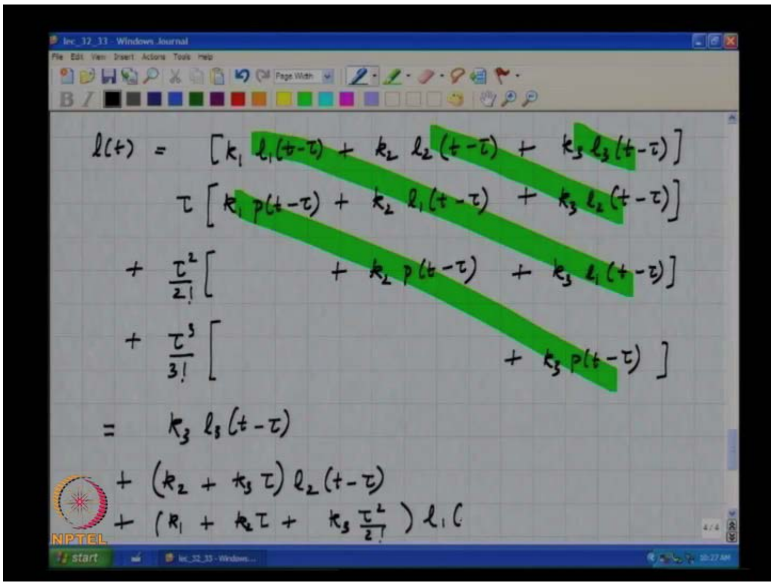

So, what the I mean what is l of t minus tau t minus tau itself is nothing, but k 1 l 1 of t minus tau plus k 2 l 2 of t minus tau plus k 3 l 3 of t minus tau.

Student: Sir this we want here p of t minus tau is 1 at first sampling.

Is 1 at the first sampling instant correct.

Student: No you should not put that it is 1 I think you should still mention that as p of t minus tau it is not.

So, I will I am sorry p of t minus tau and this will also be p of t minus tau, this will also be p of t minus tau correct the derivatives of p of t minus tau will be 0 at all sampling instance you understand. So, if we simply do the addition what do you get right please note that I will add up now all terms corresponding to l 3, l 2 I am just collecting terms corresponding to l 3, l 2 and l 1 and p right. So, this must give me k 3 times l 3 of t minus tau plus k 2 plus k 3 times tau times l 2 of t minus tau.

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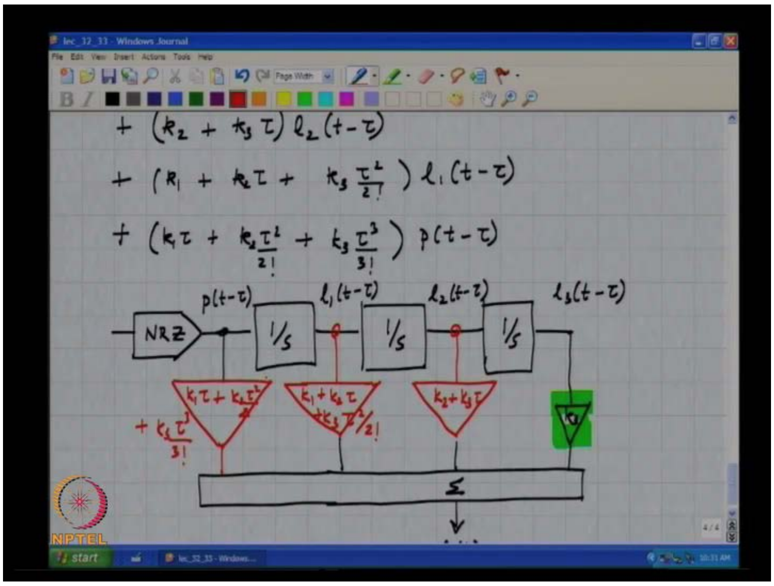

Plus k 1 plus k 2 into tau plus k 3 into tau square tau square by 2 factorial into l 1 of t minus tau plus k 1 into tau plus k 2 into tau square by 2 factorial plus k 3 into tau cube by 3 factorial times p of t minus tau correct. So, this should give you.

Student: L of t.

These samples of l of t given l of t minus tau at the sampling instant and I mean given l of t minus tau we know all it is derivatives also right. So, this is equivalent to therefore, changing the coefficients of the loop filter right or modifying the coefficients of the loop filter in the following manner, what should you do this is what we had earlier, what should I do I need please note that this will be p of t minus tau right. So, these guys will be l 1 of t minus tau l 2 of t minus tau and l 3 of t minus tau correct, so what do I need how do I need to wait l 3 of t minus tau.

Student: K 3.

K 3, so nothing happens to k 3, so what should I do with k 2.

Student: K 2 ((Refer Time: 24:04))

This k 2 must be modified as per the following, this must be k 2 plus k 3 times tau how must k 1 be modified this is k 2 plus k 3 times tau, this must be k 1 plus k 2 tau plus k 3 tau square by 2 factorial right. And what else I need a k naught, which is this k 1 tau plus k 2 tau square by 2 plus k 3 tau cube by 3 factorial right, so this is indeed the what is this, this k naught, this is the direct path around the loop filter right, so it comes out naturally out of this whole process.

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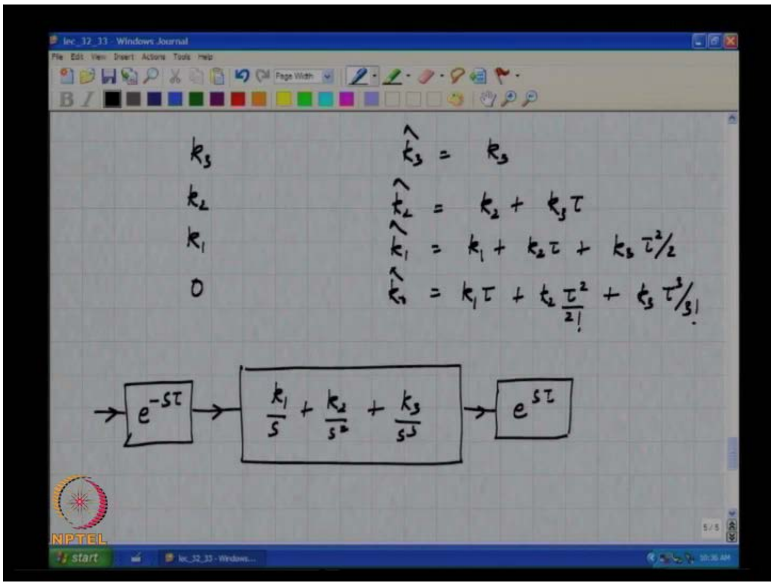

So, in other words if we had to change, so it is k 3, k 2, k 1 and 0 this is without ELD, with ELD it is k 3 hat, which is the same as k 3, k 2 hat is k 2 plus k 3 times tau k 1 hat is k 1 plus k 2 times tau plus k 3 times tau square by 2 factorial, and k naught hat is k 1 times tau plus k 2 times tau square by 2 factorial plus k 3 times tau cube by 3 factorial all right.

So, let us try and run some sanity checks the first sanity check is to make sure that.

Student: Tau equal to

For tau equal to 0 you must have the same coefficients thankfully that still stands, and we can use some the other case that we have worked out is when is for the first order where k 1 was 1 for the first order loop k 1 is 1 and k 2, k 3 etcetera are.

Student: 0

0 and what we worked out in the previous class, resulted in the direct path must have a gain of tau right. And in this example in these form set of formulae if you substitute k 1 equal to 1 and k 2, k 3 equal to 0 the direct path gain is indeed tau all right, please note

that the exercise we have done only ensures that the samples are correct, this does not mean that the waveform is the same as what you would get without excess delay right.

Only this samples are right, it turns out that there is an easy pneumonic to remember this, and that is the following you can I mean we had a loop filter, which was k 1 by s plus k 2 by s square plus k 3 by s cube and, so on. And we had a pulse shape p of t, but it was delayed by e to the by a delay tau which in the Laplace domain corresponds to e to the minus s tau right. So, if you want to get rid of the delay what should you do I mean in principle you should cascade it with plus e to the s tau all right.

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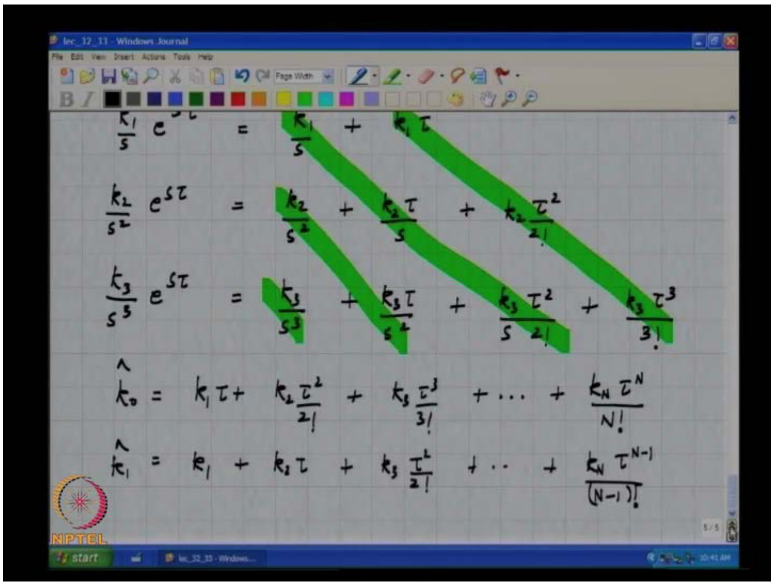

Now, so this is the within quotes new filter all right, I mean you can think of this as intuitively this make sense because, you have l of t minus tau and you are trying to look ahead to what I would get a tau later. So, if you are trying to look ahead it seems to make sense that, there is an e to the power plus s tau, but clearly it is a per s tau is non quasi right. So, you had to interpret this in a proper way and the interpretation is as follows.

So, I will simply expand e to the s tau as a Taylor series and this is 1 plus s tau plus s tau square by 2 factorial plus s cube tau cube by 3 factorial and, so on. And my formula is I will multiply each of them with e to the s tau and truncate beyond in the first case for 1 by s I will truncate beyond the s term. So, if I do that what will I get I had k 1 of s times e to the s tau is what k 1 by s plus k 1 times tau right, for k 2 by s square times e to the s tau what will I get k 2 by s square plus k 2 tau by s k 2 tau square by 2 factorial all right.

Then I do k 3 by s cube e to the s tau and that is k 3 by s cube plus k 3 tau by s square plus k 3 tau square by 2 factorial into s plus k 3 tau cube by 3 factorial right. Now, I will simply combine, so I want k 1 by s plus k 2 by s square plus k 3 by s cube into e to the s tau right, and the way I have interpreted e to the s tau is what I have shown here. So, I simply need to add up all the terms in the right hand side, so these terms will give me the direct paths, these terms will give me the 1 by s terms, these terms will give the 1 by s square terms, and these terms will give you the 1 by s cube term all right you understand.

Student: This is for extremely higher terms ((Refer Time: 34:47)).

This is the pneumonic all right this is an easy way to remember those formulae is this clear all right. So, the same thing turns out I mean if you had an arbitrary higher order loop filter with an N R z pulse, if this was delayed by if the N R z pulse was delayed by tau, you would be able to calculate the new coefficients which would restore the noise transfer function by going through this formula. So, you can actually do all this calculation by hand, so the direct path coefficient will be in this case.

In general the k naught hat will be k 1 times tau plus k 2 times tau square by 2 factorial plus k 3 times tau cube by 3 factorial and, so on up to k n tau to the n by N factorial. And you can take this k 1 hat is k 1 plus k 2 times tau plus k 3 times tau square by 2 factorial all the way up to k n tau to the n minus 1 by N minus 1 factorial.

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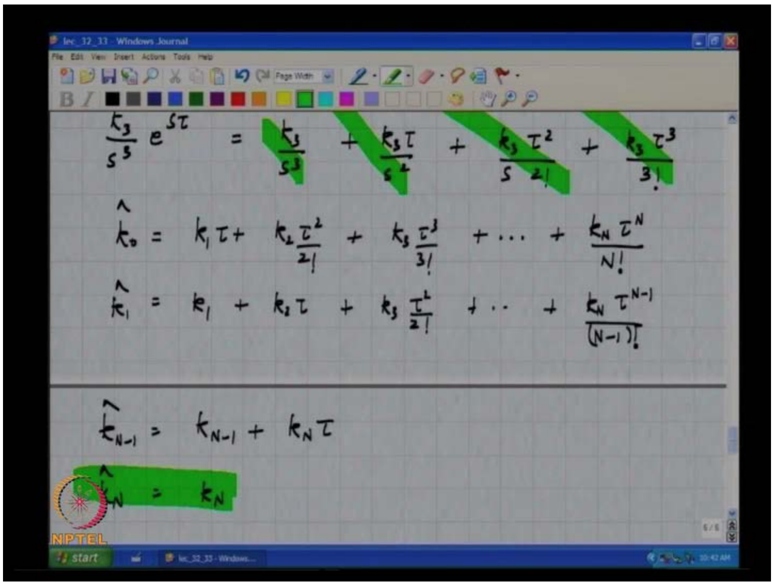

All the way up to k N minus 1 hat is k N minus 1 plus k N tau and k N hat must be the same as k N all right. So, why do you think it makes intuitive sense that k N hat is the same as k N or if you do not want to get confused with n with the third order modulator we found that.

Student: Is it because, the k N t u remains the same before and after compensation it is the last no clearly that is not right finally and the I mean that higher order terms not flexible in band noise. So, before let us say in ideal case you had the same width in ideal case as far as after compensation you have the same NDF, so; that means, you have the same required noise pole, so; that means, the coefficients are ((Refer Time: 37:40)).

So, the intuition is that with or without compensation right the in band noise flow remains the same all right. And therefore, the in band noise is dictated by the out of band I mean I am sorry the in band noise is largely dictated by the noise transfer function at low frequencies which is in turn dictated by the gain of the loop filter at low frequencies. Which means that if the NTF remains the same in both cases, the low frequency gain of the loop filter must remain the same which means that k 3 hat of the coefficient of the last integrator must remain the same right.

Now, as an aside let us say we had a loop filter which gave us some NTF, now let us say we introduced excess delay can you comment on the in band noise. You understand the question, I have a loop filter with some coefficients it gave me some noise transfer function without excess loop delay. Now, I introduce an excess loop delay and I am wandering what my in band noise will be do you have any comments on that.

Student: It will increase same as that of decreases.

It as the as to increase or decrease or remain the same right, so why do you say it increases.

Student: Because, when you are coming around the loop, so that negative feedback was suppose to cancel out. But, it is not being able to cancel out that noise.

Please note that negative feedback will cancel off the noise within the signal band only correct, which means that all that it means is that on the average the feedback DAC spectrum at low frequency since the same as the input spectrum correct. So, I mean adding a little delay in the feedback waveform does not change the low frequency spectrum. So, there should be no reason for the input noise to increase, what somebody said decrease why would it decrease.

Student: Order of the NTF would be decrease.

Well the order of the NTF increases, but the numerator order is still remains they you still have only 1 0 at z equal to 1 in the first order case, and in this case you have the number of 0's at z equal to 1 determine the in band noise correct that is not changing you understand. Because, 0's of the NTF correspond to poles of the loop filter, the poles of the loop filter are they are still 3 poles at s equal to 0.

And other way of looking at it is the in band noise is governed by the low frequency gain of the loop filter, the low frequency gain of the loop filter does not change even with excess loop delay correct. So, if the NTF is stable right the in band quantization noise will be the same it is another matter that the pole locations move, so the shape of the NTF will change correct.

But, the in band nature of the NTF will remain the same the fact that the shape of the NTF changes out of band will result in perhaps speaking in the NTF and reduced maximum stable amplitude and if you go on increasing tau it will result in an unstable modulator. But, if the loop I mean if tau is such that the NTF is stable, the in band quantization noise will remain the same is this clear all right, if the time domain way of looking at it is that the in band behaviour is governed by the output of the loop filter pulse response for large t small frequency is the same as large time right.

The fact that there is a small delay does not really effective if you are looking at the output at t equal to infinity basically right that is what corresponds to a frequency of 0 you understand. So, this is what will cause I mean, so if the loop filter is indeed stable then it will turn out that there is no impact on the in band noise, you understand now it is also possible to go and compute do this exercise for different DAC pulse shapes right.

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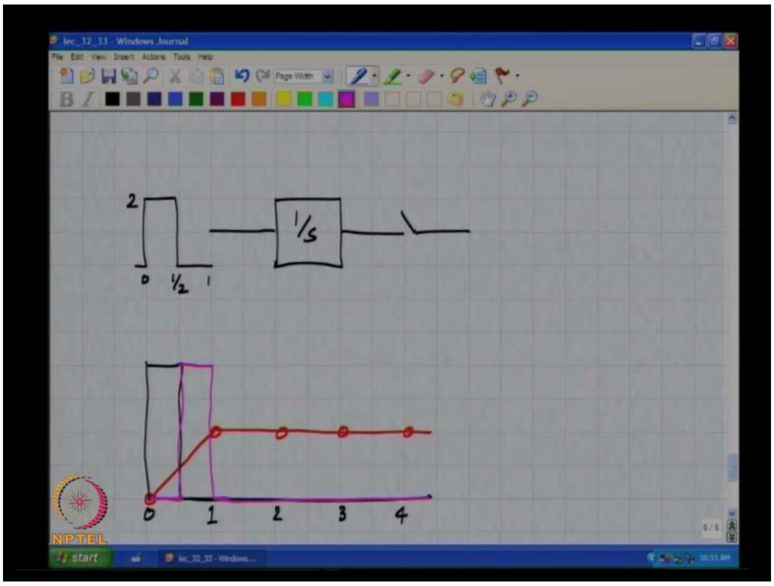

For example, let us since we are on the subject let us do this also for return to 0 DAC, so let us go back to our first order example again.

Student: Sir is it that the total noise will also will increase, but in band noise will be constant.

The total noise will I mean this has to be carefully qualified, but I would generally think the total noise will increase because, the noise transfer function peaking will increase all right. So, ideally the pulse is like this, so the output of the loop filter will behave like this and the sampled output will be these samples, now if the pulse is delayed by tau where tau is less than half right what do you think will happen.

Student: ((Refer Time: 45:17))

This height is 2, so that the area of the pulse is the same I mean that is what we discussed earlier also correct, only then the low frequency content will be the same. So, if the I must show this and sum up an error in this diagram, so with an excess loop delay of half correct, what do you think will happen to the output.

Student: Even the red one is half.

The red 1 is half.

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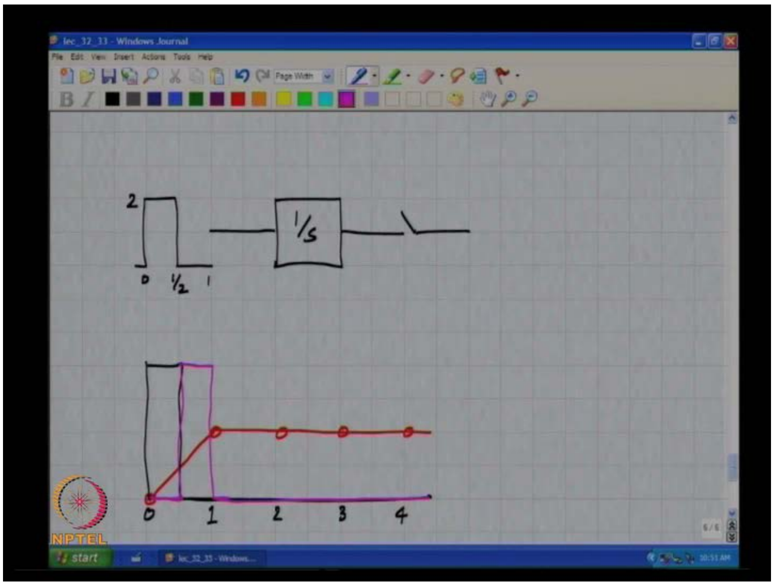

So, let me just erase the whole thing this is 2, this is half this is 1, 2, 3, 4 and, so on, so the output in the ideal case would have been something like this with samples here, the delayed version will do this and the samples will be the output waveform does this. So, the samples will still remain the same right, so what can you say as far as the R z pulse is concerned.

Student: ((Refer Time: 47:37))

So, for the first order modulator and a return to 0 pulse, the tau being less than half a clock cycle is perfectly acceptable and you do not need a direct path all right. On the other hand when the loop filter is of a high order, and the pulse shape is an R z pulse, and if the excess delay is less than half a clock period you will find that you still do not need a direct path, but you need to tweak the coefficients. And you can go through a similar set of arguments right to come up with the tweak coefficients right.

Again it is based on using the Taylor series and will give you the right I mean will give you the right coefficients which will restore the stuff back to know why do you think this make sense. See which path is actually you know most instrumental in closing the I mean or rather through which integrator does most of the high frequency energy flow through the.

Student: First integrator.

First integrator, the problem with excess loop delay is that if the pulse is delayed in the case of the N R z pulse for instance. The output of the first integrator at the end of the first sample right is smaller than what it would be without the delay correct with an R z pulse, and if the delay is less than half a clock cycle. Since the area under the pulse is the same right even if you move the pulse all the way up to by half a clock cycle, the output of the first integrator remains the same giving the that first sample that you want.

Student: For an impulsive DAC it is not an.

So, for an impulsive DAC right you will be able to if there would be no need for a direct path right even when the delay is almost the full clock cycle in practice of course, it is impossible to get a pure impulse all you can expect is a pulse with a very short duration, but a very high peak value, and you know as long as the area of the pulse right remains equal to 1 you can tolerate that amount of delay right.

So, the moral of the story is a excess loop delay causes a change in the noise transfer function, b this change can be reversed by appropriate choice of loop filter coefficients, as well as addition of a direct path. The values of these coefficients and the change in the direct path right on the and the value of the direct path depend on the DAC pulse, and you can I mean in practice the integrators will not be ideal and, so on and to determine the coefficients you can also use the that numerical technique that we discussed few classes back.

So, you in simulation you know put your real DAC pulse you measure l 1, l 2, l 3 and, so on and go and least squares fit the output to the ideal output that you wanted to see, all right thank you we will stop here we will continue on Monday.