VLSI Data Conversion Circuits
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Lecture - 20
Delta-Sigma Modulation – 2

Good morning; this is VLSI data conversion circuit lecture 20.

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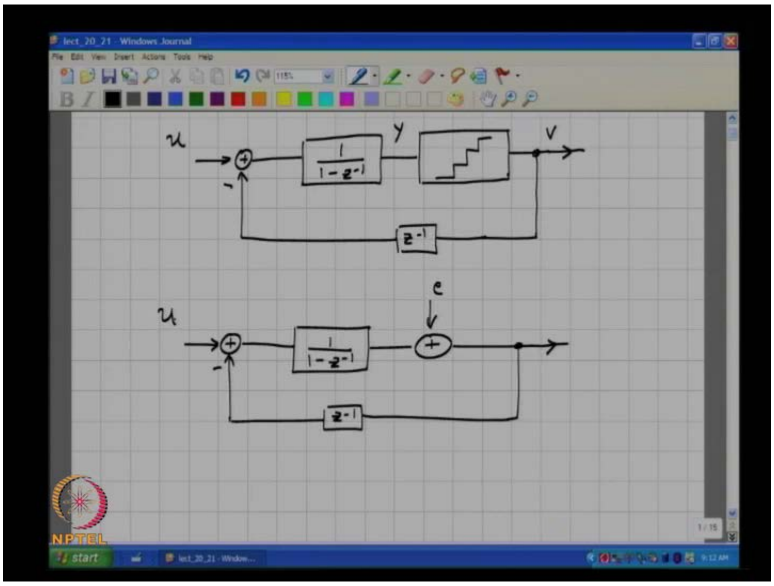

In the last class what we saw is what we called a first order delta sigma modulator. The input was denoted as u, the output sequence is denoted as v, the input to the quantizer is denoted by y. And, we also discussed why this delay makes sense. And, the motivation for choosing an integrator in the forward path; we said that if we module this quantizer by an additive noise source. Then one ends up with what is called the linear module of the first order delta sigma modulator; where instead of the quantizer you replace it by an error sequence e which is added everything else remains the same.

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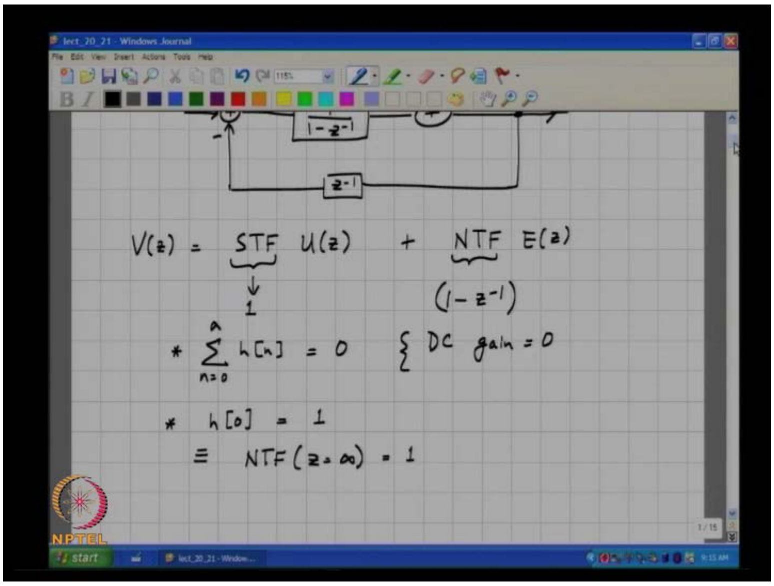

And, we saw that v of (z) can be written as STF times u of (z) plus NTF times e of (z) this is called the signal transfer function. And, if this particular case happens to be 1; this represents the transfer function from the noise source which is additive to the output and is called the noise transfer function and is denoted by. And, in this particular case happens to be 1 minus z inverse. And, as we saw last time 1 minus g inverse represents a high pass transfer function.

And, the D C gain of the high pass transfer function is 0; which means therefore that the sum of the impulse response terms corresponding to the noise transfer function is 0. Because D C gain is 0; we also saw that of fundamental thing is that h of (0) which is the first sample of the impulse response of the noise transfer function is unity. And, this is regardless of the kind of filtering network that we choose in the forward path right. And, this is equivalent to saying that the NTF evaluated at z equal to infinity is 1.

So, in general we saw that a noise transfer function is basically a high pass filter with the coefficients chosen or scaled. So, that the NTF evaluated at z equal to infinity is 1 all right.

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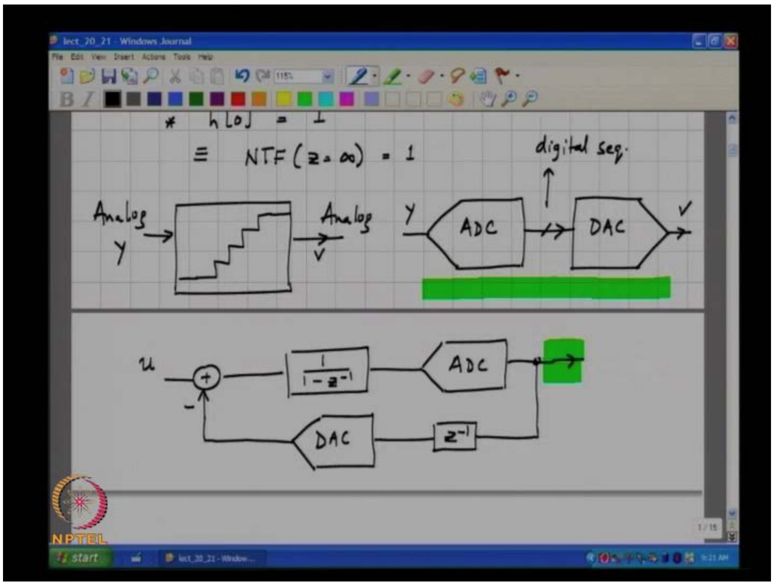

Now, a couple of minor points I think you must all be aware of this; but I completeness sake mention this. Please, note that in the loop we have been using this symbol for the quantizer right. And, if you are to close the loop around the quantizer it means that the quantities at the input of the summer and the output must all be of the same kind you understand. So, in other words if this is I mean if this is an analogue quantity; the output is also an analogue quantity accept that it is got discrete levels.

It is another story that we can now take this sequence which is got discrete levels. And, code it into a string of digital words with a finite number of bits you understand. So, as far as we are concerned I mean the quantizer represented by this diagram. And, when we now enclose this inside feedback loop essentially can be thought of is a memory less nonlinearity right; with that transfer curve like this. In practice how do you thing, you would be able to implement such a block I mean.

So, in practice what would be done is to I mean that 2 parts to this; one is to detect where the input lies in that range and generate; you know say perhaps digital word which says the input lies in this bin. But that is not good enough the output also needs to be analogue correct. So, you cannot take that digital word in feed it back it does not make sense; because the input is an analogue quantity right. So, what do you think you will do? So, you will need not only an analogue to digital converter which takes an analogue input and generates a digital sequence right; which corresponds to which bin the input lies in. But you need to take that information and generate an analogue voltage right.

So, that is what is called digital to analogue converter. So, this is y and this is v and in a practical implementation this will be y and this will be v; this is a digital sequence is this clear. So, a practical delta sigma loop will look like this; the DAC will be in the feedback path you will have an integrator here, a delay here and an A D C. So, this output sequence here is it an analogue quantity or digital quantity? It is a digital word right accept that it is coming out of very high sampling rate compared to the ((Refer Time: 08:53)) for that input signal band is this clear.

Now, this must be shift of to a digital filter whose job is to be smoothening out the sequence. And, I am basically in principle at least is a brick wall filter with a band width equal to the pi by OSR right. So, all the out of band quantization noise; please recall that since the noise transfer function is 1 minus z inverse, the quantization noise is shaped away from the signal band. So, this out of band noise will be filtered out or removed by the digital filter and after which you can drop samples. So, this that whole thing as we discussed is called decimation filter.

But as far as the system side is a concerned; I mean we know that we are going implement the quantizer as a cascade of a to d and d to a converter right. And, we need to be aware of this; but as far as all system level understanding is concerned we know what is there in this box. So, we do not have to worry about the a to d and d to a at this point; we just assume that one way or the other there is a non-linear block whose transfer curve is this step like waveform.

And, given that we know that they are going to be finite number of levels in the a to d converter. It must follow that the this step this staircase cannot continue from all the way from minus infinity to plus infinity; it will saturate beyond a certain points both on the lower side as well as the upper side depending on the number levels in the quantizer is this clear wait.

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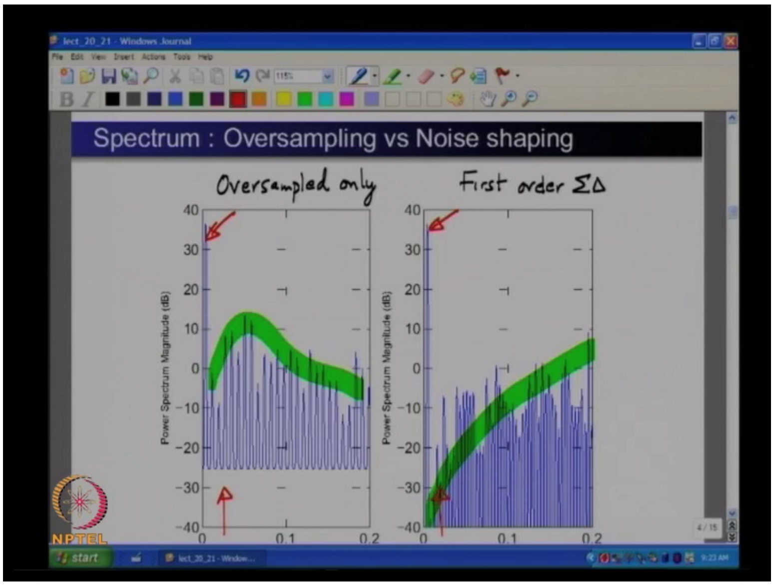

Now, let us just quickly show you some spectral pictures; the last time we saw a first order. So, this is a first order delta sigma or sigma delta modulator output spectrum; this is a simply an over sampled spectrum. And, a couple of observations since quantization is basically a deterministic phenomenon you should not expect that the spectrum will be you know wide or it will flat; you can see a whole bunch of tones and that make sense. Because it is a non-linear operation a horribly non-linear one at that and higher order harmonics will alias back into 0 to f s by 2 bands; therefore you see a whole bunch of tones sticking of ok.

And, on the same scale we show the spectrum of a first order delta sigma modulator. And, that as you can see as got a spectral shape which kind of looks like that the key point to note is that in the signal band. This is the input signal mind you this is the input in both cases with the same amplitude; you can see that within the signal bandwidth is let us say somewhere up to here you can see that the quantization noise is been shaped out of the signal band all right.

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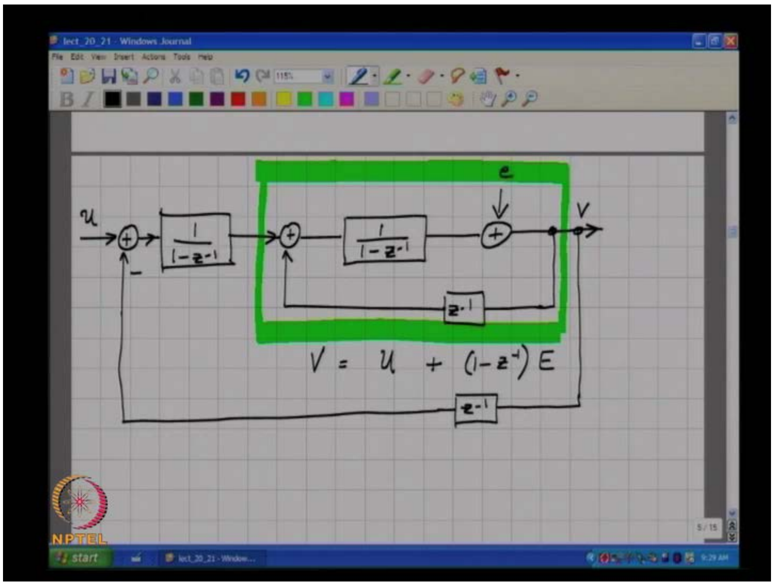

Now, that we know that feedback can do wonderful things and make a poor quantizer look like a very good one. And, now get greedy and say can we do better right; with the first order modulator we saw that doubling OSR was giving us 1 and half bits per every doubling of the OSR. So, natural question to ask is can I do better than this right. And, I mean one approach would be; and a straightforward one at that would be to take a look at the first order modulator. And, observe that v is nothing but u plus 1 minus z inverse times; rather v of (z) is u plus 1 minus z inverse times E where the upper case letters stand for the z transform. So, the respective signals correct all right.

So, you can think of this box; therefore as a quantizer where the error is you can think of this is 1 minus z inverse times E all right. And, we already know how to deal with this situation; what we were doing when we derived the first order modulator we had a quantizer where the output was when we derived the first order delta sigma modulator what did we do? What did we have? And, what how would we I mean what is the modal for the quantizer?

Student: ((Refer Time: 16:01))

Pardon.

Student: ((Refer Time: 16:03))

Ah.

Student: ((Refer Time: 16:06))

It is yeah. So, when we had the first order quantizer; all we had was the output was simply input plus E. Now, instead of the output being input plus E it is?

Student: ((Refer Time: 16:22))

Output is input plus some other noise waveform which happens to be 1 minus g inverse into E correct. And, we had when and when the output was input plus E; how do it we get rid of or reduce the amount of noise in the signal band.

Student: ((Refer Time: 16:41))

By using a negative feedback loop. So, you can say hey, now I can think of the black box the green box here; as being u plus some error which happens to be 1 minus z inverse times E. So, I can use the same thing that I did earlier that is negative feedback to reduce the error further in the signal band correct. And, therefore what I would do, would be what did we do the last time around; if you imagine yourself to not see what is there inside the green box right. But just treat the green box as output being input plus some errors then what I would do; would be to do this. And, we are understood that nothing comes out right away all right.

In other words the you can have a access to the you know the quantize signal away right. So, that is modal by the z inverse. And, what we do here now; you put.

Student: ((Refer Time: 18:05))

The same thing that we did earlier which is 1 by 1 minus z inverse; and this becomes the you knows correct. So, I am now going to remove this and see if I can make any simplifications; was block diagram and remove any redundancies if possible. And, what can I do?

Student: ((Refer Time: 19:11))

Pardon.

Student: ((Refer Time: 19:15))

No.

Student: ((Refer Time: 19:18))

What is the most straight forward?

Replace e by 1 minus.

Replace.

Student: E by.

Or remove.

Student: minus 1.

We may I mean most straight forward thing; you can think of it is notice that this signal and this signal are the same thing. So, instead of drawing this way I would draw this way is this clear. And, of course one can go and find what v is by solving the block diagram here.

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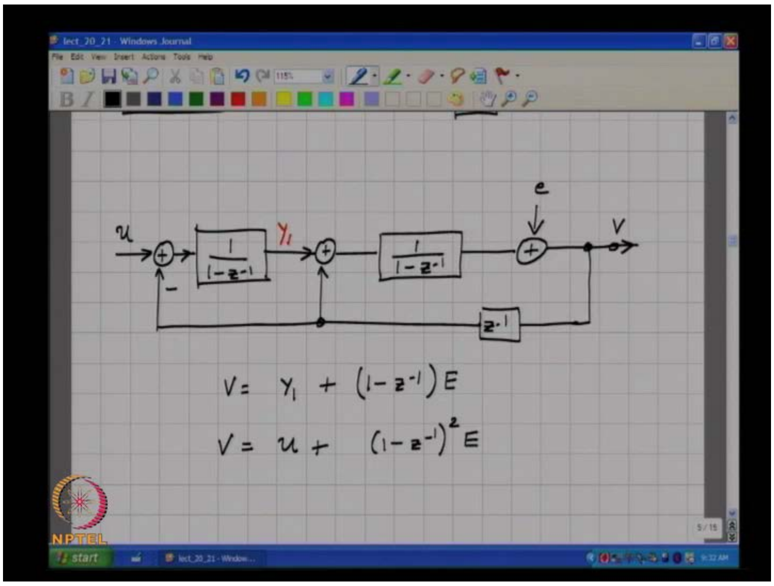

But a lot simpler way of understanding this is to realize; the fact that we derived this how? This signal say let us call this y 1. And, we are related by what?

Student: ((Refer Time: 20:21))

v is nothing but y 1 Plus 1 minus z inverse times e right which means and how are I mean; therefore what how is v related to u? Let me derive first order delta sigma modulator all these had was output was nothing but instead of being 1 minus z inverse times c it was e correct. And, once we enclose this in a first order delta sigma loop what happened to the e?

Student: ((Refer Time: 21:07))

It became 1 minus z inverse. Now, the only departure from that is that instead of having e to start off we have.

Student: ((Refer Time: 21:13))

1 minus z inverse times e to begin with. So, what you must get therefore will be.

Student: ((Refer Time: 21:21))

v must be u.

Student: ((Refer Time: 21:24))

Plus 1 minus z inverse times 1 minus z inverse times e; which is 1 minus z inverse square times e all right.

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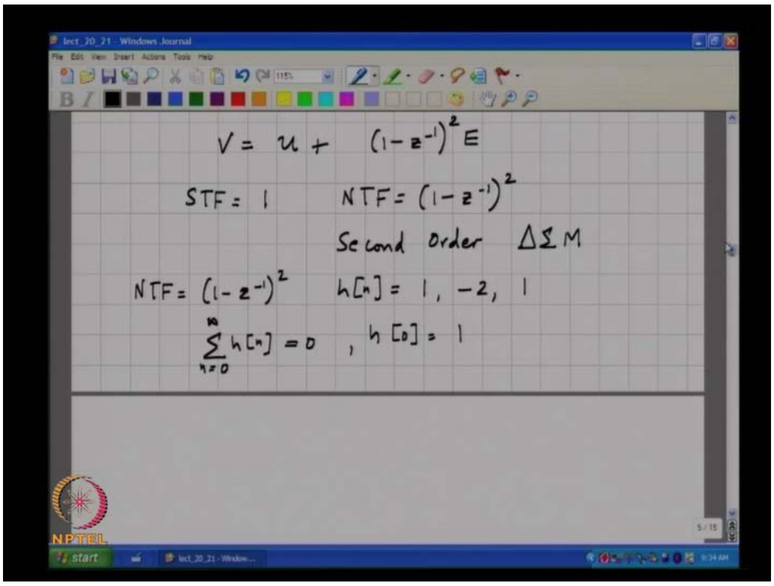

So, what is the signal transfer function now is 1 and the noise transfer function is 1 minus z inverse the whole square. So, this corresponds to a second order delta sigma modulator. So, let us quickly verify some other the facts that we are already use to expecting. So, the impulse response sequence right; NTF is 1 minus z inverse the whole square and h of (n) is what? h of (n) the sequence corresponding to this impulse response is what? 1 minus 2 and 1. So, sum of h of (n) must be 0, h of (0) is 1.

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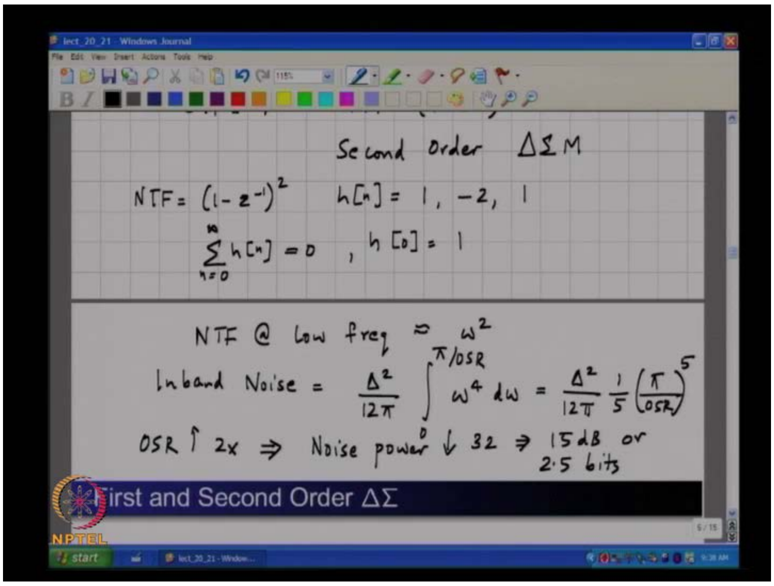

And, what about the I mean what we expect? Should be expect that we are doing much better in band or much worse in band compare to a first order modulator? We expect do much better; because the noise transfer function at low frequency is what? It goes as omega square why? So, z inverse is e to the minus j omega for small omega e to the minus j; j omega is 1 minus j omega. So, 1 minus z inverse goes as j omega the whole square the magnitude square goes as omega square all right. So, the in band quantization noise is nothing but delta square by 12 pi integral 0 to pi by OSR omega.

Student: ((Refer Time: 24:32))

Omega to the 4 d omega; which is delta square by 12 pi times pi by 1 by 5 times pi by OSR whole to the power 5 correct. So, OSR going up by 2 x implies in band noise power going down by 32 which corresponds to 15 d B or how many bits? 2 and a half bits for every doubling of the o sampling ratio is this clear. And, you must understand; of course that the assumptions made in the integral are that the NTF goes as omega square that is only true for small omega. So, as you can see increasing the order of the modulation has in fact cause the in band signal to quantization noise ratio to increase significantly for large over sampling ratios all right.

In other words this is much better high pass filter correct; the job of the noise transfer function is to remove or shape away noise from the in band reason. And, that it can only do that if it is a very very good high pass filter and what is an ideal high pass filter? It suppose to have 0 magnitude response over a small band right at low frequencies. And, let through all noise or all signal at high frequency; obviously, it is not possible to get make a high pass filter with 0 transmission over a continuous band.

And, these are all you know for a given order these are all various approximations of making of realizing high pass filter. So, it is not surprising that with second order you are able to do better then you are with first order modulator is this clear.

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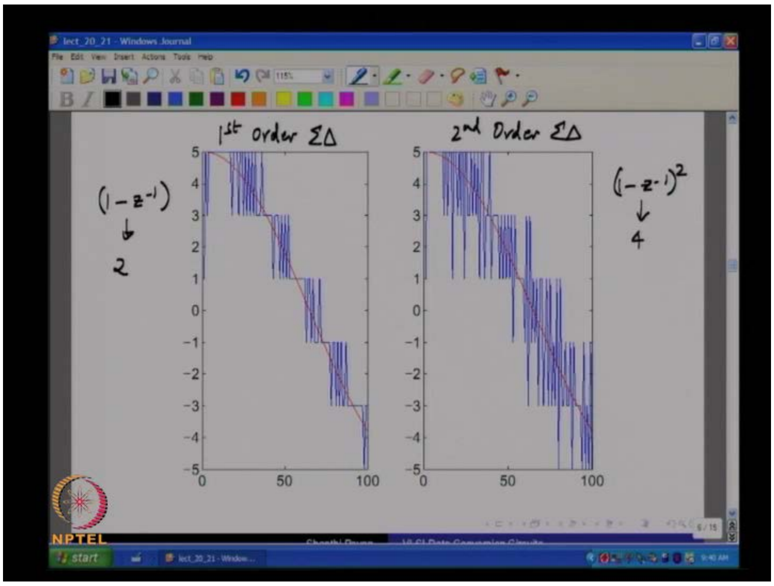

So, let us take couple of minutes to see what happens with respect to time domain waveforms all right. So, one of these graphs corresponds to a first order modulator; while the other corresponds to the output sequence of a second order modulator the red curve in both these graphs is the input. So, can you tell me which is the first order design and which is the second order design? The one on the left is a first order and this is a second order. And, why does that make sense? I mean how do you figure this out?

Student: ((Refer Time: 28:37))

So, good; so, the out of band gain or the gain at pi for the first order design is.

Student: ((Refer Time: 28:49))

2; whereas for the second order design is 4 ok.

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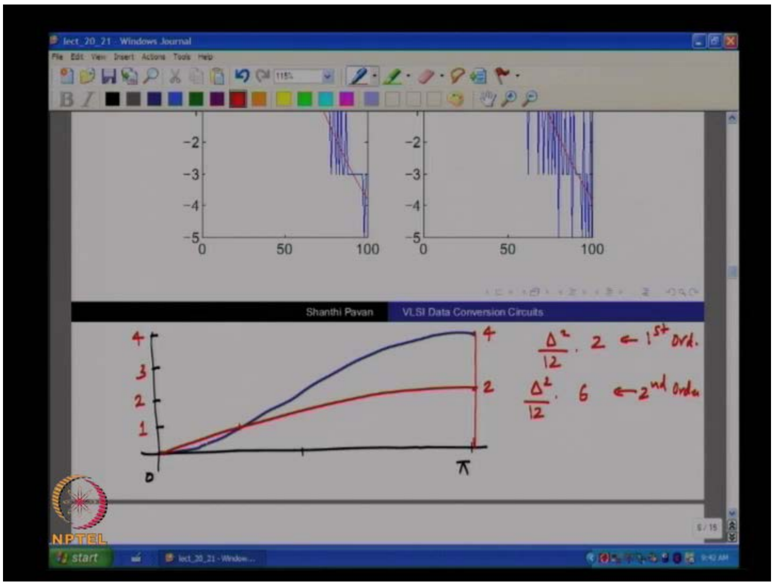

So, if you plot the noise transfer functions how will the second order noise transfer function look? It look like omega square where as for the first order 1 will go look like this is 2, this is 4 all right. When we computed the total noise not just the in band noise for a first order modulator what it be compute that to be delta square by 12 times sigma h n square which happens to be 2 this is for the first order. Now, for the second order what do you expect? 1 plus 4 Plus 1. So, it I will be 3 times larger than what you had before.

So, it can see that the total noise throughout the signal band; not just within the signal band I am sorry, the total noise over 0 to pi not just within the signal band is actually much higher in the second order case.

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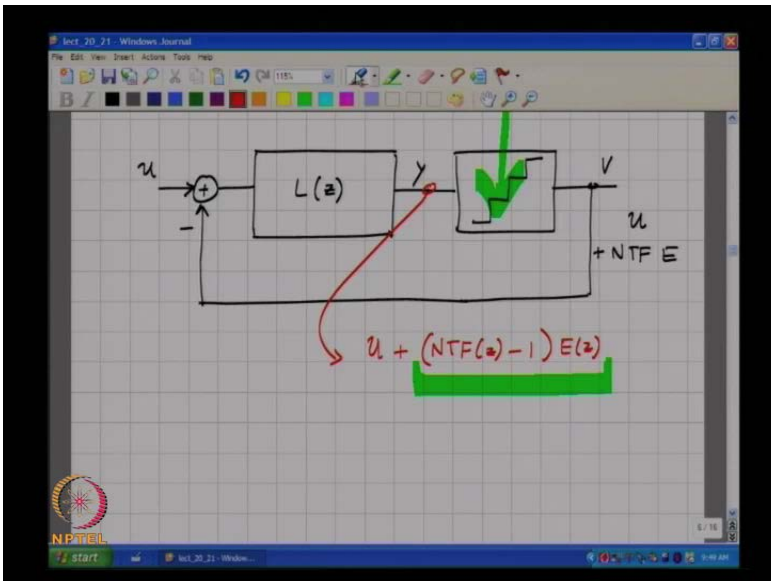

Let me also draw your attention to another aspect of these modulators. So, let me draw a more general diagram now for example, this which is also special case of having L 1 and L 2 or different transfer functions from the input. And, the feedback branches this is the quantizer this is v, this is y. So, assuming where the quantizer is linear we can think of this as being an additive noise source e; in which case the output is simply the STF times u plus NTF times e.

And, within the signal band it is very common to make the signal transfer function equal to 1; what did we see in the first and second order cases we saw that the STF is 1. And, that corresponds to unity feedback from the output to input; at D C the gain of the forward amplifier is infinite and the feedback gain is 1. So, it follows that at D C the close loop gain must be 1 is not it. So, this is a very common thing done I mean this no fundamental reason to make the close loop gain 1; regard of fed back only a fraction of the output quantity and got an a higher gain right. But that is usual not done though it can be done there is no nothing wrong about it.

So, to keep matters clean we will just assume that the STF within the signal band or at low frequency is this 1; in which case the output will simply be u plus NTF times e. So, if the output is u plus NTF times e; what happens or what can you say about the input to the quantizer?

Student: ((Refer Time: 35:44))

L of z well no.

Student: ((Refer Time: 35:53))

Pardon.

Student: e into NTF into L of (z).

Ok that is e into NTF into L of (z) is there can you make it d 1 simpler.

Student: u minus gain plus e into NTF minus 1.

Very good right. So, a simpler way of looking at it is to simply say that y must be u plus NTF of z minus 1 times e of (z) correct I mean is this clear or is there some doubt. The output of the quantizer is y plus e and that happens to be u plus NTF times e of (z) it must follow; therefore that the input to the quantizer must be smaller by e that is all correct. So, therefore can we conclude anything about I mean what do we expect the general shape of this spectrum to be at y.

So, you should expect the input to the quantizer to consist of the low frequency input that you are trying to digitized which is u plus some noise like waveform right; which is basically e of (z) which is assume to be uniform and white. And, all the stuff shaped by a filter whose frequency response is not or whose z transform is not N T F; but NTF minus 1. So, in general do you think this how do you think this will look how do you think this will look as a function of frequency?

Yes, ok How will end e of (z); I mean if I plot the frequency spectrum of e of (z) times NTF of z how will that look like high pass shape from that if we subtract e of z how will it look? It will also look like high pass transfer function; of course now the D C gain will not be 0. But there will be minus 1. D c gain will be minus 1 right. But the key point is to observe that at high frequencies there will be still be a lot of energy is this clear.

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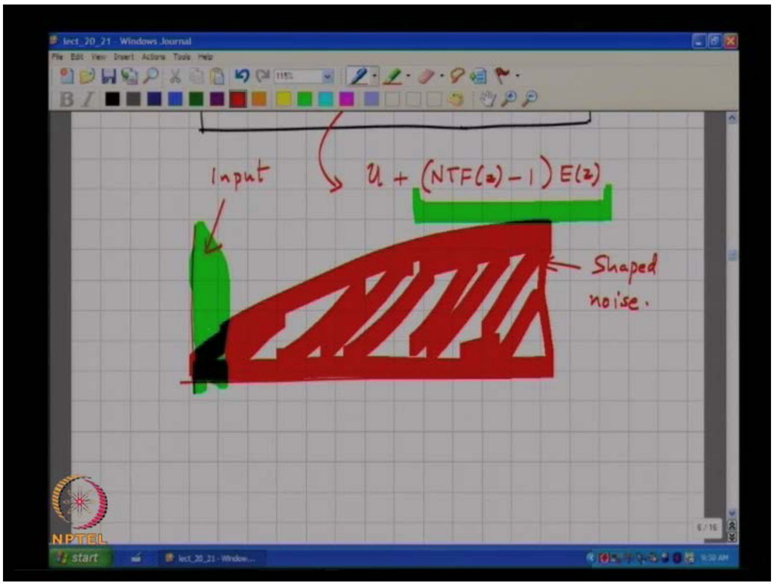

So, the spectrum at the input of the quantizer will basically look like you can expect it look like this where this corresponds to the low frequency signal; you are trying to digitize right plus some shape noise. So, this is the input and this is shape noise is this clear. Now, that we know what to expect.

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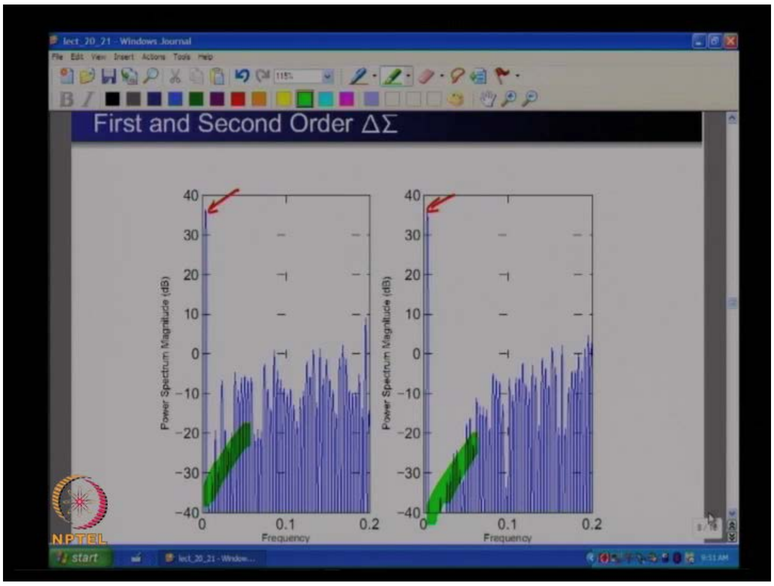

Let us take a look at spectra various points this picture shows the spectral density at the output of a first order and second order modulator designs. So, what you notice? When this is the input signal is the same in both cases; as you can see the in band noise component here is much smaller than the first order system. Of course, I only plotted it over a small axis; which is why you are not able to see the increased out of band noise in the second order case all right.

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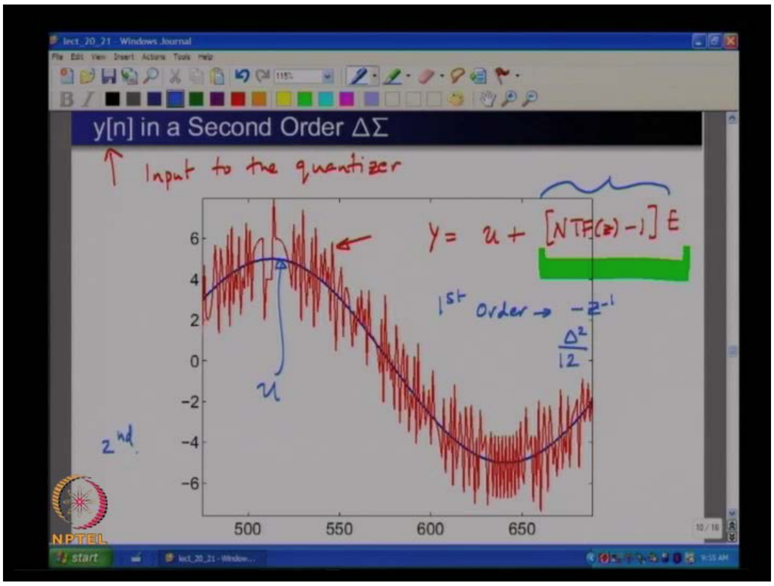

And, also good opportunity taken. And, look at the input to the quantizer; please note that y stands for the input to the quantizer. And, the blue curve here happens to be the modulator input u. And, as we expect the input to the quantizer; do you think it has got discrete levels or it has got continuous levels. It will have continuous levels right; because u is got continuous levels you understand. So, the input to the quantizer as you can see can be thought of as some kind of high pass noise which in English means that is got it vigils very rapidly. And, we can see that it is high pass noise riding over the input is this clear all right.

So, what, can you say about the variance of the noise at the input of the quantizer y is u plus N T F of (z) minus 1 times e. So, can we comment on the variance of the noise which is the noise component here?

Student: ((Refer Time: 43:55))

This is the noise component; can we comment on the variance of this noise in both first and second order cases? The first order case what is N T F?

Student: ((Refer Time: 43:14))

1 minus z inverse. So, what is the NTF of z minus 1? z inverse. So, what is the variance of the quantization noise riding over the input signal?

Student: ((Refer Time: 43:36))

It is simply be N T F minus 1; in this case simply happens to be minus z inverse. And, whose magnitude is 1 right. So, by parts of all the mean square value of this noise sequence must be delta square by 2.

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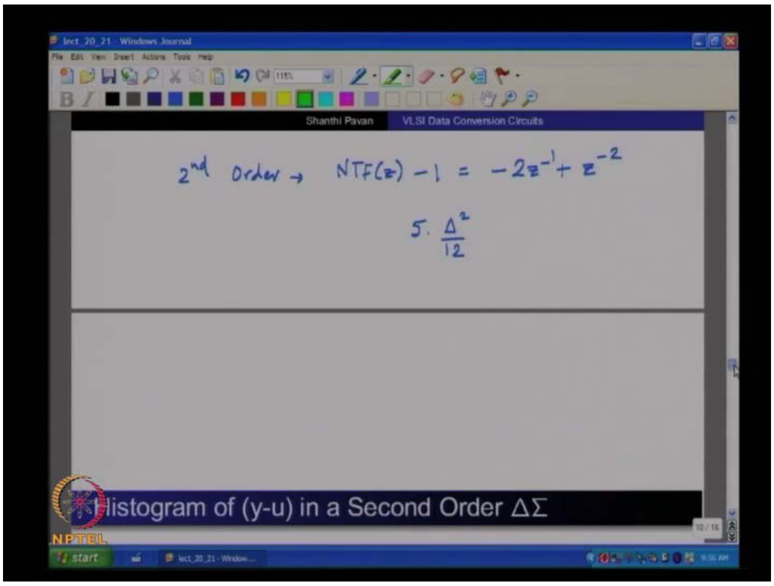

When you have a second order modulator NTF of z minus 1 is given by minus 2 z inverse; z to the minus 2 and this must be. So, what is the variance of the noise? Do the math carefully; it is simply 5 times what? Delta square by 12 is this clear all right. So, let us; I mean that is basically the variance of this part ok.

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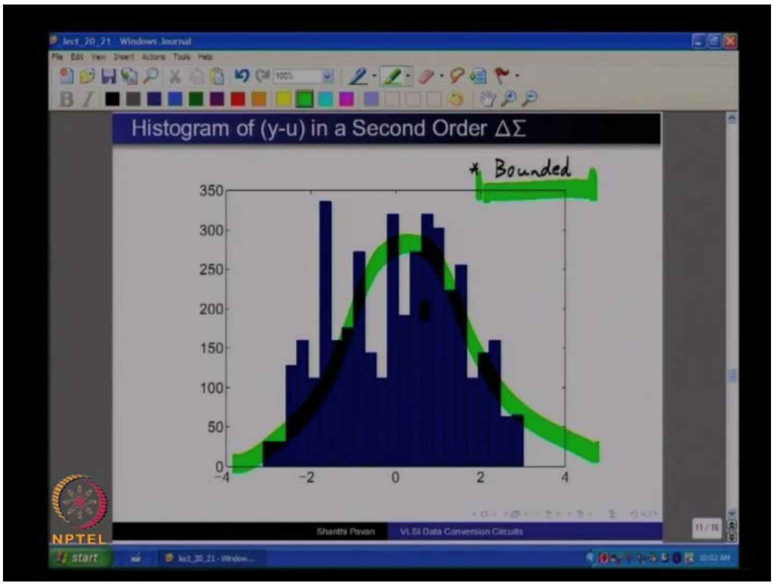

Now, let us also spend some time taking at the look at I have also plotted histogram of y minus u and what is y minus u? Yeah, it is nothing but the histogram of the noise process riding over the input at the; at which location at the input of the quantizer correct; this must be a 0 mean sequence why? What do you expect it to be 0 mean?

Student: ((Refer Time: 46:43))

Because e of (z) is 0 at which is driving a filter you can think of this noise process is being generated by taking a 0 mean sequence of e of (n) which is are assume modal for the quantizer. And, passing it through a filter which is you know NTF of (z) minus 1; since e of (z) has got 0 mean you expect the output also to be 0 mean. So, if you plot a histogram it kind of you know over around 0 all right. And, well I mean do you have any comments on this shape; no, no I mean or rather if this not a trick question anything just take a look at it and tell me what you think of. First of all it seems to be bounded correct. So, why do you think that make sense.

Student: ((Refer Time: 48:03))

Pardon.

Student: ((Refer Time: 48:05))

Which output?

Student: ((Refer Time: 48:08))

And, y?

Student: ((Refer Time: 48:13))

E of n is bounded correct.

Student: And, filter transfer function does not have a gain I mean infinite; it has finite gain. The gain is not going to infinity anywhere.

Correct. So, the filter is stable; the input is bounded because the quantization error is bounded by plus minus delta by 2. So, it indeed make sense that the error which is in error I mean which is the quantization error sequence convolved with a filter whose transfer function is NTF of (z) minus 1; must also be bounded you understand what else can you notice. So, let me put that down here; no, your first impressions or rather can you approximate this by some distribution that you know.

Student: ((Refer Time: 49:31))

Well, that is true right. So, clear this cannot be Gaussian because is bounded. But Gaussian is a nice thing simply because; I mean everything that is probably there to understand about the Gaussian process you know as probably been done before right I mean people have been working on Gaussian processes and this stuff from you know for the past probably 100 years. So, everything you need to know about the Gaussian is probably been discovered.

So, if you assume that this is Gaussian I mean you can now immediately dive into that whole body of knowledge which exists about these processes. So, it is a convenient assumption to make right; it is like saying you know I mean some guy lost a ring somewhere at location x. And, was searching for it in location y when somebody said what are doing he said I am searching for my ring and when asked where did you lose it he says location x. Then, they why are you looking at location y he is looking at searching in location why simply because there is light there you understand right; not because I mean this is a same thing here right.

The reason why we assume Gaussian is because a so much light is been thrown on Gaussian and all it is properties right. And, even if that is not exactly; we use it I mean it just what you call small side thing. So, again you know with this fitting stuff I mean you can fit anything to anything so right. Now, the movement I draw this yellow curve I am sure all of you say yeah now it is looks Gaussian you understand right. If you look at it far away anything can be make look like anything else anyway it is convenient assumption all right.

So, in the next class we will spend couple of minutes looking at we understand it is bounded. And, let us try and see what I mean; if we can come up with some upper bound for this for the you know the shape noise at the input to the quantizer you understand is this clear that this is bounded. So, then I question what are the bounds; they are clearly must depend on the magnitude of the input which we know which is e of (z) and the coefficients of the filter right.

So, in the next class we will quickly a derive bound for this. And, that I will that I will give us also some intuition about what to expect next which is I mean. So, from we have gone from first order to second order; we have seen a big improvement in the in band signal to noise ratio. So, now what is the next step you thing? It is seems like from first order from 0 th order to first order; we have got a huge improvement from first to second we got a big improvement right. So, it seems like you can go third order and forth order and fifth order and you know any order.

And, basically get infinite resolution by simply shaping out a making the high pass filter better and better and better and better right. And, clearly when something sounds too good to be true; it probably is too good to be true. So, you know it turns out that you cannot go on doing this; because there will be a problems in the loop. And, all this is you know are some observations which will help us understand the I mean; the phenomenon of instability in the delta sigma loop.