VLSI Data Conversion Circuits
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Lecture - 03
Sampling – 2

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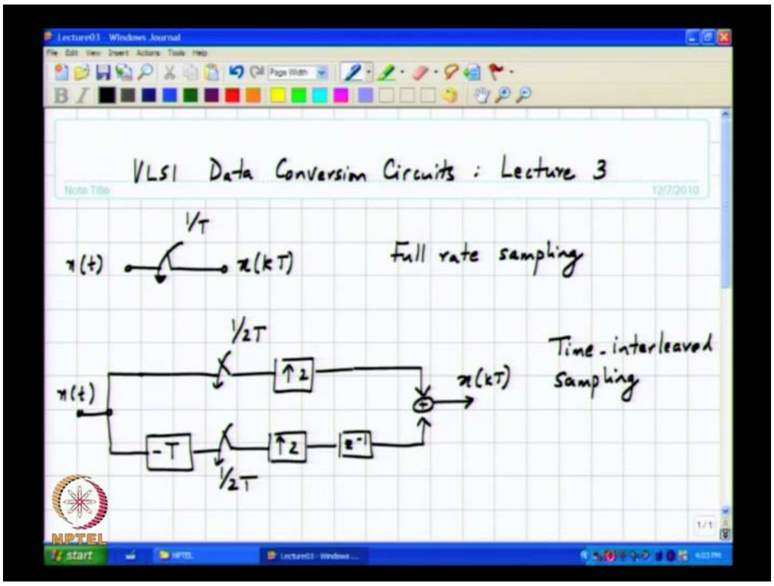

Welcome to lecture 3. In the last class, we were looking at the equivalence between full rate sampling and time-interleaved sampling. The motivation to consider time-interleaved sampling is those applications where implementing a full rate sample and hold might just not be possible. So, just like if you are not able to work hard enough if you hire more people to do the same job. Here the idea is to use 2 sample and holds sampling at half the rate and working in parallel. And somehow you stitch the outputs together in order to get an output sequence which is what the full rate sample and hold would have given you understand. And. So, I have shown here this is the equivalent to the full rate sample and hold right. And this shown below is a mathematical equivalent of time-interleaved sampling.

Please note the words mathematically equivalent physically it will not be implemented like this. What makes more sense to implement in practice is that the top sample and hold channel will be sampling at all even multiples of the sampling. Sampling clock period and the bottom channel will be sampling at a lot multiples of the sampling period which is T. To analyze it does not make a difference if we have a mathematical equivalence which is why I simplified things by sampling both of them at the same instant of time. However, 1 is artificially been advanced by a time T.

And if we put this whole contraction inside a box the system inside the green box should in principle be not distinguishable from a full rate sample and hold right. And during the last class we wrote down the expressions for spectra at various places in the system what are important are. Let us get some more intuition on this by actually plotting the spectrum last time we had equations for the various spectra. Let us now plot and get some intuition; let us just quickly remind ourselves what is the maximum bandwidth of the continuous time input signal x of T which will allow us perfect reconstruction.

Status is.

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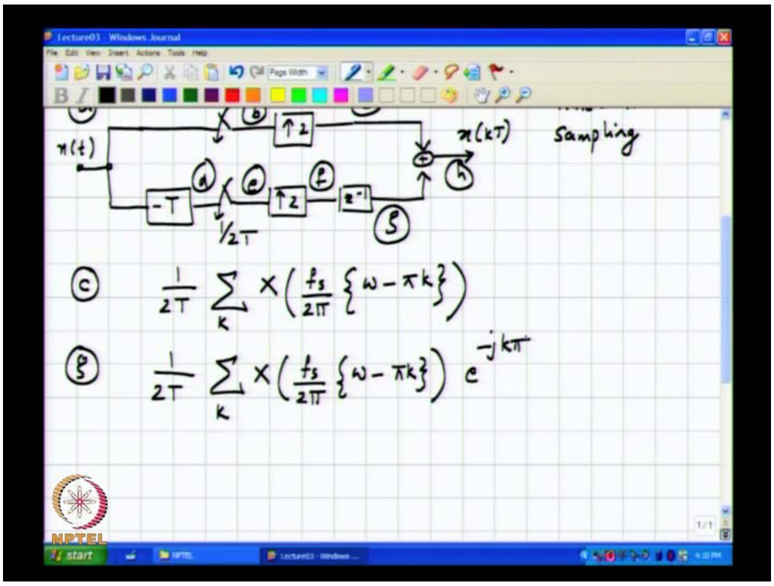

I mean please note also that we in the last class we denoted 1 by T by by f s correct. So, let us quickly remind ourselves again what is the maximum bandwidth if x of T which will allow us to enable perfect reconstruction?

F s by 2.

F s by 2 and in the interleaved sampling system, what are we sampling each channel at.? No, please note this signal x of T can have a maximum bandwidth of f s by 2. And in each in these in this interleaved channel each of these channels is sampling at what rate?

F s by 2 .

It is also sampling at F s by 2, you understand. So, the signal. So, as far as the each of these channels is concerned the input signal bandwidth is f s by 2 and the sampling rate is also f s by two. So, what do you think happens in each of these individual channels.

Aliasing.

They will be aliasing in each of these individual channels; however, from end to end we saw that the system is equivalent to a sampling system operating at f s. So, if there is aliasing happening here and aliasing happening in the lower channel. But when you add the 2 if it is equivalent to something where there is no aliasing what must be happening?

Transforming .

What must be happening is somewhere along the line the.

Alias .

Alias components are.

Cancelled.

Cancelled; so, let us simply the you know take a look at this spectra and and plot them again and plot them this time last time we saw this through the math's. Let us actually draw pictures and convince ourselves that that is indeed happen. So, can somebody please remind me what we call this to be consistent with notation used last time what did we call this point?

C.

So this was called point c all right what about this point?

J.

G for god.

So, we called this g and what did we I guess we must have called this a then and then b c d e f and g.

H.

And h is the final output, am I right?

Yes.

So, I am not going to go over all the math again at c the spectrum must be of the form 1 by 2 T sum over all k x of f s by 2 pi times omega minus.

Pi k pi k.

Pi k and at g we saw last time that the spectrum is 1 by 2 T sigma over all k x of f s by 2 pi times omega minus pi k times.

E to the power.

E to the power j.

Final minus j.

Minus j.

K pi k pi.

K pi does it make sense? This are results that we derived the last time along.

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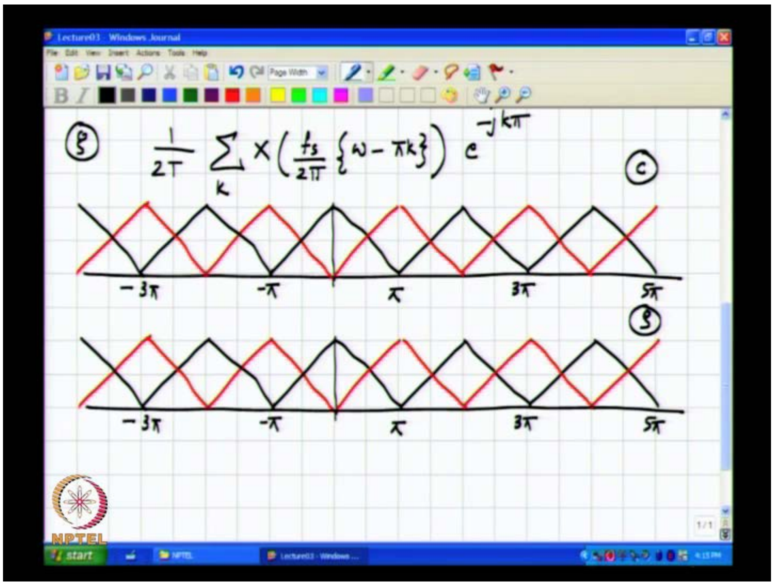

So, now let us sketch these how do you let us say this was the continuous time signal and let us assume that the continuous time signal is occupies the entire f s by 2. So, if this signal was ideally sampled at f s then they would be no aliasing correct. However, how does this look like?

It is shifted horizontally.

So, let us remind ourselves again if this signal was sampled at f s, what would the discrete time spectrum look like?

Opening.

What do we do to get from continuous time to discrete time? 2 pi multiples of 2 pi and this is the continuous time signal first you make copies of a.

Copies and paste at what intervals.

2 pi 2 pi.

At f s right. So, if we had sampled the original signal at f s this is what you would have got. And then we replace the x axis with we scale the x axis we multiply this by you know 1 over T and we scale the x axis. So, what does f s by 2 become pi pi. F s by 2 becomes pi this becomes minus pi and. So, this becomes 3 pi 5 pi and so on. What is this in the discrete time domain? This is what we would have got this is omega minus 2 pi k the stuff that we already know. But what is this expression? What is the difference I mean what is the difference between this expression and this expression?

Superior and 2 pi.

One is repeated at.

Pi.

2 pi where as the other one is repeated at.

Pi.

At pi; so, how do you think this expression will look like?

Margin t

You will have.

Aliasing aliasing.

Aliasing terms now repeated at.

Pi pi.

You will have images repeated at pi. So, you will have right. So, this corresponds to the spectrum at sorry at c, is that clear? Now, I need to find this or draw the spectrum at g, but before we go there please note that indeed we see aliasing in the top channel right the spectrum at c itself is.

Alias.

Alias; so, this makes sense because the signal bandwidth is f s by 2 and we are sampling at f s by 2. Now, let us draw the spectrum at g and to do that I will copy and paste, what is the only difference between the spectrum at c and the spectrum at g?

Ones.

The ones which you have shifted by.

Pi.

Pi are now inverted in psi. So, instead of having I am sorry.

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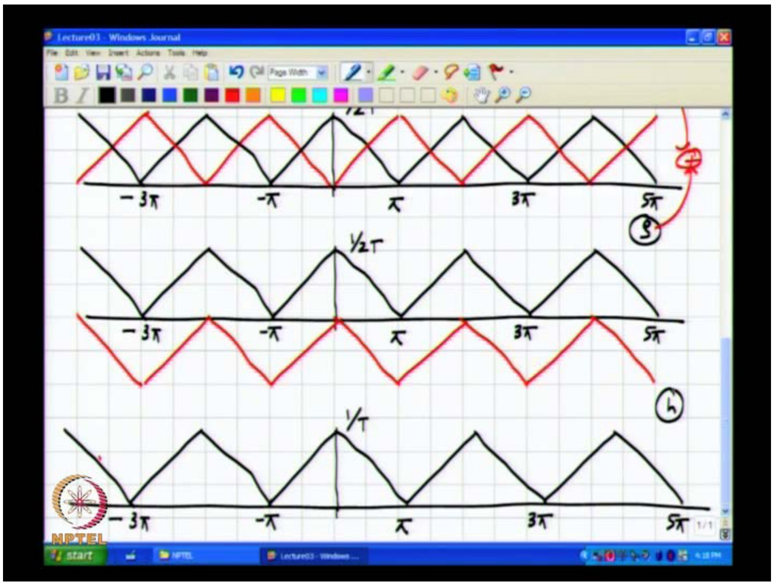

Does it make sense here also we note that that is aliasing correct, because the sum of the spectra is is the black guy plus the.

Red guys.

Red guys so, there is aliasing in both channels. However, when you add c and g together what is happening into the step and repeat happening at odd multiples of pi get cancelled. Whereas, those at the even multiples of pi reinforce each other and what we get eventually is this. And we also understand that the strength is 1 by 2 T the strength is also 1 by 2 t. So, when you add these 2 the strength will be at 1 proportional to 1 by T. And you get the resultant h is a spectrum like this which is what we expected in the first place given that this whole system is equivalent to a system which samples at rate f s it is clear. Now, the next thing is to figure out what happens when there are non idealities here.

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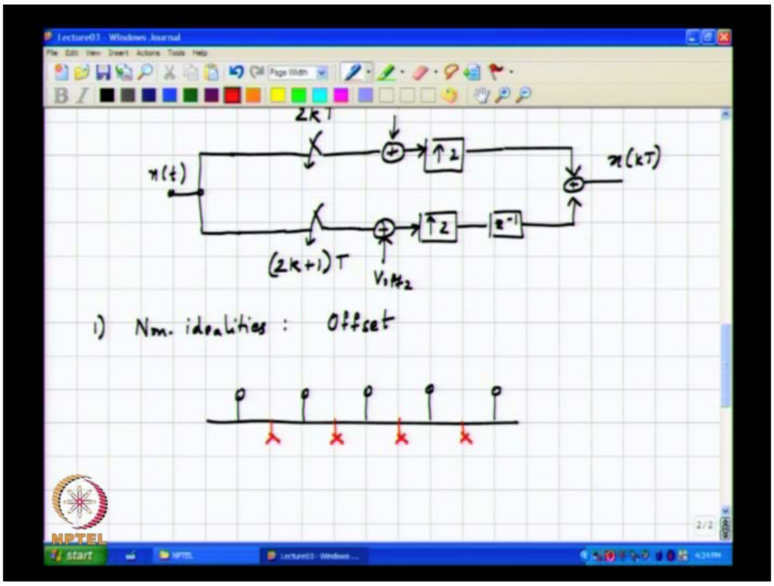

So, let me copy this again as I said a practical realization of this approach would basically have 2 sample and holds 1 sampling at all odd multiples of T and 1 sampling at all even multiples of t. Then you have these 2 sequences and you interpolate up by a factor of 2 by inserting zeroes. Then move one of the sequences by 1 sample and add the 2 the several practical non idealities which come in which we never accounted for in the derivation for 1 as I mentioned that day the signal is passing through different channels. And each channel can add offset in other words the output here for example, is not simply the sampled output plus some offset which we denote by v offset 1 and v offset 2 right. So, the first non-ideality we consider is offset now let us try and understand intuitively what we should expect for the output sequence if each of these channels had different offsets. Or before you go to different offsets may be we tried figure out what happens when there are the same offset. So, when both of them have the same offset what do you think will happen to the output?

Corresponding same.

If all the channels have the same offset then the output will simply be.

Will be ideal.

The ideal output plus.

Offset.

The offset that each channel is add correct now let us try and figure out what happens when the 2 channels have different offsets what do you think will happen?

Minus c cancellation.

How do you think the output will I mean let us try and understand this by putting 0 input. If you put in a 0 input to the system ideally what should you see at the output?

T.

Ideally you we should see 0. If both channels have the same offset, what would you see? You would see a constant d c value of v offset. Now, if both channels had different offsets v o offset 1 and v offset 2, what would you see?

P s a 1 offset.

So.

Offset.

Very good. So, in all even samples you would see.

Offset.

1.

1 all right and all odd samples you will see.

Offset.

2 mind you the offset stays constant. So, every alternate samples value will be the.

Same.

Same correct. So, if you look at this sequence, what can you decompose this as can you see something in the sequence can you comment on the spectral content of the sequence? This is a discrete time sequence.

Two different figure it seems different same period.

You please note that this is a discrete time sequence by definition it only exists as a sequence. So, there is no I mean the again the. So, may at the a and that additional additional 2 1 at what frequency? Frequency of time difference is t. So, 1 by T is please note that this is a discrete time sequence. So, it is there is there is no you know there is no question of you know frequencies f s you understand that was continuous time after sampling you only have a sequence. So, whatever frequency you come up with must be within the range 0 to pi correct. So, now, can you look at this sequence in general it is very difficult to look at a sequence, And tell I mean say what its spectral content is unless you have a natural Fourier transform at your eyes you understand.

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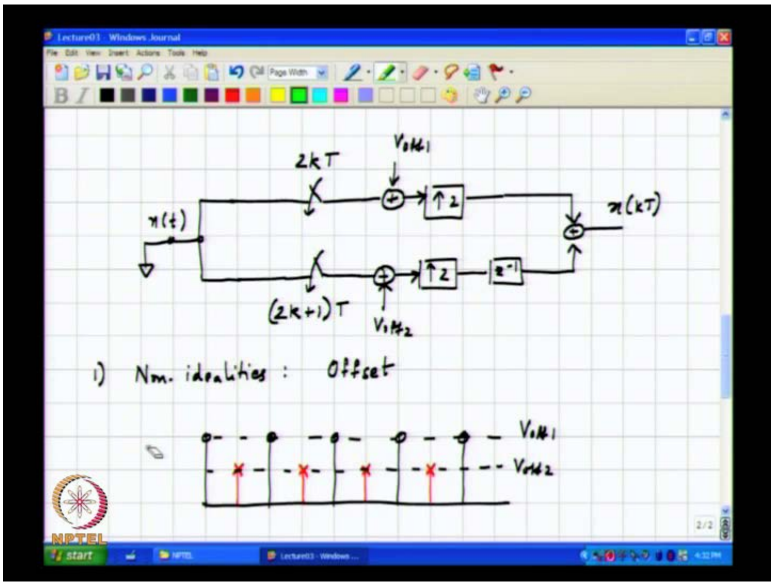

But for this special simple sequence, we should be able to look at it and say. And this is 1 channel offset; this is the other channels offset and this is v offset 1; this is v offset 2; this is clear.

Sir these are the amplitudes or this is the signal I mean v offset 1 of n.

V offset is a constant which is added to every sample in that channel correct see if the input was grounded what would you expect ideally at the output?

0.

All zeroes, now what is happening is that each channel is adding a constant, but fixed offset I mean its I am sorry its its constant offset to all samples. So, so instead of being instead of the output of this channel on the top being 0 0 0 it is v offset 1 v offset 1 v offset one. But these are only the even samples this channel on the other hand also adds offset, but its offset is not the same as the offset added by the first channel. And this can happen in practice you have to mismatch in the circuits that I use to implement these systems. So, let us not worry at this point about where these offsets are coming from and why they are different we just take them for granted that this is what would happen in practice. Now, this side the lower channel also adds offsets which is v offset 2 v offset 2 v offset 2 and so on. And since we are interleaving this, these 2 sequences it follows that the output of the composite system.

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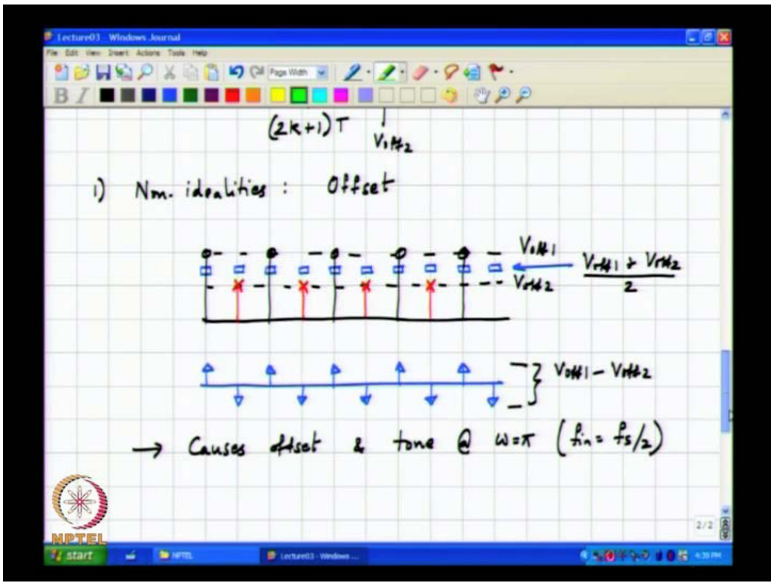

When the input is 0 will be a sequence like this is this clear to everyone. Now, all that I am saying is that we can think of this sequence as the sum of 2 sequences 1 which is.

V offset.

The average drawn in the blue squares correct plus another sequence which is what how does the other sequence look different. The difference here will denote that by triangles here what is the difference in the second sample.

Negative.

Negative and these 2 will have the same magnitude correct because by definition I have removed the average. So, this is the average which is v offset 1 plus v offset 2 by 2. And the amplitude of this sequence which as we can see is periodic with the period of 2 samples is the peak to peak amplitude is v offset 1 minus v offset 2. Since the discrete time tone is periodic with 2 samples the in the frequency domain it must be at.

Sinusoidal.

A sinusoidal to omega equal to pi is this clear. So, now, if we trace this back to the continuous time domain for ease an understanding. We can think of this tone at omega equal to pi as coming from a continuous time sinusoid operating at. It is like if we see a tone in the discrete time domain at a frequency omega equal to pi that must be coming from a. We would normally think that that would be coming from the continuous time input at what frequency.

F s by 2.

Correct, because if we put this whole thing in a black box. And we look at the output sequence and the output sequence seems to show d c plus a tone at omega equal to pi. And as far as we are concerned this is coming from a sample and hold operating at a rate of f s which basically means that the input must be coming from must have a tone at d c. As well as f s by 2 in another words offset in each of these channels will make us believe that the continuous time input has components at f s by 2. As well as d c while the real continuous time input is 0 so; obviously, this is a some kind of error correct causing us to get confused about the nature of the input in continuous time. Now, if the input was not 0 what do you think will happen?

So, this causes offset and a tone at omega equal to pi which corresponds to f in equal to f s by 2. Now, let us discuss what happens when apart from the offsets there is a regular input coming into the time-interleaved sampling system what do you think will happen? What do you think the output sequence will be this superposition the input and the offset? Very good. So, if the input was not grounded, but was a regular input that you would normally use then the output sequence apart from containing the ideal output sequence x of k T would also contain this extra sequence. Because this simply adds to what whatever was suppose to come out ideally correct. So, what is the conclusion now? Therefore, what is the conclusion?

Super position.

Yes. So, basically if you have offsets in each of these channels the output spectrum will be change of will be the ideal spectrum plus d c offset plus a tone at.

Pi.

Omega equal to pi. Now, let us think what would happen if instead of having a 2 way sample and hold. We had a 4 way sample and hold what do you think will happen to offsets?

Pi by 2 pi by 2 I think.

Pardon.

At pi by 2.

Is it only at pi by 2.

Pi by 2 pi pi by 2 pi.

Very good. So, if you have a 4 channel sample and hold system you will see tones due to offset at.

Pi by 2.

Pi by 4 2 times pi by 4 3 times pi by 4 and actually I am sorry no if what would I say we assumed what was this? So, I am sorry if we have a 2 channel sample and hold then we see tones at.

2 times pi.

At at pi. So, in other words its 2 pi by 2 if we have 4 channels we would see.

4.

Tones at.

2 pi.

2 pi by 4.

2 pi and 3.

And 2 pi by 2 and as well as d c you understand. So, in other words if you translate this into the continuous time domain a 2 channel system would make it look as if the input contained d c. As well as a tone at f s by 2 if you have a 4 channel system it will look a like d c f s by 4.

F s by.

F s by 2 and 3 f s by 4 which is the same as f s by four. So, in general if you have an n channel sampling system you will see tones at f s by.

N

N and its multiples and that makes intuitive sense because every n-th sample is the same, because the first channel processes the I mean the first sample its again exercised only after n after all after you gone through all the other channels. So, you come back every n-th sample is the same right and the samples from 0 to n minus 1 are all different because they are processed through different channels. So, you can always expand this as a Fourier series with the fundamental period equal to n samples which corresponds to f s by n does it make sense. So, in other words we already see that using time-interleaving has its problems means nothing comes for free. So, we thought we would make our life easy by using 2 slow sample and holds and stitching their outputs together. And we already see that because of offset there is. Artifacts in the sampled output which would not have been there if the sampling rate was f s I mean was it was a true full rate sample and hold, is that clear? Now, the next non, any questions?

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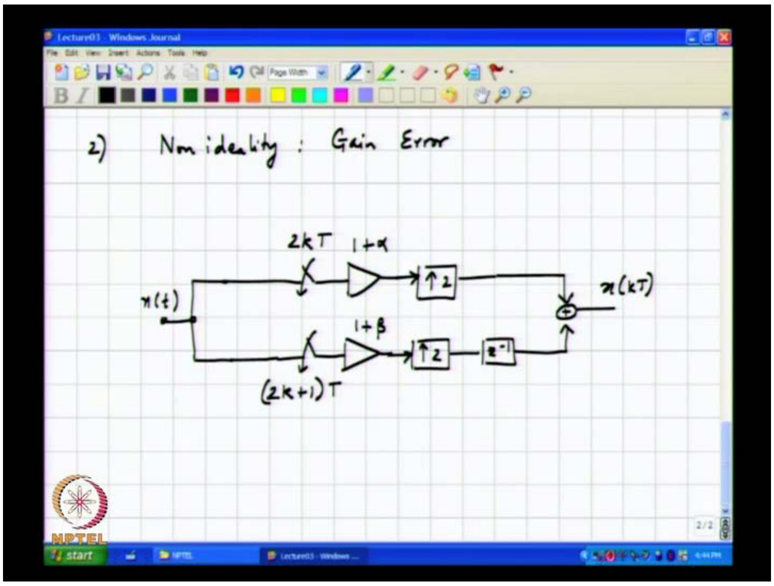

The next non-ideality is gain error. So, let me again copy this and paste now just like each channel introduces its own offset. Because these 2 channels are built using different circuitry though they are nominally identical it will turn out and there will be a small.

Gain.

Gain error in other words the gains of all the channels will not be the same you understand. So, let us now consider gain error separately in a in the. In other words the nominal gain is 1 plus alpha and the nominal gain is 1 plus beta in the lower channel ideally alpha equal to beta equal to alpha equal to beta equal to 0. Both are very very small numbers in practice alpha will not be equal to beta again. Because of random mismatch between both the channels, what do you think before we get into the math? Let us try and understand intuitively what we should expect? What do you think will happen if alpha was equal to beta? What do you think will happen?

Scaled.

If alpha is equal to beta there is no real problem in the sense that it will appear as if the entire sequence of the output is in has a gain error of 1 plus alpha I mean has an error of gain error of alpha correc. Because it is like the even samples getting multiplied by a constant 1 plus alpha and the odd samples also getting multiplied by a constant 1 plus alpha. So, when you stitch the 2 sequences together it is pretty much like the original sequence except that all numbers have been multiplied by this by this extra factor 1 plus alpha. So, there is no real problem now, what do you think will happen when alpha is not equal to beta. There is normal term.

Performance.

Very good. So, let us recall that what were we depending on. So, if the channels had identical gains then we recall that we are adding the spectra at. And we understand that each of the sub channels is not sampling at nyquist.

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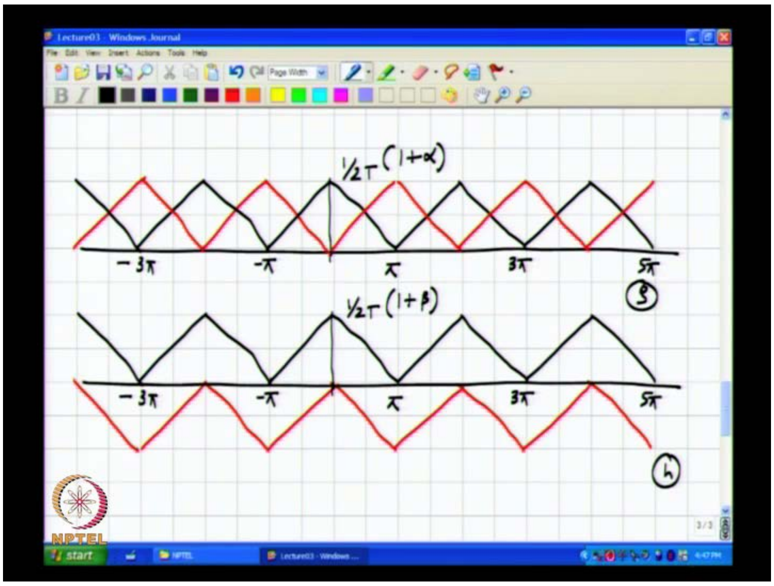

So, they will be aliasing in each of these sub-channels; however, when we stitch the signals of the sequences together magically the aliases are getting canceled off correct. Now, if there was a gain error in both these channels in other words this if you recall g is the output of the channel on top h is the output at the spectrum at the output of the lower channel. So, if the gains were different this is 1 by getting multiplied by 1 by 2 T into 1 plus alpha. Whereas, the lower channel is getting multiplied by 1 by 2 T times 1 plus beta, is this clear? Because all that we have done we have already computed the spectra at this point and this point correct.

Now, if there was a gain error in this channel it is simply taking the spectrum at c for instance and multiplying it by 1 plus alpha. In a similar fashion if there is a gain error in the lower channel it is like to compute the spectrum at g we just take the old spectrum and multiply it by 1 plus beta instead correct. So, since the picture is worth a thousand words what were we seeing earlier if alpha was equal to beta was equal to 0. Then which of these components were which of these colors were was getting cancelled out. The images in red were getting were cancelling exactly where as the images in black were adding up. Now, what do you think will happen if alpha is not equal to beta?

Beta will cancel

There will be A red component in others words there will be an.

Alias component.

Alias component. So, if I ((no audio 41:01 to 41:45)). So, the strength of the black images will be 1 by T plus 1 by 2 T times alpha plus beta the red ones. However, will be this is somewhat exaggerated, because the cancelation is not perfect any more does it make sense. And what will be the height of these images it will be 1 by 2 T into alpha minus beta clearly when alpha is equal to beta again the image is cancelled. And there is simply a gain error correct, but when alpha and beta are not equal we see that there are alias components.

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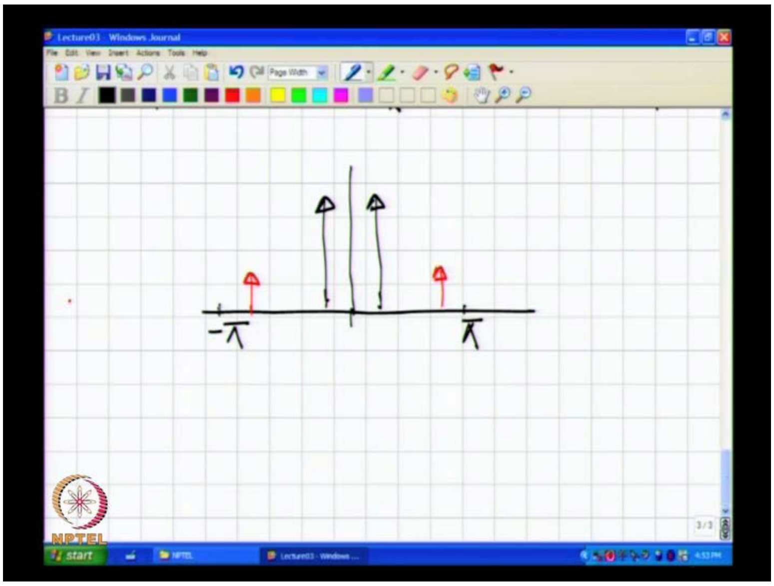

Now, specifically if the input consisted of 2 sinusoids or consists of a sinusoid what do you think will happen if the input consisted of a sinusoid? The discrete times spectrum would ideally have done something like this. Now, what do you think will happen? I mean I do not really need to I mean and draw the periodic extension, because I know its periodic between minus pi and pi. I want to get rid of the scaling factor also just to make the diagram a little clearer and what else what other tones would be present. So, there would be something here and something there correct. So, what is I am sorry I made a. So, if the input was continuous time and had a frequency of let me erase and redraw.

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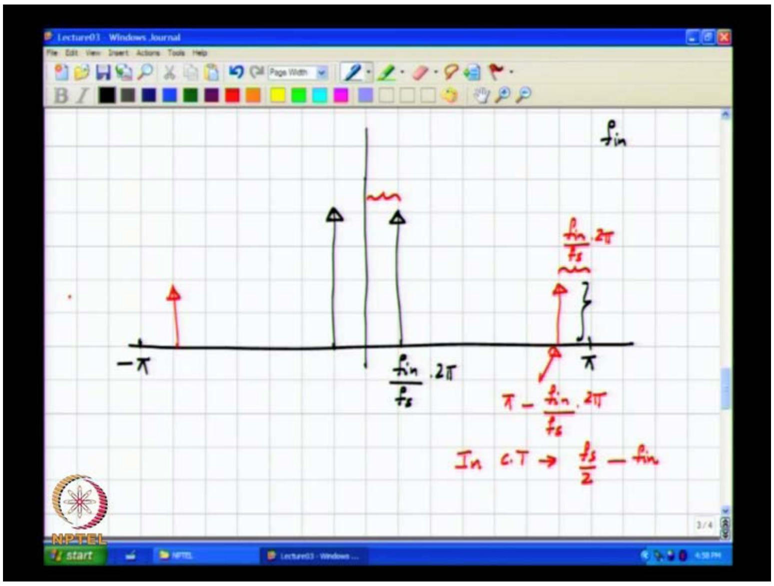

It clearly I am going up just that region between minus pi and pi what we would have got in something like this. And the red part would have done something like this is minus pi; this is pi. Now, what I am asking you is what would happen for the specific case of the input being a tone at some frequency f in right in which case the input spectrum would. For instance look would have 2 impulses here and if the input was at f in and the discrete spectrum where would they occur at.

f in .

F in by.

F s.

2 pi.

Times 2 pi correct.

Yes.

And because these alias components do not cancel they will be something there all right and this is f in by f s times 2 pi what is this distance.

.

Is also f in by f s times 2 pi correct. So, ideally if we had just a sample and hold running at the full rate what would which of these colors, would you see.

Black.

Only the spectral components shown in black could appear now, because of time-interleaving and gain mismatch. We see that there are other artifacts in the spectrum and they are at what frequency?

High frequency.

At high frequency and what is that high frequency here?

F s by 2 and.

F s by 2.

If this distance is f in by f s times 2 pi.

It is pi minus f in by f s times 2 pi translated into the continuous time domain it is like having an input tone at.

F s by 2.

So, in continuous time it is like having f s by 2 minus f. In minus f in or equivalently you can say is f s f in f s by 2 minus plus f in, because whether you have a tone at f s by 2 minus f in or f s by 2 plus f in when you sample it at at at f s. They will you understand intuitively why does this make sense is. So, if you have a tone at f s by 2 minus f in what can you tell me, why does the this intuitively make sense?

The sample f f s by 2 aliasing would happen at.

Yes.

Now, is that aliasing part is not? If you alias if you sample this f in f in tone.

At f s.

Yes.

We would aliasing would happen exactly at f in by f s by 2 minus. No how is that I means the question I am trying to get at is how is that if we would samples sinusoidal tone at a frequency f in with a single channel sampling at the rate f s. We see only these 2 tones in black; however, when you we use a 2 channel sample and hold with gain mismatch. We see additional tomes please note that the amplitude or the strength of these tones is proportional to alpha minus beta correct. So, why does this make sense? No in each of the channels you are actually things I mean.

Yes.

Aliasing is happening.

Correct.

Because there is a gain mismatch.

They are not cancelling.

Cancelling.

All right I mean you know that is the math in words, but is there more intuition to this then.

Sir in each channel you are sampling at f s by 2. So, there is a multiplication of a tone at f in and I mean some frequency components at f s by 2 and multiples of it. So, multiplication in time domain is nothing but in frequency domain you have the sounds and differences of those frequency components.

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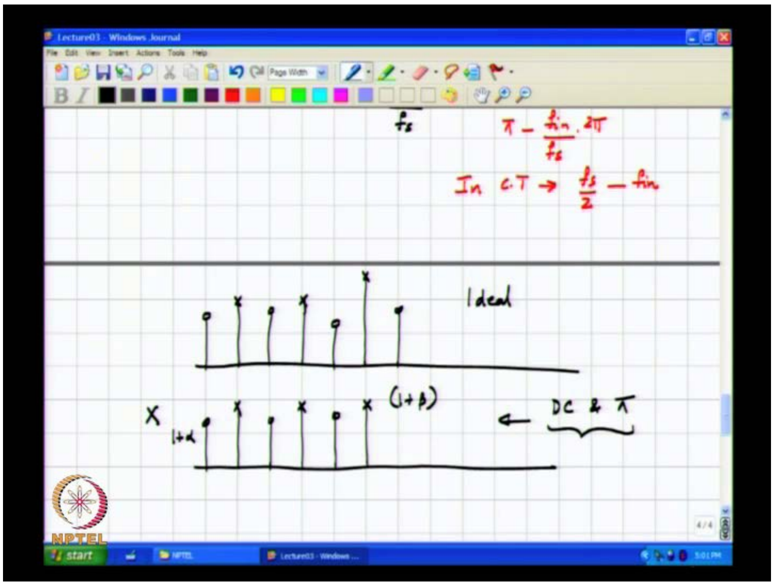

So, here because f s by 2 and f in are there we have sums and differences. Well, you kind of there, but let us see. So, if you think of what is really happening right ideally we were expecting. Let us say some sequence the crosses correspond to 1; 1 channel and the circles correspond to the other. This is let us say the ideal sequence expected out of the whole system because of gain error what is happening every odd sample is getting multiplied up by some constant which is different from unity. And every even sample is getting multiplied by another constant which is not the same as the constant which was multiplying the odd samples. So, you can think of this as taking this ideal sequence and multiplying it by a sequence which is you understand what is this sequence 1 1 plus alpha. This is the ideal output the actual output can be thought of is taking the ideal output and multiplying it by a sequence where the even samples are.

1 plus alpha.

Odd ones are.

And the odd samples are.

1 plus beta.

1 plus beta correct and just now, we said that this kind of sequence what does the spectral contain.

C and c 1.

In consist of d c.

And.

A tone at.

A tone at at pi which in the continuous time domain maps to.

F s by 2.

F s by 2. So, if you take a sequence and multiply it by another sequence which has got both a constant term and a tone at pi what would you expect for the output sequence.

Sum sum and difference on.

You will see sum and.

Difference of I mean a tone the tone at pi will beat with the input tone and give you the sum and difference. You understand that is why it makes sense that you see components at I mean we see the original spectrum plus some components at pi minus f in by f s times 2 pi. The intuition is that you multiply the ideal sequence with a sequence which is got a constant value plus a tone at omega equal to pi which was you see other artifacts. Now, taking this further to an n channel system, what do you think we will see? You will have n n minus 1 components v c. So, at if n equal to 2 we have components around f s by 2 correct if we have n equal to 4, you will have components around.

F s by 4.

Right and its multiples you understand, is this clear? So, we will stop here for today and we will continue tomorrow.