VLSI Data Conversion Circuits
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Lecture - 26
Loop Filter Architectures

Good evening. This is VLSI data conversion circuits – lecture twenty four.

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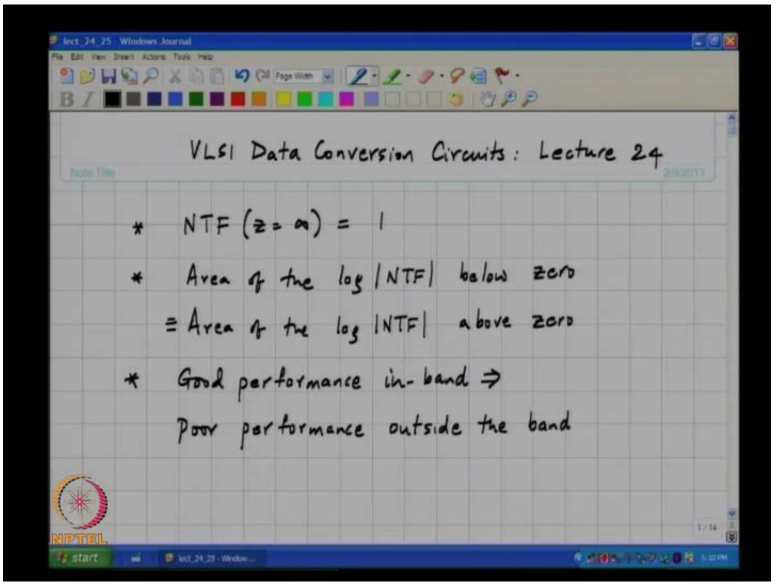

In the last class, we learnt several properties of noise transfer function. Basically, a noise transfer function is a high-pass filter, but with a special property that, noise transfer function evaluated at z equal to infinity must be 1. And, this in turn means that, if the noise transfer function is plotted on a log scale and the poles of the noise transfer function are within the unit circle; and, the zeroes lie utmost on the boundary of the circle, which corresponds to practical situations, where you want to stable noise transfer function and with zeroes of the noise transfer function located on the unit circle; then, the fact that, NTF evaluated at z is equal to infinity is 1 is equivalent to saying that, the area of the log magnitude of the NTF below 0 is the same as the area of the log magnitude of the NTF above 0. This therefore means that, good performance in-band implies poor performance outside the band. And, why we have worried about performance of the noise transfer function at out of band frequencies?

Student: Because it will over load ((Refer Slide Time: 02:53))

Correct. So, as we saw over the last two or three classes, increasing the out of band gain basically means that, the input to the quantizer, which consists of the input to the modulator plus shape noise will now start to have?

Student: Large

A large noise riding on top of the input; and, since any real quantizer will have saturation, we saw that, the moment the quantizer starts to saturate, the effective gain of the quantizer for this quantization noise, which is going round and round the loop, is starting to reduce. If we have a high order loop filter, which is presumably what you will have in order to achieve the required low in-band quantization noise, the saturation of the quantizer frequently will cause its gain to reduce; thereby causing the poles of this system to go or to the unit circle; thereby causing the modulator to become unstable; where all these in-band noise being small and all that are no longer valid. So, the shape or the gain of the noise transfer function outside the desired signal band is of paramount importance, because you are not only interested in reducing the in-band quantization noise, you are also interested in making sure that, the useful range of the modulator is a reasonable fraction of the quantizer range. We all understand that, the maximum stable range at the modulator input cannot be the same as the quantizer range, because the quantizer range must be occupied by the input plus some shape noise. So, that range can never be the same as the input range. But, even though we accept this, we want to make sure that, the stable range is a reasonable fraction of the quantizer range. We do not want the input range to be 5 percent or 10 percent of the quantizer range, because that is going to reduce the maximum stable amplitude by a large factor. Is this clear?

Now, the next thing is to see what one must do in order to have a noise transfer function, which goes as omega to the n in the signal band while the out of band gain is not 2 to the n, but smaller. And, the last time around we saw that, one can actually move the poles of the transfer function 1 minus z inverse whole to the n, whose poles are all sitting at z equal to 0. If we move the poles closer to z equal to 1, then the gain at omega equal to pi will reduce and the gain at z equal to 1 or omega equal to 0 will increase. And, we were wondering how one can go about systematically designing a noise transfer function.

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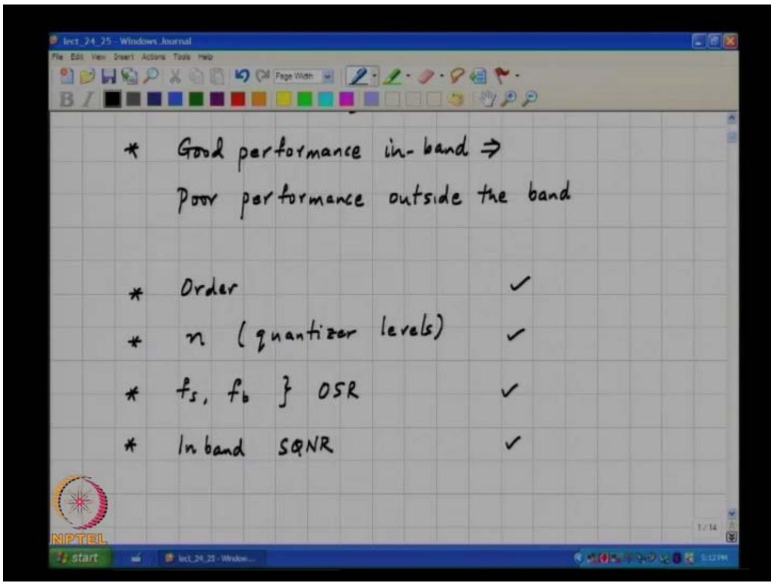

So, what we know are the following. So, presumably, we know the order of the loop that we want to design; we know the number of levels of the quantizer; we know our signal band width, which is equivalent to saying that, for a given f s, I know f b, which means that the OSR is also known. So, order is known; n is known; the over sampling ratio is known. And, of course, one should also have obviously an idea of what signal to quantization noise ratio you want to achieve in the signal band. So, we also know the desired in-band SQNR. So, given this and given that, a noise transfer function is nothing but a high-pass filter transfer function. One need not go ahead and start reinventing all of high-pass filter synthesis. The design of IAR high-pass filters is a very well-known thing, which is been beaten to death by the DSP people. So, we just ride on all the knowledge that has been accumulated in this area over the years.

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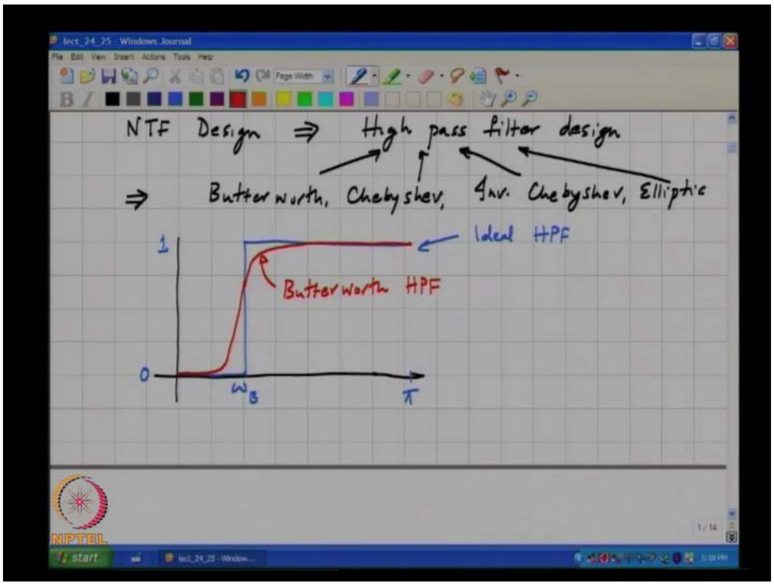

So, NTF design is the same as high-pass filter design. So, the steps… Since we know that, we want say a third order, fourth order NTF, we start looking at third or fourth order high-pass filters. And, there are many families of high-pass filters in the literature. So, you pick your favourite, for example, you can have a Butterworth or a Chebyshev or an inverse Chebyshev or even elliptic high-pass filters. These are all various families, where the properties of the high-pass filter transfer function have been worked out. And, a tool like MATLAB for instance will give you the coefficients of these filters energy free without us having to do any work at all.

And, please recall that, MATLAB does not know that you are a delta sigma modulator guy. What it will do when asked to generate a high-pass filter transfer function is to approximate the ideal high-pass characteristic, which is 0 in some band and 1 in some other band. So, this is the ideal HPF. And, depending on the nature of the approximation you choose, you will get different high-pass filter transfer functions, whose frequency response approximates this ideal brick wall, where it is 0 in some bandwidth omega B and goes to 1 in the rest of the band. So, for example, if you wanted to synthesize a Butterworth filter, what you will get is perhaps something like this. And, depending on the order, the transition band of the filter will become sharper and sharper. So, do you think this can… For example, if I give you this transfer function, by looking at it, can you give me a simple argument to say that, this cannot be a delta-sigma modulator noise transfer function.

Student: Area ((Refer Slide Time: 11:51)).

Let us… That is very good. So, you know that, if you plot log magnitude of noise transfer function, the net area under the log magnitude curve must be 0.

That can never happen if the gain of the of the high-pass filter transfer function is always smaller than 1. Correct? So, as such, even though the shape of the high-pass filter is similar to that of the noise transfer function, as such this simple argument will tell you that, the high-pass filter transfer function, which approximates the ideal brick wall, cannot be a noise transfer function. Is this clear? So, this is for example, Butterworth high-pass filter. If you have a Chebyshev high-pass filter, there will be some ripple in the pass band; and, the transition band would be sharper. If you have any inverse Chebyshev high-pass filter; in the stop band of the high-pass filter, there will be notches; and, the pass band would be maximally flat. An elliptic filter will have ripple in both the pass band and stop band; which means there will be notches in the stop band as well as the ripple in the pass band. And, for a given order, in the Chebyshev and the inverse Chebyshev cases, there is a trade off between the ripple and sharpness with which the filter rejects or discriminates between low and high frequencies. These are all well-known tradeoffs in filters. And, I am not going to go into the details. But, given this, let us try and figure out a systematic way in which one can go and figure this out.

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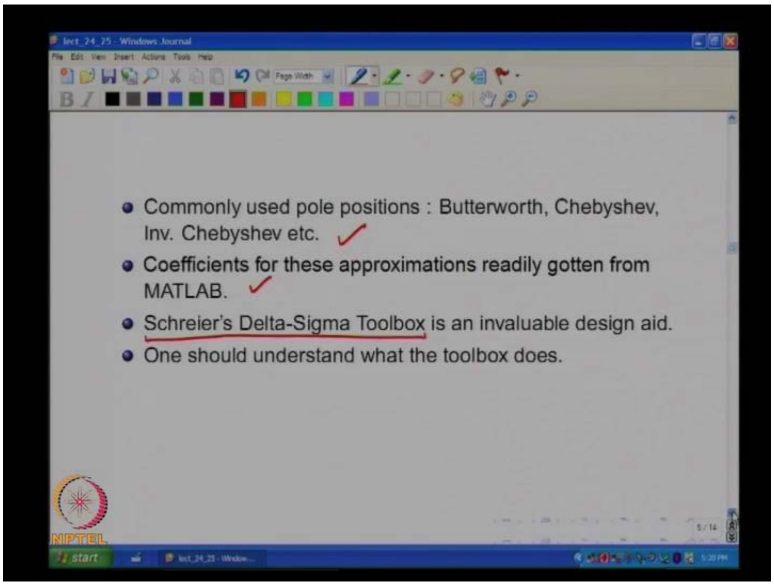

As I said, you can pick the favourite pole locations. So, you either choose Butterworth, Chebyshev, inverse Chebyshev or elliptic or whatever you like. The coefficients for these approximations are readily gotten from MATLAB. And, I will talk about this special toolbox, which has been written for delta-sigma modulator design by Richard Schreier from analog devices. This is an invaluable design aid, where lot of these mostly common; I mean all the stuff that you want to do when you design a delta-sigma toolbox has been coded into a bunch of very nice routines, which fit together; and, allow you not only to design, but at this stage also, allow us to learn and examine the behaviour of delta-sigma modulators without having to go through the pain of having to write code ourselves. So, like any tool, you must understand what the tool does in order to be able to use it efficiently. So, we will go over this a little later.

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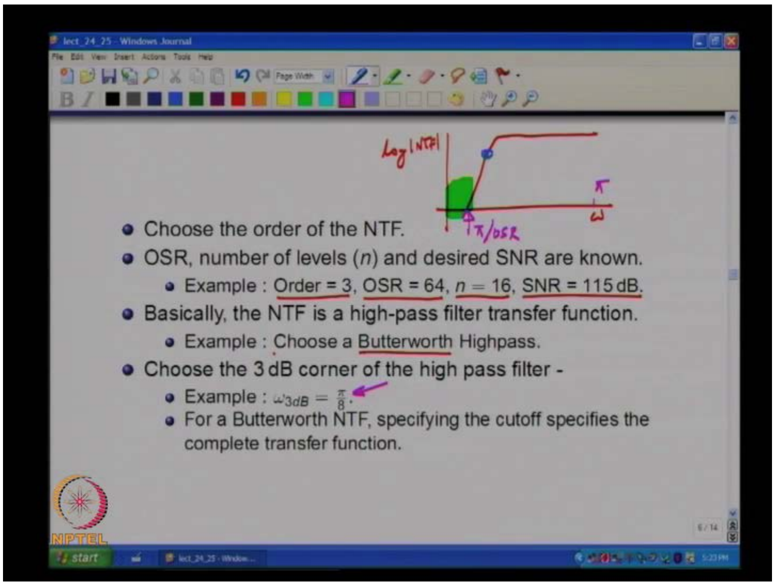

But, for the time being, let us start here. Let me take an example. So, we choose an order say 3; an oversampling ratio of 64; a 16 level quantizer; and, an in-band desired signal to noise ratio of 115 dB. So, as I just mentioned, I will just choose the Butterworth family of high-pass filters to illustrate the process. And, since the signal band… So, for us, the desired signal is sitting at low frequencies. So, if this is log magnitude of the NTF and this is omega, the desired signal is sitting here; correct? So, if I move the high-pass filter corner to the right… Which is the high-pass filter corner? This frequency here. So, if I move this to the right, what do you think will happen to the in-band response of the NTF.

Student: ((Refer Slide Time: 16:46)) reduction.

You are moving the high-pass filter corner to the right; which means that, the gain at DC will decrease. Correct? And, from this simple picture, if you want to reject a lot of quantization noise in the signal band, you cannot choose a high-pass filter corner for the Butterworth filter to be… Pi by OSR is sitting here; do you understand? So, to begin with, you need to choose some corner for the high-pass filter. And, the reasonable choice is something which is much larger than?

Student: Pi by OSR.

Pi by OSR. Is that clear? So, at this point, since I do not know anything else, I will just choose a corner of pi by 8. We will just… Which is somewhere here and this is pi. All right? So, you plot this into MATLAB. Once you say the high-pass filter is Butterworth, then there is only 1 degree of freedom, namely the cut-off frequency, because the order is fixed; and, the fact that you want it to be Butterworth means that, the relative locations of the poles is also fixed; the only thing that you can do is vary the cut-off frequency. So, once you specify this – that omega 3 dB is pi by 8, you are done. So, you plot this into MATLAB.

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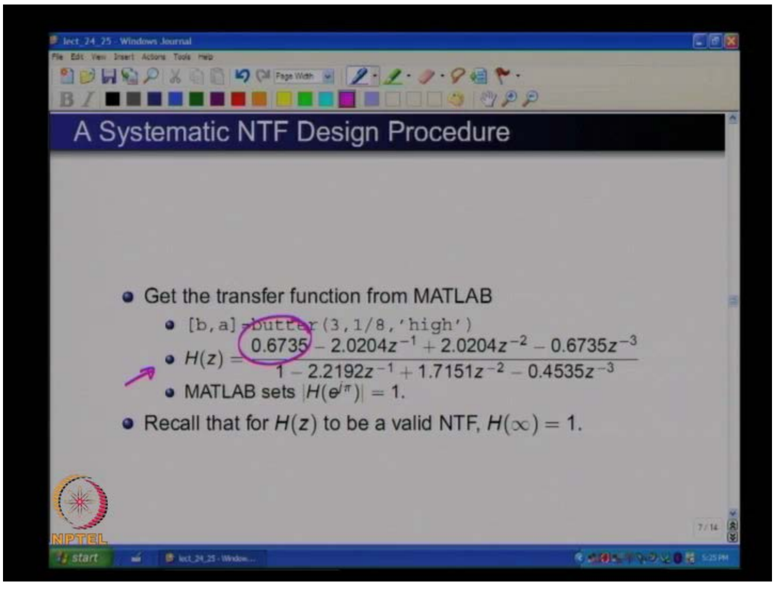

And, this happens to be the command. And, you see that, it gives out H of z. And, as I mentioned to you before, the passband gain will be normalised or will be 1. And, if you want this to be an NTF, even though the shape is the same, we know that, this cannot be a noise transfer function, infinity is not 1. In this particular case, it happens to be?

Student: 0.67…

0.6735. So, to make this, a legal NTF, what one must do is to make the first sample of the impulse response equal to 1, which is equivalent to saying that, H of z evaluated at z equal to infinity must be made 1. And, that is pretty straightforward you simply?

Student: Divide H of z...

Divide H of z by?

Student: 0.6…

In this particular case, 0.6735. So, what will this result in as far as the frequency domain picture is concerned? If I divide H of z by 0.6735… The frequency response earlier was going from 0 at DC to 1 at omega equal to pi. Correct? Now, if I divide this by 0.6735, what will happen?

Student: It will become 1.3…

The gain at omega equal to pi will simply get multiplied by 1 by 0.6735, which is about 1.5 or something like that; I am sure enough.

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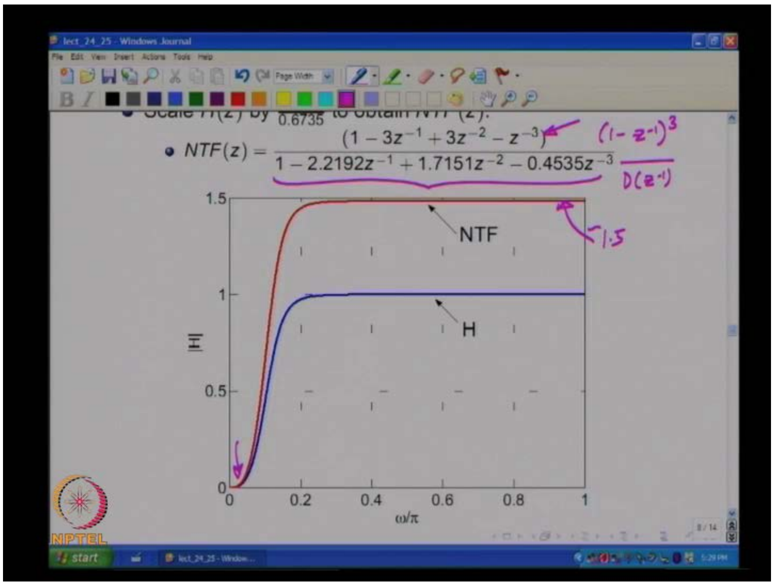

We get this. All right? And, please recall that a Butterworth transfer function has all its zeroes; a high-pass Butterworth filter has all its zeroes at?

Student: ((Refer Slide Time: 20:57))

z equal to?

Student: 1

1. So, it is no surprise that, the numerator of the NTF you get after removing the constant is 1 minus z inverse the whole cube. Is this clear? As a digression, how do you think these transfer function coefficients are calculated? Have you done this before or… Any way it turns out that, there is lot of knowledge about these transfer functions in the s domain in continuous time. I do not know if you people have done impulse invariance and that kind of… or bilinear transforms and that kind of thing in the DSP class. Have you or…

Student: No sir, ((Refer Slide Time: 22:06)) no filters are not covered…

Anyway. So, all these are derived from the so-called analog prototypes. And, analog whatever filters and analog Chebyshev transfer functions have been worked out long time ago; and, the digital filter – the IAR digital filter responses are basically taking the continuous time Butterworth, Chebyshev or elliptic or whatever filters that there are; and, converting them from continuous time into discrete time using what is called the bilinear transformation. And… So, whatever happens at 0 in the high-pass continuous time Butterworth case happens at z equal to 1 in the discrete time case. And, a high-pass continuous time Butterworth filter is of the form s cube by d of s. So, all the zeroes are at s equal to 0. And, when translated into discrete time, the zeroes all land up at z equal to 1. So, 1 minus z inverse the whole cube and you have some d of z inverse; where, clearly the poles of the denominator are not at?

Student: Constant

They are not at z equal to 0. Correct? So, we have indeed managed to move the poles away. And, that is also reflected in the out-of-band gain of the noise transfer function. If the poles were at z equal to 0, you would get 8. And, now, you can see that, the gain has significantly fallen. So, you do have the omega cube dependence at low frequencies within the signal band. But, at omega equal to pi, we see that, the gain of the noise transfer function is only about 1.5 here; thereby significantly reducing the variance of the quantization noise that is present at the input to the quantizer. Is this clear? Great.

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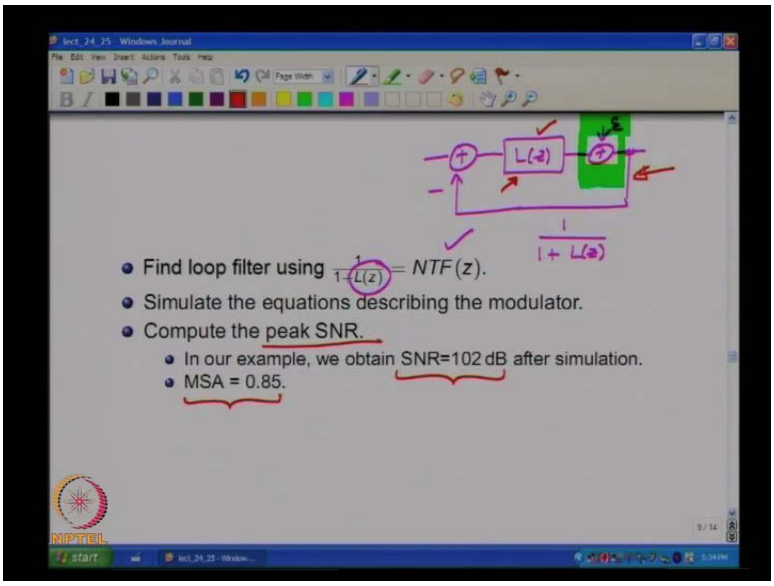

Now, once you know the noise transfer function, you can go and find the loop filter transfer function, which will give this desired noise transfer function. If the modulated topology was assumed to be of this form, then the noise transfer function is simply 1 by 1 plus L of z. All right? So, once L of z is known, all that one needs to do is simulate the system, because this is straightforward; you assume some initial conditions inside the loop filter; then, you have a quantizer here; this is actual quantizer; correct? So, you can write the non-linear difference equations that govern the whole system; the system is non-linear because of the presence of the?

Student: Quantizer.

Quantizer; correct? So, you can write the equations. You can put in a sinusoid at low frequency, where the frequency is within the signal band. You will get an output, which is a quantized version of the input and there will be shaped noise; and, you can compute the in-band signal to quantization noise ratio. As you keep increasing the input amplitude, eventually at some point, the quantizer will saturate and the modulator will become unstable and the signal to noise ratio will fall. So, you can find the peak signal to noise ratio of this modulator, which corresponds to this particular noise transfer function, where the out of band gain is – in this particular case is above 1.5. Is this clear? There are now… So, once you compute the peak SNR… This is mind you done through simulation. There are only 2 possibilities. It either meets your spec or it does not meet your spec. So, it turns out that, in this example, I did run the simulation and obtained an SNR of 102 dB; where, as I said, the OSR is 64. So, I integrate the quantization noise between 0 and pi over 64. And, using the techniques that we described last time, that is, I put in a slower amp and find the point at which the input to the quantizer just blows up; I found that, the maximum stable amplitude is about 85 percent of the range of the?

Student: Quantizer.

Quantizer. So, this is interesting, because if we are done 1 minus z inverse the whole cube without reducing the out of band gain, the maximum stable amplitude probably be about 10 percent of the quantizer range because of the amount of shape noise that is riding over the input. Is this clear? All right. So, our spec was 115 dB. We are at 102 dB. So, obviously, we are not meeting spec in this particular example. So, what do you think I should do next?

Student: ((Refer Slide Time: 28:56))

So, what I should do is this is basically because the gain of the high-pass filter within the signal band is obviously not?

Student: Low enough.

Is not?

Student: Low enough.

Low enough; correct? So, I need to reduce the gain of the high-pass filter within the signal band. And, that can only be done by pushing the corner of the high-pass filter to the left or the right?

Student: Right.

To the right. So, I will choose another 3 dB bandwidth. Last time I chose pi over 5; now, I must choose a frequency higher than pi over 8. All right?

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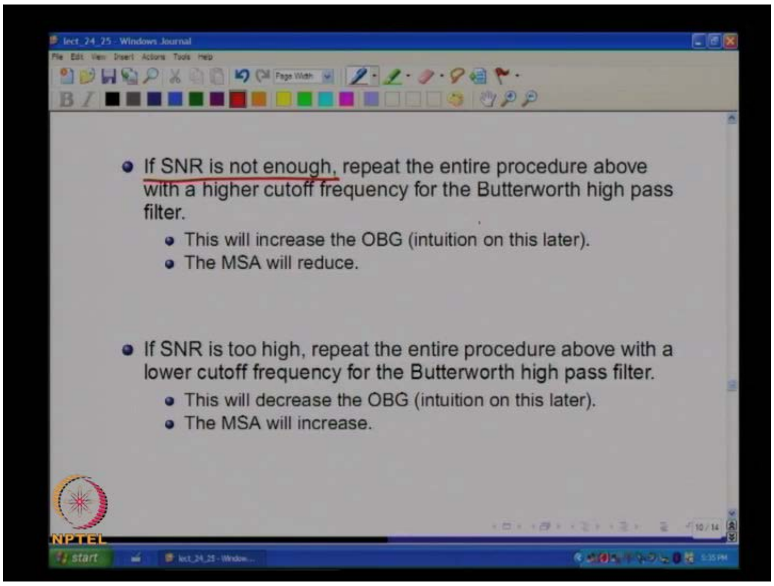

So, if the SNR is not enough, which is the case that we are facing at this point; we repeat the entire procedure with a higher cut-off frequency for the Butterworth filter. This will increase the out-of-band gain. And, why does this make intuitive sense?

Student: ((Refer Slide Time: 30:10)) Because the gain in the in-band ((Refer Slide Time: 30:15)) So, the area below 0 dB line is increased. So, the area above also should increase.

Great.

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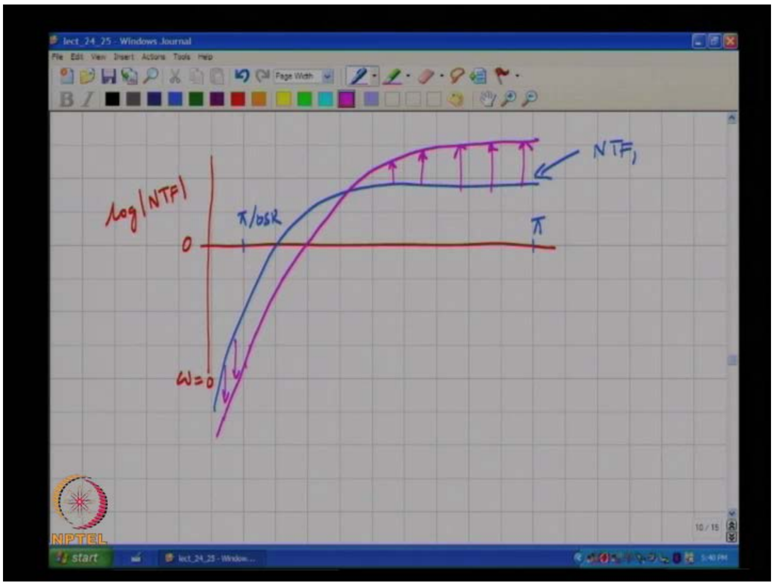

So, the intuition is that, earlier… I will make this as pi and let us say we had Butterworth transfer function like this. Please note for all these Bode’s sensitivity integral type plots, the x-axis is on the linear scale; it is only the y-axis, which is on the log scale. So, this is NTF 1; and, this is say pi by OSR. All right? Or, let me just remove this, so that there is no confusion. All right? So, obviously, the in-band SQNR is not high enough, because the average gain within the signal band is too high. So, what one would do would be to attempt to increase the out-of-band gain or decrease the in-band gain by pushing the cut-off frequency of the Butterworth filter towards the right. And, by the intuition, we gained about the Bode’s sensitivity integral, where the in-band performance and the out-of-band performance are correlated. Pushing the in-band response down will cause the out-of-band response to go up. So, we have pushed this down. And, not surprisingly, this goes up.

Now, if the out-of-band gain goes up, what do you think will happen to the maximum stable amplitude?

Student: It will…

It will?

Student: Reduce.

Reduce; but, hopefully, the reduction of in-band quantization noise will be?

Student: Much larger…

Much larger than the factor by which the MSA has fallen, so that you get a net improvement in the peak SQNR. Does it make sense? The peak signal to the quantization noise ratio is ratio of the signal power to quantization noise power. If the signal power is made too high, the modulator will become unstable and the quantization noise in-band will go up. On the other hand, if the signal power is too small, then the signal to noise ratio will be small because the quantization noise remains constant and the signal power is very small. As you go on increasing the signal power, the SQNR will go on increasing at first, because the numerator goes on increasing while the denominator remains constant. The moment you start to saturate the quantizer, the numerator increases a bit slowly, but the denominator just increases dramatically, because the modulator has gone unstable. So, the peak signal to quantization noise ratio is achieved when the input amplitude is equal to the maximum stable amplitude of the modulator loop. All right? And therefore, if you make the noise transfer function more aggressive… And, when I say aggressive, it means that, the in-band gain is small or reduced; automatically, the out-of-band gain becomes large; thereby causing a reduction in the maximum stable amplitude. At this point, we only know that, the MSA should be expected to reduce. It is not immediately obvious by how much it will reduce. And, for this, we just resort to simulation; it is quite straightforward to figure that out. Is this clear?

So, in this particular example, the signal to noise ratio is not enough. So, we need to make the noise transfer function more aggressive. And, as we just discussed, this will increase the out-of-band gain and will also thereby cause the maximum stable amplitude to reduce. If the opposite was true… In other words, if our desired SNR was not as high… Let us say our desired SNR is only 90 dB and then we chose some cut-off frequency for the Butterworth and found that the SNR was 110 or 115 dB; clearly, we are doing… We are overdoing it. So, what one can do is…

Student: ((Refer Slide Time: 36:25)).

Say we do not need the SNR to be so much. So, you can?

Student: Reduce the ((Refer Slide Time: 36:32))

Reduce the cut off frequency; thereby increasing the in-band gain of the noise transfer function, which will automatically cause the out-of-band gain to decrease; thereby mildly pushing up the?

Student: OSR.

OSR; you understand? So, whether we end up with this scenario or that scenario, depends on the spec. And, we repeat this process until the in-band SQNR is equal to the spec that we are shooting for; and, that is when we are done. This is an iterative process.

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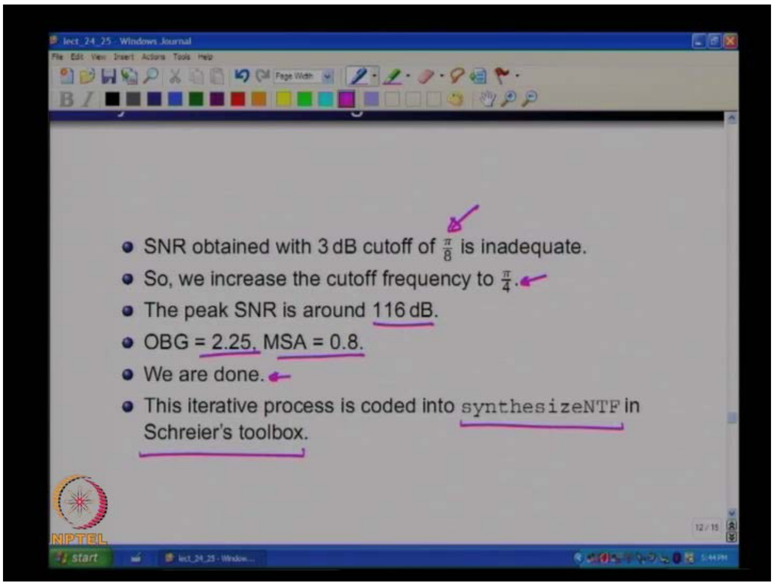

So, in our example, choosing omega 3 dB is pi by 8 is inadequate. So, the next thing to do is to increase the cut-off frequency. Quite arbitrarily I chose pi by 4. All right? And then, recomputed the high-pass filter transfer function and normalized it, so that it is evaluated at z equal to infinity; it reduces to 1. And, when I did that, it turned out that, the out of band gain has gone up now to.

Student: 2.

2.?

Student: 25

25; it should be larger than 1 and a half. Correct? And, the maximum stable amplitude when ran simulation, has fallen down from 0.85 or 85 percent of the quantizer range to 80 percent of the quantizer range. All right? This is also in the right direction, it makes intuitive sense. And, the peak SNR is now upon simulation turns out to be 116 dB. It is close enough to 115 dB and I decide that, I am done and I am not interested in iterating to get exactly 115 dB; and this is ok. All right? So, as you can see, systematic design of the noise transfer function to achieve a given in-band signal to noise ratio is an iterative process. All right? And, such things are easily done in software. So, in this toolbox, which I will talk about later, there is a routine, which does this automatically. While we have a tool to do it, it always is better if you know what goes beyond the tool and what the intuition behind the process is.

Now, let us say for argument sake that, the spec is so tight that I go on increasing the cut-off frequency of the Butterworth filter and I still do not seem to be meeting my spec. What do you suggest?

Student: Increase the outer order…

So, one thing you can do is increase the order. What do you think will happen if I increase the number of levels, where factor of 2 – same order; but, number of levels have gone up by a factor of 2. What do you think will happen to the peak in-band SQNR.

Student: ((Refer Slide Time: 40:19))

Increase the number of levels from say 16 to 32.

Student: Increase the ((Refer Slide Time: 40:26))

It will?

Student: Increase

Increase by how much?

Student: 6 dB

By 6 dB, because delta has gone down by a factor of 2; which means that, increasing the number of levels to from 16 to 32, is only going to increase the SNR by 6 dB. All right? And, what do you think will happen to the maximum stable amplitude? Do you think it will get higher or it will get lower?

Student: Higher; it will get higher.

It will get?

Student: Higher

Higher; and, why does that make sense?

Student: Variance of noise…

Very good. So, variance of noise riding over the input is delta square by 12 integral 0 to pi NTF e to the j omega minus 1 [FT] the whole square. And, clearly, if the quantization noise variance goes down, the variance of the shape quantization noise also goes down; thereby causing the MSA to increase. Though when you go from 16 to 32, the increase will be very small. The intuition for this is that… Let us say for 16 in this particular example, the maximum stable amplitude is 80 percent of the quantizer range. So, the 20 percent of the quantizer range is getting eaten up by?

Student: Shape noise

By the shape noise. Now, if the shape noise becomes one-half its original value, same noise transfer function just increasing the number of levels will cause the shape noise variance to simply…

Student: One-fourth.

Becomes one-fourth of the standard deviation of the variance to go down by a factor of 2. So, if this is occupying 20 percent before, it will now occupy 10 percent. So, the MSA will go from? From 0.8, you can expect it to go to?

Student: 0.9

0.9. So, if you are off from the spec by a large amount, clearly increasing the number of levels in the quantizer, is only going to give you?

Student: 6 dB

Approximately about 6 dB for every doubling of the number of levels in the quantizer; in which case, you conclude that, this is not good enough and you shoot for… You switch order, where you get an additional omega term in the noise transfer function; and, that pushes the gain of the noise transfer function down. Does it make sense? All right?

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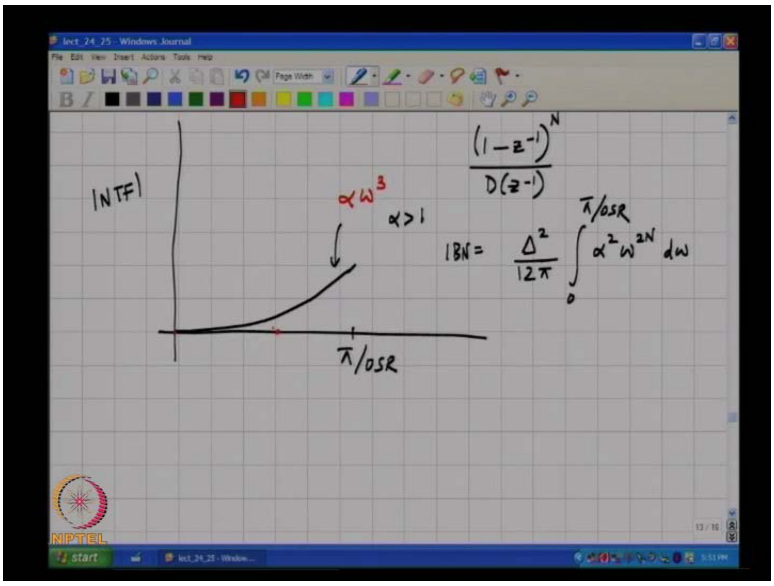

Now, one thing that I have not discussed so far is the fact that, all the NTFs so far, have had zeroes at the origin. In other words, the noise transfer functions have been of the form 1 minus z inverse whole to the N divided D of z inverse. So, at omega equal to 0… So, let me blow up the region between 0 to pi by OSR. We find that, this will simply go as… So, if this is mod NTF, this will go as?

Student: Omega ((Refer Slide Time: 44:07))

Omega to the N. And in fact, it will be of the form some alpha into omega to the N; where, alpha is greater than 1 or less than 1?

Student: It should greater sir.

It should be?

Student: Greater than 1.

Greater than 1. All right? So, the in-band SNR will be of the form – integral is delta square by 12. So, the in-band noise is delta square by 12 pi integral 0 to pi by OSR times alpha square omega to the power 2 N d omega. Now, it is not really necessary to have all the zeroes at DC; it turns out that, you can spread the zeroes over the pass band of the modulator. In other words, the stop band of the noise transfer function. And, get a lower in-band quantization noise. In other words, what I am saying is that, instead of having this; let us for argument sake, say this is alpha omega cube. All right? You can in fact, choose one zero at DC and move two zeroes to some location inside the pass band.

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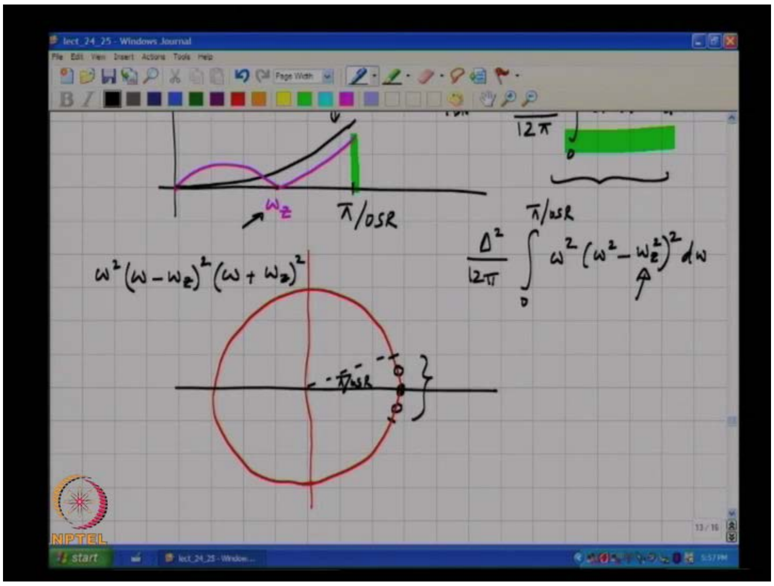

In other words, in the pole-zero diagram, all the zeroes being at DC, is equivalent to having all the zeroes there. Pi by OSR in a grossly exaggerated sense is this. This is pi by OSR. All right? Now, all that I am saying is that, one could move the zeroes from DC and spread them around in the pass band so as to minimise the?

Student: In-band noise

In-band noise. So, the intuition is that, it is possible stands from the fact that, in this integral, where do you think I am picking up most of my noise? Where is most of the contribution to this integral coming from in the region 0 to pi by OSR?

Student: ((Refer Slide Time: 47:18))

Fine most of the contribution to this integral is coming from this region. So, then I will say what is the point in trying to put all the zeroes at DC, where a zero is basically attempting to make the value at and around that point equal to 0. Correct? So, you might say, if I chose 0 smartly… In other words, I moved the zeroes away from z equal to 1 to say somewhere here. For example, please recall that, the zeroes must be complex conjugate. So, if you had a third order noise transfer function; if you move one zero away, automatically, another zero also must go away. So, you can have a pair of complex conjugate zeroes, which are on the unit circle; and, one continues to be at?

Student: z equal to 1.

z equal to?

Student: 1

1 or omega equal to 0. In the frequency response, the intuition being that, you will get… If you have one zero at omega equal to 0, clearly, the NTF will be 0 here. If you have a pair of zeroes at some other frequency omega z, what will happen to the NTF?

Student: It is 0 ((Refer Slide Time: 48:56))

It is 0 at origin and it will do this. And, around omega z, how will the noise transfer function look?

Student: It will again ((Refer Slide Time: 49:11))

It will again pick up and then do this. All right? You understand? So, now, what do you see? You see that, since most of the contribution to this term was coming from frequencies in the neighbourhood of pi by OSR; it makes sense that, if you move the zeroes around in the pass band and you can reduce the?

Student: In-band noise

In-band noise. So, in the third order case, you can actually go and compute the location of omega z, which will minimize the in-band quantization noise. And, that by minimizing, it turns out that, you will be able to get 8 dB more in the third order example. I will give this as an exercise. You can do this; work out the math. So, in other words, choose the location of omega z that will minimize the quantization noise, which is now of the form delta square by 12 pi integral 0 to pi by OSR. The 2 NTF square in the third order example will be of the form omega square only, because there is one zero at the origin. Correct? NTF must be of the form?

Student: ((Refer Slide time: 50:48))

NTF square must be of the form omega square times omega square minus omega z square the whole square d omega. If omega z is 0, you must get omega to the 6. All right? The intuition is that, around this point, this must behave like a parabola. Correct? So, the NTF square around omega z must be of the form omega minus omega z the whole square. There must be a similar kind of expression because of the conjugate or the negative frequency. So, that must be omega plus omega z the whole square. And, around the origin, mod square of the NTF must go as omega square. So, you combine all these together and you get omega square times omega square minus omega z square the whole square; integrate from 0 to pi by OSR will give you the noise. And, you can go and differentiate the value of this integral with respect to omega z; and, you will find the location of omega z in… And, that will obviously be in relation to the band edge. So, it will be a fraction of pi by OSR. And, in the third order example, it turns out that, the SNR you evaluate for the optimal location of omega z, is about 8 dB lower than what you get with all zeroes at the origin. Is this clear? You must be doing better is that, intuitively at least clear. All right.

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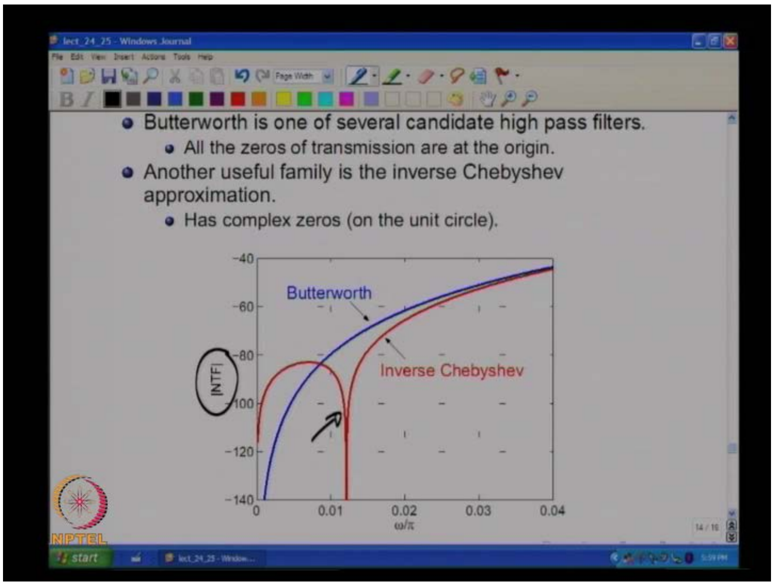

And the NTF will look… For example, if I plot the magnitude of the NTF, for the Butterworth, it will go to minus infinity dB only at 0. The inverse Chebyshev approximation is one approximation, where the zeroes are complex. And, there you will have notches in the stop band and you can also optimize the zeroes according to the approach I just mentioned. And, you will find that, the NTF now does something like this. So, complex zeroes will result in a lower in-band quantization noise for the same out-of-band gain.

Student: It should be we slightly lesser than 1 inverse because the…

Because the zeroes have slightly moved. That will be so small that, you would not notice. All right?

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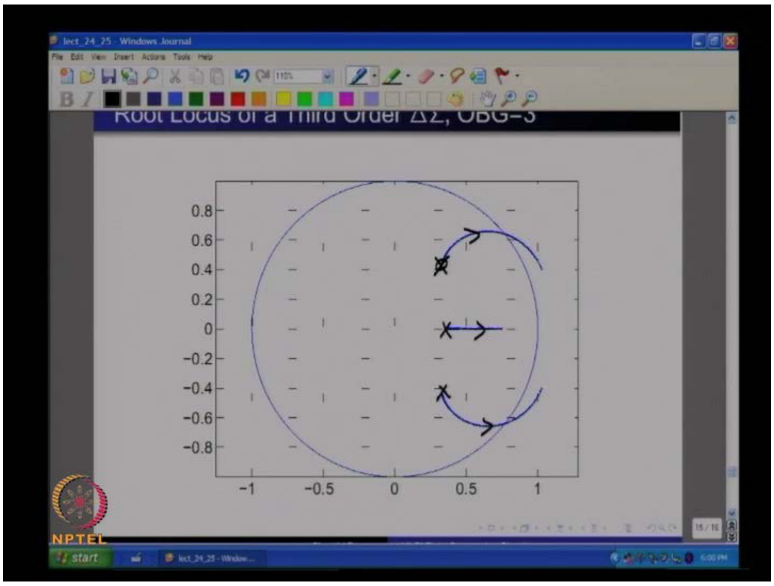

Now, once you compute the out-of-band gain, which will meet your desired SNR spec; you can go and plot the root locus of the modulator again as the k goes from 1; which is what happens when you are not overloaded to lower values; which is what will happen when the quantization gets overloaded. And, at k equal to 1, the poles will start off here. And, just like in the case, where all the poles were at z equal to 0, reducing the gain will cause the poles to move like this. And eventually, the modulator will also become… This modulator will also become unstable. What I want to point out is that, in either case, if we have a high-order loop filter; whether you choose a high out-of-band gain or a low out-of-band gain, modulator will become unstable. The only thing is how much I may say you get. All right? So, we continue in the next class.