VLSI Data Conversion Circuits
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Lecture - 28
Implicit Antialiasing

This is VLSI Data Conversion Circuits lecture 28.

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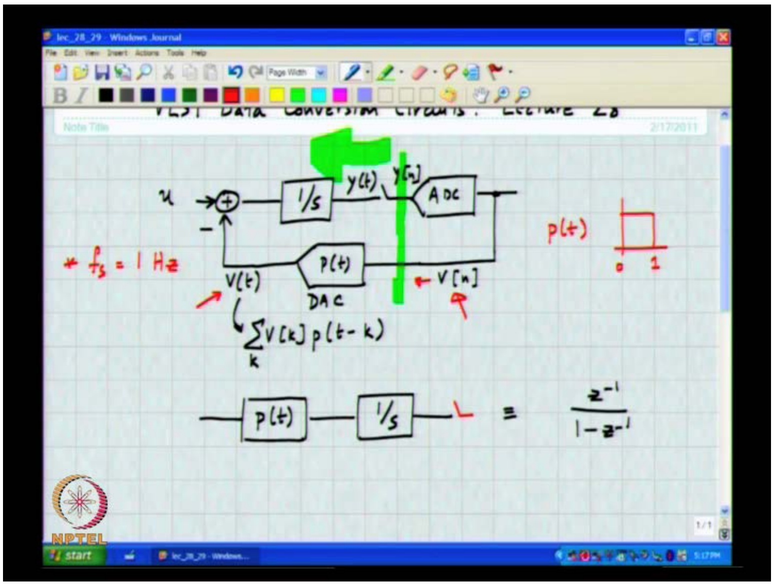

In the last class, we were looking at a way of implementing the loop filter in continuous time. So, this was the system you are examining towards the end of the last class, where the basic idea is to make a continuous time filter, mimic the behavior of a discrete time loop filter. And as we saw the last time around, what we want to do, is to make sure that the cascade of the DAC pulse shape P of t along with 1 over s behave equivalently to z inverse by 1 minus z inverse.

This way the loop gain of the loop, if you break the loop here for example, and you inject an impulse and see what sequence comes back. Then, if the loop filter is chosen to be 1 by s for the first order case, the whole system will be indistinguishable from having a discrete time loop filter, whose transfer function is z inverse by 1 minus z inverse. So, as far as the noise transfer function is concerned, the last time around we looked at a pulse shape P of t being the energy pulse and I had like to remind you again that, sampling rate is normalized to 1 hertz.

It can always take the values of the components or whatever you get in a 1 hertz normalized schematic and scale them so that, it gives you the same noise transfer function at some other sampling rate. The advantage of scaling everything to 1 hertz is that, the numbers all become, all component values will have the same order of magnitude. And you do not have to worry about f s and things like that in the formulae, which will prevent you from making errors.

This is a standard thing to do in all filter work, you always assume that, your normalizing cutoff is 1 hertz or 1 radiant per second as the case may be. Work everything out and then, scale this prototype to whatever the other frequency that you want. So, the basic idea is you can see, is to make sure that, the samples of the loop filter driven by the DAC pulse must be exactly equal to the impulse response of the loop filter from the port V to 1, you understand.

In other words, to the left of this green line, what one would traditionally find would be a discrete time loop filter and all that is happening now is that, this continuous time system. Please note that, P of t the pulse shape of the DAC is instrumental and transforming the input sequence V into a continuous time output wave form V of t. This is a...

Student: Discrete

Discrete time sequence, V of t on the other hand is a continuous time waveform, to transform V of n into V of t, one needs to have a pulse shape, which is being provided by the DAC. As I discussed the last time around, there are several possible pulse shapes or common thing to use is, what is called the non return to 0 pulse shape, which stays put at 1 all the way from 0 to 1. You can also have the so called return to 0 pulse, which is height 2 and a width of half and similarly, an exponentially decaying pulse and many other pulses also.

But, for simplicity, we will consider the non return to 0 pulse in the beginning, so P of t convolved with the impulse response of the loop filter, the continuous time loop filter must mimic after sampling the impulse response of the discrete time loop filter.

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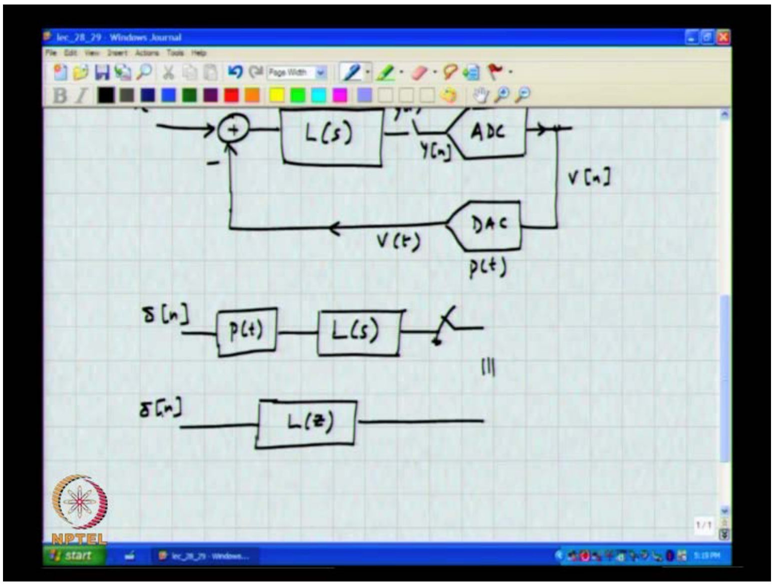

So, in general therefore, if I assumed ASCII DSM of this nature then, what it means is that, P of t exciting this loop filter and sampled should be exactly identical to L of z, where L of z is the...

Student: Discrete time

Discrete time loop filter that one wanted to realize and L of z is obtained from the noise transfer function from as...

Student: 1 by s L r

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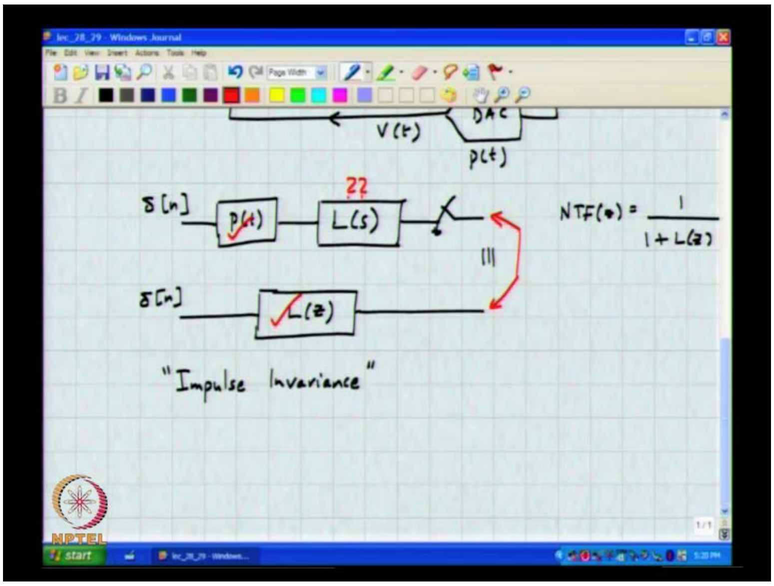

Once you know the noise transfer function, you can determine L of z using the relation NTF of z that is, 1 by 1 plus L of z. Now, this is what is called the principle of impulse invariance, you must have seen this in your DSP classes. So, the job at hand is to figure out given L of z and P of t, what is L of s, which enables these two sequences to be identical, when excited by an impulse, a discrete time impulse mind you at the input. For the first order case, we found it was quite straight forward to do.

We will now go forward and see, what one can do, how one can compute this in the more general case, where the NTF of z is not simply 1 minus z inverse, but something of the form 1 minus z inverse to the N divided by some d of z inverse, which has the usual kind of noise transfer function one deals with.

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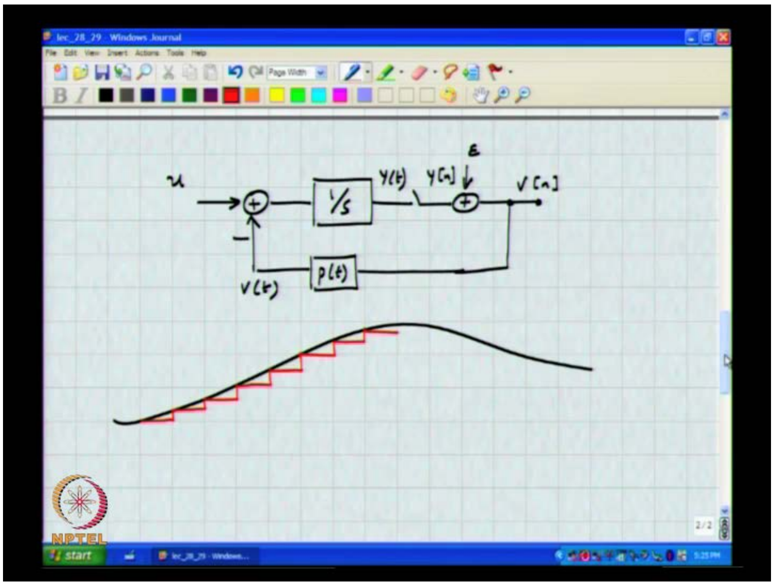

But before that, let me also point out an interesting property and I will illustrate that, using the first order continuous time delta sigma modulator, I have replace the ADC DSC combination with an additive noise source and this is u and this is V of n, this is V of t, Y of n. May be there analyzing the response of this modulator to continuous time inputs, the last time around we said that, if the input is slowly varying, which is definitely the case.

Because, by construction this whole thing is supposed be an over sampling system, so the rate at which the input varies from clock cycle to clock cycle is very small, in which case one can replace the continuous time system. The continuous time input signal with a stepped approximation, in which case you can think of the input as being sampled convolved with a impulse response P of t before getting into the continuous time loop filter.

And then, we isolated the 1 over s along with the pulse shapes and then, determine that its equivalent to a discrete time loop filter, which was z inverse by 1 minus z inverse. So, for all practical purposes, this is exactly equivalent to, the noise transfer function is a same as a first order discrete time modulator. The signal transfer function is also the same as within the signal band at least, the signal transfer function is also the same as a first order delta sigma modulator.

So, it seems as if there is no difference between the two, however as far as the signals are concerned, this stepped approximation is only valid when the input signal is varying slowly. one thing that we had find interesting is that, what happens to this loop when the input is not at dc, but is a sinusoidal at f s in this case at 1 hertz. In a regular A to D converter for example, either an Nyquist state converter or in discrete time delta sigma modulator, if the input was as the sampling rate, what would happen to the output sequence.

Student: Will be a dc, it is a constant.

Any sinusoid at multiples of the sampling rate will alias to...

Student: Dc

Dc, so taking this a little further, frequencies around the sampling rate will alias to...

Student: (((Refer Time: 14:22?))

To frequencies within the signal band, so let us here there is a difference in the way the A to D converter's bridge, because the sampling is not happening upfront, but it is happening inside the loop that is, at the output of the loop filter. So, we had like to, we had be interested in figuring out, what happens to the, how does a loop respond to an input at high frequencies or at an input which is at multiples of the sampling rate.

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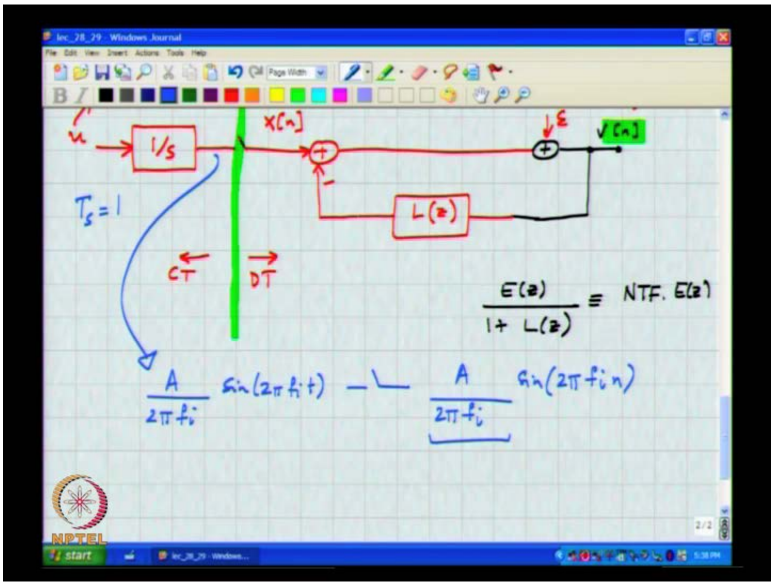

To see what the loop does, let me manipulate the loop a little bit, for starters what I am going to do is, to move this 1 by s through the summer. So, what I will get will be this, this is V of t now, is a transformation from here. All that I have done is, move the integrator or the loop filter in general, through the summer, this is mathematically equivalent. Do you think this is a practical way of implementing the loop, all of you agree that, this is mathematically equivalent.

So, do you think this is a practical way of, can I implement this loop, this is an aside and not directly relevant to our discussion, but I am just out of curiosity I am asking you, do you think it is practical to implement the loop like this.

Student: No

Why?

Student: ((Refer Time: 17:25))

The integrator is outside the loop and we all know that, an integrator working open loop will saturate. But, as far as analysis is concerned, it is perfectly free to do this, because mathematically it is equivalent. Now, that I moved the integrator outside the loop, I also like to move the sampler through the summing node. So, to speak, so the next point in the evolution is to copy and paste and move this way. So I mean, you might wonder, why one would do this kind of manipulation, the reason for this is the following, to analyze this loop, is kind of difficult, why.

Student: ((Refer Time: 19:39))

The problem with this is that, it consists of both continuous and discrete time systems and there is a feedback loop to boot. So, it is not I mean, it is not impossible to do, but it seems somewhat messy to do. So, one way of kind of approaching the problem is to try and split this up, where the continuous time part is separate and the discrete time part is separate. Then, we can analyze the discrete part using z domain techniques and the continuous time part by s domain techniques and some of stitch the two and figure things out.

Since mathematically the whole thing is equivalent, the final answer we get must be correct. So, as a first step, we push the continuous time part out then, we moved the sample and hold also in such a manner that, this part is now completely can it be interpreted as a...

Student: Discrete

Discrete time system why, because this by definition or by choice of the integrator, this is nothing but, L of z you understand. And once we replace the cascade of P of t, 1 over s and the sample and hold by L of z, this system here is completely.

Student: ((Refer Time: 22:00))

So, on this side it is continuous time or rather to the left of the sampler, it is continuous time, to the right of the sampler it is discrete time, you understand. Now, what is it the, given this sequence, what is the transfer function from e to the output, what is the transfer function from this sequence to the output.

Student: ((Refer Time: 22:48))

It is not surprisingly that transfer function is E of z times 1 by 1 plus L of z and the way we have chosen L of z is that, this is nothing but, the NTF that we want times E of z. What we are interested in is the response from u to the output sequence V, so how do we attack this, what is the response from here to V.

Student: ((Refer Time: 23:44))

It is...

Student: NTF

The same as the NTF, because you can think of this quantization error as the input to this loop. So, if I call this intermediate signal say X of n then, the transfer function from X to V is nothing but, 1 by 1 plus L of z is V of z. But, what is the transfer function from u to X?

Student: ((Refer Time: 24:43))

If I told you that, u was a sinusoid at a frequency f and an amp had an amplitude A, so A cos 2 pi f i times t, how will you figure out what V is.

Student: ((Refer Time: 25:21))

Yes.

Student: ((Refer Time: 25:25))

No, not 0 to t, this is continuous time integrator, it is not...

Student: Minus infinity

Minus infinity to t, that is correct.

Student: ((Refer Time: 25:36))

Yes, so you will get a.

Student: ((Refer Time: 25:45))

You will get discrete and sinusoid and once you know that, the discrete time transfer function from X of n to V, so...

Student: We can write the differentiation

You can just write the, you can just plug in the appropriate value for z, which will be e to the j and determine the magnitude of the sinusoid at V given u. So, in other words, this signal here is of the form A by 2 pi f i sine 2 pi f i into t. So, after sampling, what will this result in A by 2 pi f i times sine 2 pi.

Student: ((Refer Time: 27:16))

F i times n, because I assume that...

Student: T s is 1

T s is 1, let me put that down here, the sampling rate is 1 hertz, so as far as X of n is concerned, it is a discrete time sinusoid with some amplitude, which happens to be in this case A by 2 pi f i. Now, this goes through...

Student: 1 by 1 plus L of z.

1 by 1 plus L of z, which is in this particular example, a noise transfer function, which is 1 minus z inverse. So, to determine, please note that, our aim is to determine the

amplitude of the sinusoid at V, which is the output of the modulator, given the amplitude of a continuous time input sinusoid at u. And because we have split our system up as a continuous time system, cascaded by a discrete time system, we propagate the sinusoid through the continuous time system and the output of the continuous time system.

That is, here will simply be a tone at the same frequency, but how the amplitude and phase will be changed depending on the transfer function of the loop filter, that is sampled and where is aliasing occurring?

Student: At the output of the sinusoid

The sampling action is what is responsible for the aliasing action, so if f i was f s plus delta f for example, then after sampling it will look like.

Student: ((Refer time: 29:24))

It will look like a sinusoid at...

Student: Delta f

Delta f you understand, so this sinusoid here because of aliasing due to sampling, will always be such that, it is frequency lies between 0 and pi. Now, we know the transfer function from X to the output which we said, is the same as the noise transfer function. So, given the amplitude of the sinusoid at X of n, it is very straight forward to find the amplitude of this sinusoid at V, because we know the transfer function.

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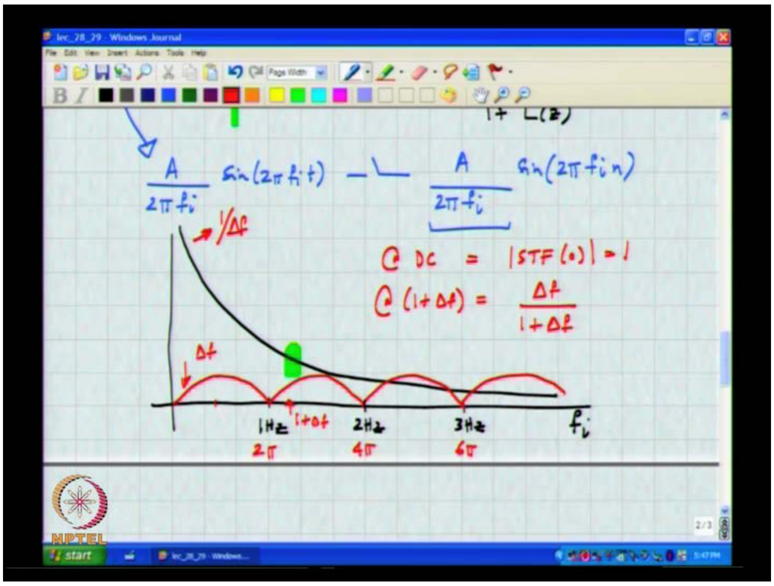

So, if I plot L of s as a function of or the magnitude of the continuous time loop filters gain as a function of input frequency, how will it look like, at d c it will be...

Student: ((Refer Time: 30:56))

No, linearly.

Student: ((Refer Time: 31:02))

No, what is the gain from u to the output of the loop filter as frequency is changing, it decreases at d c it is...

Student: Infinite

It is infinite and it falls of as...

Student: ((Refer Time: 31:19))

As 1 by alpha, say 1 by 2 pi f i is the correct expression, but at low frequency what I want to point out is that, it goes as 1 over f i. Now, if f i exceeds 1 hertz, I have just chosen some random X axis, so let us assume that, we are now interested in dummying the response of this loop to frequencies, which are at multiples of the input sampling rate. If the input is at 1 hertz plus delta f, you can see that the amplitude of the input is, after the loop filter is something like that that is, the amplitude of the discrete time alias tone after sampling and that has to be multiplied by...

Student: NTF

NTF and so, to show it in the same picture, the sampling rate corresponds to...

Student: ((Refer Time: 33:00))

How many radiance in the discrete time domain...

Student: 2 pi

2 pi, so this is 4 pi, 6 pi and so on, at pi what is the gain of the NTF?

Student: ((Refer Time: 33:15))

1 minus z inverse it is 2, at 2 pi what is the gain?

Student: ((Refer Time: 33:26))

This will be periodic like, it must be exactly the same, does it make sense, so an input tone at dc, let us understand what happens. We know the answer, if the input is dc, what should the output be...

Student: ((Refer Time: 34:11))

In this modulator, the input is dc, what would one expect V of n to be?

Student: It must be the same.

It must be the same dc, now we should expect to get the same answer, because this block diagram is nothing but, a manipulation of the original one. So, let us make sure that, we get the, we should expect a d c gain of 1 even after we analyze this. So, at dc, this goes as.

Student: ((Refer Time: 34:51))

Or if you consider a small frequency delta f, this goes as 1 by delta f and what about, what happens to this guy.

Student: ((Refer Time: 35:06))

This goes as...

Student: Delta f

Delta f, so when you multiply the two you get...

Student: 1

You get 1 you understand, you might wonder what happens to the 2 pi, since you are always dealing with frequencies rather than radiance per second, the 2 pi is also canceled out. Otherwise, the gain of 1 over s is proportional to 1 by omega, the NTF at low frequencies is proportional to omega and the two cancel out and you get a dc gain of 1. Now, let us see, so at dc the magnitude of the STF is 1, now at a frequency delta f or rather at a frequency 1 plus delta f, what do you think will be the gain will be...

Student: ((Refer Time: 36:14))

No.

Student: ((Refer Time: 36:16))

Pardon.

Student: ((Refer Time: 36:25))

No.

Student: ((Refer Time: 36:28))

Student: Delta f by 1 plus delta f

It is delta f divided by...

Student: 1 plus delta f

Delta f divided by 1 plus delta f and how do you get that?

Student: ((Refer Time: 36:42))

So, we can see that, at a frequency input which is 1 plus delta f, where delta f is a small number, what will happen. A cos of 2 pi, the 1 plus delta f times t, at the output of the integrator what is the amplitude.

Student: ((Refer Time: 37:25))

A by...

Student: ((Refer Time: 37:28))

1 plus I mean, A divided 2 pi times 1 plus delta f, but after sampling what is the frequency.

Student: Delta A

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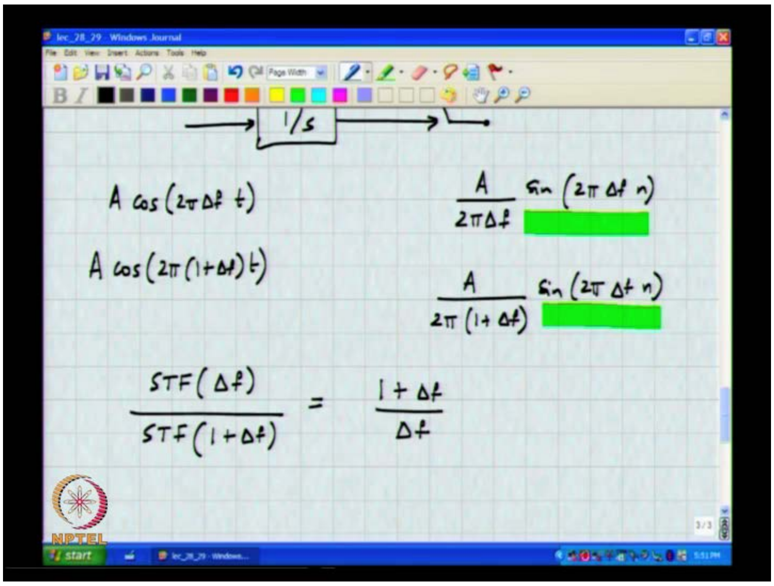

So, even though these 2 tones, so let me write that down here, so we had the loop filter which is 1 by s. We consider two cases, A cos 2 pi delta f times t and A cos 2 pi into 1 plus delta f times t. After sampling, what happens to the first tone, the amplitude is A by 2 pi delta f sine 2 pi delta f times n, what happens to the second tone.

Student: A by...

A by...

Student: 2 pi

2 pi into...

Student: 1plus delta f

1 plus delta f...

Student: Sine 2 pi

Sine 2 pi ....

Student: Delta f

Into 1 plus delta f times n, which is exactly the same as 2 pi delta f times n, so both these tones alias to the same discrete time frequency. This is well known, because sampling is involved, we will have multiple tones aliasing to the same discrete time sequence. However, you see that, there is a big difference in the amplitudes, so the discrete time system following this is the same in both cases.

So, both these tones alias to the same frequency in the discrete time domain, after which the gain in the discrete time domain is the same for both. So, what is the gain of an input, so the ratio of the STF at delta f to the STF at 1 plus delta f must be...

Student: ((Refer Time: 40:10))

Must be in this case what...

Student: One by 1 plus delta f

It is simply 1 plus delta f by delta f, you understand and note specifically that, if the input is exactly at the sampling rate, what happens to the output.

Student: ((Refer Time: 40:47))

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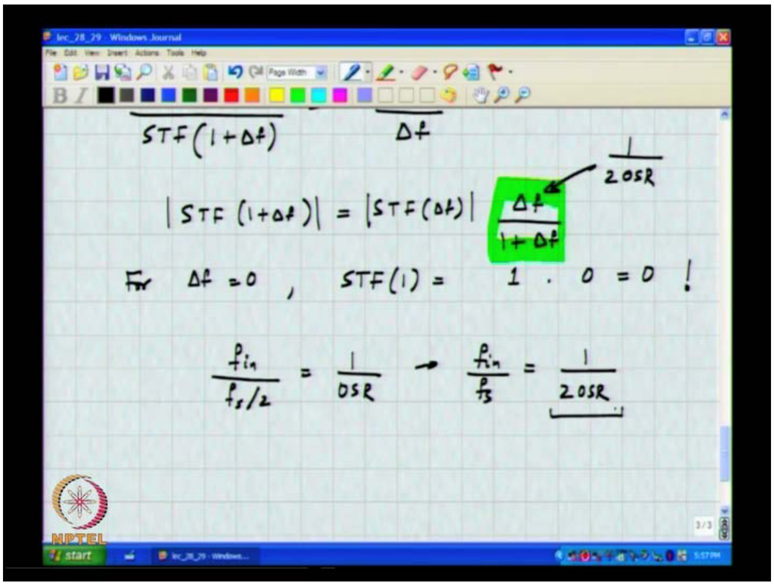

STF at 1 plus delta f is nothing but, STF at delta f multiplied by, in this particular case delta f divided by 1 plus delta f. If delta f is 0, in other words if we put in a tone at the sampling clock frequency, which in a normal converter would cause...

Student: ((Refer Time: 41:33))

Severe aliasing, because the same tone will appear, a tone at the sampling rate will appear like dc. So, it will simply alias, unless you put a very strong anti alias filter, which removes tones at beyond f s by 2, you understand. We saw this in the very beginning of this course that, if you have to have a sample and hold to prevent aliasing, you must have a very sharp filter, which eliminates tones which can alias into your signal band. So, at delta f equal to 0, we see that so, for delta f equal to 0, the magnitude of the signal transfer function at 1, which is the sampling rate, is STF at 0 times 0. STF of 0 is what?

Student: ((Refer Time: 42:40))

The STF at dc is 1, so we see that, we now have the remarkable property that, at multiples of...

Student: Sampling rate

The sampling rate in this first order example, there are nulls in the...

Student: STF

STF, which is great news, because what is this mean?

Student: ((Refer Time: 43:17))

This means that, in principle we do not need an anti alias filter, because at multiples of the sampling rate, the signal transfer function goes to...

Student: 0

0, so as if we have something which eliminates all tones at f s, similarly at frequencies which are close to the sampling rate that is, small delta f, the alias rejection is or the response to tones at 1 plus delta f is, this delta f divided by...

Student: 1 plus delta f

1 plus delta f, which is approximately...

Student: Delta f

Delta f itself and delta f by design will be a small number, because this is an over sampling system. The input frequencies that we are interested in are such that, this delta f will be of the order of pi over OSR or it will be very small compared to 1. So, for frequencies in the neighborhood of...

Student: f s

F s, you will get an STF, which is...

Student: ((Refer Time: 44:39))

Which is close to 0, at f s of course, the signal transfer formation is actually 0, where frequencies close to f s, you will get a signal transfer function, which is of the order of delta f. So, the delta f will be of the order of 1 by OSR. So, if it is actually it will be 1 by 2 OSR, because f in by f s by 2 is 1 by OSR. So, the delta f numbers that we are talking about in practice, will be of the order of 1 by 2 OSR.

So, with a first order delta sigma modulator I mean, is this clear why this is 1 by 2 OSR, no. So, f in by f s by 2 is 1 by OSR which means that, the input frequency divided by the sampling rate is 1 by 2 OSR. So, if we are talking about signals aliasing into the desired band, it must follow, that this delta f must be smaller than 1 by 2 OSR, does it make sense.

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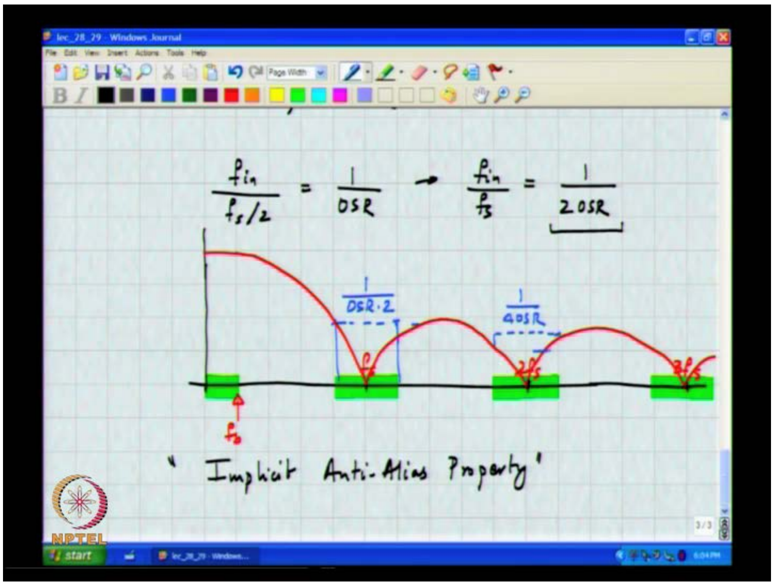

Which means that, for frequencies which are around the sampling rate, please note that I am closely exaggerating here. If this is the desired signal band and this is f s and this is 2 f s and this is 3 f s and so on, where the alias bands.

Student: ((Refer Time: 47:12))

F s plus minus...

Student: f b

F b, the STF at dc is...

Student: 1

What is the STF at d c...

Student: 1

1 and at low frequencies, you can expect it to be close to 1, so dc or in the signal band, the STF is close to 1, at f s what is the STF?

Student: 0

At multiples of f s it is 0, at frequencies close to f s it is...

Student: ((Refer Time: 47:55))

Close to 0, it turns out that and how will this fall off...

Student: 1 by f

It must fall off as...

Student: 1 by f

1 by f and why is it 1 by f, because the signal transfer function is nothing but, the magnitude response of the integrator, which falls off as 1 by f multiplied by this periodic noise transfer function. At dc, the noise transfer function is going as omega, the loop filter is going as 1 by omega, so the two of them canceled and you get one. At high frequencies, the noise transfer function at frequencies, which are 1 plus delta f or at f s plus some small frequency, the noise transfer function is still going as omega.

But, the response of the integrator is reduced significantly which means that, the gain must fall. And what I am saying is that, in this special case of the first order system we have been discussing, this alias rejection which is the maximum gain in this band will be at least 1 by, attenuation of these signals will be at least 1 by OSR times 2. What about the attenuation here in the second alias band?

Student: ((Refer Time: 49:56))

It will be at least...

Student: Delta f by 2

Delta f by 2 plus delta f, delta was very small compared to 2, so it is roughly 1 by...

Student: 4 OSR

4 OSR and 1 by 6 OSR and so on, so intuitively this makes sense, because the loop filter treats a sinusoid at 1 plus delta f very differently from a sinusoid at delta f and the loop filter output is being sampled. This is in contrast with the discrete time delta sigma modulator, where sampling is happening right upfront, so the treatment given to a

sinusoid at 1 plus delta f is exactly the same as the treatment given to a sinusoid at delta f.

So, in other words, a discrete time modulator as we expect, cannot distinguish between a sinusoid at 1 plus delta f and a sinusoid at delta f. It will have the same magnitude for the STF for both these sinusoids, which is why you need an anti alias filter upfront, which will attenuate components in the alias bands to a sufficiently large degree so that, you do not make incur alias errors. Whereas, in a continuous time modulator, by virtue of continuous time operations of the loop filter we see that, at multiples of f s, the STF is indeed 0 and for frequencies which are close to f s, the STF is...

Student: Very small

Is small, thereby it appears as if you have cascaded a anti aliasing filter upfront, which greatly reduces the gain for the input signals in the alias bands. So, this property is called the implicit anti alias property of a continuous time delta sigma modulator. So, in the next class, we will generalize these two high order loop filters and see, what the signal transfer function is in that case.