VLSI Data Conversion Circuits
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 30
High Order CTDSMs

This is VLSI data conversion circuits – lecture thirty.

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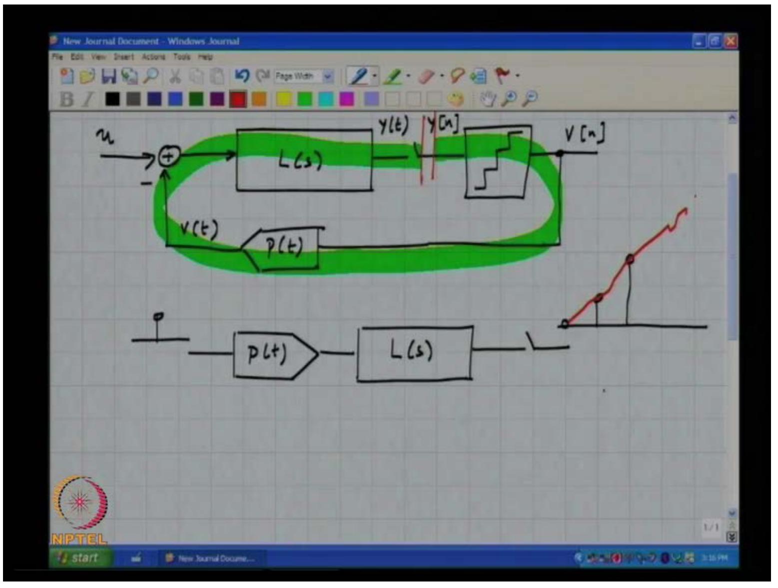

In the last class, we were wondering how to choose L of s in this particular modulator let us say. So, the basic idea behind continuous-time delta sigma modulation is to make the loop filter in continuous time and sample its output, so that as far as the feedback loop like this is concerned; if we break the loop here; the loop would not know the difference between a discrete-time loop filter or?

Student: Continuous-time…

A continuous-time filter. For the trivial case of a first order noise transfer function, where NTF is 1 minus z inverse; and, assuming for normalization sake that, the sampling rate of the continuous time delta-sigma modulator is also 1 hertz, we saw that, all that you need to do as far as the loop filter is concerned is to make it an integrator with the transfer function 1 by s. This way, the NTF we saw; we calculated to be 1 minus z inverse. And, this is assuming that, the pulse shape of the DAC is an NRZ pulse. Now, the process therefore, is to try and figure out what L of s one must use, so that if I put in a discrete-time impulse here, the sampled output here is giving me the right values. The first sample by definition must give you 0. Is that clear to everybody? Because if it does not give you 0, then there will be a?

Student: ((Refer Slide Time: 03:12))

A delay-free loop, which is not acceptable.

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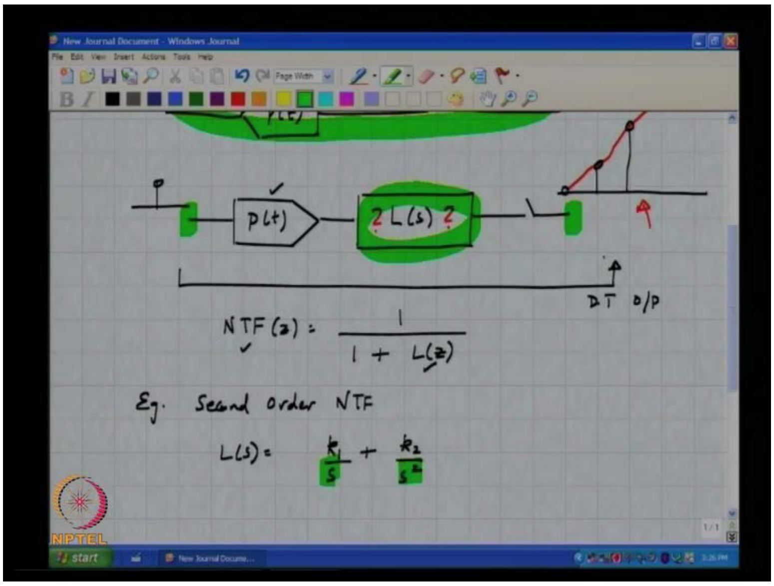

So, you know it probably gives you some continuous-time waveform like this, whose samples must give you L of z. So, this is a system with discrete-time input and discrete-time output. And, the impulse response of the cascaded version of the DAC pulse shape and the loop filter after sampling must give you L of z, which is given by 1 plus?

Student: ((Refer Slide Time: 04:08))

L of z is computed by the equation NTF of z is nothing but 1 by 1 plus L of z. So, if NTF is known, we know this. And, once we know p of t and the discrete-time impulse response we need between this point and this point, hopefully, we can go and figure out what this L of s is. For the second-order example, we said we are going to use a cascade of?

Student: Two integrators

Two integrators; and, in general, if you have a cascade of two integrators, the loop filter transfer function is of the form k 1 by s plus k 2 by s square. And, we know the pulse shape. So, we excite this loop filter with a rectangular pulse; in this case, an NRZ pulse; look at the output response; the output response will obviously be a function of k 1 and k 2. And, we go and try and fit k 1 and k 2 such that, the impulse response as what we want. One thing which seems rather surprising is that, this is a problem, where there are only two unknowns and a lot of equations. Correct? And, somewhat surprisingly, we see that, regardless of which two equations you take, the solution you get is always the?

Student: Same

Same. Now, let us try and extend this to a high order NTF.

Student: Sir, does that mean for any L of z, you can always find an equivalent L of s; is that what it means? Because we were getting consistent… Let us say that, we have infinite number of equations;

Correct.

Student: Let us say that, you are not getting the consistent solutions if you take any set of… You want two unknowns.

Sure.

Student: You take two equations.

Correct.

Student: You get one solution.

Correct.

Student: You take another set of two equations. Suppose if you get another two solutions;

Correct.

Student: That means no L of s is possible for a given L of s.

No; please note that, the L of s is somewhat restricted in the sense that, we have already assumed the locations of its poles. Correct? All we are trying to do is trying to find in more general terms that, denominator polynomial of the loop filter is already been fixed by our choice of two integrators. The only thing we are worrying about now is we are trying place the zeroes of L of s such that it matches the?

Student: L of z

The L of z. So, if the solutions do not match for any two choices of equations that you choose from this infinite set, all that it means is that, this choice of poles is not.

Student: Appropriate

Appropriate; that is all. Is that clear? I was hoping to take this up a little later; but, since he brought it up… What we are seeing here is that, in spite of having an infinite set of equations in the sense that, this sequence here is an infinite sequence all the way from n equal to 0 to n equal to infinity. At first, it might seem a little puzzling that, even though there are lot more equations than the variables you need to solve for namely k 1 and k 2, taking any two equations is good enough to give you the?

Student: The solution

The solution as a solution is a unique one whether you take n equal to 1024 and 6752. If you take those two equations and solve for k 1 and k 2, you will get exactly the same solution as you would get for n equal to 1 and n equal to 2. And namely, that solution is k 1 is 1.5 and k 2 is 1 if you want to realize the close loop noise transfer function of 1 minus z inverse the?

Student: Whole square

Whole square; correct? At least I would be somewhat puzzled that, this is happening. So, please note…

Student: The pulse shape ((Refer Slide Time: 09:14))

Clearly, the pulse shape does have a bearing on the?

Student: k 1 and k 2

On the k 1 and k 2. The last time around we did see that, when we had an impulsive DAC, the k 1 and k 2 are?

Student: 1 and 1

1 and 1; whereas, if you had an NRZ pulse shape, it is?

Student: One and half

One and half and?

Student: 1

And, 1. You can do the same kind of math for a different pulse shape namely, the return-to zero DAC pulse. And, you will find that, k 2 will remain the same. We saw the intuition for that the last time around. At low frequencies, the noise transfer function goes as omega square; which means the loop filter transfer function must go as 1 over omega square; which means that, k 2 must go as?

Student: 1

Must be equal to?

Student: 1

1; however, k 1 keeps varying depending on the DAC pulse shape. If you have a more exotic pulse shape – return-to zero or impulsive or nonreturn-to zero pulses, are not the only pulses in the world. Very common pulse shaped to have is what the so-called exponentially decaying pulse shape. For that, the math will be a little more involved, because it is an exponential, but it is still easy to compute. In principle, how you need to do is put the pulse, measure the outputs of the loop filter, express them as functions of k 1 and k 2, and go and fit k 1 and k 2 to get the right L of z.

Student: For any pulse shape, you can write down the exponential.

Yes; in principle, for any pulse shape, you should be able to find out the stuff. And, because it is possible to have an infinite number of loop filters, whose samples are the?

Student: Same

Are the same; for example, if I took a loop filter and translated it in frequency; let us say I multiply all the impulse responses by say a number say cos omega t. Cos omega t now goes to 1 at 0 and multiples of?

Student: ((Refer Slide Time: 11:46))

At less, if I multiply it by cos 2 pi 1 hertz times t, this waveform will go to 1 at multiples of 1 second; correct? So, if I took an impulse continuous-time filter; multiplied its impulse response by cos 2 pi into 1 hertz into t; while the absolute time waveform is different, the samples of this waveform are the same; correct? which means that, if I am able to synthesize a continuous-time filter, whose impulse response is this L of t that we found here and multiplied it up by cos 2 pi into n times t; where, n can be any arbitrary integer; then, even though the absolute impulse response will look totally different, the samples of the response will look?

Student: ((Refer Slide Time: 12:52))

Exactly the same. So, this is telling you that, it is possible to have.

Student: Any number of continuous…

Any number of continuous time filters, whose sampled response is?

Student: ((Refer Slide Time: 13:04))

Is what you want.

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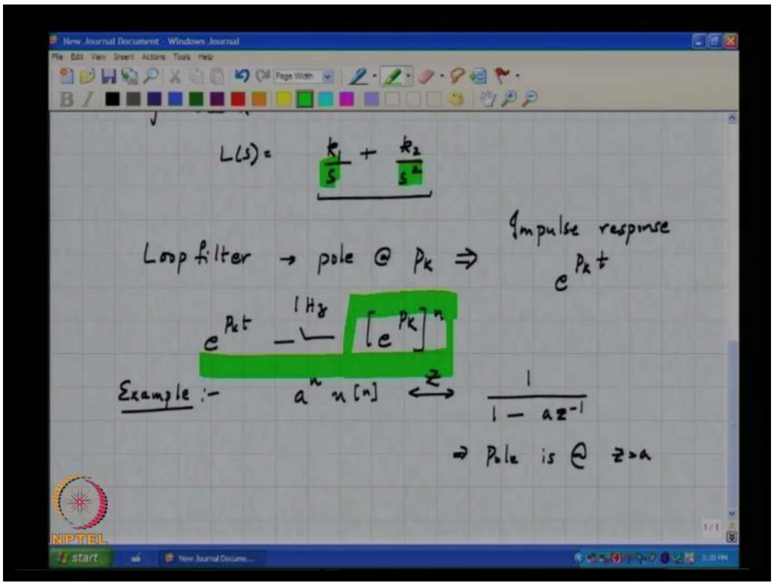

So, in any case, the intuition so far as to choosing the poles of the loop filter has been… In the discrete-time domain, there are two integrators in the loop filter; correct? because at low frequencies, you must have 1 by 1 minus z inverse the whole square minus something. That is what L of z is. Then, you say maybe therefore, the continuous-time loop filter must also have two integrators and then the weights with which you multiply the outputs of these two integrators are treated as free variables and we have solved for k 1 and k 2. And magically, we seem to be getting the same k 1 and k 2 for whichever set of equations we saw. It turns out that, this is not surprising. And, the intuition behind that is the following.

So, let us say we have a loop filter in continuous time with a pole at p k. So, if a continuous-time linear system has a pole at p k, it is impulse response must be of the form?

Student: e to the power of p k t

e to the?

Student: p k t

So, the impulse response is e to the power p k into t. And, it will be multiplied by some constant, which depends on the details of the excitation and locations of other poles and other ((Refer Slide Time: 15:06)) But, the key point is to understand that, if you have a linear system with a natural frequency or a pole at p k, then its impulse response will contain a term of the form e to the p k times t. Now, what we are doing is sampling the output of the loop filter at?

Student: 1 hertz

1 hertz in our example; correct? So, if you sample this e to the power p k times t at 1 hertz; it will correspond to a?

Student: ((Refer Slide Time: 15:43))

Discrete time sequence, which looks like e to the p k times to the power n. Does it make sense? So, what does this say as far as the pole of the discrete-time sequence is concerned?

Student: Discrete time ((Refer Slide Time: 16:15))

Let me refresh your memory and mine too. So, if a discrete-time sequence a to the power n times u of n is there; what is the z transform of this?

Student: 1 by 1 minus ((Refer Slide Time: 16:43))

Is 1 by?

Student: 1 minus…

1 minus?

Student: a z inverse

a z inverse; so, this means the pole is at?

Student: z equal to a

Z equal to a. So, in other words, a to the power n times u of n corresponds to a sequence… If you think of it as an impulse response of a filter, it corresponds to a filter, whose pole is at z equal to a. So, by the same token, this therefore, corresponds to the discrete-time system, whose pole is at?

Student: e power…

e to the power p k.

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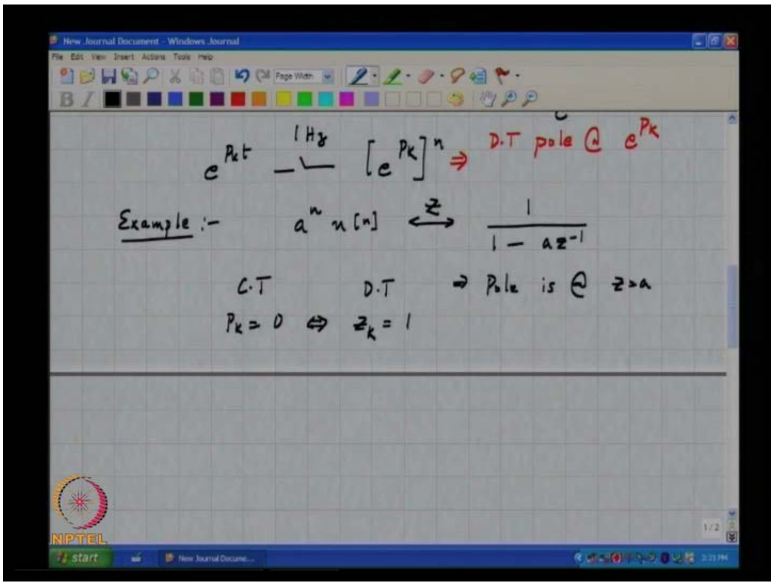

So, discrete-time system with a pole at e to the power p k. So, if the pole is at 0 in the continuous-time system, what is it corresponds to in discrete-time?

Student: z equals to 1; 1

Is 1 and vice versa. So, if you want to emulate a system with a discrete-time system, whose impulse response has components, which correspond to z k to the power n, all that you need to do is to find p k using the relation?

Student: e to the p k…

e to the p k is equal to?

Student: z k.

z k; does it make sense? So, now, if a system has a pole at…

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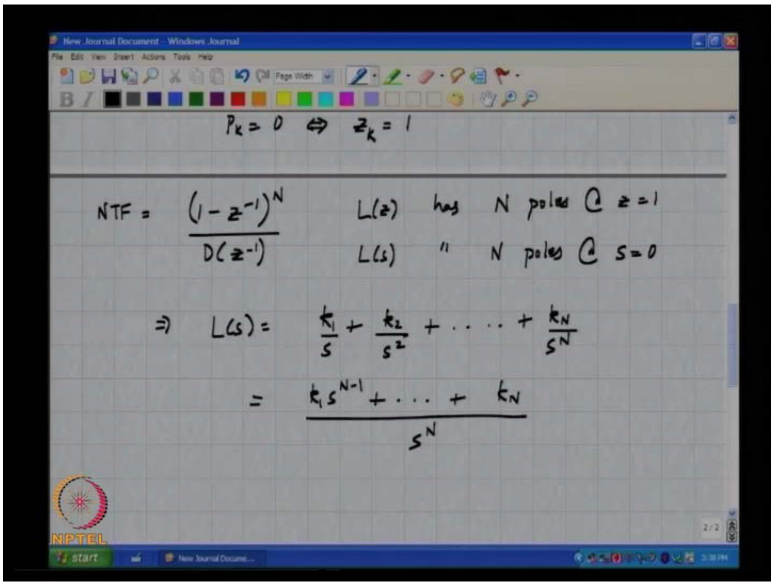

Therefore, if a noise transfer function is of the form 1 minus z inverse whole to the power N by D of z inverse, what can you say about the loop filter poles?

Student: 1 by 1 minus…

Pardon

Student: ((Refer Slide time: 19:25))

No, what can we say about the poles of L of z?

Student: L of z equal to 1; it is 0 ((Refer Slide Time: 19:33)) NTF

So, what can you say about the poles of L of z?

Student: N poles at z equal to…

The loop filter in discrete-time has n poles at?

Student: z equal to 1

z equal to 1; which means the continuous-time loop filter must have?

Student: N poles at s equal to 0 ((Refer Slide Time: 19:55))

Student: s equal to 0.

s equal to 0; correct? which means that…

Student: ((Refer Slide Time: 20:12))

We will… In general, you can think of this as… which means L of s must be of the form k 1 by s plus k 2 by s square plus [FT] k N by s power N. In other words, you have N poles and the denominator is arbitrary. We have assumed that, the denominator is fixed; we have N poles all at s equal to 0; and, the nominator is arbitrary. So, in principle, it can vary from see… Please note that, this is equivalent to k 1 s to the power N minus 1 plus k N divided by?

Student: s power N

s power N. Why have I not said k naught times s to the power N in the numerator?

Student: Because ((Refer Slide Time: 21:30))

So, the only reason for not having k naught s to the power N is because there is... You cannot have a?

Student: Delay…

Delay-free path; is this clear? For all practical purposes, you can also assume that, k naught s to the power N and find k naught through k n by going through the same process. Now, you will have how many unknowns in this particular example?

Student: n

We have n unknowns; all you need to do is?

Student: We just solve

Solve N equations and get the coefficients k 1 through k N. And, the fact that, the continuous-time system and the discrete-time system are chosen in such a way that, e to the p k is equal to z k; that is, the relationship between the location of the poles of the continuous-time and discrete-time system are chosen such that the exponential of the pole of the?

Student: Continuous-time…

Continuous-time system is the?

Student: The pole…

The pole location of the discrete-time system; allows us to… Without getting into questions of – will these cases exist or are they unique; how do I know that, if I solve these any N equations, I will get the same values, etcetera. This should put those doubts to rest, because we know that, the discrete-time system with poles at z 1, z 2, z 3, z N will produce in the impulse response sequences of the form z 1 to the power n, z 2 to the power n, z N to the power N; is that clear? And, if we choose the continuous-time system in such a way that, e to the power p k is the same as z k; then, the impulse response of the continuous-time system will be of the form some constant times e to the power p 1 times t plus some other constant times e to the power p 2 times t and so on. And, these constants of proportionality will depend on the locations of the zeroes of the continuous-time system. However, the samples will be some constant multiplied by e to the power p 1 times N plus some other constant times e to the power p 2 times N and so on. And, by design, we have chosen e to the power p 1 to be z 1, e to the power p 2 be z 2, and so on. So, it follows that, on this side, you have some constant times z 1 to the power N plus some other constant times z 2 to the power n, and so on. On the right hand side, you have something, which is a linear combination of z 1 to the power n, z 2 to the power N and so on. So, it is just a matter of matching these?

Students: Coefficients

Coefficients. So, you have N equations in… How many ever variables, you will always have a unique solution; is this clear?

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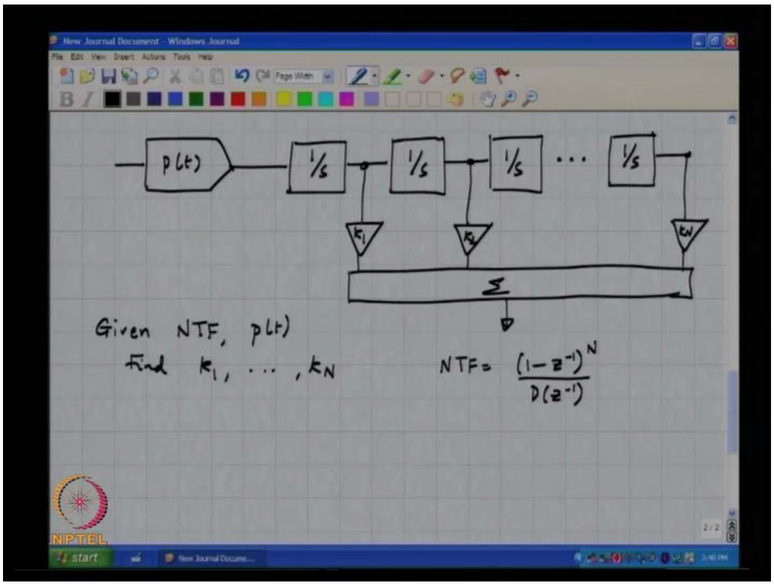

So, given that, we understand this; the procedure for finding the k 1 through k N is very straightforward. So, what will you do? You have p of t; let me just take an example. This is our generalized loop filter; I can think of this as a chain of integrators. Do you understand? So, can you now tell me how you will find k 1 through k N? In general, given the NTF, you want to realize given p of t; find k 1 through k N. So, how do you think I will go about doing this?

Student: ((Refer Slide Time: 27:24))

It is understood first that, the NTF is of the form 1 minus z inverse?

Student: Whole power N…

Whole power N divided by D of z inverse; correct? Only then will all the poles of the continuous-time system?

Student: Be at s equal to 0

Be at s equal to 0. Now, that this is true, what do we do now? Can you tell me a practical way of going about this from scratch?

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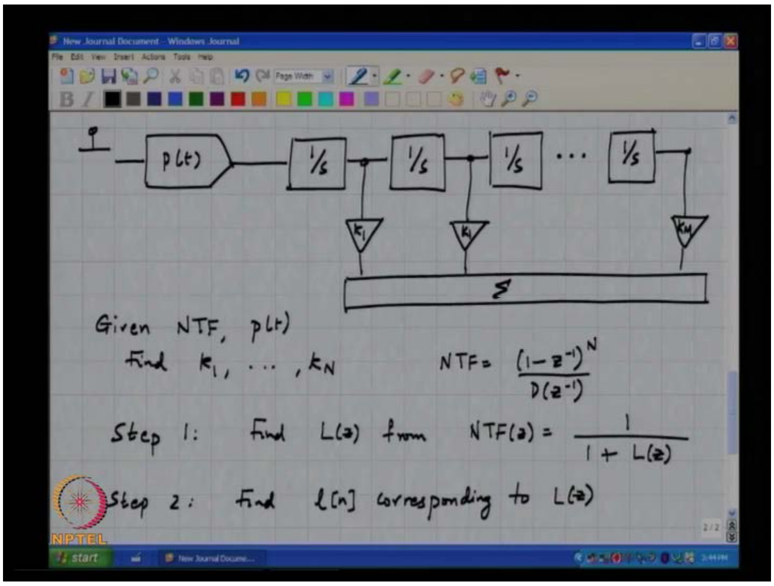

What is the first step?

Student: Find L of z

Find L of z from?

Student: From NTF of z

From NTF of z equal to 1 by?

Student: 1 plus L of z

1 plus L of z. Then, what do you do next?

Student: In the impulse response…

Find l of n corresponding to L of z. And, what next?

Student: Sir, assuming pulse shape and you may get ((Refer Slide Time: 29:15))

So, what I will do is notice the following. I am trying to find the sampled output of the impulse response of p of t cascaded with the chain of integrators, where the weights of the integrators have been are at this point, variables that we need to solve for. Correct? So, I can always move the sampling before the summation. Correct? So, in other words, I can say this is equivalent to… I sample the outputs of this chain of integrators and then from the discrete-time sequence, which is the sum of the samples of the outputs of these integrators. Correct? So, let me call this l 1 of t, l 2 of t all the way up to l n of t. Therefore, the sampled values will be l 1 of n, l 2 of n – all the way up to l n of n. Is this clear? Now, what should I do?

Student: ((Refer Slide Time: 30:57))

Actually, let me just… A couple of things, which are messy and make this notation bad. Let me do one thing; let me make this… I am going to…

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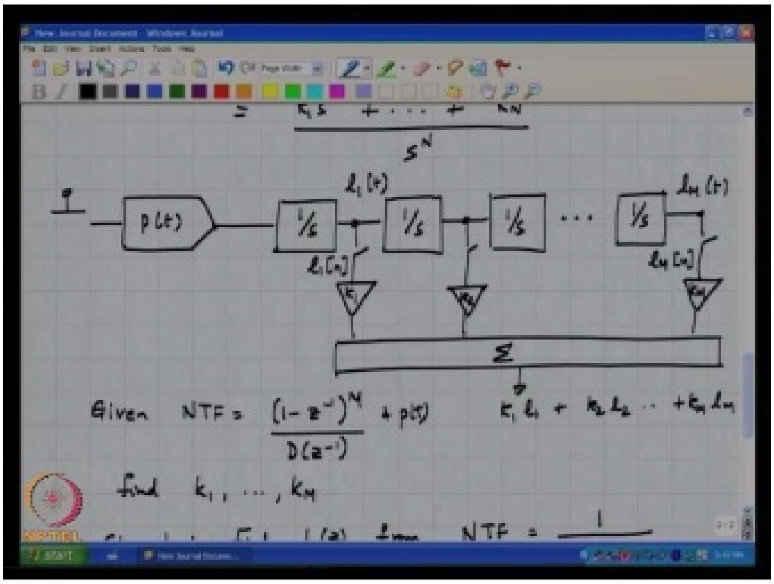

So, let me see. So, given that, the NTF is of the form 1 minus z inverse whole to the power M divided by D of z inverse; and, given the DAC pulse shape p of t; the problem therefore, is to find k 1 through?

Student: k M

k M such that the continuous-time loop filter is indistinguishable from a?

Student: Discrete-time…

Discrete-time loop filter, which gives you the same noise transfer function. Now, how would one go about doing this? First thing would be to find L of z from?

Student: ((Refer Slide Time: 32:39))

NTF is 1 by 1 plus L of z; correct? Step 2 is to find l of n corresponding to L of z. And, now, we want to try and find what combinations or what values of k give us the right sampled values. And, we can see clearly that, the sampling can be moved before the summation and before the waiting of the outputs of the integrators. Correct? So, what I am going to do therefore is to move the sampling operation here. So, this will be l 1 of t; this will be l M of t. So, this will be l 1 of n; this will be l M of n. So, what is this output sequence?

Student: ((Refer Slide Time: 34:24))

What is the output sequence?

Student: ((Refer Slide Time: 34:31))

It is k 1 l 1 plus k 2 l 2 and so on plus k M l M. Is this clear? So, what are we trying to do?

Student: ((Refer Slide Time: 34:51))

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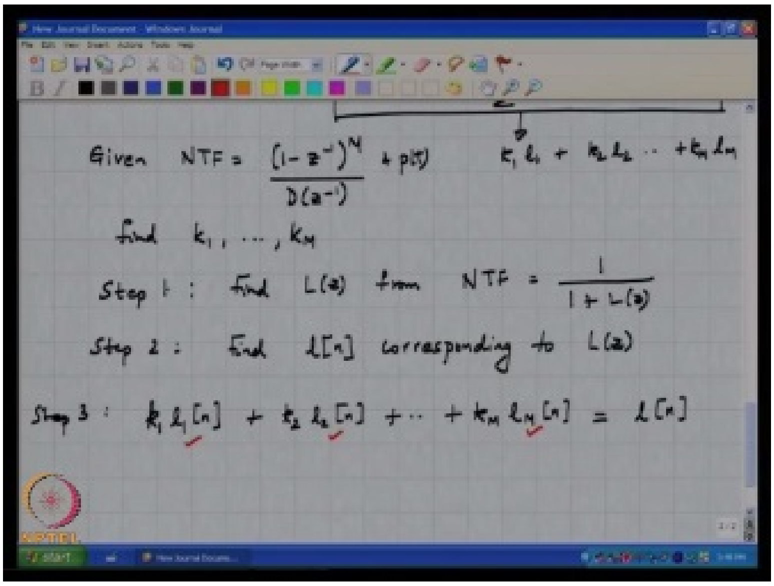

You want to make sure that k 1 l 1 of n plus k 2 l 2 of n plus k M l M of n is?

Student: l of n

l of n; so, all these are known. Why are they known?

Student: There is a pulse shape…

We can know the pulse shape; and, these are nothing but integrals of the?

Student: Pulse shape

Pulse shape; is this clear?

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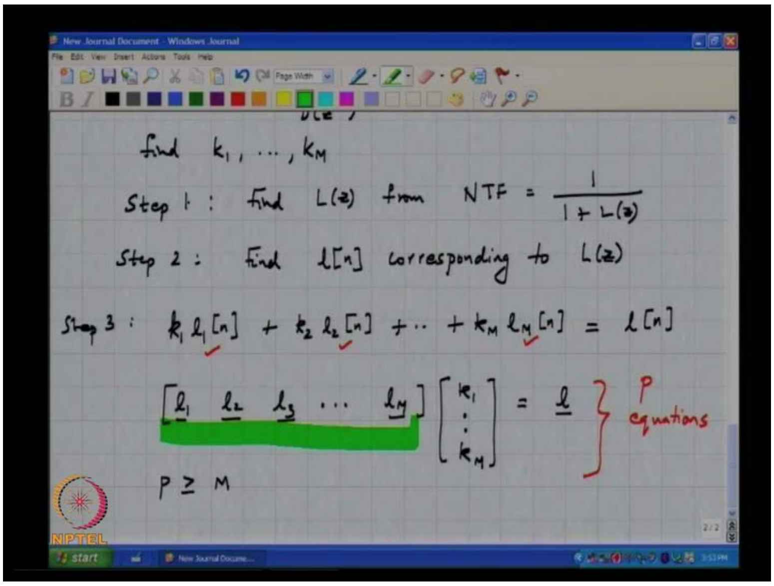

So, if l 1 through l M and l itself are expressed as column vectors; correct? Then, we can write this in matrix notation as l 1; I will put an underline there to denote that, it is a?

Student: 1 cross…

Column vector; correct? You can truncate the column vector wherever you want. Ideally, l 1 through l M, all run from 0 to infinity. So, let us say you truncate it at your favorite point, which at this time is arbitrary – l 1, l 2, l 3 and so on up to l M times k 1 through k M equals?

Student: l

l; and… So, in principle, how many equations are there?

Student: N

Let me call this… I am losing out some subscripts here. Let us call this – p equations; where, p is the point at which you have truncated all these impulse responses. So, you need to find M numbers you have p equations. So, what is the minimum value of p you need?

Student: ((Refer Slide Time: 37:41))

So, p must be greater than or equal to M. And, because of the arguments we made about the choice of the poles of the continuous-time loop filter, it follows that, we choose…Any M equations in the set p, you will get the?

Student: Same…

Same solution. So, clearly, if you choose p much much greater than M, there are obviously lot more equations than you have variables. But, you can actually find a unique solution k 1 through k M. And, that solution will remain the same irrespective of which set of M equations you choose. Another way of doing this is to say I am just going to do least squares and this is of the form a x equal to b; I will find the least square solution. But, in this particular case, it so happens if the least square solution you get will be the same regardless of how many equations there are; which means that, this in… In linear algebra terms, what is this? You are trying to find that linear combination of these vectors l 1 through l M; which will give?

Student: l

l. And, because the poles of the continuous-time system have been chosen in an appropriate manner, it will follow that, regardless of the number of equations p… In other words, regardless of the length of the column vectors l – l 1 through l m and l, the vector on the right-hand side, that is, l will always lie in the column space of l 1 through?

Student: l M

l M. If the poles are not the same; if the poles have not been chosen according to e to the power p k equal to z k, this will not happen. Do you understand? And, you will find that, depending on the choice of equations that you use, you will get different values for?

Student: k 1 through k M

k 1 through k M. And then, you are now at a loss as to figure out which values of k 1 through k M to use. Is this clear? So, anyway, for the time being, let us not worry about it – about that situation.

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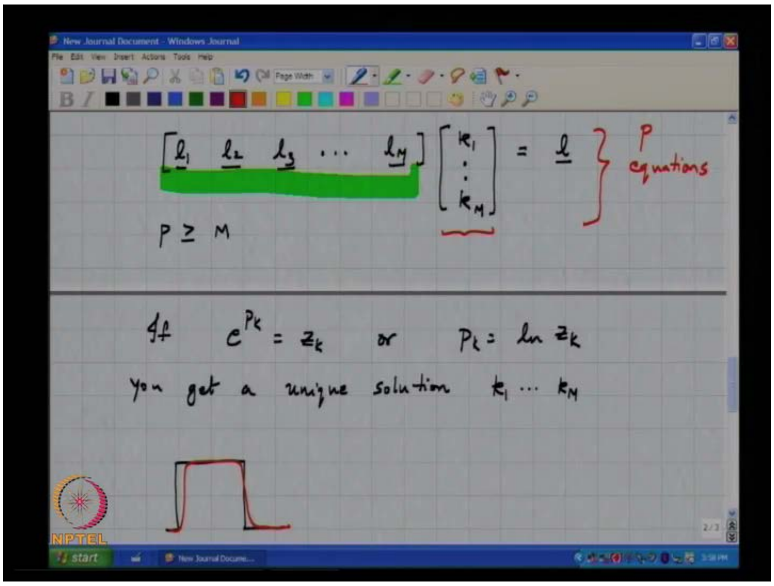

So, if e to the power p k has been chosen to be equal to z k or p k is chosen to be l n of z k; then, you get a unique solution for k 1 through k M. Do you understand? So, this is a very straightforward way of doing things. If of course the pulses has got some simple form like it is a combination of rectangular pulses or whatever; then, you can even do this with hand. Of course, if you have a lot of equations, it gets messy, but is fairly straightforward. If the pulse is messy; not because you want to deliberately make it messy, but because the DAC the way it is designed; let us say the pulse is not really a rectangle. Ideally, let us say this is the NRZ pulse that you want. In practice, for instance, no edge can be?

Student: Sharp

Perfectly sharp. So, a practical NRZ DAC pulse will probably look like this. Now, it is impractical to go and try and find the waveform and do all these analytically. It is simply easier to generate the DAC pulse; have a cascade of integrators; you pump this pulse through the set of integrators in simulation; and, in which case, these waveforms are known; in which case, these will then be known; in which case, you can go and determine the case. Is this clear? And, that will result in p equations with only M unknowns, which thankfully can be solved to give you a unique solution.

Student: Sir, if all the poles are not at s equal to 0 ((Refer Slide Time: 43:43))

Pardon; come again.

Student: If all the poles are not at s equal to 0…

No, you start with the discrete-time system poles. So, if you have discrete-time system with poles at z k; you will choose the continuous time system with poles at chosen to be at p k is equal to l n of z k. So, if all the poles is at discrete-time system, are at z equal to 1; they will all translate to continuous-time system poles being at s equal to?

Student: 0

0. Now, as we have seen earlier, it is possible to choose an NTF such that, the zeroes of the noise transfer function are not all sitting at z equal to 1, but are spread in the signal band of the noise transfer function. I think in third order example, you have already done something in your home work; that means on the unit circle, the poles are not at z equal to 1; but, you have some poles, which are complex conjugate. They are on the unit circle, but they are not at z equal to 1. By taking the logarithm of those poles, they will correspond to complex conjugate poles in the?

Student: Continuous-time system

Continuous time system also, which will be close to?

Student: ((Refer Slide time: 45:10))

If the poles…

Student: ((Refer Slide Time: 45:12))

…in the NTF in the discrete-time system are close to z equal to 1, the poles of the continuous-time system will be close to?

Student: 0

z equal to 0; they will not be equal to 0, but they will be complex conjugate and be close to?

Student: s equal to 0

s equal to 0. So, you need to choose a continuous-time system with poles at those particular locations. Does it answer your question? So, this is a clean way of finding the coefficients without going through a whole lot of hassle. Another popular way of finding the coefficients is to have a dictionary for example; where, you have the equivalent discrete-time impulse response. Please note that all we are doing is linearly combining the impulse responses of p of t driving 1 over s; p of t driving 1 over s square; p of t driving 1 over s power?

Student: N

N. So, if we sat and computed these responses… Think of each of these systems. The input is at discrete-time impulse; the output is at discrete-time sequence; correct? So, you can always find the z transform of that sequence and tabulate your results; correct? So, you say p of t driving 1 over s and sampled at 1 hertz looks like…

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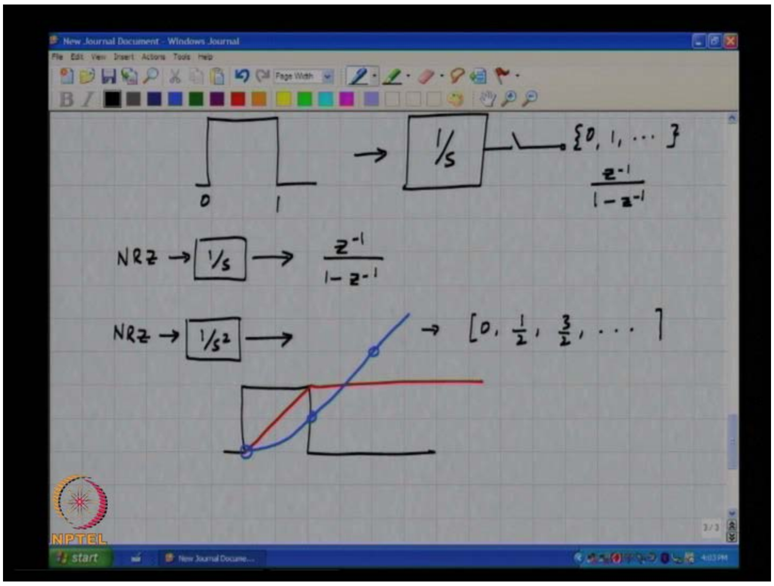

A case in point being an NRZ pulse driving an integrator translates to and sampled.

Student: ((Refer Slide Time: 47:31))

0, 1, etcetera, which is… In the z domain, what…

Student: ((Refer Slide Time: 47:39))

Student: z inverse…

What is the z transform of the sequence?

Student: z inverse by…

z inverse by?

Student: 1 minus z inverse

1 minus z inverse; so, I make an entry, which has p of t 1 over s in the discrete-time domain after sampling corresponds to z inverse by 1 minus z inverse. Similarly, an NRZ pulse driving 1 by s square gives?

Student: ((Refer Slide Time: 48:25))

Student: Is inverse of 1 minus z inverse…

It is tempting to say it is z inverse by 1 minus z inverse the whole square; but, it is not quite right, because…

Student: ((Refer Slide Time: 48:35))

See you cannot take the individual responses and then multiply them; you cascade them in continuous time; does not mean that the sampled response is cascaded. So, you have to be a little careful. Admittedly, it is quite messy; but, this is what people have been doing for a long time. So, I will just do it for completeness sake. So, if an NRZ pulse goes into 1 by s square; how does the output look like? So, the continuous-time output looks like this. And, after integrating it once more, what do you get?

Student: Parabola ((Refer Slide Time: 49:25))

You will get a parabola; and then, you get a straight line . So, the samples of this animal are this guy, which are 0.

Student: 1 by 2

Half

Student: 3 by 2

3 by 2 and so on. We continue with this in the next class.