VLSI Data Conversion Circuits
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Lecture - 23
High Order DSMs

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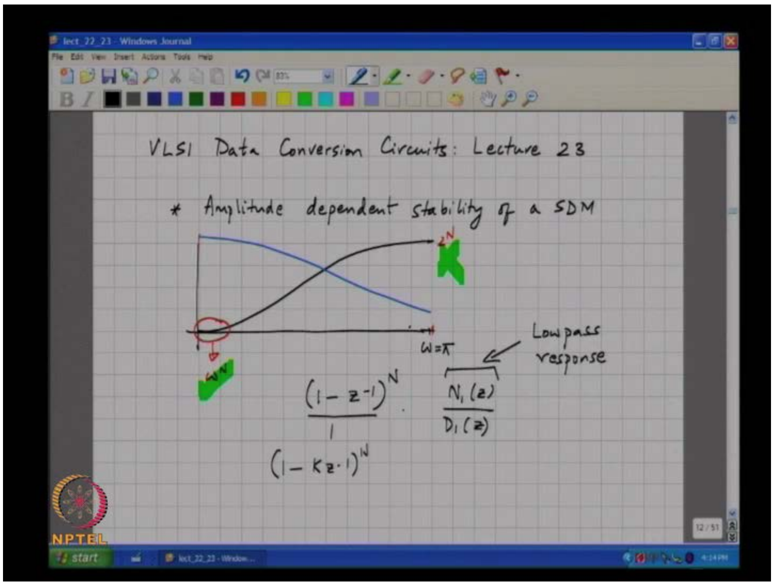

This is VLSI data conversion circuits lecture 23; in the last class we saw the intuition behind the amplitude dependent stability of a sigma delta modulator, right. And, the key point was that the input to the quantizer consists of the input to the modulator plus quantization noise which is being shaped by a filter; whose transfer function is NTF of z minus 1, right. And, if the gain of the noise transfer function at high frequencies that is at frequencies around omega equal to pi becomes very large. Then, the variance of the noise riding over the input which is what is present at the input to the quantizer becomes very large causing the quantizer to saturate; if the quantizer saturates the gain for the quantization noise falls down. If you have a high order loop filter which is what you will do or you will tend to do in order to push the in band quantization noise low; the closed loop system will become unstable. So, in other words the noise transfer function is doing something like this; this is problematic it is too high, correct. But this goes as omega to the power N and this will be 2 to the power N. So, we like this but we do not like this part. So, the question is what do we do or can we do anything at all to fix the problem?

So, any suggestions? I mean you want something you do not want something; so what will you do? So, the basic idea is to say if I pass this instead of having if my NTF was not looking like this. But if I multiplied my NTF with a function of the form which emphasized lower frequencies more than higher frequencies; then it seems that I will you know it is like having my cake and eating it too. So, I will be able to have the nice omega to the power N dependence of the in band quantization noise; while at the same time attenuating some of the high frequency components of the noise does it make sense. So, instead of having a noise transfer function of the form 1 minus z inverse to the power N right; I could for example, multiply this by some N 1 of Z divided by D1 of Z; where this is the a low pass response. I mean we do not know much about the low pass response other than the fact that we want it to be high at low frequency; and kind of taper off at high frequencies right. So, clearly doing this seems to you know address the problem. However, you must understand that cascading 1 minus z inverse to the N with another low pass filter will basically increase the order of the loop filter. So, the question is can we do anything, can we achieve the same effect without increasing the order of the filter you understand? So, any suggestions?

Student: ((Refer Time: 04:55)).

Pardon.

Student: ((Refer Time: 04:59)).

Where?

Student: In that loop function.

Oh; well one suggestion is why do not you make this 1 minus K z inverse; but the problem with this is that if K is not 1; what happens to the the responses of NTF at D C? What do you want for the NTF to do at D C 0?

Student: ((Refer Time: 05:24)).

It must be 0. So, if K is not 1 then the noise shaping is lost correct. So, that is not a workable idea all right.

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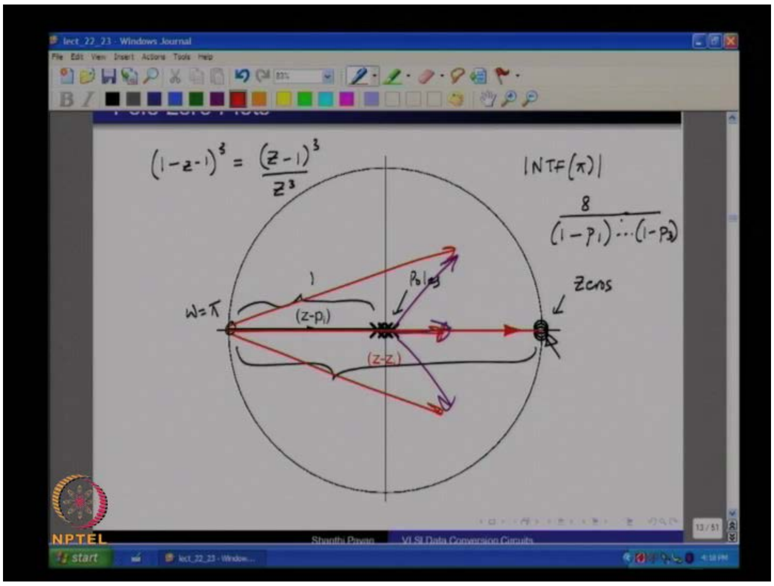

Let me again get back to this pole 0 plot; the gain of the noise transfer function at omega equal to pi is at this point you draw vectors from omega equal to pi; which is what I have shown here to the 3 poles. And, where are the 3 poles in this examples I have chosen 1 minus z inverse the whole cube which is nothing but z minus 1 the whole cube by z cube. So, the 3 poles are at the origin and the 3 zeros are at Z equal to 1; does it make sense? The question now is we want to reduce the gain of the noise transfer function at omega equal to pi right; what is it right now?

Student: It seem to be 8.

It is 8 because it is the this part is 2 correct; that cube is 8, this happens to be always equal to 1. So, the gain at omega equal to pi is 8; we want to be able to not increase the order of the loop filter. So, we still want 3 poles and 3 zeros and we want to reduce the gain at.

Student: Pi.

At pi right; there is an additional constraint that you cannot move the zeros. Because if you move the zeros away from Z equal to 1 you will not have.

Student: Noise shaping.

Noise shaping correct. So, the only I mean this is this is a good thing because then you only have to worry about where you are going to place the.

Student:Poles.

Poles, does it make sense? So, if you want the gain of the noise tranfer function to decrease at omega equal to pi what will you do with the poles?

Student:((Refer time:08:03))

We know that the gain at pi is basically the location it is 8 divided by.

Student: ((Refer Time: 08:19)).

Pi minus whatever.

Student: P 1.

P 1 into blah blah blah pi minus P 3 correct. So, if you want to reduce the gain at omega equal to pi it is sorry it is 1 minus p 1, 1 minus p 3; if you want to reduce the gain at omega equal to pi what should you do?

Student: Try to push.

I mean you cannot change 1 So, you must change.

Student: P 1, P 2, P1.

The locations of P 1, P 2 and P 3. So, where will you put them?

Student: Focus to ((Refer Time: 09:26)).

So, if you push these poles in this general direction right; the low frequency part of the NTF will still be proportional to z minus 1 the whole cube. But the gain at omega equal to pi will be.

Student: ((Refer Time: 09:29)).

Now,

Student: Less than.

Dependent on the lengths of these vectors which is obviously larger than.

Student: 1.

1. So, they will reduce the gain at high frequencies, does it make sense?

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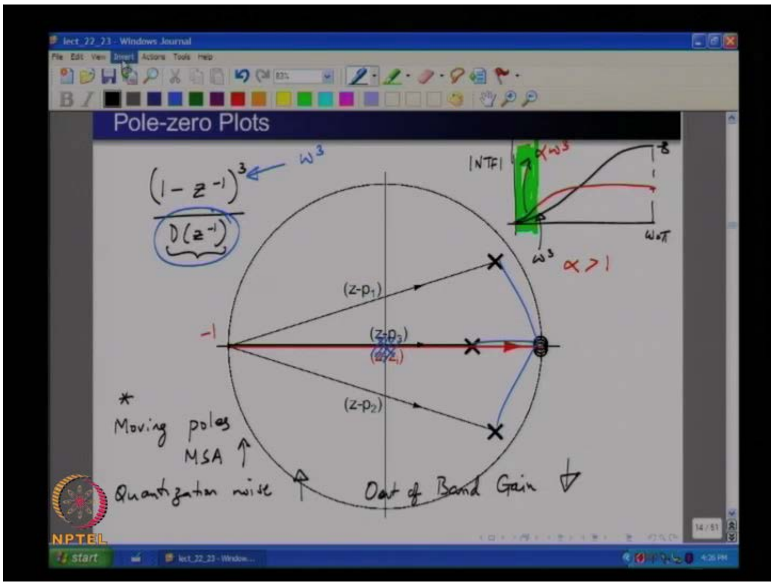

So, for example if I moved the poles from where they were originally that is at the origin towards the right. Then, the magnitude of the vectors joining omega equal to pi or z equal to minus 1 to these poles has increased significantly; what was originally one seems to have gone to a number closer to close to 2 now; obviously it is smaller than 2. But it is significantly larger than 1. So, in other words the noise transfer function is now of the form 1 minus z inverse the whole cube divided by D of z inverse earlier D of z inverse was simply 1 right. Now, D of z inverse is some polynomial in z inverse of order.

Student: 3.

3 right. And, if you choose the poles of the NTF; in other words the roots of D of z inverse properly by I mean when I say properly I am it means that in the generally in the.

Student: ((Refer Time: 11:25)).

You know towards the right then the magnitude of the denominator at omega equal to pi is larger than 1. Therefore, reducing the gain of the noise transfer function at high frequencies. And, this is possible without increasing the overall order of the noise transfer function; does it make sense? So, what was originally something like this; this is mod NTF, this is omega equal to pi, this is 8 all right. Now, for this pole constellation what do you think the gain will be at omega equal to pi is smaller than.

Student: 8, 8.

Smaller than 8. So, let us say it is something like this; what can you say about the gain at the origin at omega equal to 0 or z equal to 1.

Student: 0.

Of course at z equal to 1 it is 0 right; for small omega how does this go the original NTF 1 minus z inverse whole to the 3.

Student: Omega power omega power 6.

It mod NTF.

Student: ((Refer Time: 12:56)); omega power cube.

Omega cube; now how does it go?

Student: Positive omega constant.

It will be it will go as constant times.

Student: Omega.

So, at the origin this guy will have gone as omega cube. Now, because of the poles being pushed close to the 0; what do you think will happen to the gain at low frequencies? It will still go as omega cube because you have 3 zeros at.

Student: ((Refer Time: 13:45)).

At z equal to 1. But will it go faster I mean will it go as 1 times omega cube or will it go as 5 times omega cube or will it go as 0.1 times omega cube? Look at the picture that is the key to ok.

Student: Smallest smaller than 1, smaller than 1.

No.

Student: Larger.

No, if it is not smaller it got to be larger right.

Student: Larger means the whole now it has come inside for distance is less than.

Correct so.

Student: Denominator is less than 1 means it will be larger larger than one.

Great that is absolutely right. So, earlier the poles were sitting here at the origin right and at z equal to 1 correct; this in the neighborhood of z equal to 1 always evaluates to omega cube right. Now, the only point of contention is what does this do at small frequencies correct? And, this is clearly earlier what was one these vectors are now reduced in magnitude which means that.

Student: Denominator is less than 1.

The denominator is less than 1. So, this must go as.

Student: Less.

The red one must go as K times omega cube; where K or alpha times omega cube since we have been using K all over the place alpha is definitely greater than 1 right; whereas the original NTF would have gone as simply omega cube; does it make sense all right? So, in English what does this mean, what has happened now by moving the poles in this fashion, what has happened or what do you hope has happened to the maximum stable amplitude?

Student: MSA ((Refer Time: 16:16)).

MSA has gone hopefully.

Student: Up.

Up right; what do you what can you say about the in-band quantization noise?

Student: ((Refer Time: 16:41)).

Has gone up or gone down.

Student: Gone down in band or.

In band, in band quantization noise.

Student: Up sir.

The in band quantization noise is the this integral of NTF square in some band here.

Student: Gone up.

It has gone up. So, quantization noise is gone up what is happened to the out of band gain?

Student: ((Refer Time: 17: 11)).

Has gone.

Student: Down, down.

Down. So, what I did not like to bring to your attention is the following; the in band quantization noise has gone up. And, the out of band quantization has gone I am sorry the out of band gain has gone.

Student: Down.

Low or gone down right. And, this is not just a coincidence.

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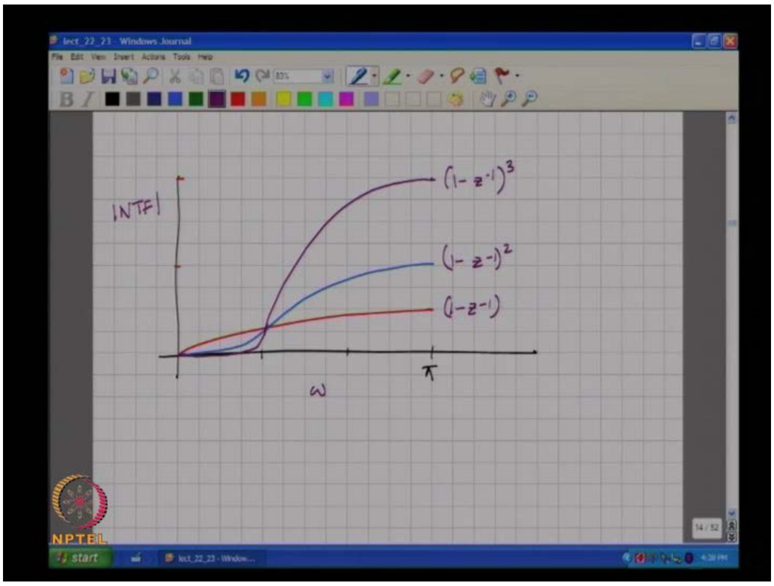

Let us look at the NTS that we have seen at pi all right. So, for the first order NTF how does it look, how does the NTF look at the origin?

Student: Suppose it goes.

It goes as omega right and what is it at out of band?

Student: 2, 2.

It is 2. So, it looks like this; for the second order how does it look?

Student: Near the origin omega square.

Near the origin it is omega square and at pi.

Student: 4. it is 4.

It is 4 all right. Now, what happens for the third order modulator?

Student: Omega cube, omega cube.

Omega cube and then this goes as goes to 8 all right. So, this is mod NTF and this is omega and this is 1 minus z inverse, this is 1 minus z inverse the whole square, this is 1 minus g inverse the whole cube and what trend do you see?

Student: Band in band width is.

So, whenever we are doing better in band we seem to be doing worse.

Student: Out of band.

Out of band right. And, not only when we change the order but also when for the same order we move the poles in such a way as to reduce the out of band gain; which seem to be doing.

Student: In band noise the in band noise is increasing.

The in band noise is increasing you understand; it turns out that this is a fundamental property. And, we will come to this in a couple of minutes.

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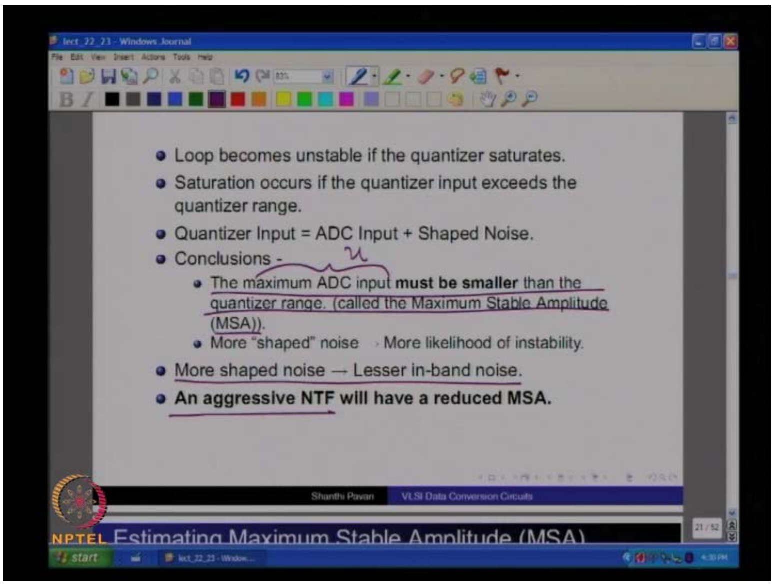

To retreat the maximum A D C input must be smaller than the quantizer range right; that this is the so called u that we have been talking about. And, it has more shaped noise then it means that there is more likelihood of instability. And, as you have just seen now we have made a passing observation that the more the shaped noise the smaller seems to be the.

Student: In band.

In band noise right now the evidence is simply circumstantial right we have seen 1 minus z inverse, 1 minus z inverse the whole square, 1 minus z inverse the whole cube as well as 1 minus z inverse the whole cube by D of z inverse. And, in all these cases it seems like if the out of band gain goes up the in band gain goes down all right; that does not mean that it is true for everything. But it turns out that it is true for all noise transfer function that we encounter in practice; I will derive that a little down the line. An aggressive noise transfer function loosely speaking is one which has much smaller in band quantization noise than a non aggressive one right; an aggressive one if you have an aggressive noise transfer function in an attempt to push the in band quantization noise lower it must follow that the out of band gain must be.

Student: Higher, higher, higher.

Higher; if you have a higher out of band gain what can you say about the maximum stable amplitude?

Student: Less, less.

It must fall. Because for the same quantizer input range if you have a higher out of band gain it means that the variance of the noise riding above the input is higher. So, in order to keep the quantizer happy you need to reduce the input amplitude thereby reducing the maximum stable amplitude of the modulator loop right all right.

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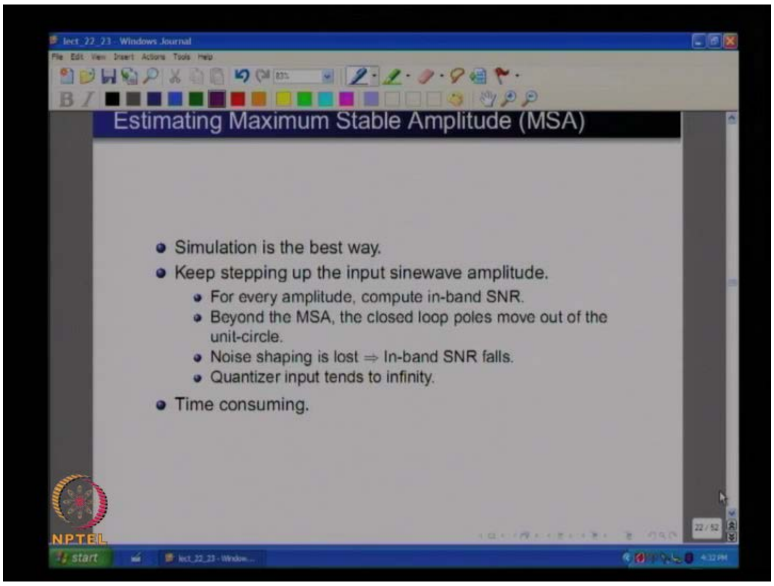

Before I go further I just like to mention you know how one might estimate the maximum stable amplitude of a modulator in practice; this is obviously something which is very relevant right. So, simulation is the most reliable way of doing it; where in the given that doing this analytically it can be a very messy affair.

So, one thing you could do is put a sinusoid with some amplitude; look at the state variables inside if none of them blow up you are fine the; this amplitude is within the stable range. You go on stepping up the amplitude and right you can do this unfortunately this takes a lot of time.

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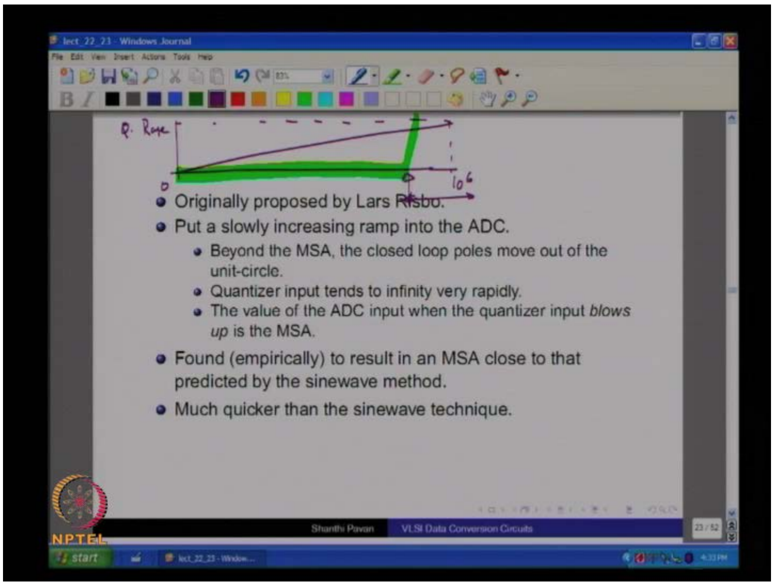

So, another technique is to take a very slowly varying ramp; by slowly varying I mean you know this varies from 0 to full scale of the range of the quantizer right over a say a million steps, correct. So, these are very very slow ramp and monitor the input to the quantizer; we know that when the modulators become unstable what happens to the input of the quantizer?

Student: It blows up, blows up.

It blows up right. So, rather than plot the input to the quantizer you can plot the log of the input to the quantizer; in which case what will happen is that it look nice and clean up to somewhere here right. And, then once you start exceeding the maximum stable amplitude; the magnitude will blow up which means that the log will also blow up. And, you will see something like this. And, then the ratio of this to 10 power 6; how long you have gone before the input to the quantizer blows up is an estimate of the maximum stable amplitude. And, people have found that this is a good way of doing things.

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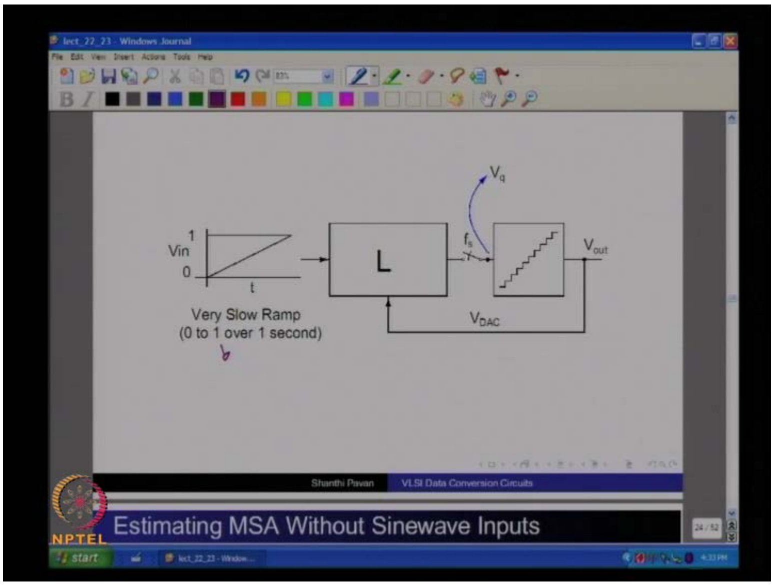

So, this is what I was talking about it is a very slow ramp; where one stands for the quantizer range right, this is a loop filter. And, you keep monitoring the input to the quantizer.

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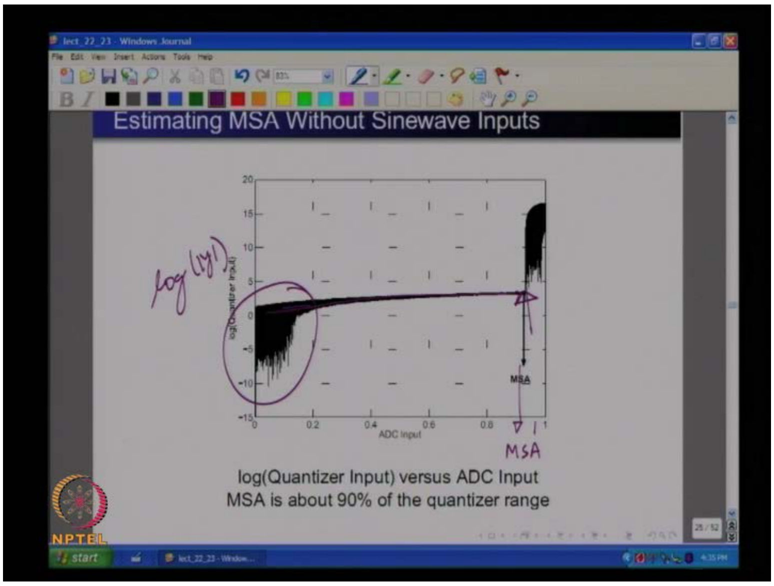

And, you will get a plot like this. So, please note that this is log of mod y; when you start at z I mean the initial part is because the input is close to 0 right. So, the y will be hovering you know up above and below 0 right. So, if you since it is 0 if you take it is log it will be negative correct. And, then as you keep increasing the input amplitude you can see that in general the magnitude of y increases. And, at a certain point in time which can also be mapped to a certain input; you can see that the log of the y just blows up which means that y is simply gone to infinity; which means that this is the maximum stable amplitude. In this particular example it is about 90 percent of the quantizer range; is this clear all right? I will come back to this a little later.

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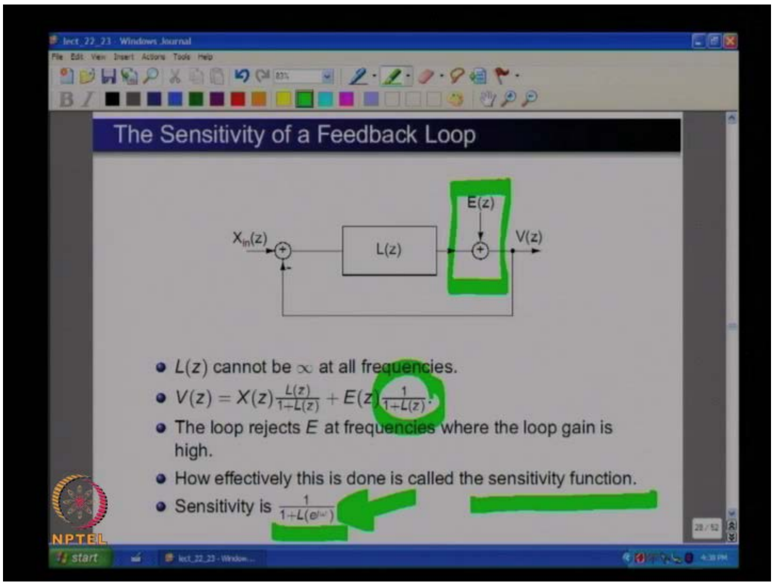

So, now let us digress a little bit. And, see if we can get any intuition about or rather we can get through more light on the fact that the in band performance and the out of band performance of a noise transfer function seem to be related; which seem to be not be able to do better in band without worsening the performance out of band. So, to understand this one needs to kind of rewind a little bit and refresh or familiarize oneself with some terminology. And, some results this is an example of a negative feedback loop; this has got nothing to do with delta sigma modulation right; this is a standard negative feedback loop. And, if the loop gain is very high correct then what is V? At all frequencies where L tends to infinity or L is very large we have find that V must be the same as.

Student: Input.

The input correct? And, that make sense because V can be written as x times L by 1 plus L plus E times 1 by 1 plus L; correct this is all well known stuff. So, in other words at frequencies where the loop gain is very large; the output is completely devoid of E. In other words the loop has completely rejected.

Student: Error.

This error injected at the output of the loop filter or the loop is extremely insensitive to E not to L to E; at those frequencies where the magnitude of the loop gain is infinite. Now, the ratio 1 by 1 plus L (Z) evaluated on the unit circle is a measure of how effectively the feedback system rejects disturbances or noise injected at its output? And, classical control this is being called what is called the sensitivity function of the loop ok. Please note that this is a function of the loop gain all right. Now, in a delta sigma modulator assuming the quantization noise is additive the sensitivity function is the same as the.

Student: NTF.

Noise transfer function correct; I mean now if somebody told you that this is a block diagram you would immediately say that this represents the additive noise of the quantizer. And, therefore you can identify the sensitivity function of a negative feedback loop in the delta sigma context by the noise transfer function of the loop, correct.

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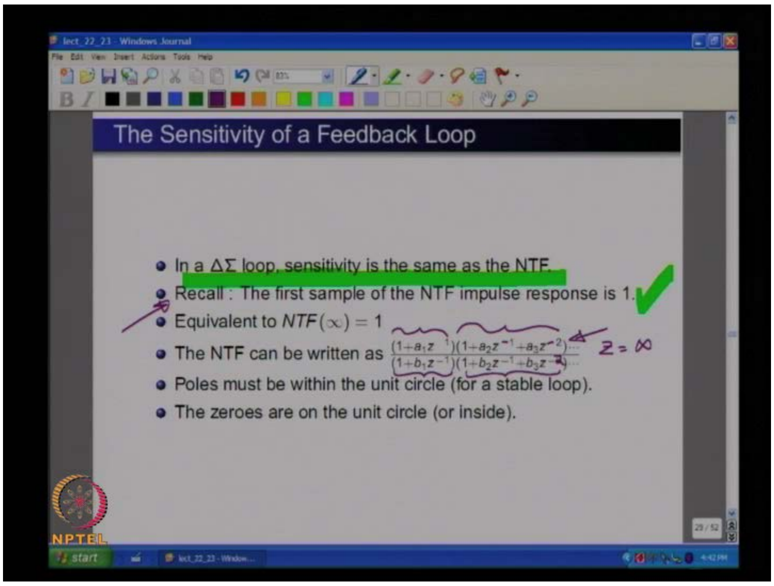

And, it turns out that since in a sigma delta loop the sensitivity is the same as the noise transfer function; some very interesting results from control can be applied. And, to get there let us recall the first thing which is the first sample of the impulse response of a noise transfer function and must be 1. And, why is this happening?

Student: Delay, delay free.

Because you cannot have a delay free loop all right; in the frequency domain this means that the noise transfer function evaluated at z equal to infinity must be.

Student: 1, 1, 1.

Must be 1 correct? Now, if the noise transfer function has poles and zeros on slash within the unit circle; of course the poles you would expect to be well within the unit circle; the zeros are either on the unit circle or sometimes it so happens that the zeros move a little bit inside. For example, when the integrators have finite gain you can show that the zeros of the loop right; instead of being z minus 1 or 1 minus z inverse will be 1 minus alpha z inverse; where alpha is slightly smaller than 1. So, you can always factor the noise transfer function into something of this form. So, 1 plus a z inverse is the first order factor; if you have complex conjugate poles you can factor them into 1 plus a 2 z inverse plus z 2 I mean a 3 z to the minus 2 and so on. And, the denominator is 1 plus b z inverse b 2 z inverse plus b 3 z to the minus 2 and so on. And, the roots of all these factors that is the roots of this factor that is factor, this factor, this and this will all be for the poles they will be.

Student: Definitely ((Refer Time: 31:03)).

Definitely within the unit circle if you want to have a working modulator right the zeros at the most they can be.

Student: On the unit cell.

On the unit cell all right. And, clearly this form of writing the NTF satisfies this fundamental condition; which is at the first sample of the NTF impulse response must be equal to 1 why? Because if I take this and evaluate it at z equal to infinity what do I get?

Student: 1.

I get 1 you understand; in other words any noise transfer function can be written in this form is this clear?

Student: Sir, the zeros can be inside these.

Yeah the 0 can be inside the unit circle.

Student: Not the outside.

I mean most of the in most practical cases you will find it they are either on the unit circle or.

Student: Inside.

Inside. So, you design them to be on the unit circle; however did you finite gain effects and so on they actually might move in other is this clear all right.

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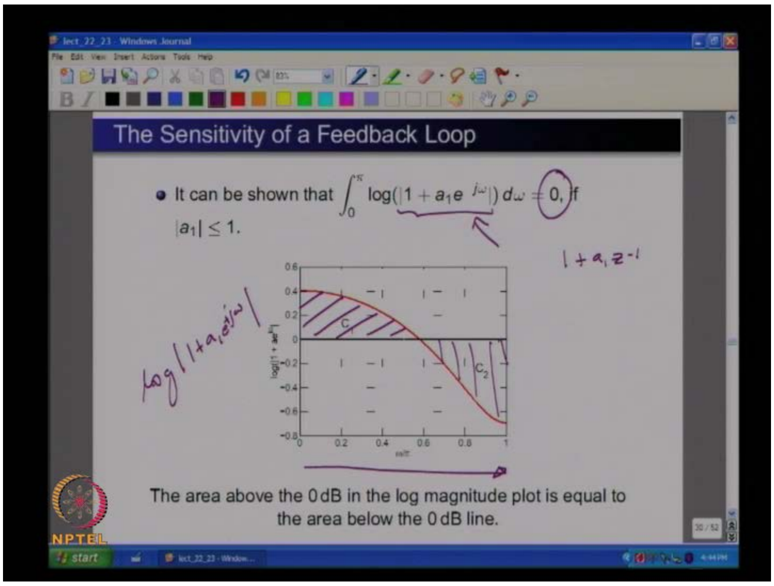

Now, it turns out that it can be shown that if I evaluate the integral from 0 to pi of the log of the magnitude of a term 1 plus a 1 e to the j omega from 0 to pi. Then, this turns out to be equal to 0; you understand? ;et me repeat this again I have a term 1 plus a 1 e to the j omega; in other words this is nothing but 1 plus a 1 z inverse evaluated on the unit circle right; I find it is magnitude only take the logarithm and integrate it from.

Student: 0 to pi.

0 to pi all right. And, it turns out that this is 0 it is not too difficult to show. Now, since a picture is worth 1000 words it is nice here to draw a picture. So, if I draw on the x-axis omega or omega by pi and on the y axis I draw log of 1 plus a 1 e to the minus j omega or a to the j omega. Then, the net area is 0; in other words the area of this log magnitude curve above 0 must be the same as.

Student: ((Refer Time: 35:16)).

The area below 0; is this clear? I mean if we have multiply this whole thing by 20 nothing changes correct and then you can express everything in d B. So, if you plot the log magnitude of 1 plus a z a 1 z inverse; where a 1 has a magnitude smaller than 1. Then, the area above the 0 d B line will be exactly equal to the area below the 0 d B line so.

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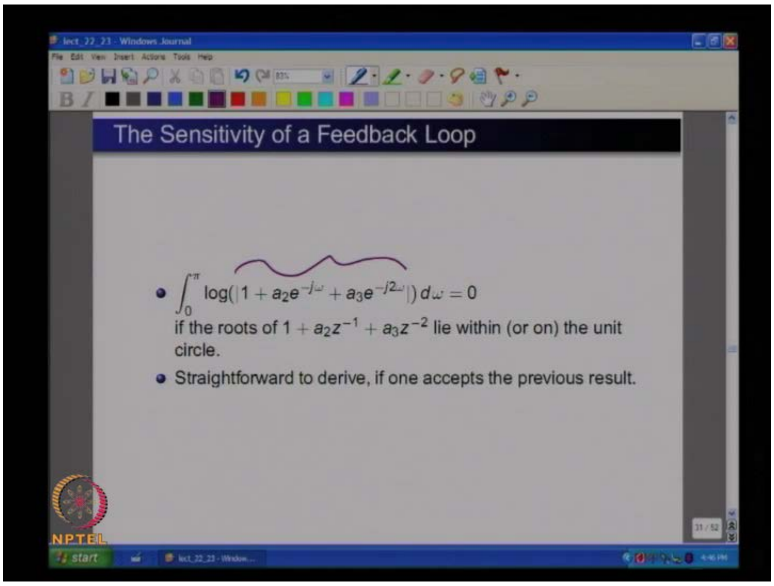

Now, consider the second order factors; if the poles lie within the unit circle you can always break this up into a product of.

Student: 2 first order.

2 first order terms where the coefficients are now complex conjugate right; if the roots are complex otherwise they are 2 real poles. And, if the if we accept the previous result; then you can show that if you plot the log magnitude of this transfer function. Then,

again the area above 0 must be equal to the area below 0 or in d B the area above the 0 d B line must be the same as the area.

Student: Below.

Below the 0 d B line, correct.

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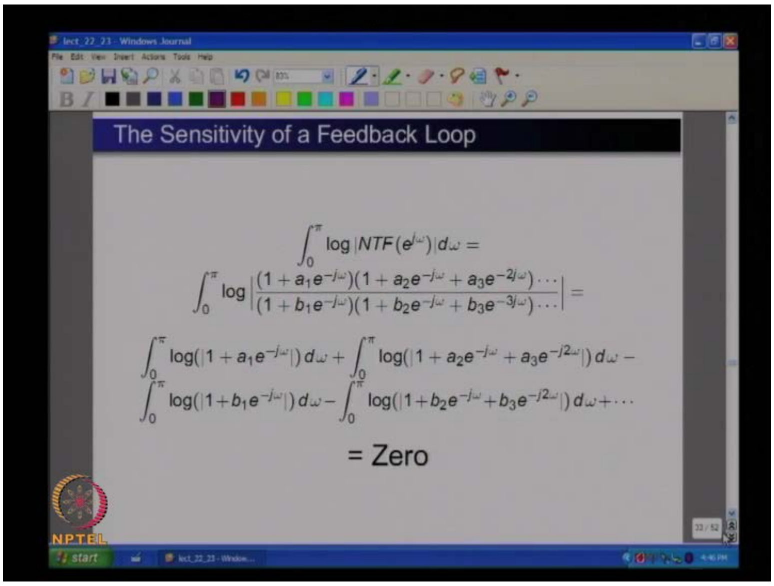

Now, that we have accepted this; our NTF is simply log of.

Student: ((Refer Time: 37:13)).

I mean whatever log magnitude of the NTF is simply nothing but log magnitude of products of terms like this correct. And, each one of them individually evaluates to.

Student: 0, 0.

0 right. So, in other words the log magnitude of the noise transfer function integrated from 0 to pi must therefore be.

Student: 0.

0 is this clear? So, in other words if you plot the log magnitude of the noise transfer function. And, the NTF is stable and the 0 of the NTF are either on the unit circle or inside; then the area of the log magnitude of the NTF above 0 must be equal to the area.

Student: Below 0.

Below 0 right; I mean this fits in well with all our circumstantial evidence namely right. You know when we attempted to increase the out of band gain correct; what would we find the in band noise is getting better. On the other hand when we reduce the out of band gain which is equivalent to saying if you reduce the out of band gain the out of band log of the gain also reduces; which means that that area above the 0 d B line is reducing; which means that the area below the 0 d B line must increase. And, where is the magnitude log magnitude less than 0? For the NTF at what frequencies is the log magnitude smaller than 0?

Student: ((Refer Time: 39:11)).

Pardon.

Student: Frequency.

At in band frequencies correct. So, right

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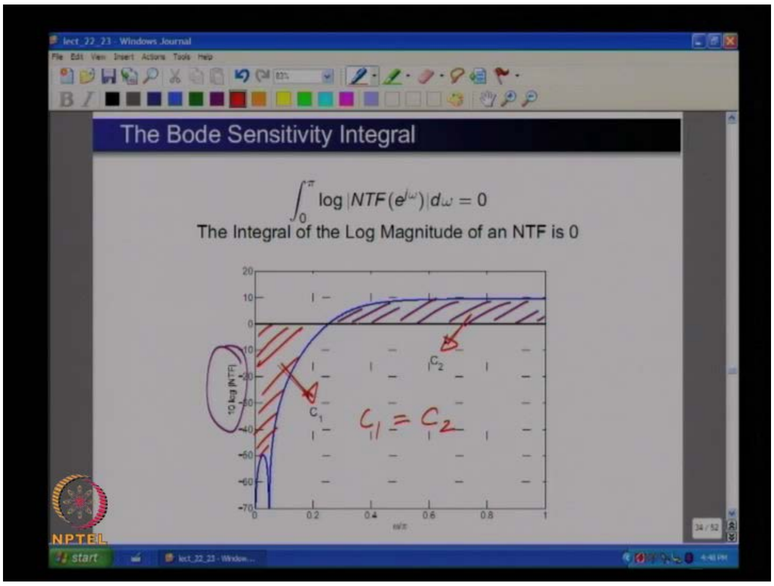

So, this area. So, if you plot say log NTF; the area of this above 0 must be the same as area below 0 right C 1 which is this area and C 2 which is this area must be the same.

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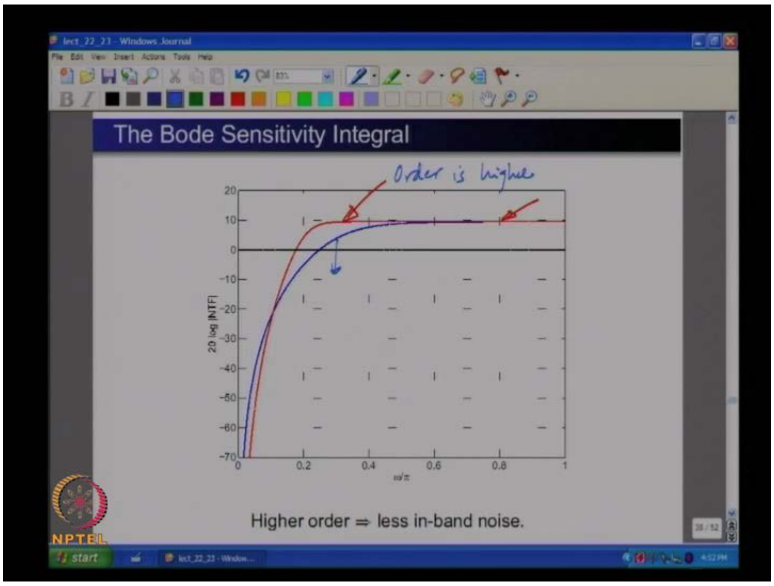

And, this is what is called the bode sensitivity integral all right. And, therefore in English this means that good in-band performance can only be obtained at the expense of.

Student: Poor out of band.

Poor out of band performance which is something we have seen all along. For example, a 1 minus z inverse has good out of band performance because out of band gain is only 2. But it is in band performance is not great; a second order modulator has a poorer out of band performance but better.

Student: In band.

In band performance and so on and so all right. So, let us keep this for the time being. So, this also I mean please note that we have not said anything about the order of the loop filter so far; this is irrespective of the number of poles and zeros in the noise transfer function. So, if we had 2 noise transfer functions with the same out of band performance right. But if one had a higher order than the other then you can do better in band and the reason is that.

Student: Because the noise transfer function is.

I will I mean let me explain what this graph shows; it shows 2 noise transfer functions the NTF in red has a higher order then the NTF in blue clearly; both these noise transfer functions have the same.

Student: Out of band out of band gain.

Out of band gain correct. But in band one of them is doing better which one is doing better?

Student: higher order red is better.

The red one is doing better; the higher order modulator is doing better why do you think this make sense? From the discussion we just had about this bode sensitivity integral.

Student: Because it has more area.

Student: Ok.

Student: Above 0 d B it has more area.

All right. So, I mean you can see that they I mean clearly the area above the 0 d B line must be must be the same as the area below the 0 d B line correct; in a high order modulator you can make the transition very.

Student: Sharp, sharp.

Sharp; this is exactly analogous to I mean your digital what you have done in DSP right; if you have a higher order transfer function you can make it.

Student: Sharp.

Sharper; you can see that the lower order transfer function wastes a lot of area in the.

Student: Transition.

Transition whereas if you had a higher order NTF since the transition is sharper right; you do not need to waste that area in the transition right. All the positive area for example can be utilized to get a larger negative area which all happens at.

Student: In band.

In band frequencies you understand ok? So, that is another piece of intuition.

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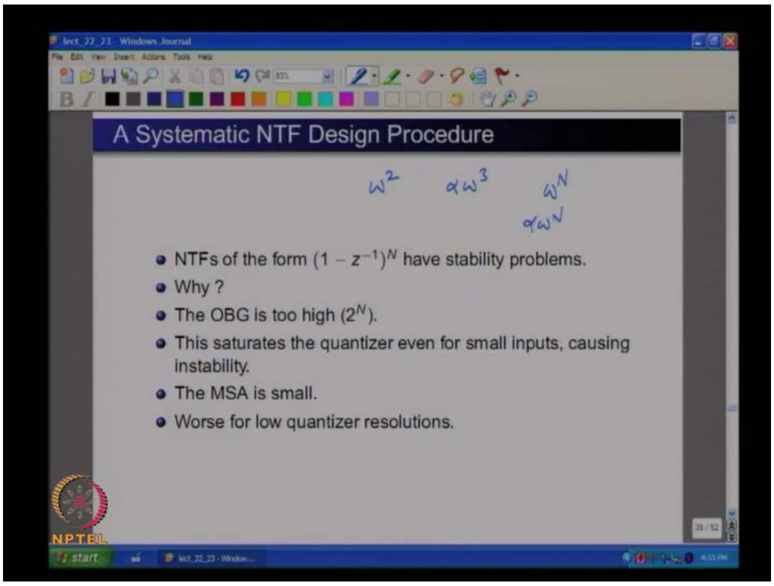

So, to summarize the discussion so far all noise transfer functions of the form 1 minus z inverse to the N have stability problems. And, the reason is that the out of band gain is too high; fortunately this can be remedied by.

Student: Moving.

Moving the poles all from z equal to 0 to locations which are closer to the zeros; thereby bringing the out of band gain.

Student: Down.

Down and this automatically means that the in band gain will go.

Student: Up.

Up somewhat. Because the out of band gain and the in-band gain are related; and you cannot do better out of band without making things in band worse. So, we now at least have a way where yeah sure we have a high order NTF right. But the I mean earlier we had an NTF with say omega to the power N with very small MSA. Now, you have some alpha times omega to the power N but the MSA is much larger right; this alpha being greater than 1 is not really an issue. Because going from I mean the what is the whole idea in going to high order modulator you want to reduce the in band noise correct.

So, let us say you had for second order you had omega square, third order you cannot have omega cube you must have some alpha omega cube; where alpha is greater than 1. So, compared to omega square alpha omega cube is much smaller right because at low frequency omega is virtually.

Student: 0.

0, right. So, the increase in quantization noise due to alpha being greater than 1 is much smaller than.

Student: Reduction.

The reduction in the quantization noise because of the extra factor.

Student: Omega.

Omega, right. So, it does indeed make sense to go for a high order modulator which of course has been designed. So, that the maximum stable amplitude is not very small; and that is done by moving the poles choosing the poles properly. So, that the out of band gain is smaller than 2 to the N correct.

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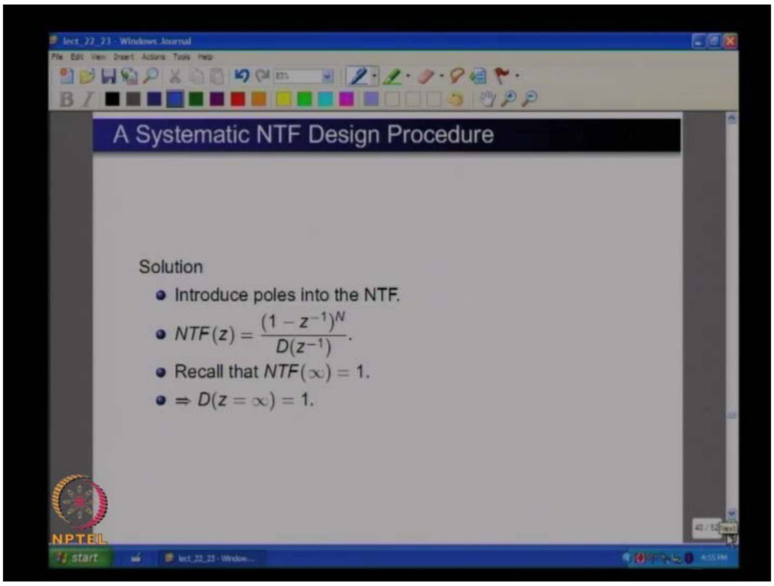

So, let me quickly go over a systematic way in which one can design an NTF. So, as we said we introduced poles in the noise transfer function is now of the form 1 minus z inverse to the N divided by D of z inverse right. So, D (z) where evaluated at is equal to infinity must be 1.

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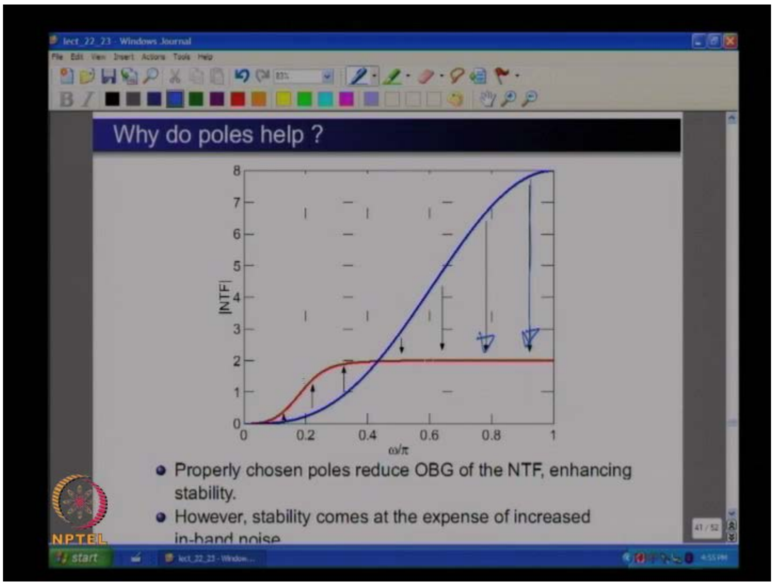

And, as we saw the out of band gain will now reduce. Because of the moment of the poles the in band gain will go up; and that is not a problem as we just discussed.

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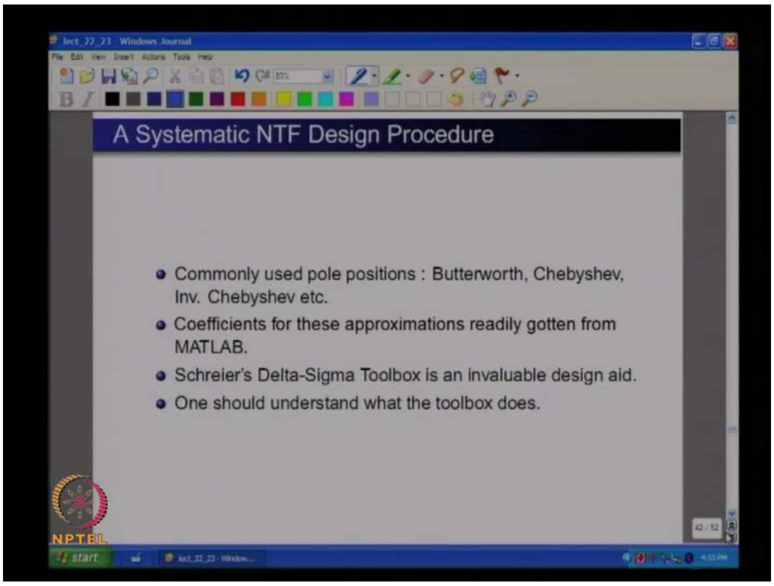

So, to systematically design an NTF what one realizes is that a noise transfer function is nothing but a high pass filter. So, the job of designing a noise transfer function is analogous to.

Student: ((Refer Time: 47:10)).

That of designing a proper high pass filter. But can any high pass filter be a noise transfer function.

Student: No.

If I gave you a high pass filter transfer function and said this is your NTF; what check will you run to make sure that this is indeed an NTF?

Student: ((Refer Time: 47:30)).

You must make sure that the high pass filters response that the transfer function when evaluated at z equal to infinity.

Student: Must be 1.

Must be 1 otherwise you will not be able to realize it. Because you for realization you must have a you must not have any delay free loops. So, and high pass filters you know the synthesis and approximations have been worked out; they are all well known stuff. So, it does not make sense for us to start inventing new high pass filters; they actually take resort to mat lab where you know all these libraries are have been around for ages. So, you pick your favorite high pass filter let us say one of you likes Butterworth, one of you likes chebyshev, one of you likes inverse chebyshev it does not matter; you pick a high pass filter family all right.

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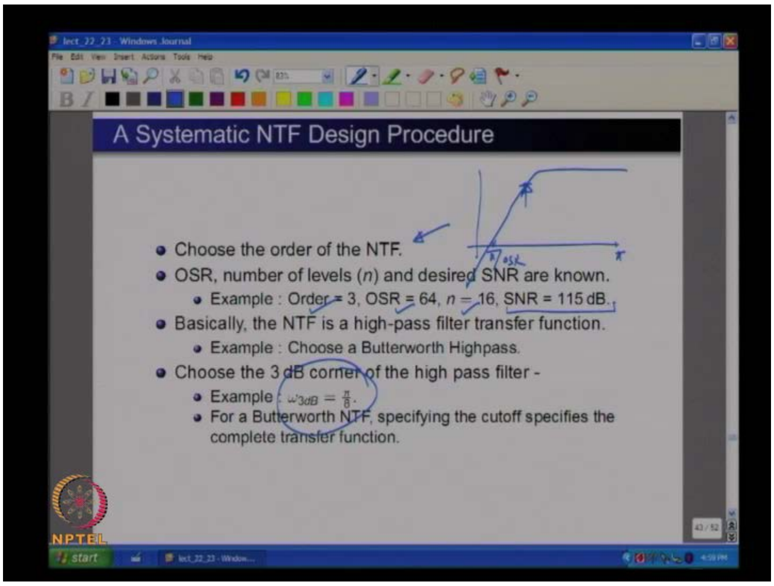

And, I mean so when you starting of let us say you choose the order right, you choose the over sampling ratio. And, you choose the number of levels in the quantizer and you choose the target SNR; this SNR is the.

Student: In band.

In band signal to noise ratio. So, as an example let me say I like Butterworth. So, I will say I will choose a third order Butterworth high pass filter. So, once I say that my high pass filter is Butterworth and is of third order; there is only 1 degree of freedom and what is that?

Student: Cut off.

There is only one you know free variable that is the cut off frequency correct. So, at since I do not know anything I will just pick some random cut off frequency; I mean. So, this is my pi, this is my signal band pi by OSR; the cut off frequencies of the Butterworth can you comment on should it be greater than pi by OSR or smaller than pi by OSR?

Student: Pi by OSR greater than.

Must be much higher than.

Pi by OSR.

Pi by OSR. Because the high pass filters transfer function is like this; you do not want to pick a cut off I mean this is the cut off frequency correct. And, your signal band must be in the stop band of the high pass filter. So, it make sense to choose a number which is much higher than pi by OSR right. So, I randomly chose pi by 8.

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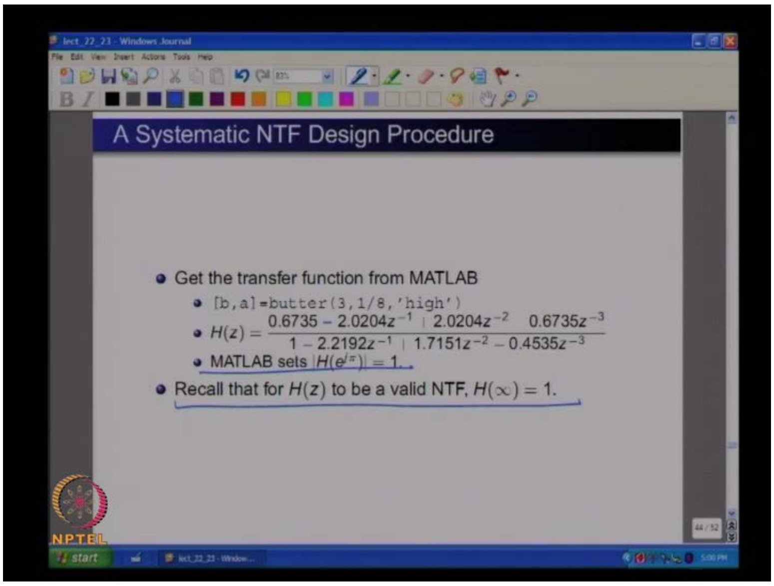

And, so we get the transfer function from mat lab; mat lab unfortunately I think this picture is missing the minuses and pluses. So, anyway mat lab does not know that you are a sigma delta designer. So, if you ask it for a high pass filter it will give you ratio of polynomial such that the gain at in the pass band is.

Student: 1, it is 1.

It is 1 correct. So, unfortunately or I mean whatever so but we do know that for a valid NTF H of infinity has to be plus 1. So, what do you do?

Student: Divide it by.

You just have to divide the first term; make sure that the first terms are the constant terms in the numerator and denominator are both.

Student: 1.

1.

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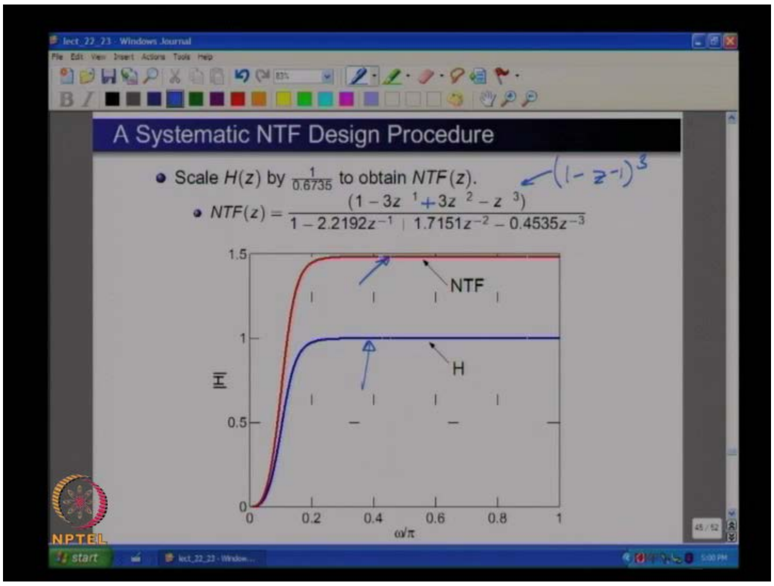

So, once you do that you will get an NTF where the numerator must be of the form.

Student: 1.

This must be 1 minus z inverse the whole cube. Because the Butterworth filter has got all its zeros at z equal to 1; Butterworth high pass filter will have all its zeros at z equal to 1. So, if the original H the high pass filter that mat lab gave you is like this; once you scale the coefficients you will get a noise transfer function whose magnitude response looks like this. So, in the next class we will continue and see how one goes through the systematic design procedure.

Thank you.