VLSI Data Conversion Circuits
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Lecture - 31
CTDSM Design

This is VLSI data conversion circuits – lecture thirty one.

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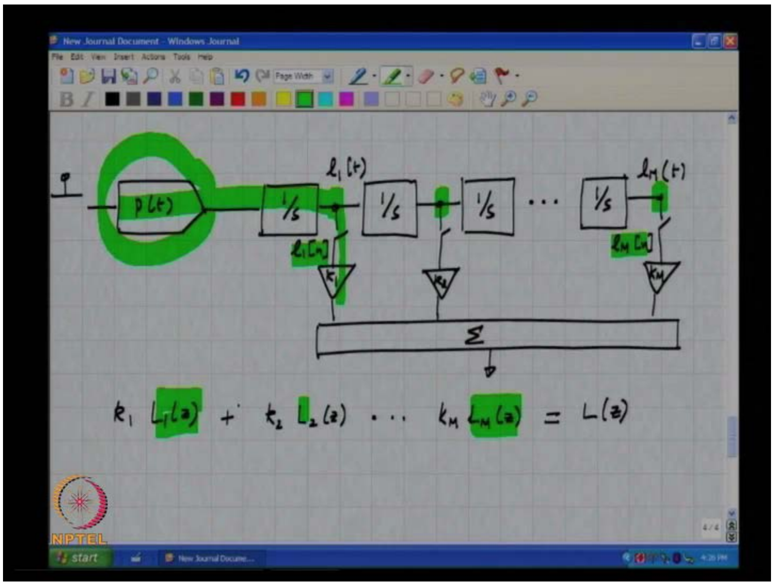

In the last class, we saw a numerical way of figuring out these coefficients k 1 through k M, which will result in a closed loop noise transfer function that we want. So, in other words, from the noise transfer function, we figure out the transfer function of the discrete-time loop filter from which we find the sequence that, this cascade of pulse shape and the continuous time loop filter – it samples must be exactly equal to l of n. So, one way of doing that was to simply either by hook or by crook, you find these impulse responses l 1 through l M of t; and thereby, the samples; and, solve a set of linear equations to find k 1 through k M. Since we have chosen the poles of the continuous-time loop filter to be such that, e to the p k is equal to z k; or, the poles with the continuous-time loop filter are the natural logarithm of the poles of the?

Student: Discrete time…

Discrete-time loop filter, which we know from the NTF; correct? In our case, since we assumed that, the NTF is of the form 1 minus z inverse to the power M, the poles of l of z will all lie at z equal to 1; which means that, the poles of the continuous-time loop filter will all lie at s equal to?

Student: 0

0. Now, the only problem is to find k 1 through k M. In the last class, we saw how one could do that numerically. Another way – a more traditional way of doing this has been the so-called z transform method; which is based on the fact that, once you know the z transforms of… The idea is the following. So, if you know L 1 of z; then, the z transform of the output of the loop filter is given by k 1 times L 1 of z plus k 2 times L 2 of z; all the way up to k M into L M of z; where, L 1 through L M of z are the z transforms of these.

Student: ((Refer Slide Time: 02:43))

Sequences L 1 through L M. And, this must therefore be equal to?

Student: L of…

L of z. Now, because we have chosen the continuous-time filter poles appropriately, it will follow that, the denominator of the left-hand side will be the same as the denominator of the right-hand side; in which case, the denominators go away and we just have to equate the numerators. Correct? And clearly, the L 1 through L M will be dependent on p of t. After all, p of t exciting an integrator is what produces L 1 of t for example. So, one way of attacking this problem, that is, the problem of finding k 1 through k M is to have a dictionary, where one has already tabulated L 1 through L M for a given?

Student: p of t

p of t; a case in point being…

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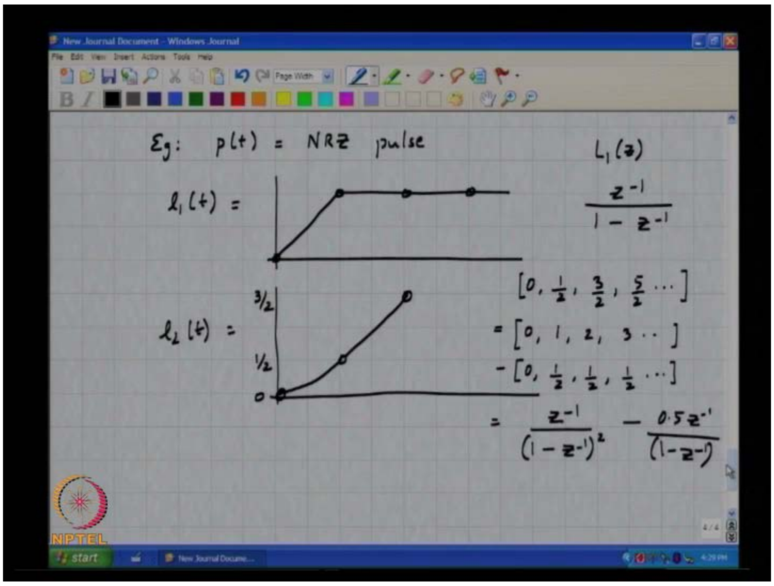

So, example – if p of t is an NRZ pulse; then, l 1 of t is nothing but this. Therefore, l 1 of n is given by these samples. So, l 1 of z is given by z inverse by 1 minus z inverse. l 2 of t is nothing but the integral of this waveform shown above. And therefore, will be 0, half, 3-halves and so on; which will translate to in the discrete time domain, a sequence of the form 0, half, 3-halves, 5-halves and so on; which is then 0, 1, 2, 3 and so on minus 0, half, half, half and so on; which therefore means that, 0, 1, 2, 3 is a discrete-time ramp. So, it is z inverse by.

Student: ((Refer Slide Time: 05:48))

1 minus z inverse the whole square minus

Student: ((Refer Slide Time: 05:56))

Half z inverse by 1 minus z inverse; do you understand? It is important to realize that, l 2 of t is not l 1 of t the whole square; do you understand? Which is what one might be tempted to conclude, because you say I am cascading two integrators; so, maybe I simply find the impulse responses of one integrator; find the z transform and multiply the z transforms. It is clearly not the same thing. So, one can come up with a dictionary for l 1 through l M. And then, use this equation here to determine k 1 through k M. And, every time the pulse changes, what should you do?

Student: ((Refer Slide Time: 07:03))

You have the whole new page in the dictionary, which will give these l 1 of z through l M of z for a different pulse shape. And, if the pulse shape is a very non-conventional one; here we were lucky to have to be able to look at it and write down the transforms. If the pulse shape was messy, then it is not clear that, you will even have an analytical solution for l 1 through?

Student: l n

l n. So, to say the least, this is a very messy process. And, in practice, anyway when you design, you will never get a pure NRZ pulse; there will be finite rise times or may be some overshoot; you do not know what. So, as a personal preference, I strongly advocate the use of the numerical way of doing it, because it is lot more practical; you can deal with a whole bunch of pulse shapes; and, you can implement it easily on a computer without any problem at all. Is this clear?

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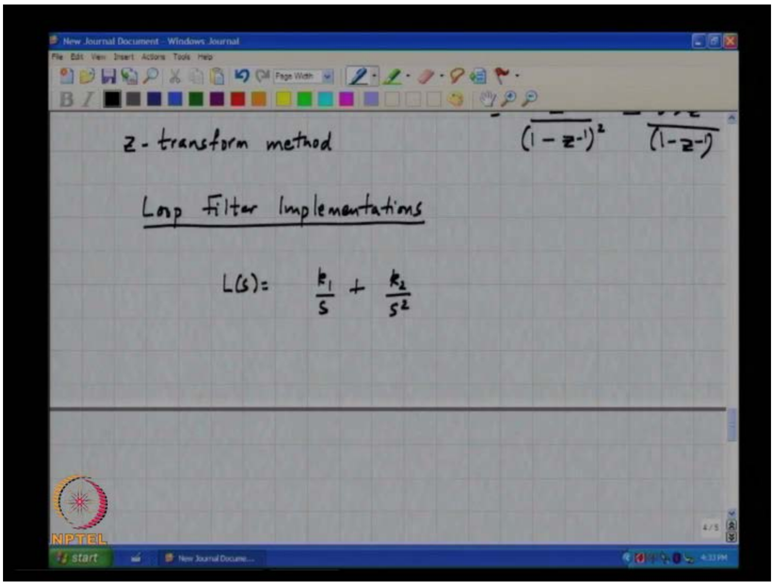


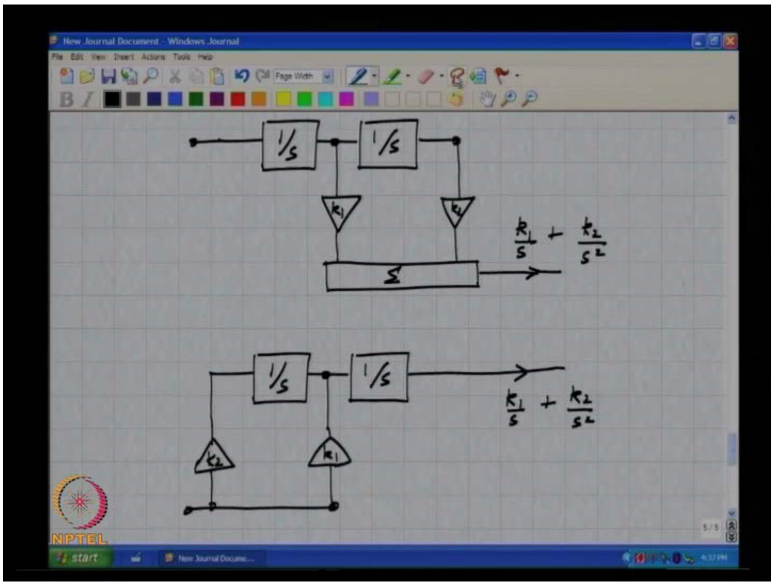
So, this is what is called the so-called z transform method. Please note that, if the poles of the continuous-time filter are not chosen properly; in this equation, you will not be able to cancel out the denominators. So, you will not be able to find a?

Student: Sir, a unique solution

A unique solution; do you understand? In fact, you will not be able to solve the equations at all, because… How are you going to solve for k 1 through k M here? The denominators cancel out; there will be M terms in the numerator on both sides and you equate?

Student: M coefficients

M coefficients. Now, if the denominators do not cancel out, then you know this simply does not work, because you have a lot more equations than 10 variables; and because of the lack of cancellation, you will simply not be able to find a unique k 1 through k M. This is by… In principle, both the methods are doing exactly the same thing; instead of equating time domain pulse samples, we are doing this in the frequency domain here. But, doing it in the time domain, if you have a unique solution, it does not matter which you do. Doing it in the time domain is a lot more convenient in practice. So, this is how one would determine the loop filter in order to get the NTF that you want. Now, let us take a closer look at the way one would implement the loop filter. So, for example, let us take a second order modulator, whose… Let us say L of s is k 1 by s plus k 2 by s square.(Refer Slide Time: 11:14)



One way of implementing the loop filter is to do what we have done so far; which is to do this. Correct? Another way of doing this is to do the following. You copy and paste this. So, in either case, this is the transfer function, is k 1 by s plus k 2 by s square. Here also it is k 1 by s plus k 2 by s square. So, these topologies are very analogous to what you see in discrete-time filters, where you call them direct form 1 and ?

Student: Direct form 2.

Direct form 2. One is a cascade of delays and then you tap the delays of... Other one is you feed them into the delays. This is a very similar idea here. So, topologically, even though the transfer function is the same, it can be implemented in many ways. And, two ways are as shown here. Corresponding to this, you will have different structures for the delta sigma loop.

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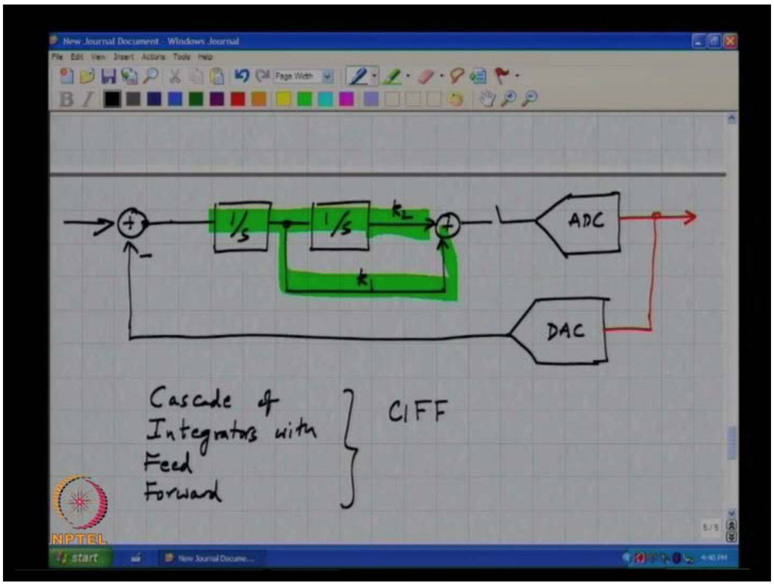

So, one is to take the output of this summer; put an ADC. I will draw the digital bits in red.

Student: ((Refer Slide Time: 15:46))

I will come to that. So, if I draw this in a less cluttered manner – in a more compact manner, which will make the diagram look rather cluttered; it will be of the form… So, this kind of loop filter realization is what is called… As you can see… So, it is a cascade of integrators. And, what is happening? The outputs of the integrators are getting added. So, this kind of loop filter architecture is called cascade of integrators with feed forward or abbreviated as CIFF. Now, here what does one do?

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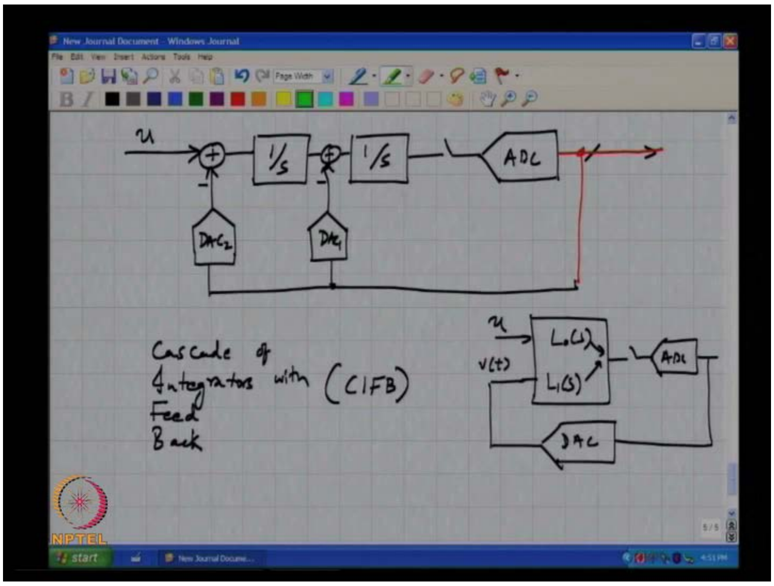

So, let me draw the quantizer. As far as the NTF is concerned, this is ok; but, we need to find some place to inject the signal to be digitized. So, what you propose we do?

Student: After DAC put summer…

Where?

Student: After DAC

There are many places after DAC. You can add something here, here.

Student: Check the…

Or, equivalently you can do this here for instance. So, here what I mean here is there must be an… Please note that, there is a negative sign here; the same thing must happen here too. And, this is what is called… This is again a cascade of integrators, but with many feedback paths into the loop filter. So, this is what is called the cascade of integrators with feedback; or, often called the CIFB. And now, another way of realizing this is to have for instance, not have k 1 and k 2; not have one DAC, but a common way of doing this is to have multiple DACs – DAC 2 and DAC 1 like this where the only difference is the full scale of DAC 2 and DAC 1; they are just modified according to k 2 and k 1. You might wonder why you need two DACs when you can do with one. It turns out that, in many cases, these feedback waveform quantities are actually… All these are implemented as currents and it turns out that, it is more convenient to have two DACs rather than one. When we see implementation, you will understand this. But, at this point, from a shear abstraction point of view, there is in principle, no difference.

And, let me also point out another difference between the CIFB and the CIFF structures. Here the signal and the quantization noise – the signal as well as the fed back quantity go through the same loop filter transfer function; and, that being k 1 by s plus k 2 by s square. Whereas, here what happens? The fed back quantity goes through k 1 by s plus k 2 by s square. But, the input goes through?

Student: 1 by s square.

1 by?

Student: s square

s square; do you understand? So, this is a case for example, where you have L naught of s, L 1 of s. This is u; this is v of t. Do you understand? So, L naught and L 1 are different; that is all that it means. In any case, if you want… At low frequencies, how should the STF look like?

Student: ((Refer Slide Time: 25:54))

You want to make it 1. And, as we saw the last time around, you can think of it as the open loop transfer function of the loop filter multiplied by the periodic extension of the noise transfer function. That is what the STF for a continuous-time delta sigma modulator is. The last time we saw it for the case where the signal and the fed back quantity went through the same loop filter. We can do a similar kind of analysis for this kind of loop filter implementation and show the signal transfer function of… and compute the signal transfer function. It is just a matter of manipulating the block diagram. And, you will see that, obviously, the STF for this will be different from the STF for the CIFF case, because the path from the input signal to the output of the loop filter has different transfer functions in both cases. So, the STF should be different; however, the NTF should be the same, because the transfer function from here to the loop filter output is the same in both cases. Do you understand? So, one can also have additional feed-ins into this path or this path, for example. And, you will all see that, they will do slightly different things to the signal transfer function. At this point, I do not want to get into any more detail on this aspect, but all I want to do is draw your attention to the fact that, there are at least two distinctive ways of realizing the loop filter; which means that, there are two distinctive topologies of modulators we can think of: one is the so-called CIFF loop filter base modulator; and, the other one is the CIFB modulator.

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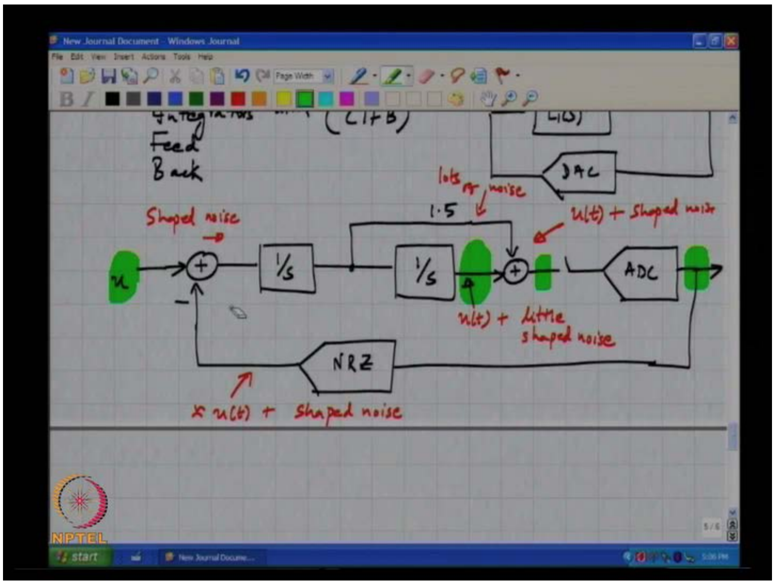

Now, let us get into some fine points regarding these modulators. I will take a second order example with an NRZ DAC pulse. Now, if u is a low frequency sinusoid, what must… that means use a sinusoid well within the signal bandwidth. In other words, is a very low frequency sinusoid, where the noise transfer function’s value is very small. How does the signal here look like?

Student: Same

There is a sequence, which is u plus shape noise. So, what is the signal here look like?

Student: ((Refer Slide Time: 30:24)) another shape noise ((Refer Slide Time: 30:27))

It is a u plus shape noise; it is NTF minus 1 times u. Now, we have this loop filter; correct? The input to this loop filter is what?

Student: ((Refer Slide Time: 30:48)) input and…

The output signal here is what?

Student: ((Refer Slide Time: 31:01))

This should be approximately u of t plus.

Student: Shape noise ((Refer Slide Time: 31:10))

Shape noise will pass through v of t; correct? The shape noise sequence is actually going through a loop filter – the DAC pulse shape, which is p of t. So, let me call this simply shape noise. And, why do I say it is approximately u of t?

Student: Because the loop gain is not infinite.

Because the loop gain is not infinite; only a DC is the loop gain infinite. In the signal band, the loop gain is very large, but not quite infinite. So, the output will only be approximately u of t. For example, at frequencies close to DC, it could be say 0.9999 times u. So, what is the actual signal going into the loop filter?

Student: Shape noise.

Approximately, shape noise plus a very very small input component; however, the output here is u of t plus shape noise; do you understand? And, the loop filter has got two paths: one is the path through 1 over s; the other one is the path through?

Student: ((Refer Slide Time: 32:58))

So, one path is like this; the other path is like this. Now, let us try and figure out the contents of each of these – how each of these paths is contributing to the output. Can I have any comments? The input to both these to the loop filter is practically shape noise plus some very very very very small fraction of the input. The output on the other hand is?

Student: ((Refer Slide Time: 33:56))

The full input.

Student: Plus shape noise

Plus shape noise. So, what do you think each of these paths is contributing? When you look at the input and output of this loop filter, what do we find?

Student: ((Refer Slide Time: 34:27))

We can see that shape noise is kind of coming through. What is happening to the input?

Student: ((Refer Slide Time: 34:40))

The input, which is a very low frequency signal is getting?

Student: Amplified

Amplified by a

Student: ((Refer Slide Time: 34:50))

Huge factor; correct? So, if the input is getting amplified by a huge factor, which of these paths do you think is responsible for most of the component at the output?

Student: 1 by s square ((Refer Slide Time: 35:03)) integrators 1 by s ((Refer Slide Time: 35:05))

There are two paths here: one through a single integrator; one through the?

Student: Double integrator

Double integrator; so, the double integrator path must be responsible for the low frequency component of the?

Student: Input

Output; correct? Why? Because the DC gain of the low frequency gain of the double integrator path is several orders of magnitude higher than the DC gain of the low frequency gain of the single integrator path. Is this clear? So, of this u of t, a majority of this u of t must be coming from the 1 by s square path. So, this output will approximately be the output of the second integrator here should approximately be?

Student: u of t

u of t plus there will be some shape noise; you cannot avoid it, because… What is that you are taking small input plus shape noise and integrating it? So, the amount of the high frequency part of the shape noise because of integration will?

Student: Reduce

Reduce. But, you cannot avoid it altogether. But, one thing you can say for sure is that, of the u of t appearing at the output of the loop filter, it is all getting – most of it is coming from the contribution of the 1 by s square path. Is this clear? So, let me get a little imprecise here and say a little shape noise. Little shape noise is simply because of the integration. Do you understand? Now… So, what must be the 1 over s path be contributing to?

Student: ((Refer Slide Time: 37:34))

So, most of the shape noise must be coming from… Simply because beyond s is equal to 1, the gain of the 1 by s is?

Student: ((Refer Slide Time: 38:03))

If you plot the gain of 1 by s square versus 1.5 by s beyond a frequency of about some 2 by 3 or square root of – I think 2 by 3 or something like that. You will find that, the gain of the 1 by s is larger than the gain of the 1 by s square; which means that, at high frequencies, most of the signal is coming from the 1 by s path rather than the 1 by s square path. At low frequencies, it is the?

Student: ((Refer Slide Time: 38:39))

Opposite that happens. Most of the signals must be coming from the 1 by s square path and little from the 1 by s path. So, to take it a little further, can you comment on the low frequency content at this point?

Student: ((Refer Slide Time: 39:03))

I mean can you comment on the low frequency content at that node?

Student: ((Refer Slide Time: 39:12))

It will be?

Student: Delta ((Refer Slide Time: 39:17)) Very small.

Very?

Student: Small

Small; why?

Student: Because that is that is why ((Refer Slide Time: 39:22))

No, can you give me a more compelling argument?

Student: When the output is second ((Refer Slide Time: 39:42)) is 0…

Correct.

Student: So, the input ((Refer Slide Time: 39:45))

Right; it is a lot more… What he is saying also right, but this is a lot more compelling argument. So, you see that, the output of the second integrator must contain at low frequencies, u of t. But, the gain of the second integrator at low frequency is some ridiculously high number; which means therefore that, the low frequency content at the input of the second integrator, which is also the output of the first integrator must be virtually?

Student: 0

0 at low frequencies. So, this must largely contain the integrated version of shape noise. And, the output of this must contain an integrated version of the integrated version of shape noise. So, it will be a lot smoother than the shape noise. Integration is basically a smoothing operation. And, it will contain u of t. One might wonder how is it possible to generate u of t from nothing at the input of the loop filter. It is not really nothing, there is a?

Student: Small…

A small delta because the loop gain is not infinite; and, that is getting multiplied by the loop gain at that frequency; and therefore, generating u of t. Now, I can extend this to a high order CIFF loop filter. So, if you had many integrators in cascade, what can you say about the signal contributions of… Can you comment on the outputs – low frequency outputs of these integrators? Should I repeat the question? Let us say I had a fourth order NTF realized as a CIFF structure. Now, there are four integrators in cascade with outputs of each integrator being weighted by some factors, which you have determined by the techniques that we have discussed. The question that I am asking you is – can we make any comment at all on the low frequency contents of the outputs of the four integrators?

Student: ((Refer Slide Time: 42:15)) first integrator to the last integrator.

The low frequency content will go on increasing from first integrator to last integrator. That is sincere… That is correct, but the output of the fourth integrator must contain.

Student: Full signal

The full signal. So, the input of the fourth integrator must contain?

Student: ((Refer Slide Time: 42:43))

Virtually, no signal at all. And, the input of the third integrator must consist of… Virtually nothing at low frequencies and so on all the way up to the front. So, barring the last integrator, the outputs of all other integrators will have a?

Student: ((Refer Slide Time: 43:05))

Very very small?

Student: Input

Input component. Do you understand? That is the property of the CIFF structure – the way the integrators are arranged. Do you understand? So, all the integrators except the last one only process… It is fair to say therefore that, all integrators other than the last one only process shape noise. So, their outputs will all be integrated versions of the shape noise; only the last integrator will consist of the input signal. Does it make sense? Another… Let me just… I will bring that up later.

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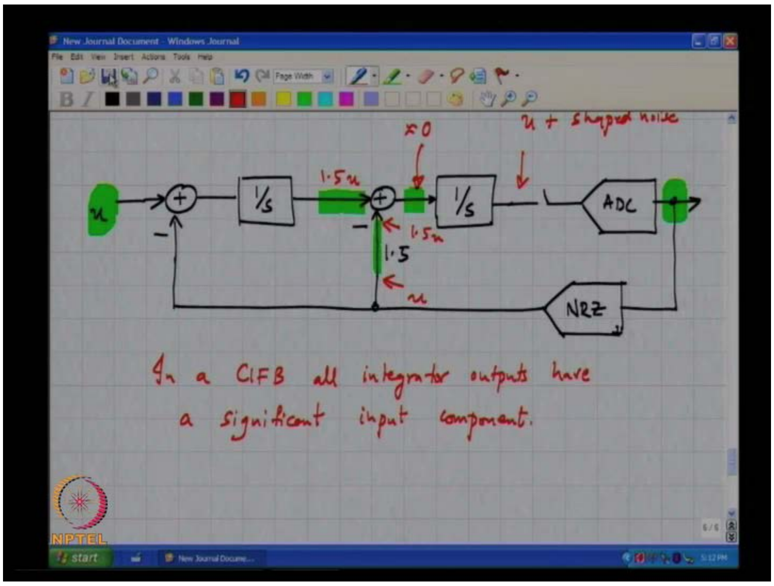

Now, let us see what the situation is with a CIFB structure being go through the same analysis or reasoning. Let me for argument sake say we have the same DAC; and, we have k 2 by s square plus k 1 by s in this fashion. So, what happens here? This must consist of?

Student: ((Refer Slide Time: 46:21))

u plus shape noise. So, what is the low frequency content here? If the output of the integrator consists of u at low frequencies, the input…

Student: Is almost ((Refer Slide Time: 46:43))

So, the input low frequency content is approximately?

Student: 0

0; correct? If the input is 0, at low frequencies, it means that, the difference between what is the input here at low frequencies? It is the low frequency – difference of low frequency signals here and here. But, what is the low frequency content here? Here it is u. So, here it must be?

Student: ((Refer Slide Time: 47:15))

1.5 u. So, which means that, if this has to be 0; if the difference has to be 0, it must follow that, the low frequency content of the output of the first integrator must be?

1.5.

1.5; correct? So, you can see that, the low frequency content of the output of the loop filter of the first integrator consists of the input signal that you have applied in addition to shape noise. Please note that, this first integrator is also integrating shape noise plus any difference between the input and output of the modulator. So, now, if you extend this to a high order CIFB structure, what can you say?

Student: ((Refer Slide Time: 48:17))

A high order CIFB structure will simply be copy-paste; you will have multiple… At the input of every integrator, you will have a feedback path from the output; which means that, every integrator output?

Student: Will have ((Refer Slide Time: 48:36))

Will have a component, which is u of t. Does it make sense? So, in a CIFB, all integrator outputs have a significant input component; whereas, in a CIFF, only the last integrator has got an input component. So, with this, we will stop. We will continue in the next class.